Physics through Computational Thinking
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Lecture 31
Driven Oscillations using the Improved Euler Method

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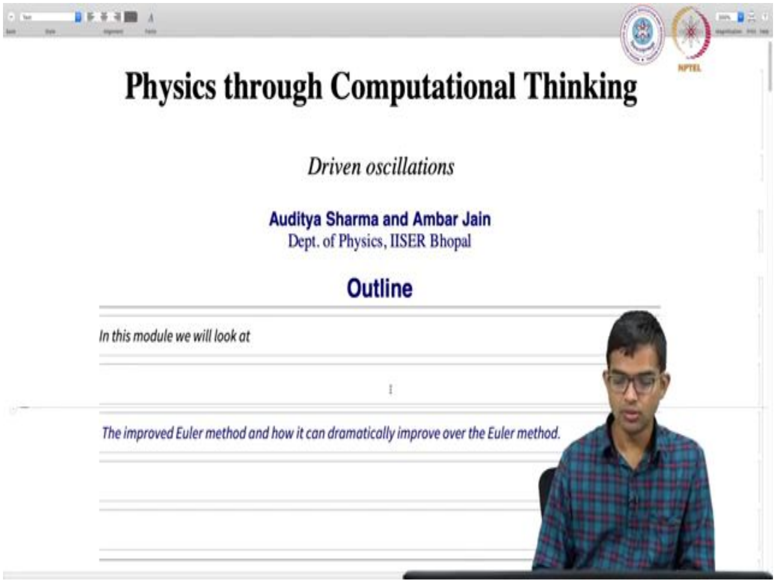

Okay. So, this is a quick module, where I want to show you how using the Improved Euler Method in place of the Euler Method can actually dramatically improve the quality of the results, right.

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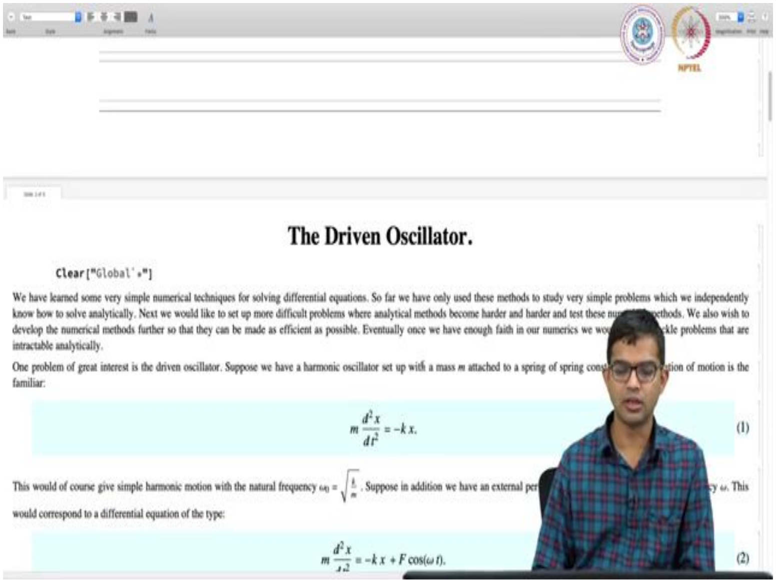

So the quick recap is about how we looked at the Driven Oscillator. You know, we went through all this analysis, non-dimensionalization, et cetera.

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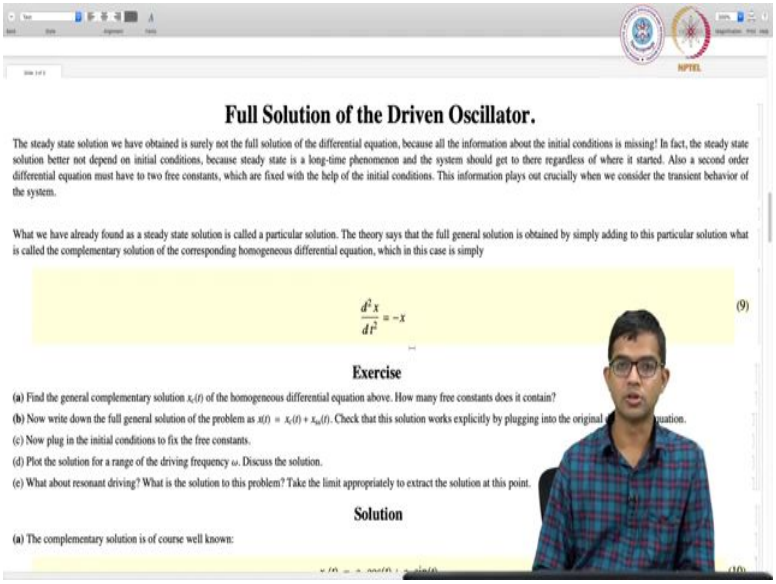

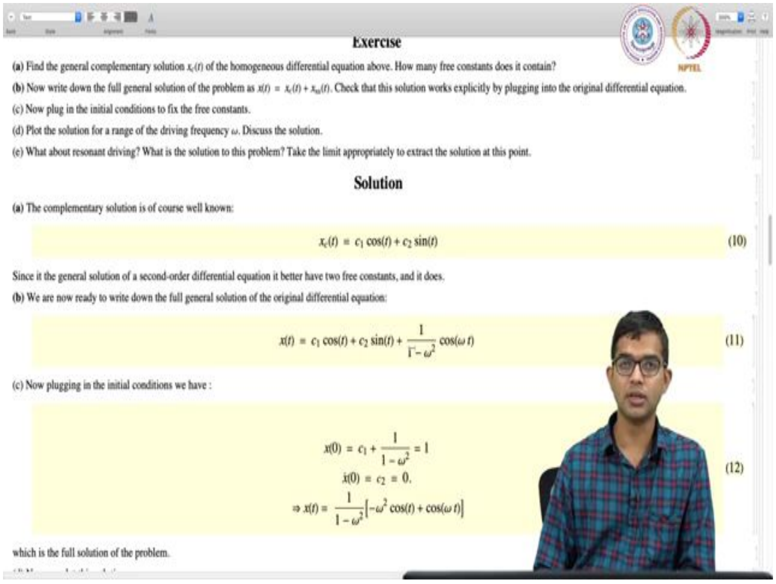

We wrote down the steady state solution. Then we worked out the transient solution, which comes out from finding the general solution of the homogeneous equation.

Then we stitched them together right down the full solution, which includes inputs from the initial conditions as well. right.

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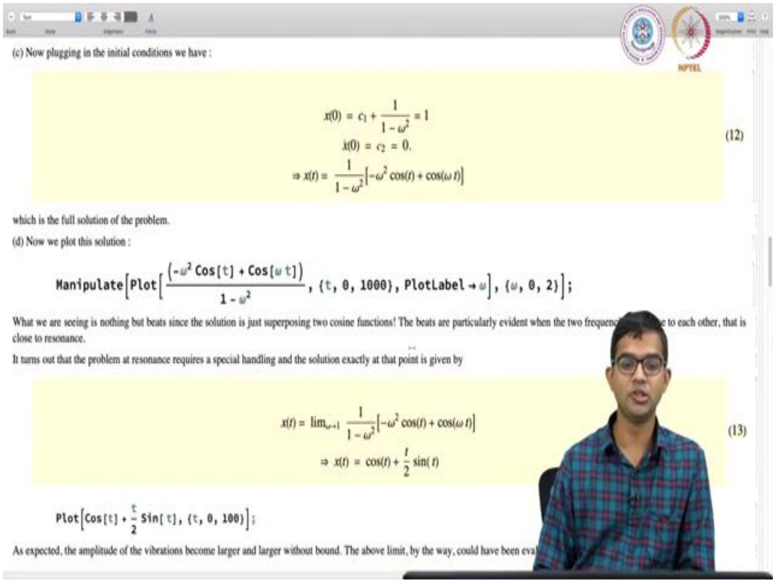

Then we visualized the data, we visualized the solution. Then we argued about the subtleties associated with the resonance point omega equal to 1. And we saw how the solution is of a different kind there. And then, we basically reproduced the data using the Euler Method.

So, let us, so, I have to do this clear global, so that there is no memory of the system, so that I do not need this. I will go on to Euler Improved.

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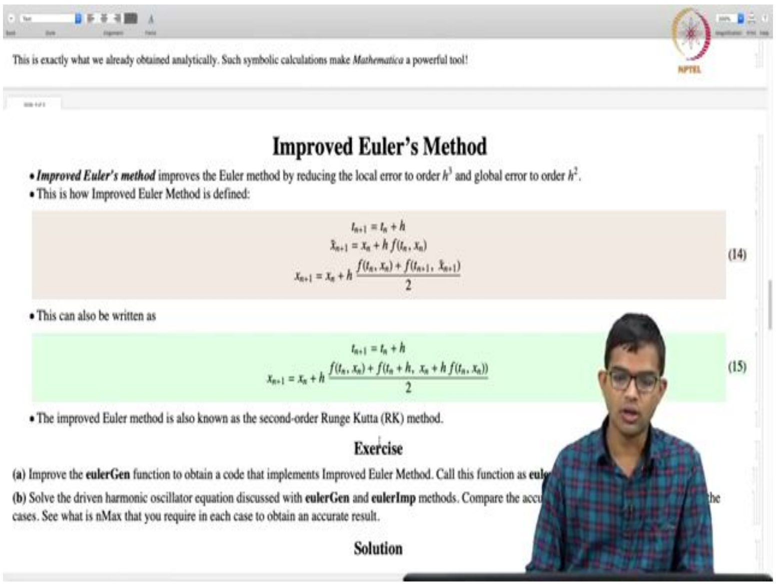

Okay, so the Improved Euler Method is like the Euler Method. But it just introduces one in between step. So, in the Euler Method, what do you do? You just go from x_n to x_{n+1}. Here too, that is what you will eventually do. But you do not directly go from x_n to x_{n+1}. You go to an x_{n+1}, which is like the Euler Method, right.

So, $x_n + h *$. So, you try to go to the next step. But then, you try, you use the information of the derivatives at both the points, right, so there is a certain, so in the Euler Method, you go to x_{n+1} using the derivative information at x_n. And that is it, you have moved to the next step. But here what you do is, you pretend to go to x_{n+1}, but you call it only $\tilde{x}_{n+1}$.

And then, go there and figure out the derivative information there. And then use the derivative information at this tilde point and then, along with the derivative information at xn itself and take the average of these two. That is the 3rd step. And it turns out that, this method of finding the derivative is going to give you much better results.

So, technically, this becomes a, it reduces the local error from h^3 to order h^2. So, the details you do not have to understand at this point. So, there is a notion of local error, global error and all this which you might see in a more sophisticated numerical methods course.

But so, we are here interested only in the implementation of this. And you can directly see by comparing with your numerics of using this method and the other method, that this is better. Okay so, this can also be written as in this, more compact way. And the Euler, Improved Euler Method is also known as the 2nd order RK Method, Runge-Kutta Method.

So, that is a more powerful 4th order Runge-Kutta Method, which also we will discuss at some point or maybe we will give it to you as a home work. So, if you want, you can pause and take the code from last time, where the Euler Method is implemented. And make some small modifications to it. And then you can, you can get the code for the Improved Euler Method, right.

And then go ahead and use the Improved Euler Methods to solve the driven harmonic oscillator problem. We have already seen how to solve the driven harmonic oscillator with the Euler Method. And then you can compare the accuracy of these two, doing some test. So, that is what I am doing here.

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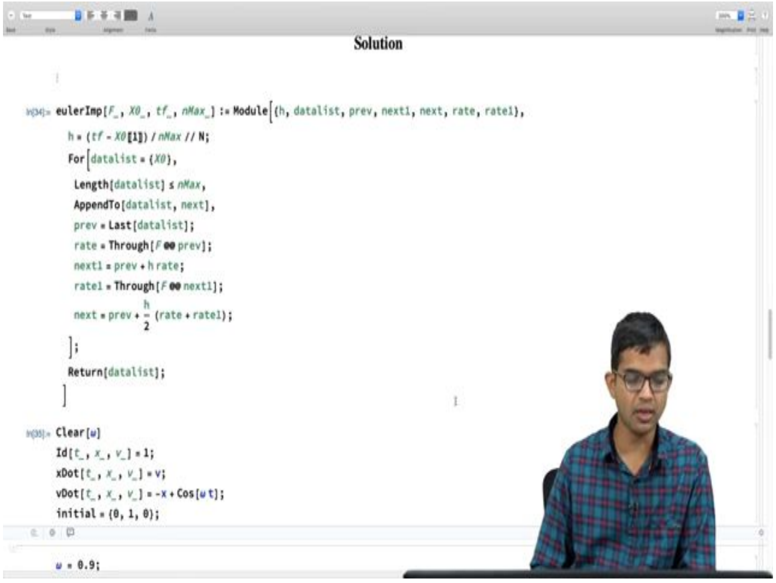

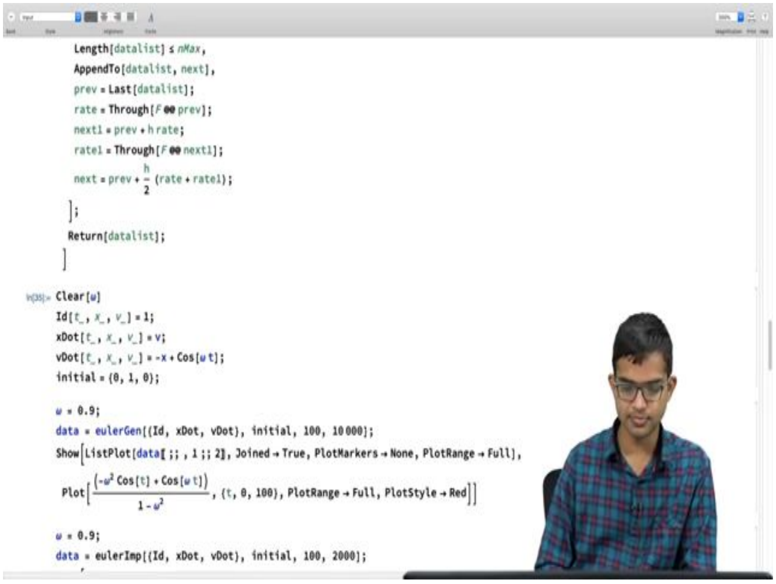

So, this is my implementation of Improved Euler Method, right, there are only some small modifications. There is this in-between step, which comes in, which is also implemented in here. But once again, I urge you to find your own solution, before you look at my solution. Maybe you have a better solution. Who knows, right?

So, we will clear ω, then once again I have, you know these 3 functions remain unchanged, right, and then I input all this. I choose ω = 1. So, now you see that if I use Euler Gen, what was taking me so long?

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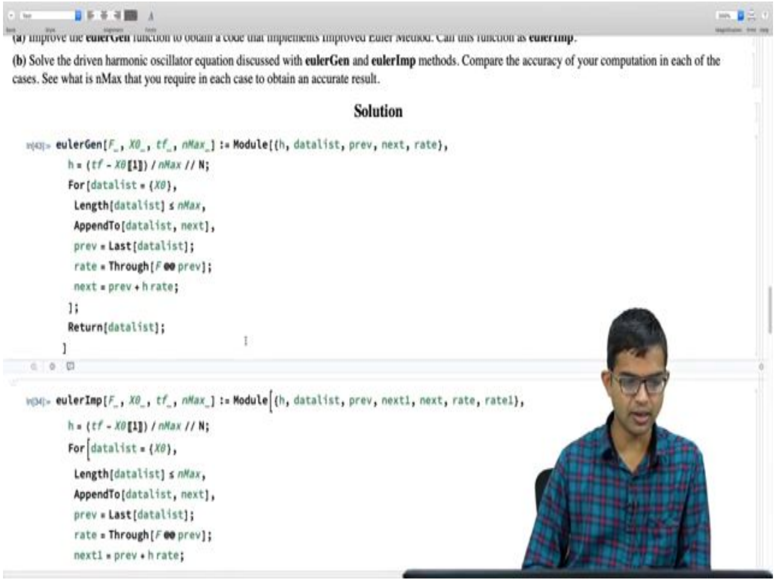

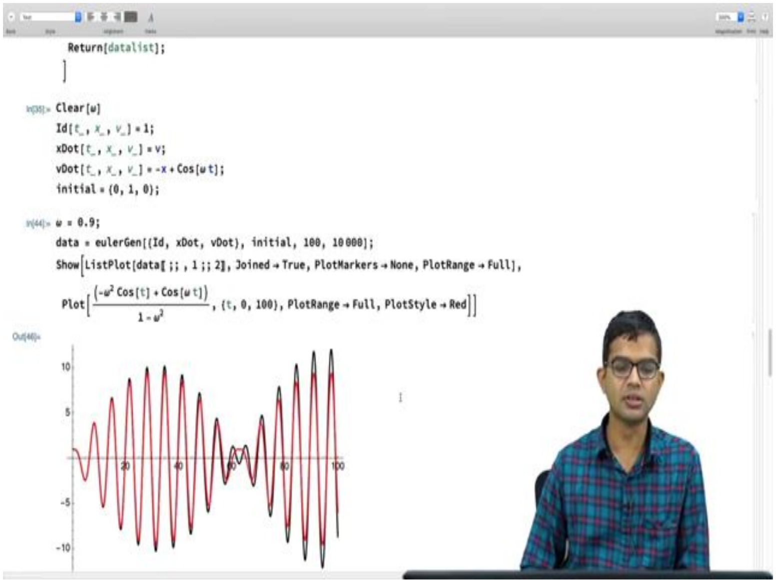

Okay, so, since I want to compare against Euler Gen, I have copied the old code as well for comparison here and I have copied and pasted it. I will run it. And then, if I go ahead and run this, so, this should work out alright. So, there you go, this is data that we have already seen. This is a plot that you have already seen. But now I want to re-do the whole calculation, but with the Improved Euler Method. There you go.

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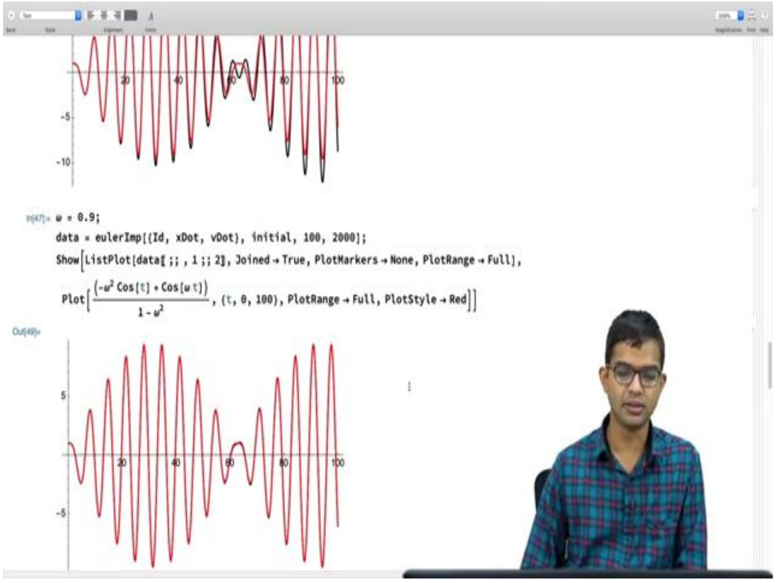

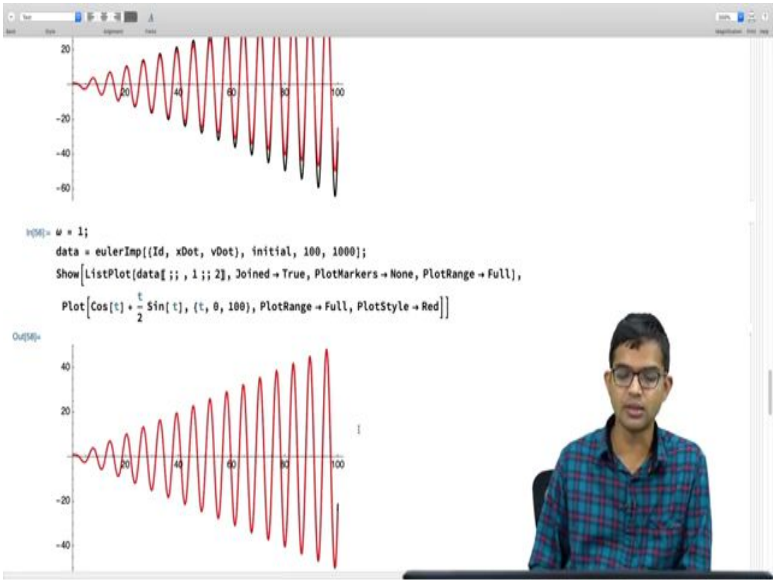

So, you see that the agreement is much better, not only is the agreement much better. But look at the time steps involved. I have only run it for 2000. There I ran even though I ran it with 10000, the agreement was not so great. Now here, it is like practically indistinguishable the red and the black are sitting on top of each other.

If I run it for 10000; and which I do not even need to do, 2000 already is good enough. If I go to the resonant case, so, let us see what happens with the resonant case. This is the old one. So, we saw that for larger times, it failed by a bigger and bigger margin with Improved Euler

Method once again, even just with keeping this as 1000, this parameter as 1000, already the agreement is excellent.

So, when you go from just the Euler Method by a small modification to Improved Euler Method, already you see that the results have improved so dramatically. But if you go from RK 2 to RK 4, the results will be even more dependable. And so, in fact RK 4 is the standard method, which is very reliable. Even there are higher order methods, but there is always a cost.

You know you pay in terms of the complexity of the code for the accuracy that you get. But it turns out that a nice balance between cost of the algorithm and accuracy is found in the RK 4 method, which will be described later on. Okay; so, that is what this module was about. But I want to finish this with some fun and games.

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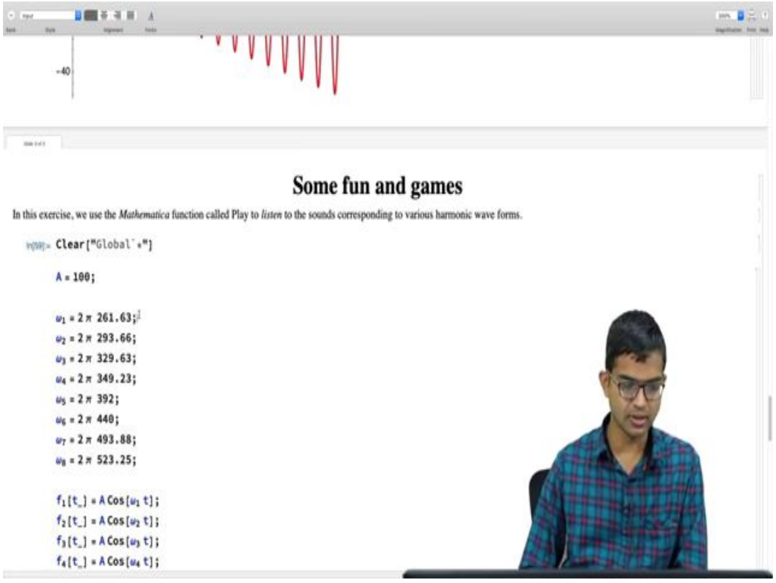

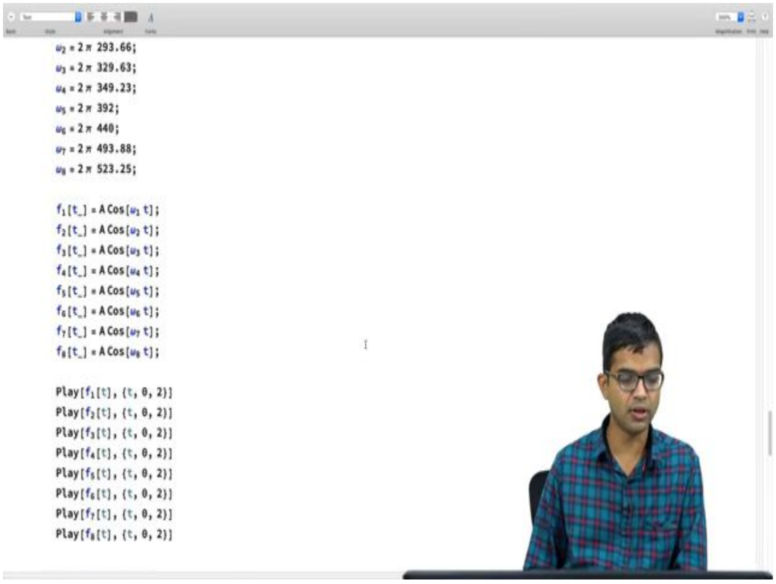

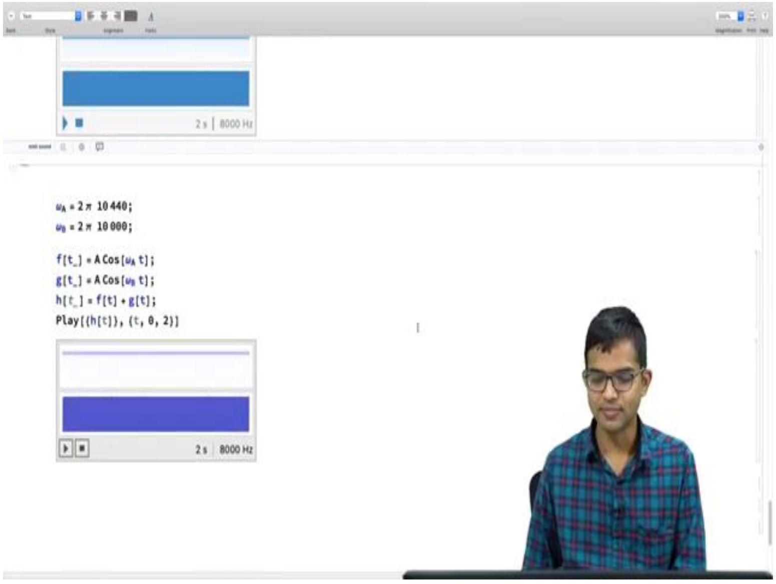

So, it turns out that you can actually, literally play sounds on Mathematica. So, there was this previous lecture, where I talked about oscillations and how you can superpose oscillations you know in terms of the linear superposition principle and so on. So, it turns out that you can go to Mathematica and create a wave, like this.

And not only visualize it, but actually you can even, you can hear it, right so, you create a wave of this kind, A cosine of omega 1 t, as if chosen a bunch of frequencies. And you will see in a moment, why I have chosen these. And there is this comment called Play. You can go ahead and play these various frequencies, right.

So, I have chosen some; you know a set of frequencies very carefully. You will see in a moment, what they imply. So, if I play this (Playing frequencies) there you go. Right, so, I hope you recognized what that was.

In fact, you can go ahead and create your own small groups of nodes and then you can superpose various waves and play them together. And see if you can make some nice discoveries or some happy sounds can come out of this. Okay, so, that is what this module was about. Thank you.