Physics through Computational Thinking

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Lecture 03

Plotting Simple Functions

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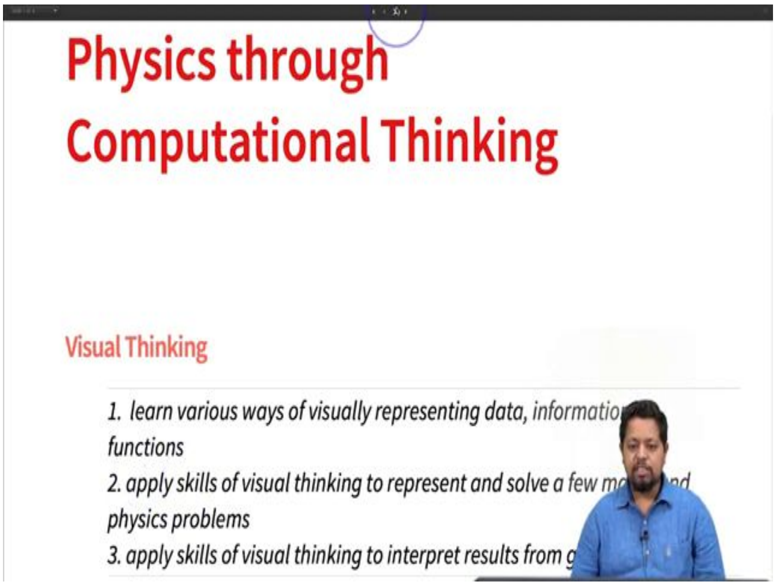

Okay guys, welcome back. From this video onwards I will shift to my own version of Mathematica installed on my computer. You can continue to work on the web version of Mathematica on the wolfram cloud or you can use a natively installed version on your desktop.

In todays video, we will talk about Visual Thinking. Visual thinking is about learning how to represent the information, data or our understanding, our knowledge in the form of a picture or a graph or a plot. It is also useful in solving problems. We will also learn how to take some piece of the problem and apply visual thinking to it, so that we can represent piece of the problem in form of a graph, so that our understanding about the concept of the problem can improve.

And finally, the third use of visual thinking is to interpret the data and improve our understanding of the problem. We represent the final results in terms of a graph or a plot and then we use that to understand and infer from that. Okay so, let us get started with today's lecture.

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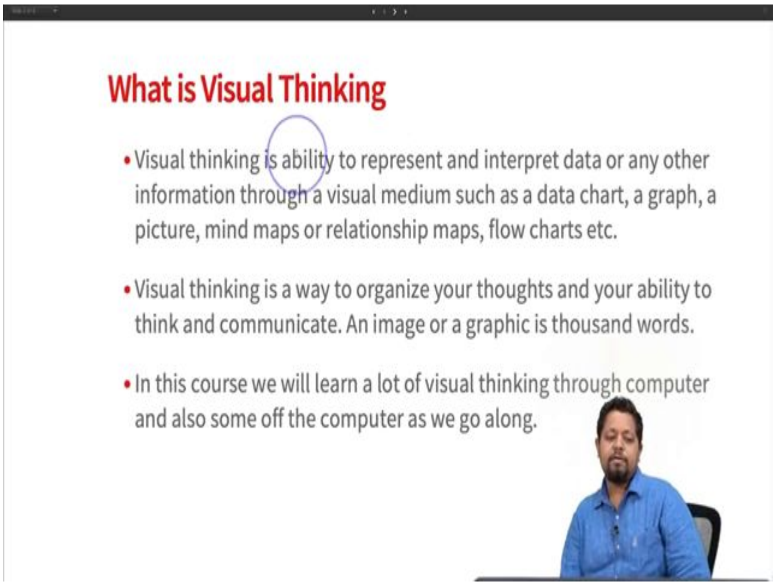

So, what is Visual Thinking? Visual thinking is ability to represent and interpret data or any other information through a visual medium, such as datachart, a graph, a picture, a mind map, a relationship map, flow charts, etc. So, visual thinking can be any of these many things. Visual thinking is also a way to organize your thoughts and ability and think and communicate.

Remember that picture is a 1000 words. A picture can say a lot more than what you can write in form of text or even express through equations. In this course, we will learn visual thinking throughout the course and how to do visual thinking on a computer, how to take the information that we have from problems and how to represent it on a computer. So, visual thinking is a very wide word, it can mean many things.

It can also mean representing your thoughts and ideas through an abstract painting. But that is not what we are referring to, we are referring to visual thinking as something where we take physics and math problems and represent them on a computer through visual medium.

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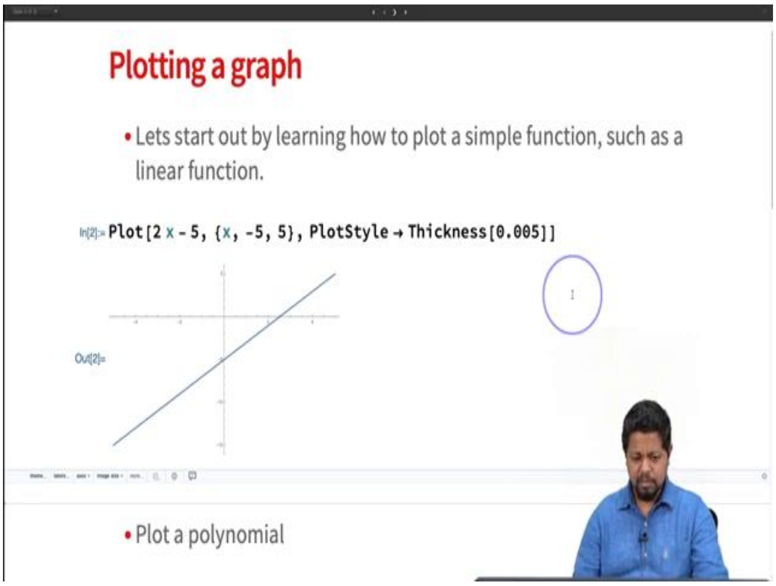

Let us move ahead and get started with something. So, we will learn how to plot a graph. I have already given you some introduction, so I will move through this very fast. Let us start by a very simple function, such as a linear function. The linear function in question here is 2x - 5, I want to plot 2 x - 5.

And as discussed last time, you can simply use the Plot command, express the range of the variable. The variable is x over here and the range of x is -5 to 5, and you can say a PlotStyle. And then you can go ahead and execute the command.

When you execute it, this is the output you get. What I can do is, I can actually make this thicker, maybe by adjusting the thickness. So, these little tricks we will keep on learning as we go along in the course. So, this was simply plotting a linear function. Let us go ahead and make our life a little bit more complex and plot a polynomial.

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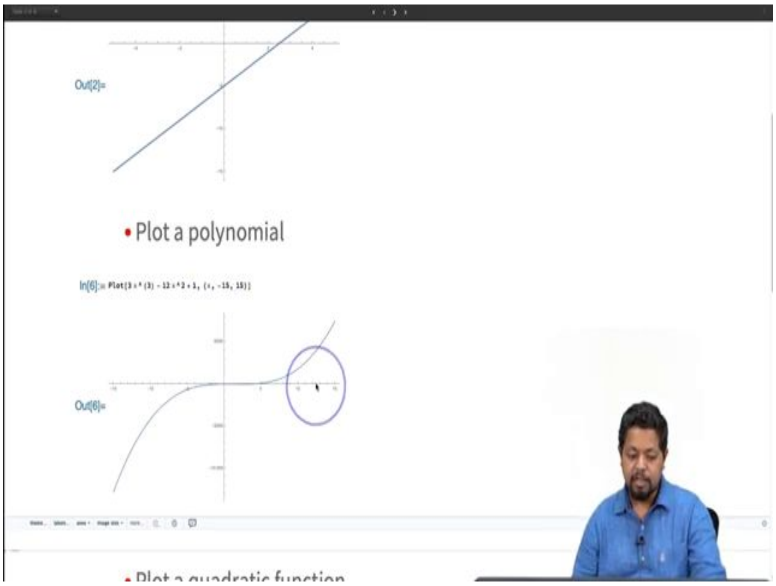

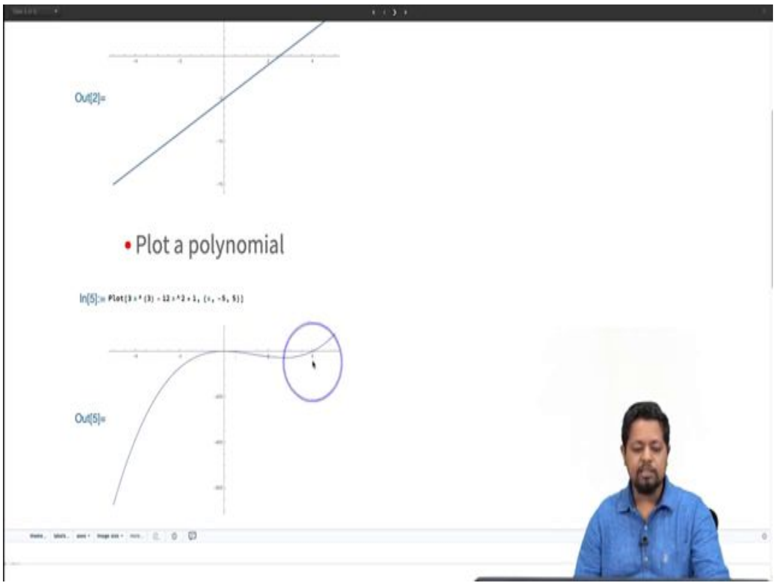

So, here is an example of a polynomial $x^3 - 4x^2 + 1$. When I execute it, I get this plot. Let us change this. Let us make it $3x^3$ or maybe we can increase this, make it 12. Now, you see, whenever there is a zero for the polynomial, there is a zero crossing with the x axis.

The plot crosses the x axis at 2 points, and that tells us this is the place where there are 0s. You can go ahead and quickly check $3x^3 - 12x^2 + 1$, where it's 0s lie, and confirm that they agree with this plot, which suggests that there are two 0s and those 2 zeros are at x = 0 and x = 4.

We can go ahead and change the range and we can make it -15 to 15. To see what is the asymptotic behavior of the function that is when x goes large how does the function behave? And in this zoomed out plot we are looking at the plot from far away. We see that the function diverges at negative x and diverges at positive x.

So, as x goes to -∞, the function goes to -∞ and as x goes to +∞, the function goes to +∞. And there is some detailed behavior going on in this region, which we can, of course, look more closely by zooming into that region. And from here the range -5 to 5 will give the more details, which is what we just saw.

Alright, let us go ahead and make our life slightly more complicated, this time we will plot a quadratic function. In fact, we will plot a bunch of quadratic functions.

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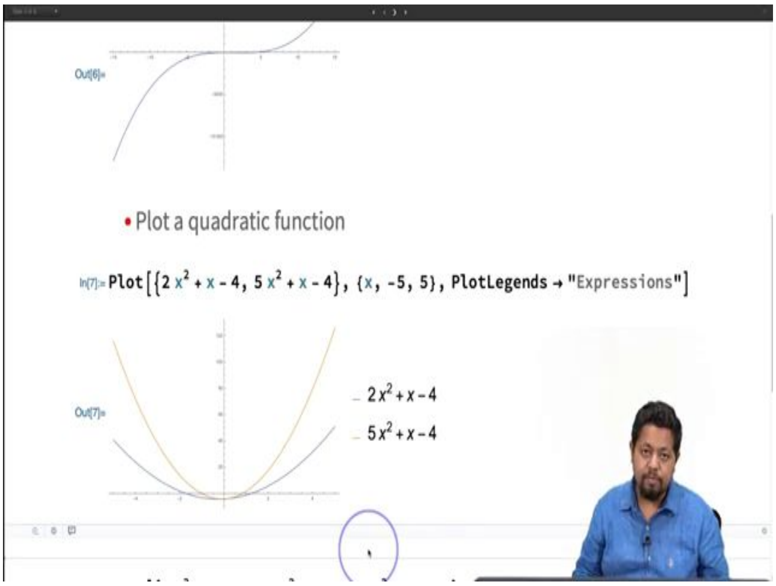

The functions I am interested in here are: $2\ x^2 + x - 4$, and $5\ x^2 + x - 4$. These are just some examples. You can go ahead and try your own examples. You can pause this video at any time and simultaneously work on the things that you are learning. It is a really good idea to keep working together. As I explain things here and you want to try something out. Go ahead, pause me and do your thing.

So, let us go ahead and plot this and see if I mentioned PlotLegends is Expression, so that I can actually identify which plot corresponds to which function. So, I get this quadratic function in orange and another one in blue. The difference between the orange and the blue, if you notice carefully the orange is more curved up. Both of them have a bowl shape facing up, but the orange one has a higher curvature than the blue curve.

This is something that even if you did not know, by looking at these functions, we could plot it and quickly see it. The curvature of quadratic functions is always related to its coefficient of x^2. Since the coefficient of x^2 is larger for the orange curve than the blue curve, the curvature is also bigger for the orange curve and that is something we can see here visually.

It is very important to understand or even predict what is going to be the functions behavior before we plot it. Usually we are not accustomed to thinking like that and we blindly use computer to plot it. But it is a good exercise to anticipate what the function is going to look like when you plot it.

And then when you plot it, you should examine, what does it look like? Is it agreeing with your expectations? Is it what it is that you expect it to see, or is something different? If it is something different, you can zoom in, zoom out and play around with the range of the plotting functions, choose other options in the plotting and you can understand the function better.

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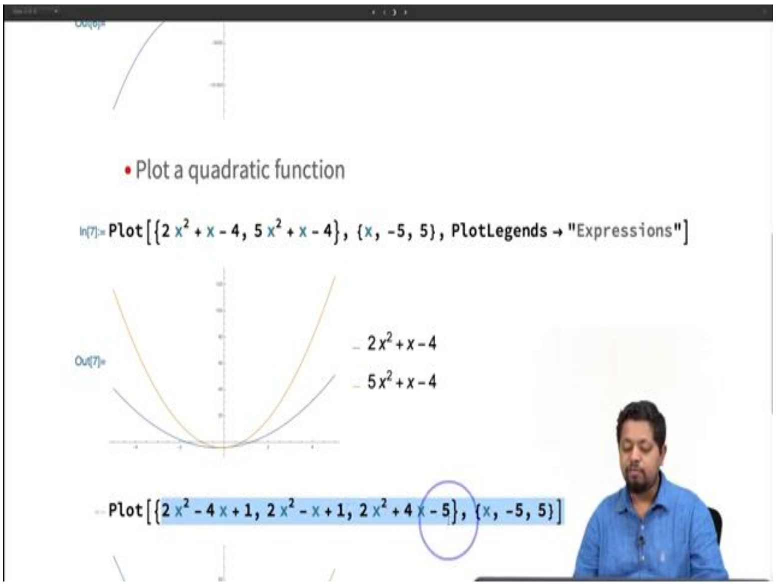

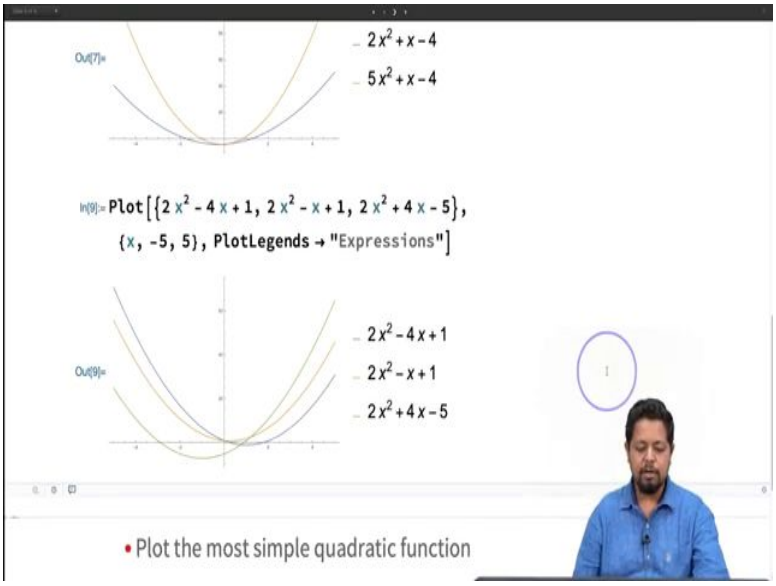

Ok. Let us go ahead and do some more plotting in this example over here. I want to plot these 3 functions. So, these are: $2x^2 - 4x + 1$, $2x^2 - x + 1$, $2x^2 + 4x - 5$ in the range x from -5 to 5. Go ahead and execute it. These are the 3 functions. These are the 3 plots that I get.

Now, it is hard for me to say which one is orange, which one is green. So, let me go ahead and add the options PlotLegends. All the options are always added after the main arguments of the function are being done. So, Plot function's minimum requirement is 2 arguments. This is the first argument. The second argument is the range of x. After that, I can add as many options as I like.

And in order to see the options, I will say PlotLegends and I will click here Expressions. And that is it. So, here are my 3 functions. Why are these 3 functions different? Green function has a minima over here, the orange one has a minima over here, the blue one has a minima somewhere over there, the minima's are different. Their curvatures appear to be the same and the minima's are located at a different value and their minimum value is also different. So, why is that happening? We want to understand that.

So, in order to understand that let us go ahead and plot the most simple quadratic function, which is x^2.

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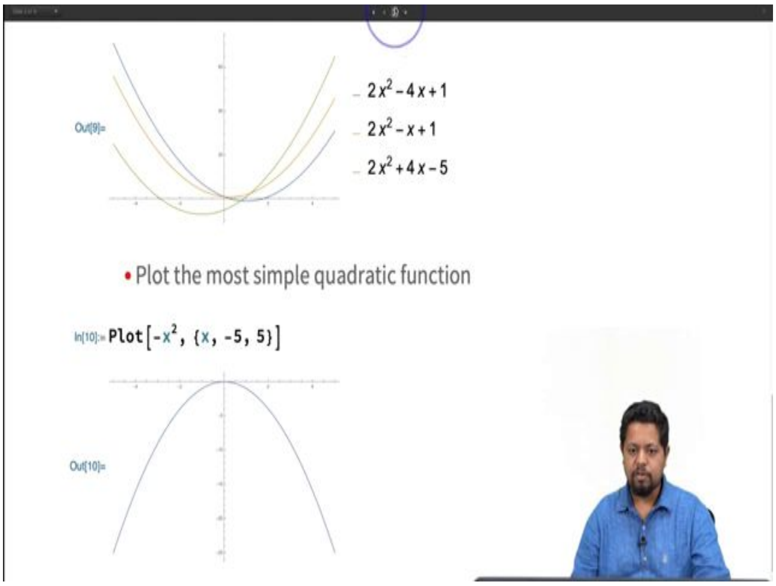

x^2 simply looks like a bowl shape curve, so what we noticed by comparing this shape and that shape is that essentially according functions as a bowl shape curve. It can curve upwards or downwards depending on whether the coefficient of x^2 is positive or negative. If I do $-x^2$, I get that. Now, all these other variations of quadratic function, what are they doing? They are just shifting the 0s.

Or if the coefficient of x^2 is different, they will make the bowl curve more or curve less. Apart from this, the basic behavior of quadratic function is all the same. The reason we are discussing quadratic functions here is because quadratic functions are something which are extremely important in physics they show up everywhere.

One of the reasons for quadratic functions appearing in many places is because anytime you combine 2 functions to form a minima, that minima can be approximated by a quadratic function.

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Understanding Quadratic Function

- Lets vary the parameters of a general quadratic function using **Manipulate** construct and see what is the effect of each parameter on the function.

$$a\,x^2 + b\,x + c = 0$$

```
Manipulate[Plot[a x^2 + b x + c, {x, -5, 5}, PlotRange → {{-5, 5}, {-10, 10}},
  Frame → True], {{a, 1}, -5, 5}, {{b, 0}, -5, 5}, {{c, 0}, -5, 5}]
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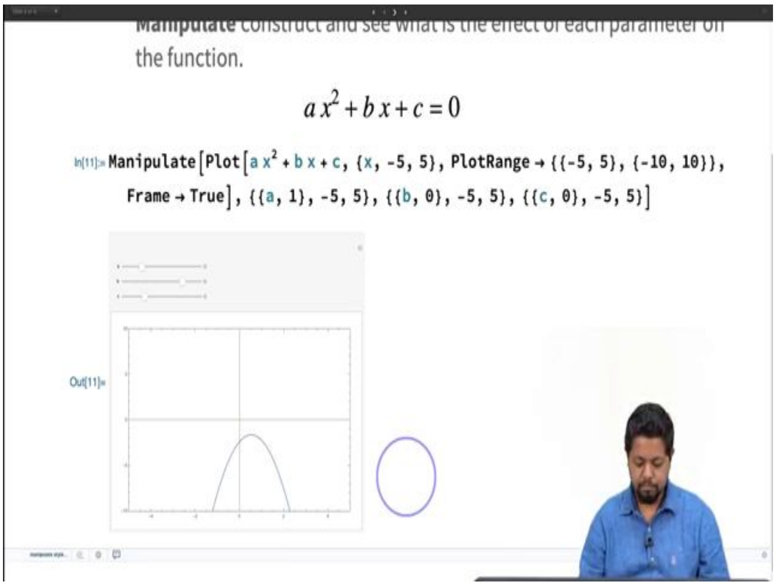

Alright, let us go ahead and understand this further by understanding quadratic function. So, what we will do in this particular exercise is we will take the the quadratic function a x^2 + b x + c, which is the most common quadratic function, 0 is not required, 0 will say that find the roots, but we just want to plot the left hand side of this equation, that is "a x^2 + b x + c".

And I want to see what happens when I change a, b and c. This is how I can understand what these different parameters do for the function. In order to do that, I will use the Manipulate command. And remember last time we discussed this at great length, how to use Manipulate command, so this is an example where we will use Manipulate command to understand how these 3 parameters, i.e varying a, b, c, works.

So, let us go ahead and actually build it up slowly. So, this is the function, if I execute it, the outcome will be a plot with 3 sliders for a, b, c. As I change a, as I change b, my plot will change. In order to learn, let us construct this whole thing step by step. And this is a good exercise and thinking how to actually construct complicated functions.

At first look when you look at this piece of code, you may be scared of what is going on over here, but we will break it down for you in simple terms and we will build this complicated looking expression step by step. So, let us go ahead and do that.

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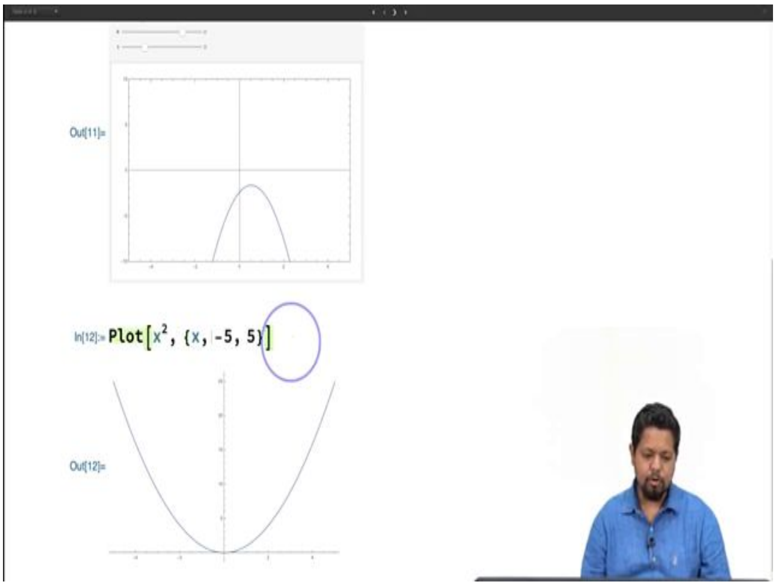

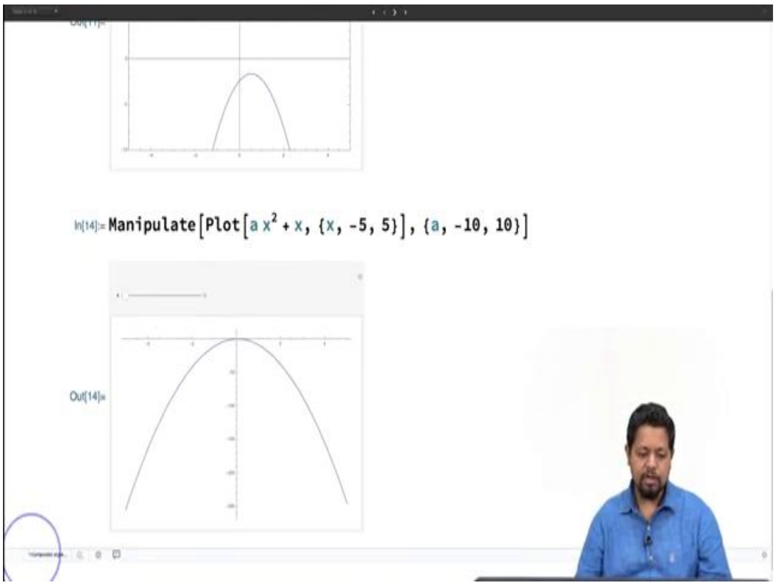

I want to simply so first plot x^2 in the range -5 to 5. Let me get that first. Once I have control of this, I will say I want to make it more complicated. Maybe I want to add x, and then see what happens. Great, now that we have got this under control, what we will do now is we will, so this is my fundamental function that I want to use.

I will wrap this into the Manipulate layer. I will take a Manipulate function and wrap this thing inside that. So, I will go ahead. I will say Manipulate, so there is this Manipulate. The auto completion allows you to complete some of these things really quickly. Now, for Manipulate I want to say: what do I want to Manipulate? I need a parameter. I haven't put any parameter over here. So, let me add a parameter.

Let me say the parameter here is a, give a space, so a is different from x^2, this is a x^2. And over here in the round brackets, I say that 'a' is a parameter for manipulate. So, I say 'a' goes from let us say -5 to 5 okay. Let us not, so that we are not confused. Let us make it -10 to 10 okay, x is -5 to 5 and a is -10 to 10. Let us go with that. Now I will execute this.

So, what does this do? It says that Manipulate the Plot function for 'a' between -10 and 10. Now, what is going to do is not to pick up some initial value of 'a' so that we can actually make some plot, it can execute the plot command for some value of 'a'. Otherwise, if 'a' is not defined, Plot command will not work. So, this will automatically pick a value of 'a' and in this case we will just pick the first initial value of 'a' that is -10.

So, as a consequence, you get an inverted parabola, an inverted bowl because 'a' it has chosen as -10. If I click on this +, it expands it out over here and I see that there is a -10, 'a' is set to -10, I can move the slider and change the value of 'a'. And as I slide and change the value of 'a', you see that pretty much nothing much is going on and then suddenly at around over here when 'a' was small, close to 0, the shape flips.

But when it appeared, nothing was going on what was actually happening was this scale on the y axis was just changing. The scale on the y axis is changing as I move the slider, the scale changes. Notice that again, I will move the slider. As I move the slider, the scale changes and the curve appears to be the same, this curve appears to be non-changing. And that is because scales are adjusting. So, I need to fix these scales, so that I can see actually the function is changing.

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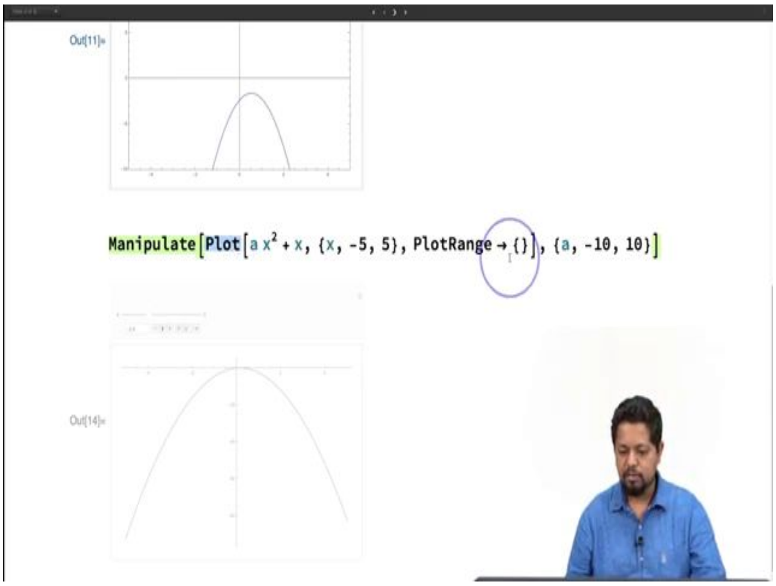

So, let us go ahead and do that by adding ‘a’, adding an option to Plot function, notice that I am adding option to the Plot function. This is the square bracket that is closing over here. So everything inside this square bracket and this square bracket is the arguments for the Plot function. The option I want to use here is PlotRange. Now, you might start to feel that where do you find all these options?

All these options are available in the manual of Mathematica that is in the help section of Mathematica. All you have to do is just select this and then click F1 on your computer and you will be able to see the information about PlotRange. If you want to find the options for a function, you select the function and press F1 on your computer, open the help for that function. And in that functions description, you can go and find what are the options available to do various kinds of manipulation.

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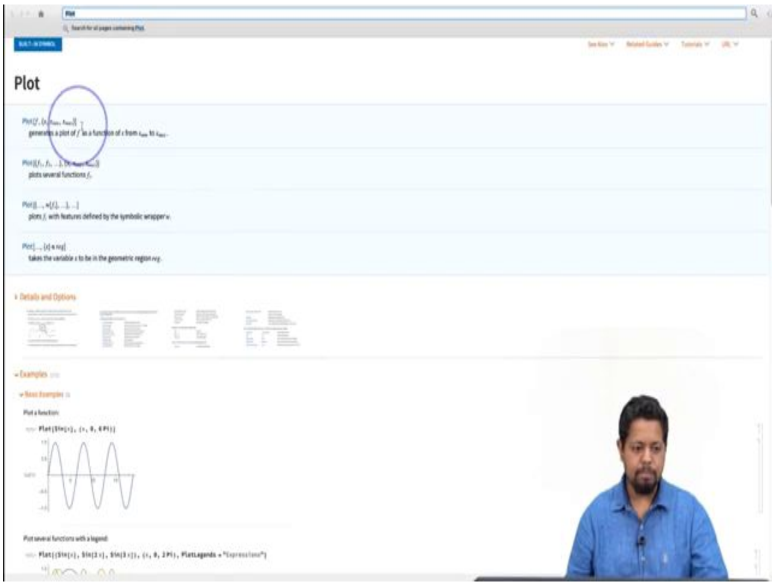

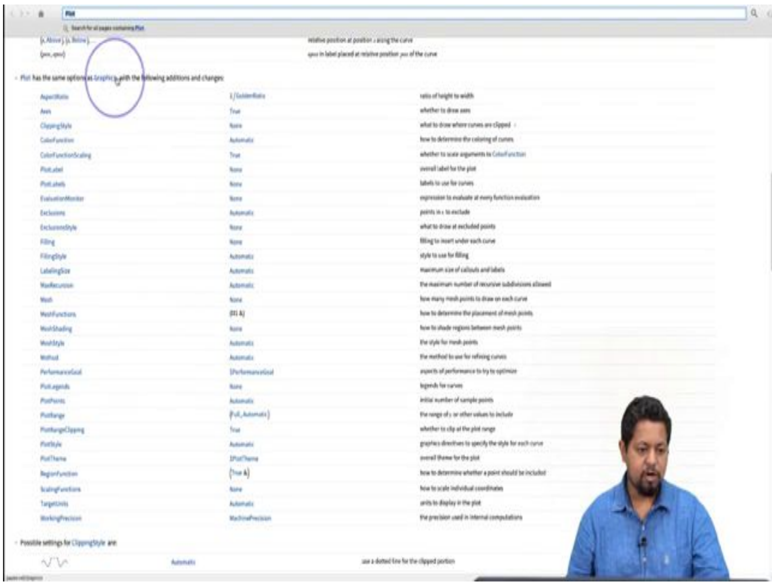

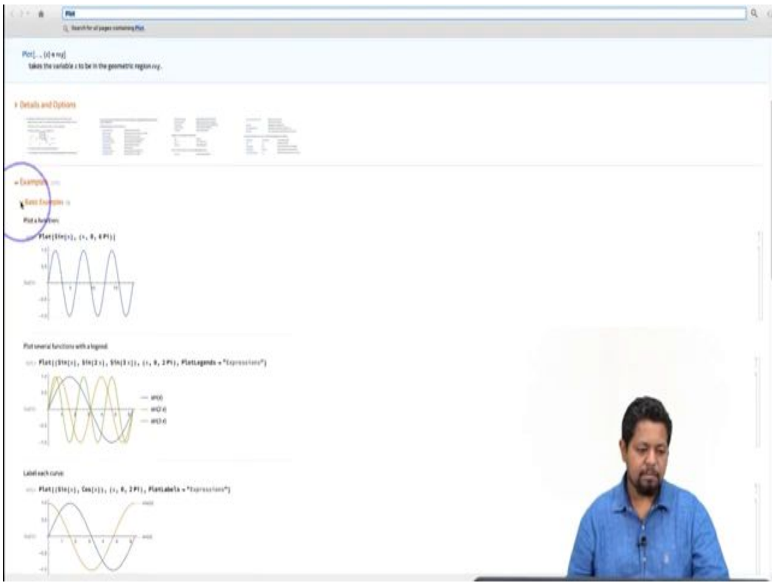

So, let me give you a quick introduction to that. If I press F1 on my computer, this will open a help, this shows plot over here. And then I can go into details and options. And I can go through this. This is extremely detailed and there is lots of information here and these are the options. So, this plot has the same options as graphics with following additions and changes so it gives a list of various options available for plot.

But this is not a really good way of learning, you should learn by examples. So, examples are given over here. So, you can go through basic examples or you can go through other options like the scope which gives what are the other ways of using the function.

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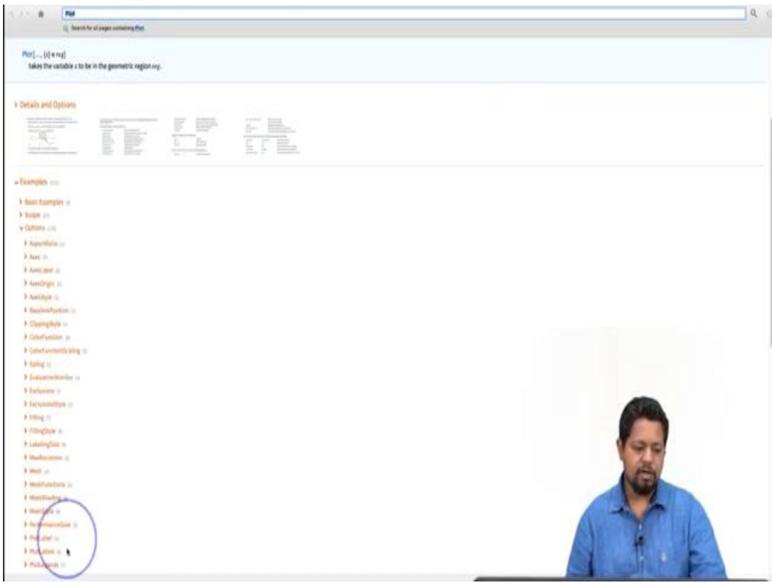

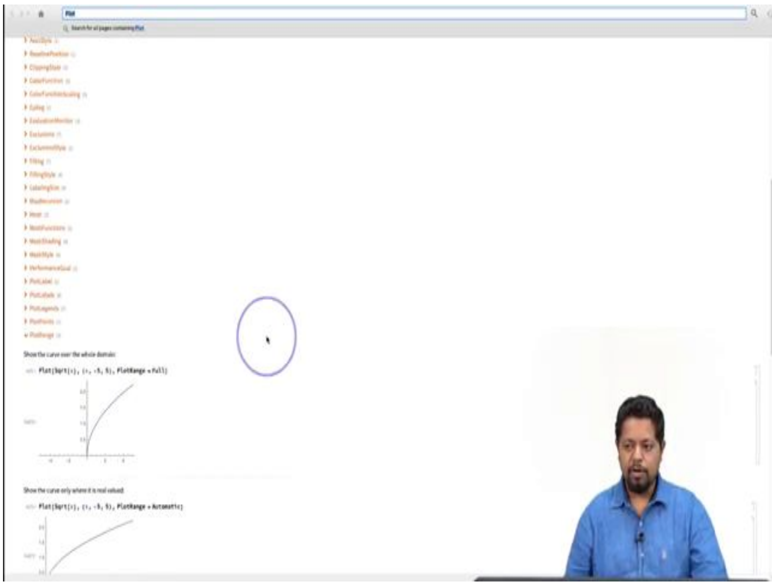

And probably the most useful at this point might be options, so that you can see examples of various options and see how to control. So, for example, PlotRange here is the details of the option, how I can control the plot range.

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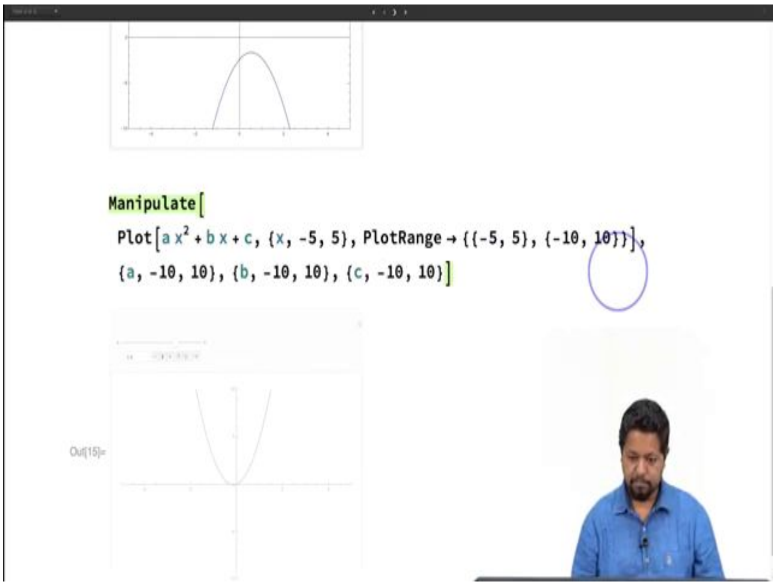

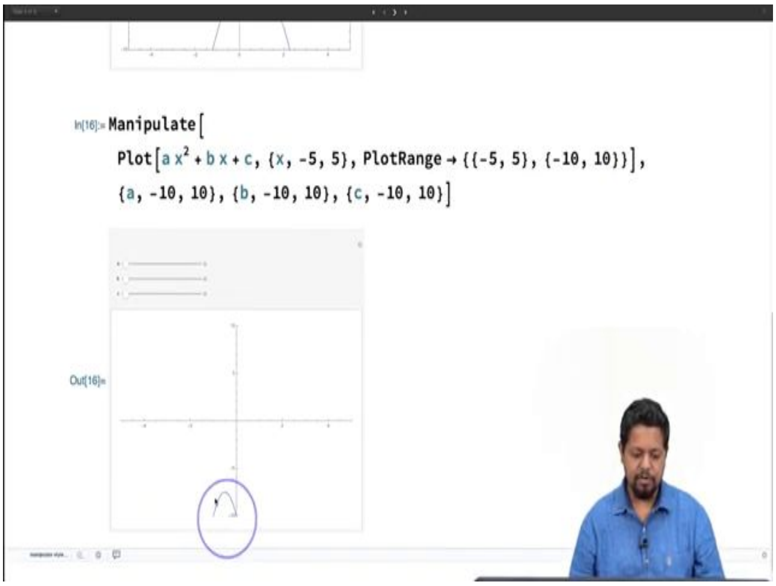

Ok. So, let us go back to our work. And here I want to introduce PlotRange. And I will say I want y-axis. I want the x-axis to be between -5 and 5. So, let me specify that -5 to 5 and then I want y axis to, in the range of -10 to 10. So, this will also freeze the frame for me. Great, so here we go. The frame is frozen.

Now, you see, as I change ‘a’, the curvature changes. At some point curvature becomes very small and then suddenly it flips over at very close to $x = 0$, and very close to $a = 0$ that flips over. And there we go. As I increase ‘a’, the curvature of the bowl/the parabola becomes higher and higher.

Great, so now that we accomplished this much, we will go ahead and add more parameters to this. I will add a parameter ‘b’ over here and a parameter ‘c’ over here and I will go ahead and specify all those parameters, comma separated arguments and I will say b is also -10 to 10, and c is also -10 to 10, I will execute that. And you see when I do that, I get a little parabola over here.

Fortunately, it did not go out of the range. I mean, these types of parameters could have been such that my plot may have ended outside the frame. For example, if I make this c is -50. I will see no plot because my parabola is way down. I do not see it. So, do not get surprised if something funny happens with your choice of parameters. Go back, think about it and fix it. All right, so I have got these 3 parameters here. I will go ahead and expand them by clicking on these little pluses and minuses.

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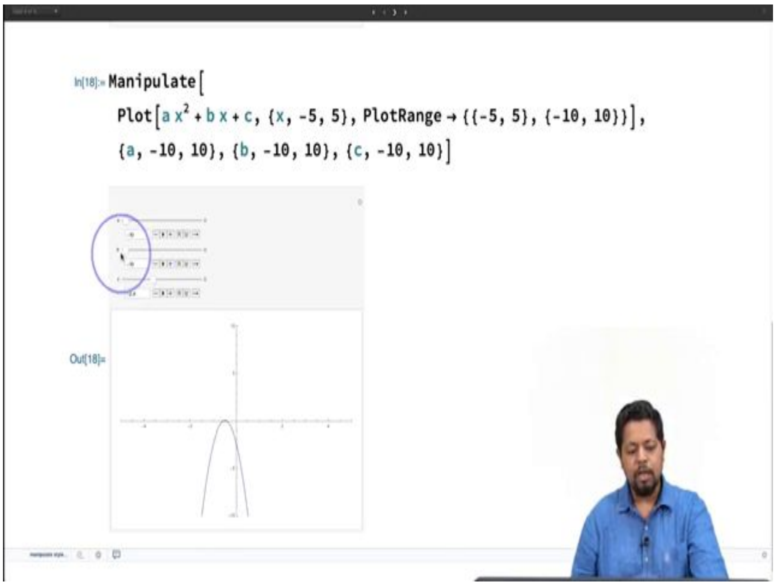

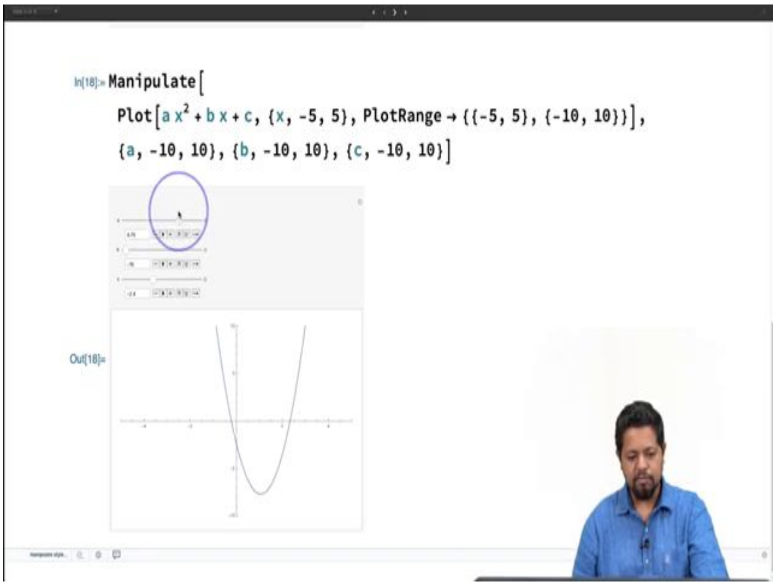

And now, I will go ahead and first increase c, let me pick it up, bring it up into the center over here. Now, I will change b, see what b does, b does something interesting, isn't it? Alright, 'a' you know what it does? 'a' changes the curvature and at some point it will change the curvature from negative to positive. Great, now that we know what 'a' does, let us just leave an over there. Figure out what c does? What does c do? It is a very simple thing, c simply leaves the entire parabola up and down. It just adds a constant value to the whole function.

So, all it does, it lifts the entire parabola up or it brings it down. Now, b does something interesting, b changes the position of the minima, it changes the 0s. Well of course, c also does that by moving the entire function up and down, the zeros change. The zeros are the crossing of the parabola with the x axis, this and that. As I change c, zeros change. As I change b, also zeros change but you see just moving up and down, the minima is not moving simply up and down.

It is also making some sort of curve, can you find out what is that curve? So leave that as a homework exercise for you to figure out. How is the minima changing when I move b? You see, do you have a guess. You should make an educated guess and then go back and test your theory whether you can find out how the minima changes or moves around what, as you change b that is what is the trajectory of the minima of a parabola when you change the parameter b.

Can you work over its equation, plot it on the same graph and then move b around and see that minima is actually moving on that. Okay I will leave this as a homework exercise for you. And we will talk about its solution at a later time. Great, so let me just quickly recap what we learnt here. Changing ‘a’ changes the curvature, changing b shifts the minima left and right, up and down as well. And changes the zeros, changing c, only shifts the minima up and down and changes the zeros.