Physics through Computational Thinking
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Lecture 19
Linear Superposition of Oscillations

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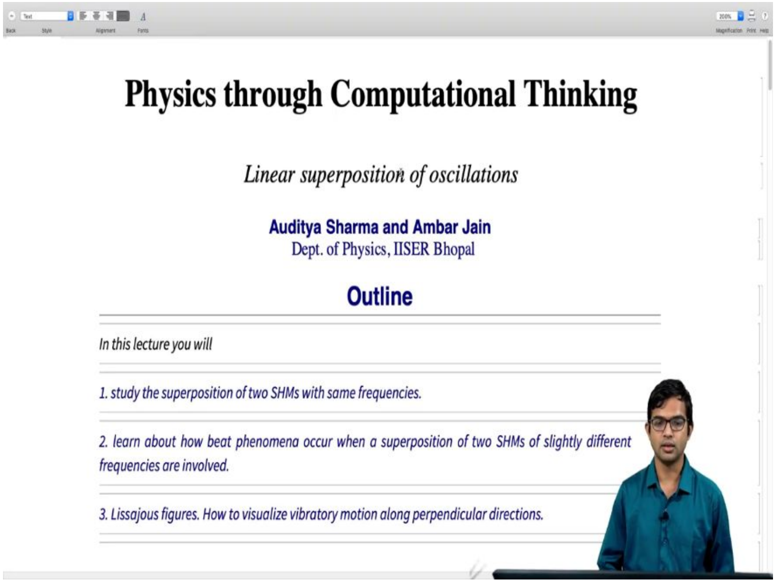

So, in this lecture we will look at the notion of linear superposition, we will look at how we can combine together oscillation and what kinds of signals that can come out of linear superposition. So, we will first start by recalling some ideas of linear superposition which perhaps you are already familiar with and then we will see how with the help of Mathematica we can visualize some of these ideas.

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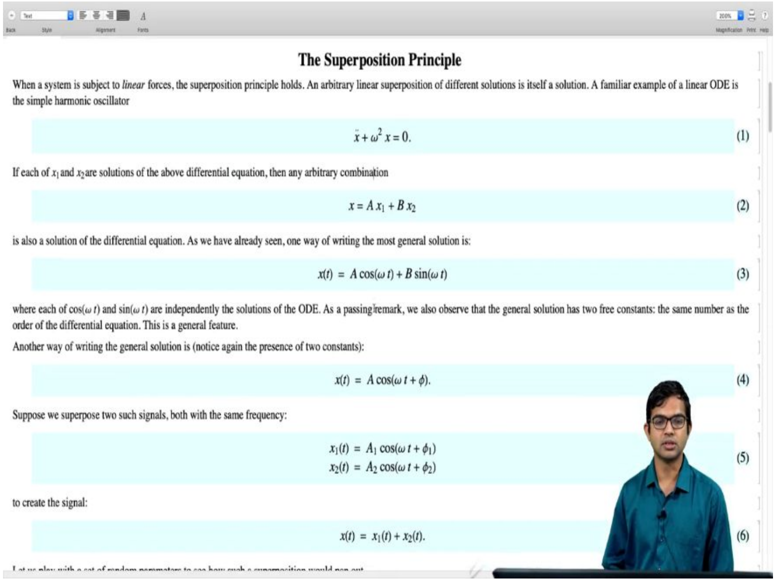

Alright, so, the superposition principle says that if a system is subjected to a bunch of linear forces, so in some sense, this is actually a definition of what constitutes a linear force. If superposition principle holds, then it is linear in nature and if system is linear then superposition principle holds.

So, a familiar example of a linear system is a linear ODE, which comes from the simple harmonic oscillator right? So, we know that in such a system if x_1 and x_2 are two independent solutions of such a differential equation, then an arbitrary combination of these two solutions like $A * x_1 + B * x_2$ is also going to be a solution of such a differential equation.

So, this is a very useful property and a lot of physical phenomena you know have this feature underlying them and it simplifies their analysis substantially. Of course, there are also, as opposed to linear systems, there are also a lot of systems which are nonlinear in nature and life is very hard whenever nonlinearities are involved.

And that is a whole discipline in itself, nonlinear systems. So, but here, so let us just recall this principle, which is that you can have linear combinations also being valid solutions. And then what happens to this particular example, the simple harmonic oscillator is that of course, we know that Cos(ωt) is one solution, Sin(ωt) is another solution, as you can verify by explicitly plugging in these functions into the equation.

And so in fact, the general solution for this x(t) is going to be A*Cos(ωt) + B*Sin(ωt). So, a completely equivalent way of writing down this general solution for this particular problem is A Cos(ωt + φ). So, as you can see once again, you have two free parameters. right

So, this is a second order differential equation and it is linear. So, the theory tells us that in fact, there would be two free parameters, it could be either A and B as in equation three or it could be A and φ as in this form and these two are completely equivalent ways of writing out the general solution.

So here, we want to ask what happens if you know you take two signals right. So, it could be signals which have come out of a solution of a differential equation like this or you may be thinking of preparing you know such signals in the lab and you ask, what happens if you simultaneously have both of these kinds of signals present? So, then these two signals would add to create a signal which is a combination of the two like here, $x(t) = x_1 + x_2$. So, here we want to use Mathematica and plot various functions and see how these combinations would play out.

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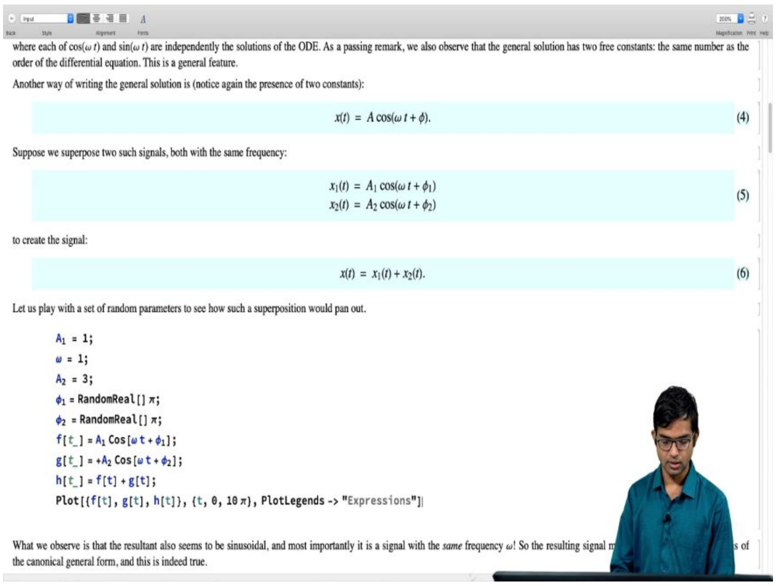

So, for this purpose I have considered $A_1 = 1$ and $A_2 = 3$. I have just randomly taken some numbers. ω is, so I am adding two signals, both of which have the same frequency right so later on, we will consider other kinds of super positions. For now, let us say I am considering two signals with the same frequency ω.

So, for simplicity I am taking $\omega = 1$. And then I am playing with these phases, φ_1 and φ_2. I am defining all these quantities here. And then I have this function f[t], which is $A_1 Cos(\omega t+\varphi_1)$ have another function $A_2 Cos(\omega t+\varphi_2)$.

So, this is the mathematical equation which I am trying to add and this is the syntax to make them all you know for Mathematica to understand them as functions here and which can be plotted. So, I am just going to create a new function h, which is the sum of these two functions, all very intuitive but there is also syntax in here. So, notice that I have used this function called random real.

Random real will give you a number between 0 and 1 and drawn from a uniform distribution right. It is a useful function which Mathematica has inbuilt. And here I am trying to generate phases which are truly random. So, I am just taking it to be some arbitrary multiple of п. I mean, I could have taken it to be an arbitrary multiply of 2п right. So, but you can play this game. So, let us see what happens.

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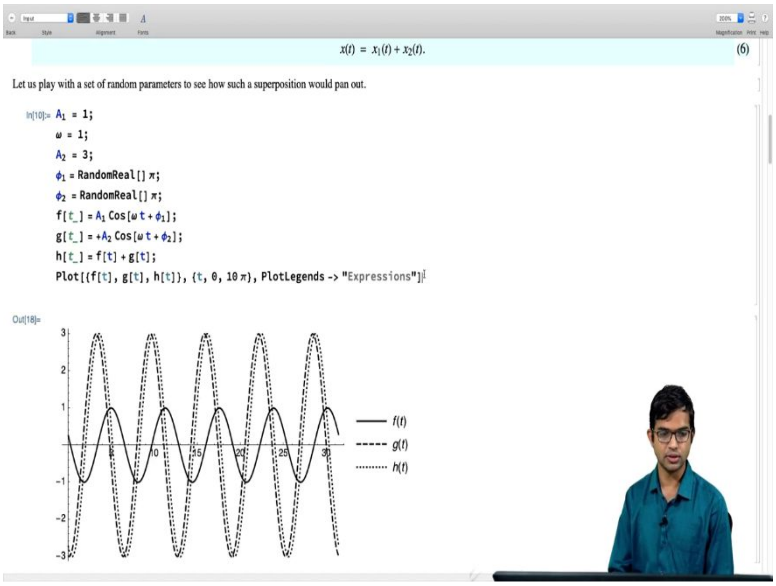

$$x(t) = A\cos(\omega t) + B\sin(\omega t) \quad (3)$$

where each of $\cos(\omega t)$ and $\sin(\omega t)$ are independently the solutions of the ODE. As a passing remark, we also observe that the general solution has two free constants: the same number as the order of the differential equation. This is a general feature.

Another way of writing the general solution is (notice again the presence of two constants):

$$x(t) = A\cos(\omega t + \phi). \quad (4)$$

Suppose we superpose two such signals, both with the same frequency:

$$\begin{aligned} x_1(t) &= A_1\cos(\omega t + \phi_1) \\ x_2(t) &= A_2\cos(\omega t + \phi_2) \end{aligned} \quad (5)$$

to create the signal:

$$x(t) = x_1(t) + x_2(t). \quad (6)$$

Let us play with a set of random parameters to see how such a superposition would pan out.

```
In[19]:= A1 = 1;
ω = 1;
A2 = 3;
ϕ1 = RandomReal[] π;
ϕ2 = RandomReal[] π;
f[t_] = A1 Cos[ω t + ϕ1];
g[t_] = +A2 Cos[ω t + ϕ2];
h[t_] = f[t] + g[t];
Plot[{f[t], g[t], h[t]}, {t, 0, 10 π}, PlotLegends -> "Expressions"]
```

```
g[t_] = +A2 Cos[ω t + ϕ2];
h[t_] = f[t] + g[t];
Plot[{f[t], g[t], h[t]}, {t, 0, 10 π}, PlotLegends -> "Expressions"]
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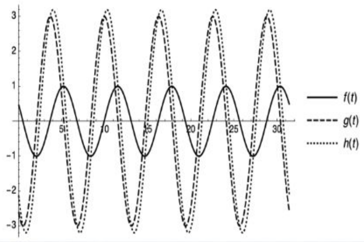

What we observe is that the resultant also seems to be sinusoidal, and most importantly it is a signal with the *same* frequency ω! So the resulting signal must also ... ble in terms of the canonical general form, and this is indeed true.

Homework: Show that $x(t) = A\cos(\omega t + \phi)$, where $A^2 = A_1^2 + A_2^2 + 2A_1A_2\cos(\phi_2 - \phi_1)$ and $\phi = \phi_1 + \beta$ where β is calculated from the expression $A\sin(\beta) = A_2\sin($...

As a quick check, we can look at the simple particular case when the amplitudes of both the signals are the same. Then we have

$$\begin{aligned} x(t) &= A_1[\cos(\omega t + \phi_1) + \cos(\omega t + \phi_2)] \\ &= 2A_1\left[\cos\left(\frac{\phi_1 - \phi_2}{2}\right)\cos\left(\omega t + \frac{\phi_1 + \phi_2}{2}\right)\right] \end{aligned} \quad (7)$$

implying that

```
g[t_] = +A2 Cos[ω t + ϕ2];
h[t_] = f[t] + g[t];
Plot[{f[t], g[t], h[t]}, {t, 0, 10 π}, PlotLegends -> "Expressions"]
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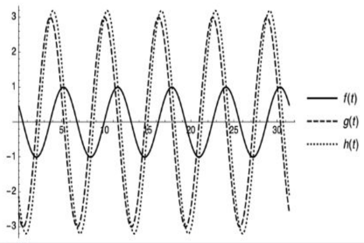

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implying that

So, if I plot this, I have function 1, function 2, and also the sum of these two functions. So, if I do it again, it looks something like this. If I do it a third time and so on. I can play this game. So f is the solid line, g is this dashed line, but bigger dashes, the smallest dashes are the final sum.

So, what is most important to observe here is that in fact, all three functions have the same period right, which is not a surprise if you think about it for a moment. You are adding two signals, each of which has the same frequency. So, it is not a surprise that the sum of these two signals also has the same frequency.

So, and it is sinusoidal in nature. You added two functions, which were sinusoidal, which were of the same nature. So, the final signal also must be expressible in canonical form, and which in fact is true. So, this is homework for you to work this out to show in fact, how to get this overall function in the canonical form.

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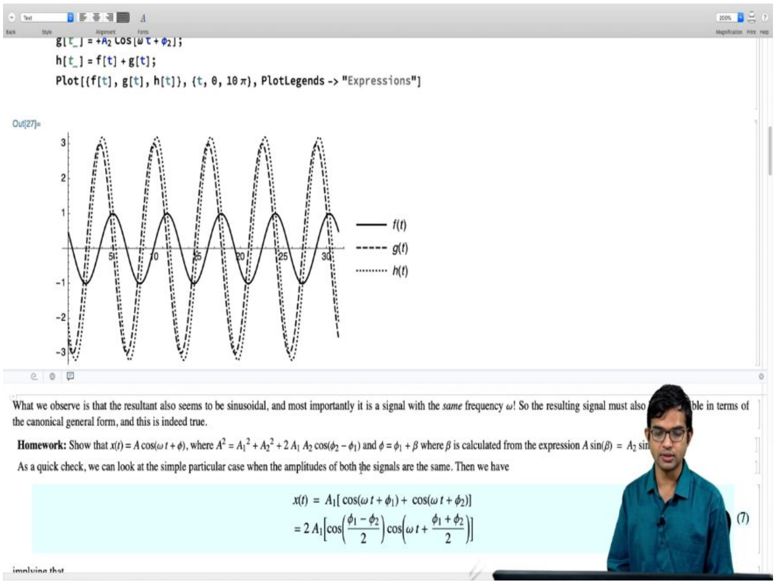

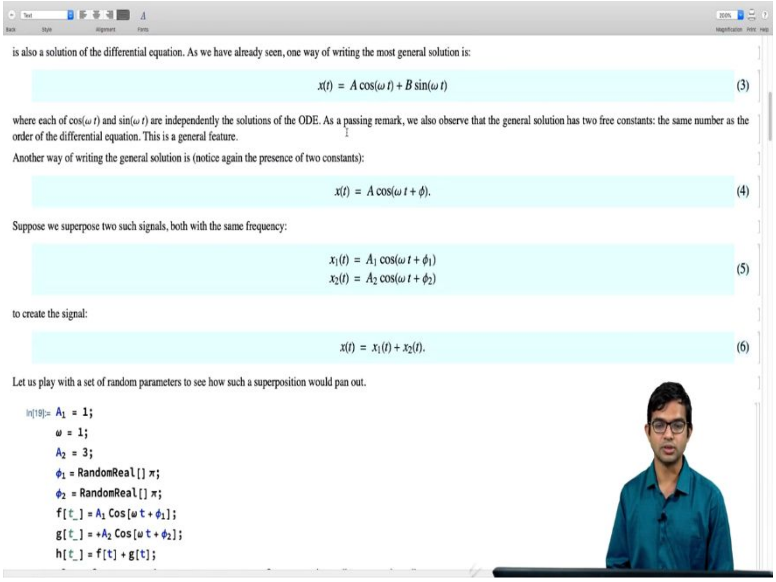

So, I said that $A*Cos(\omega t+\varphi)$ is one form for this function, but you could also write it as, you know, there are other forms one of them is like here A*Cos(..) + B*Sin(..), but let us say that you want to get to this for expression $A*Cos(\omega t+\varphi)$. So, you have to extract these constants A and φ and rewrite them in terms of A_1, A_2, φ_1, φ_2, these are the norms and you need to get to A and φ.

So, it is an exercise, a relatively simple one for you to show that these expressions which are flashed here is for A^2 and φ. Using them you can actually work out what A is and what is φ. Ok. So, in order to quickly check that what I have flashed is reasonable, we can take some special cases. So, suppose you take the case where both the amplitudes A_1 and A_2 are the same.

So, then of course, you can write x(t) as A_1, a common A_1 outside times cosine of first argument + cosine of the second argument. And with some basic trigonometric identities, we can rewrite it as $2A_1 [Cos((\varphi_1 -\varphi_2)/2) * Cos(\omega t+(\varphi_1+\varphi_2)/2)]$.

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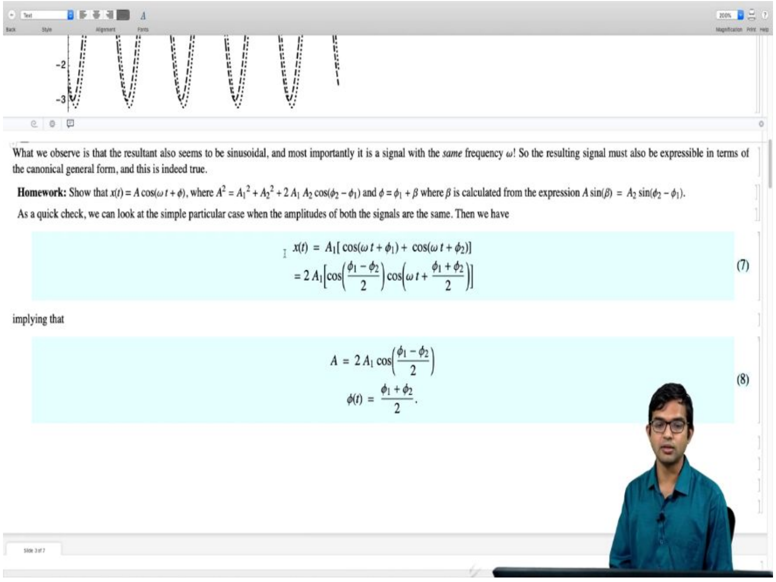

So, implying that A is nothing but $2A_1Cos((\varphi_1 - \varphi_2)/2)$ and $\varphi(t)$ is the mean of the phases of the two signals. So, this is just a quick check, but the more general expression is given here and you can work this out. So, it is a bit like one can give it a geometrical interpretation and there is a you know the cosine law, triangular law which one can exploit or you can do it by some other mean.

So, there is a whole theory of phases and all these things which you might have encountered in a elementary course on waves and optics and so on. Ok, but regardless of your background with some basic algebra, this is something that can be computed. So, let us move on to the idea of superposing two signals, but of different frequency.

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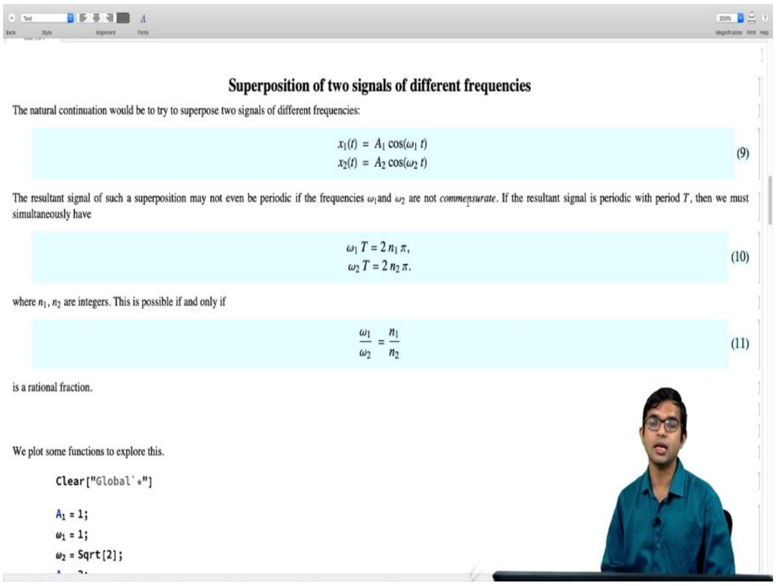

So, all this was straightforward. So, now what happens, suppose you add two frequencies, which are not the same, so you can again ask will the resulting signal be also of the same? Now, there is no common frequency. Is it going to be periodic at all right? And it turns out that the answer depends on the relationship between ω_1 and ω_2.

If ω_1 and ω_2 are commensurate. That is the key word here. If they are commensurate, then indeed the resultant signal is also periodic. And with period T, which you can exploit, it is like finding the common multiple, least common multiple of these two frequencies right.

So, $\omega_1 t$ is equal to $2n_1\pi$ and $\omega_2 t$ is equal to $2n_2\pi$, where n_1 and n_2 are integers and so this is only possible if $\omega_1/\omega_2 = n_1/n_2$. This is the key condition. So, if ω_1/ω_2 is can be written as the ratio of two integers or it is a rational fraction, then and only then will your resultant signal be periodic right. So, let us play with Mathematica and see this out.

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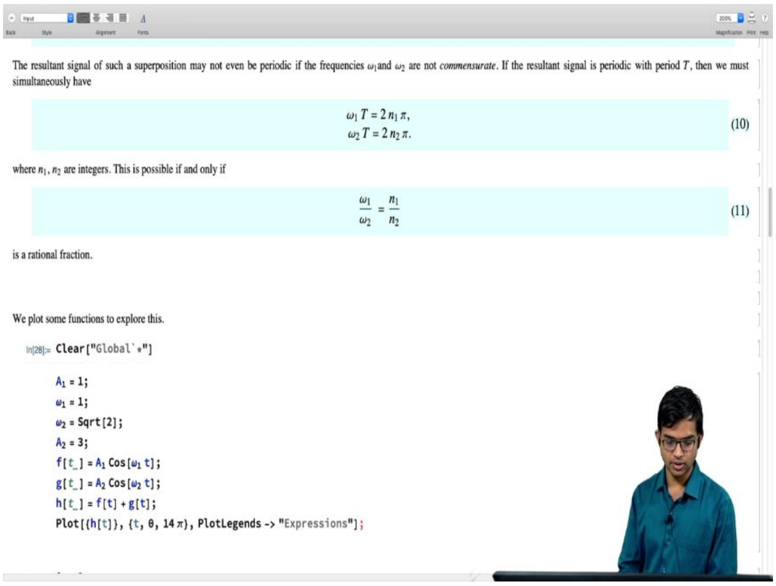

So, when you have relatively independent pieces of code that you are writing. So, we had some piece of code earlier, which we started in this session. And then if I want to write more code, sometimes some of the constants that we defined earlier, some of the parameters that we defined here here earlier might conflict with what we are going to define later on.

So, in order to avoid that, a useful method is to clear all the memory from Mathematica inside within one session. And that is to use this thing called clear of global with this very specific syntax. So, you must look at the syntax carefully and get all the fine details right and then you do shift enter.

So, then I have $A_1 = 1$ $\omega_1 = 1$. So, just to see how this thing will play out, I am taking $\omega_2 = \sqrt{2}$, which is not a rational fraction, which does not make ω_1/ω_2 a rational fraction. So, if I and then A_2, I am just simply choosing it to be 3 and then I have f and g and I am going to add them and then plot them. So, this is straightforward syntax. So, let me see what happens if I do this.

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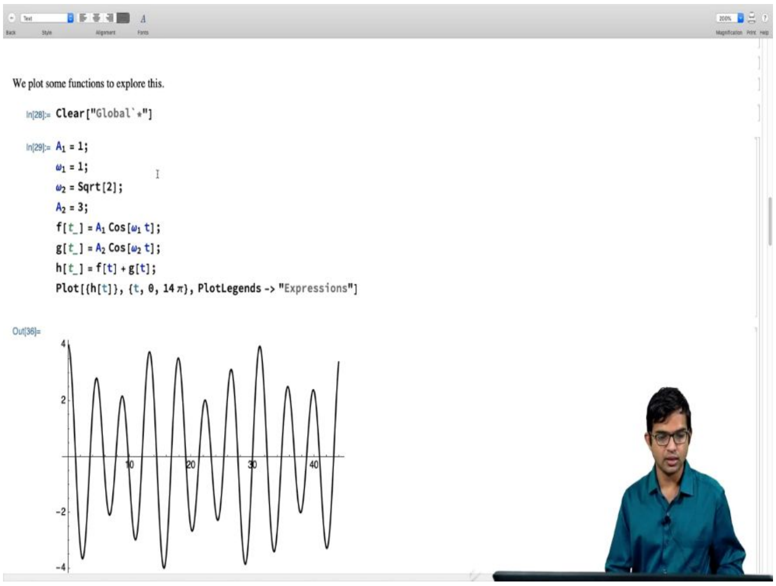

So, there you go. So, the resultant function, I can take it up to even larger. Let me take it to a larger time scale.

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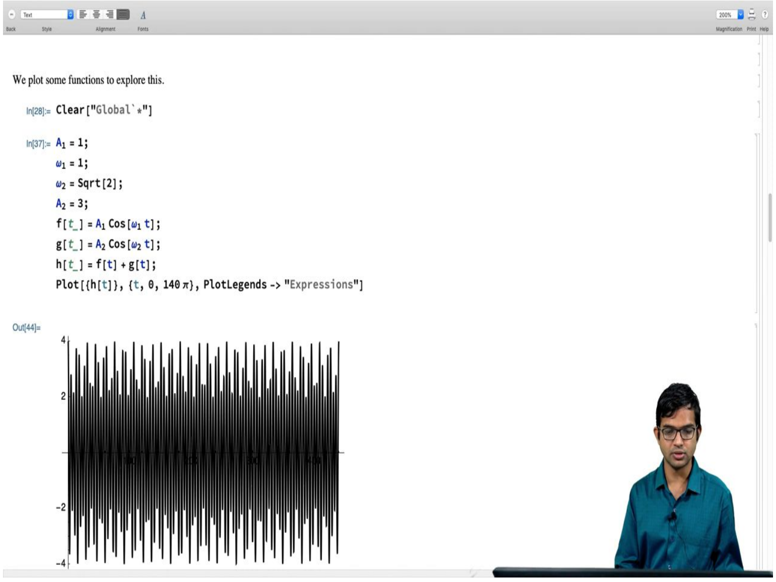

So, you see that it is, it looks kind of periodic, but actually it is not. So, there is no, there is some kind of order. It is pretending to have some order but it is because of this ω_2 being $\sqrt{2}$. So, let us take a smaller.

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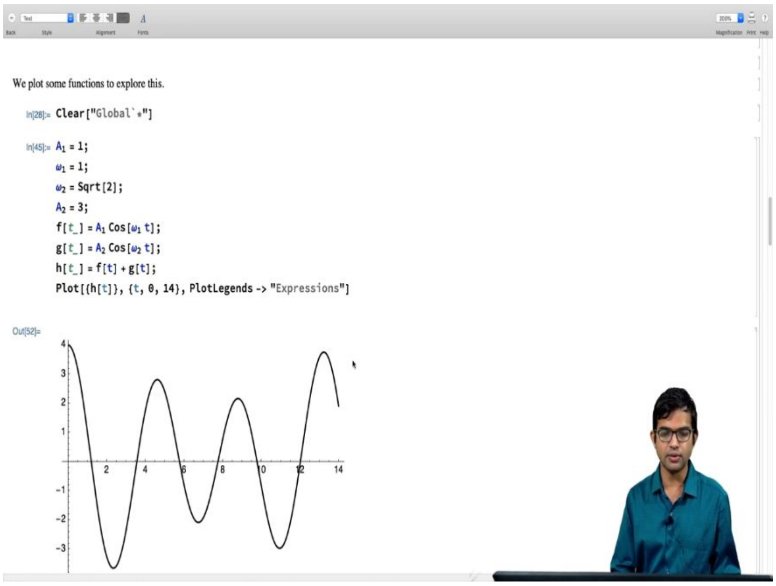

Yeah, so there you go. It is actually never really returning to where it started.

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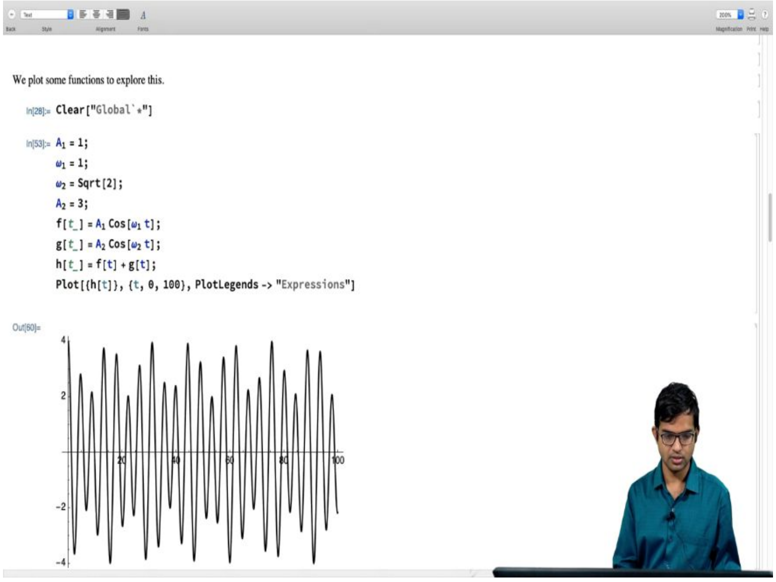

So, you will get a you know how irregular the function is, is also something that one can try to study and one can try to quantify. So, but we are not going there at this point. So, we just observe that, in fact, we do not get any recurrence, we do not get periodicity.

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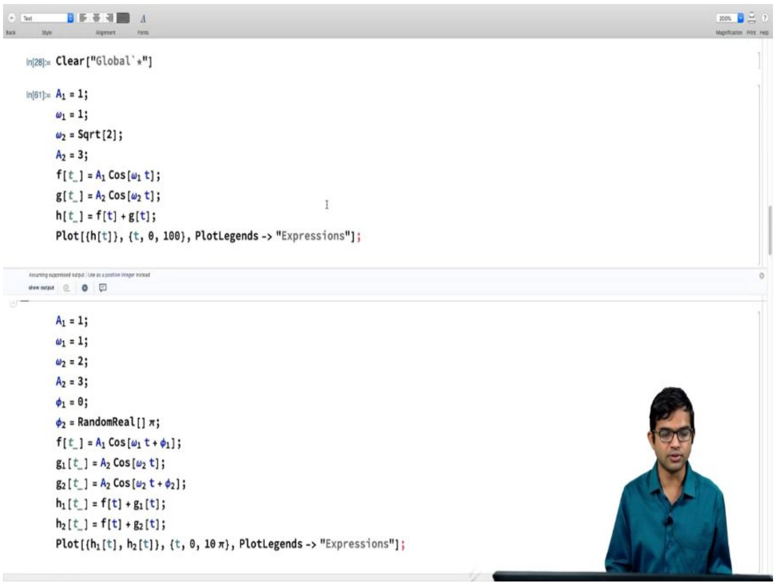

But on the other hand, if you choose A_1, ω_1/ω_2 to be a rational fraction like here, does not matter what A_1 is, does not matter what A_2 is and in fact, it does not even matter what, how these phases are related to each other, you can take them you can also play with these phases. For simplicity, I am just choosing φ_1 to be 1 as 0 and only φ_2, I will alter and let us see what happens if I do this and so I will plot this function.

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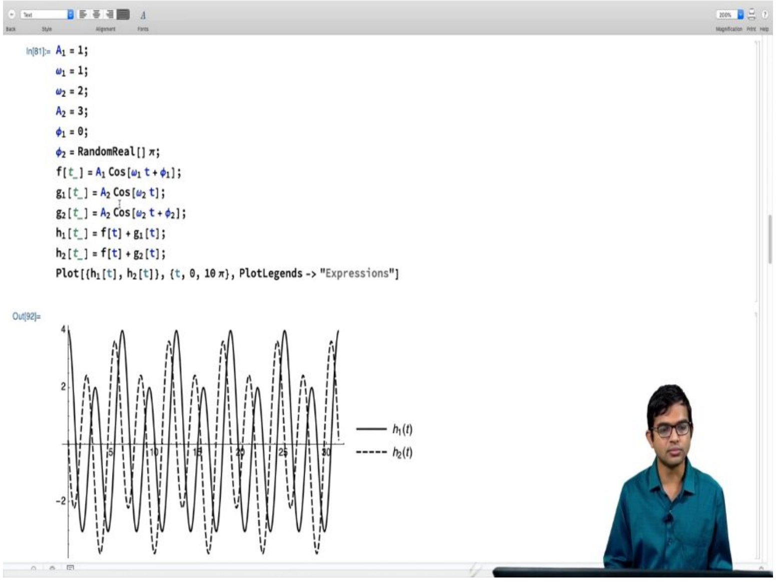

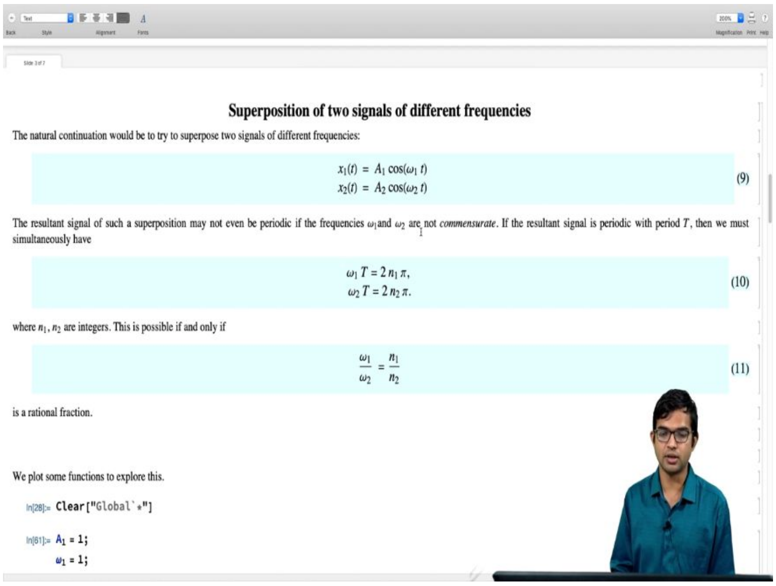

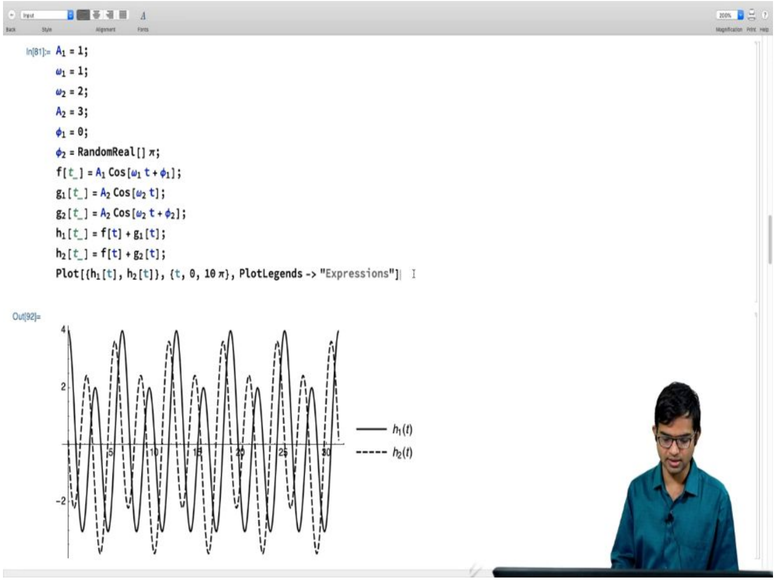

So, I am plotting both h_1 and h_2. So, I have h_1, where g_1 has no phase shift and where h_2 is composed of f and g_2, where g_2 does have this random phase shift. So, you will see that in any case, the periodicity of this overall function is not an issue at all, you will always get periodic functions, you will get some shift because of the phase.

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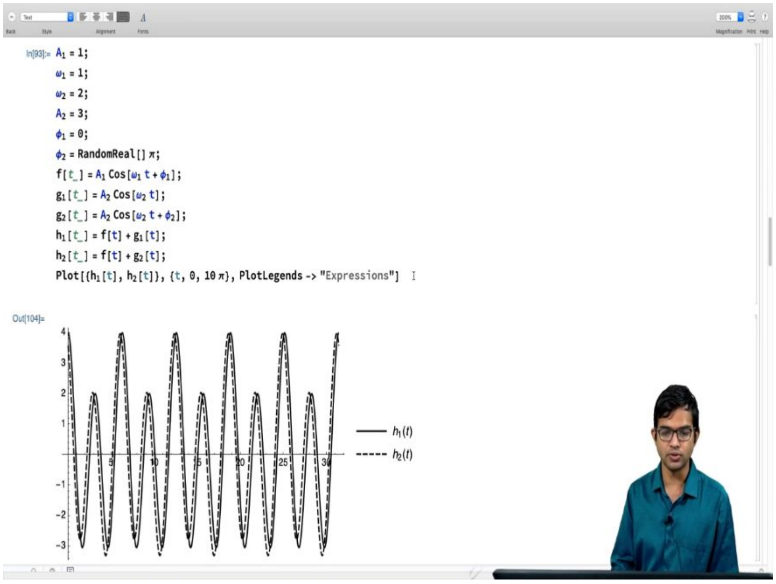

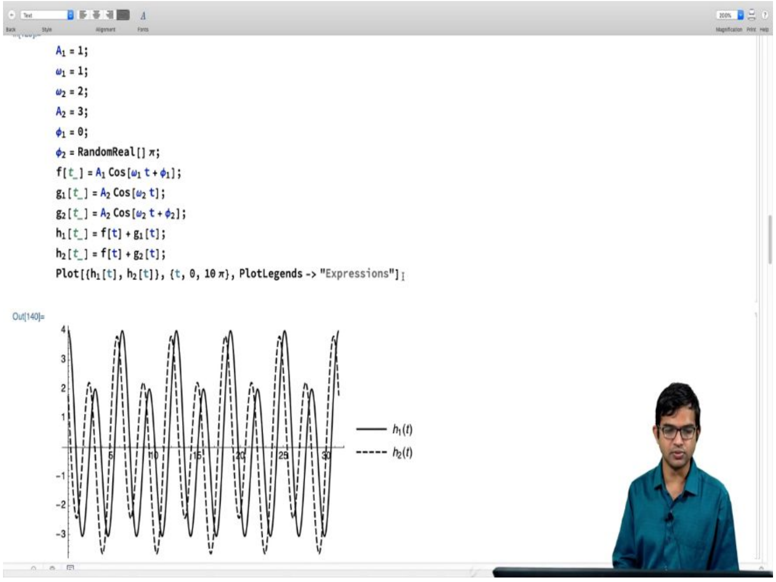

So, there is going to be some, only some details will change. But overall, it does not matter whether you look at h_1 or h_2, they are both going to turn out to be periodic right. And so the key message from here, this game that we just played is that if there are two signals with different frequencies that we are trying to superpose, the only thing that really counts is what is the ratio ω_1/ω_2. If it is a rational fraction, and for sure, you will get an overall function which is periodic, regardless of the relative phases, else you will not get it, does not matter what the phases are doing.

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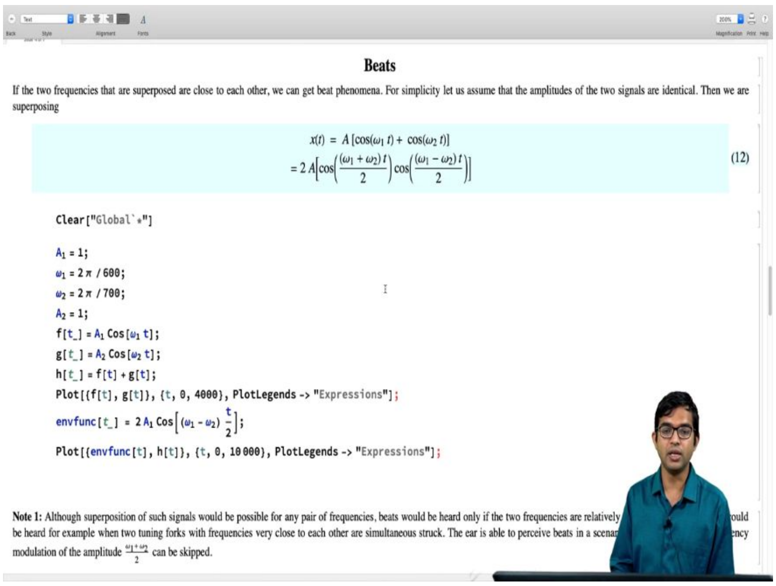

Okay, so another familiar concept in this context is that of beat phenomenon right. So, this happens when the two frequencies that are superposed are different, but not too different. If they are close to each other then we can get beat phenomenon. So, for simplicity, let us assume that the amplitudes of the two signals are identical. Of course, you can play this game with different amplitudes. And also, here I am taking the phases to be the same. So, in order to not clutter up the analysis.

So, let us say you have these superposition of these two functions $Cos(\omega_1 t) + Cos(\omega_2 t)$. So then, of course, just using the trigonometric identity, this is the overall signal. So once again, I will clear this, so I am going to go back and play this game. So A_1 I am taking to be 1 and A_2 also is 1, just like I have said both the A's are the same.

So, you see, I have taken ω_1 and ω_2 to be slightly different. So, one of them is $2\pi/600$ and the other one is $2\pi/700$. And then when I superpose these two, so this is something you can play with various ω, ω_1s and ω_2s. And you will see that when ω_1 and ω_2 are sufficiently close to each other.

So, then I am going to plot this envelope function. So, you see that I have the envelope function, which is $2A^* Cos(2t(\omega_1 - \omega_2)/2)$, you can think of it as the envelope function. And then if I were to plot on the same graph, I am going to plot the envelope and the superposition.

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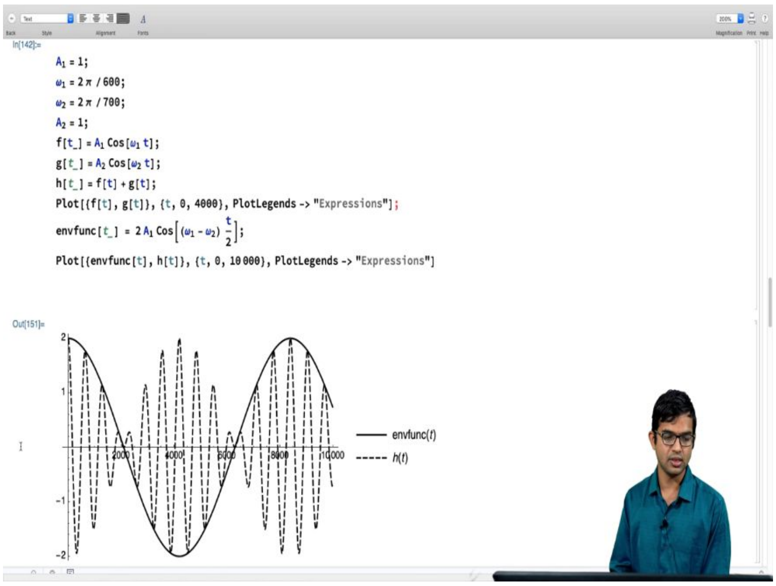

So, you see that beat phenomena involves you know increase in amplitude and decrease in amplitude but this overall envelope is given by a much smaller frequency curve. That is the curve that is perceived by the ear. So, oftentimes we have heard beats where with sounds, you have two sounds which are being simultaneously played.

And both of them have frequencies which are very close to each other. And then if you play them simultaneously you are going to hear a waxing and waning of sounds and that is going to have actually much lower frequency than these very large frequency modulation is also happening here. So, the amplitude is getting modulated, but that is not really going to be perceived as much as the envelope function itself.

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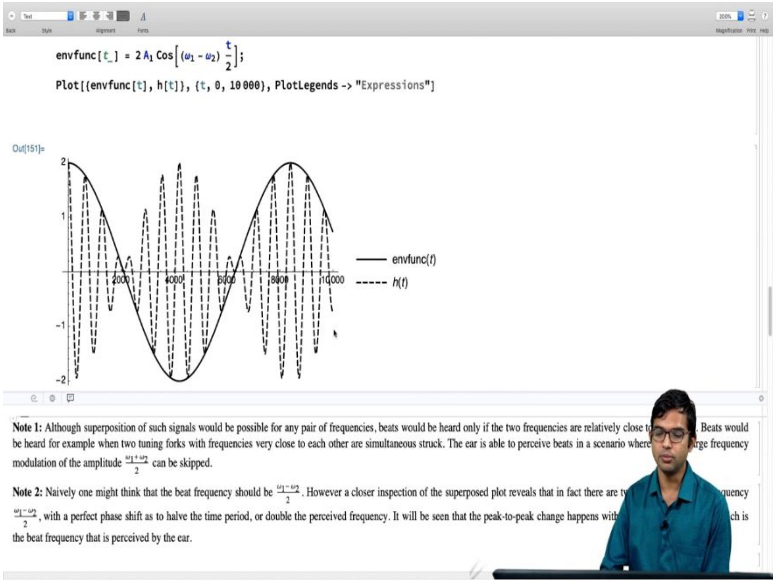

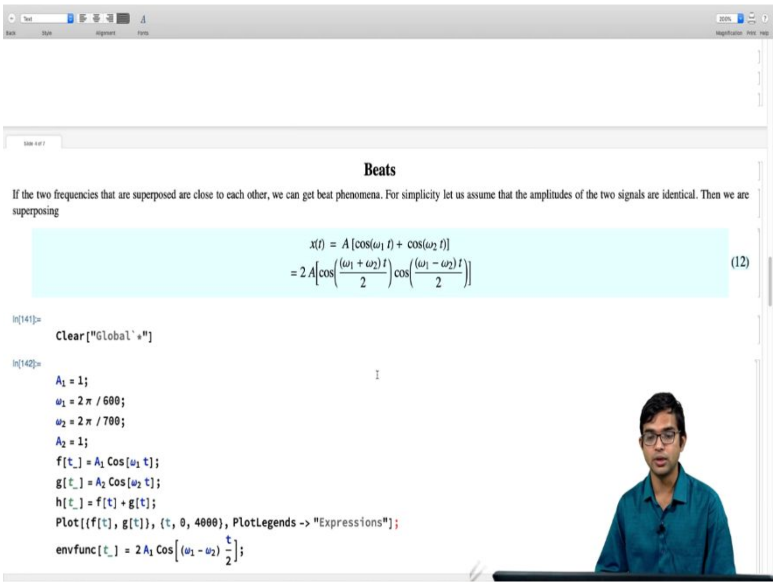

So, there are two key points to note from here, from this study of this graph. One is that although superposition would be possible for any pair of frequencies, so beats should be heard only if the frequencies are relatively close to each other right. So, I mean, this is something that you might have performed these kinds of experiments in the lab if you had two tuning folks which are tuned with frequencies very close to each other and they are simultaneously stuck.

So here is able to perceive beats in a scenario where the very large frequency modulation can be skipped, so you do not really care about the details of these fluctuations in the amplitude, the ear is able to perceive only this. And also the other very interesting point is to note that actually the beat frequency is $(\omega_1 - \omega_2)/2$, you might naively think that just looking at this expression, it is going to be $\text{Cos}((\omega_1 - \omega_2)/2)$.

But in fact, there is this envelope and then there is the other envelope as well, which I have not drawn. So, each of these is going to be separately perceived. So, if you just count the distance in some sense, the distance between two consecutive peaks or two consecutive troughs in this curve, that is going to give you the correct frequency, that is the frequency that the ear will perceive. And that is given by $\omega_1 - \omega_2$, it is not ω_1 and ω_2 by 2. So, this is not a factor of 1/2.

So, this is another example of superposition of oscillations and how you can get, you can visualize beat phenomenon. Now, we will go ahead and look at what happens if you have actually two signals which are being superposed along perpendicular directions.

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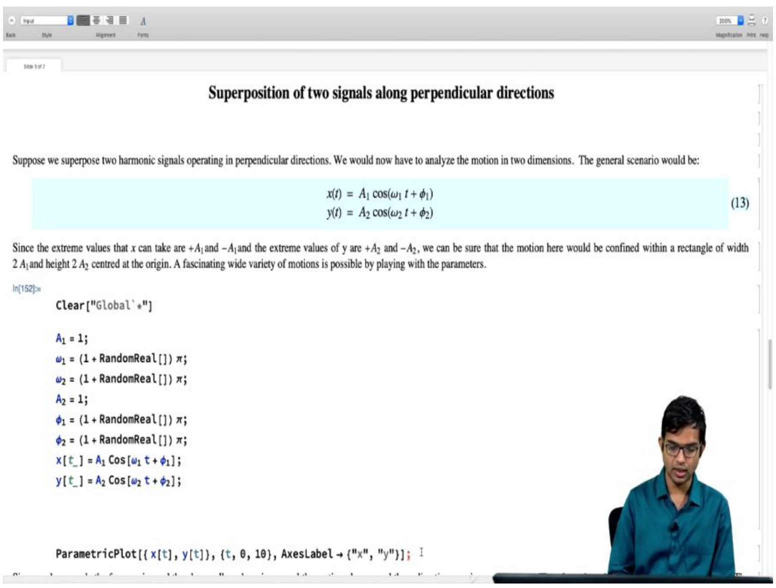

So, you have, there is a harmonic signal which is operating along the x axis and another harmonic signal which is operating along the y axis. And if you superimpose these two, you are going to get motion which is two dimensional. So, there is an x component to it and a y component to it.

And you can ask, is there some order that one can get? Is there some method to this? And so, this again, once again some simple analysis is involved and Mathematica allows us to visualize this. So, once again, I will start by clearing this global. So yeah, so one thing that we can immediately say is, of course, all motion must be confined to a rectangle, so this rectangle will have dimensions $(2A_1)/(2A_2)$ right.

Along the x axis, your maximum and minimum values are A_1 and $-A_1$. Along the y axis maximum and minimum values are $+A_2$ and $-A_2$. So, there is no question of your signal being, going outside of this rectangle. But the question here is, how much of your rectangle can get covered? Will the whole rectangle be covered or only some part of the rectangle be covered? Is there some order inside here? So, let us look at some random, all the four parameters.

So A_1 and A_2 are less significant. So, let me just start by putting them to be both 1. You can play with making them different A_1 and A_2 that will only make you get a rectangle instead of a square. It is not very, does not have any dramatic consequences. So let us just put them to be 1.

For starters, I am going to just take all these to be random ω_1, ω_2, ω_3, φ_1 and φ_2. Let us see what happens. And if I plot the superposition of the. So here, it is useful to plot this with the help of a parametric plot function. So this is this command, so this is some syntax you can look up Mathematica, you can just play with this and see what happens.

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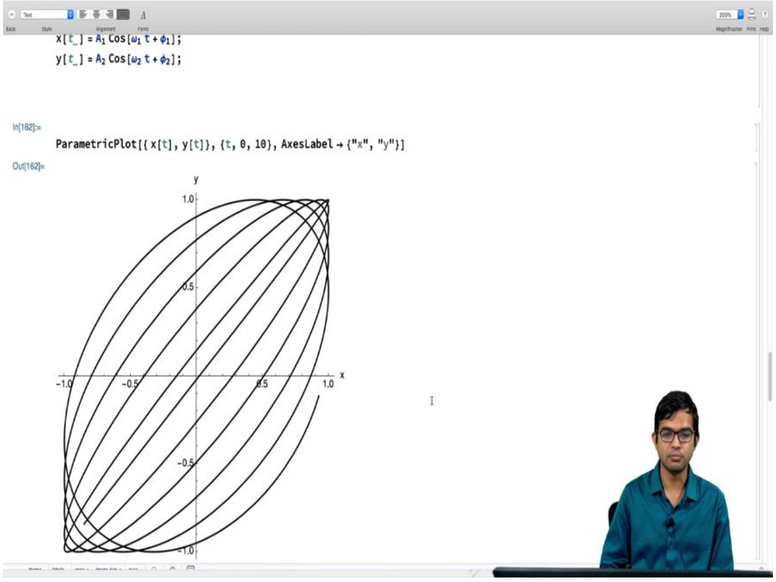

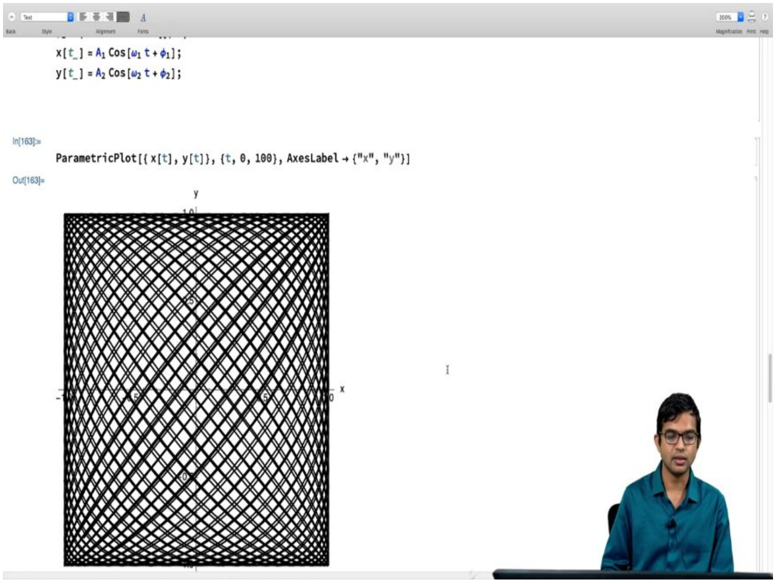

So x and y are getting superposed and you can plot them as a parametric plot. So, let me first, I have to first run this, only then come to here. There you go. So they will get this very rich behaviour, you see, as you change ω_1, ω_2, φ_1, φ_2. Let me run this again. So, in fact, let me keep the parameters as it is. So, I have run here only from time going from 0 to 10. So if I make it go from 0 to 100, let us see what happens.

So already I see that. So, of course, I told you that it can never go outside of this rectangle, which is a square in this case. It seems like it is a very rich tapestry, some pattern is coming out. And so the question is are there some forbidden reasons here? Is it like refusing to go in certain regions? So, if I run it for even longer and you see that actually, it is not the case.

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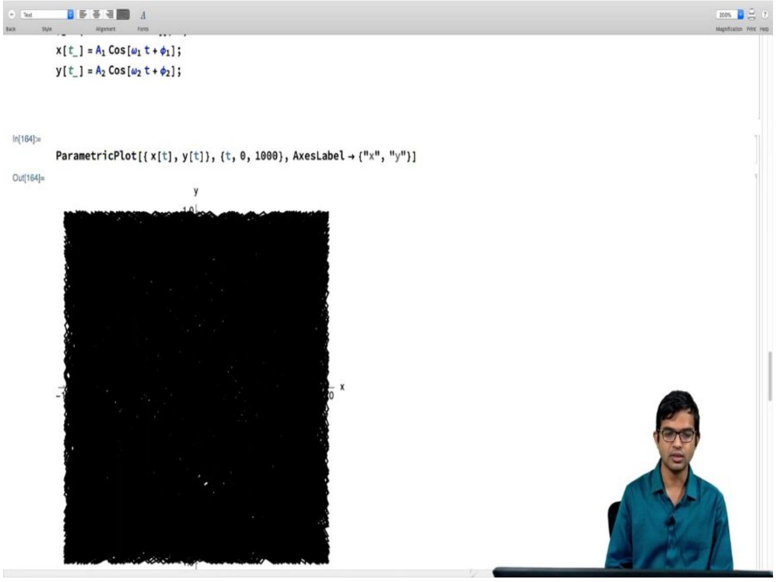

If I keep running it longer and longer and longer, in fact, I can pretty much cover the whole square. And so you can ask the question, is it a generic property? Is it always going to be covered or not right?

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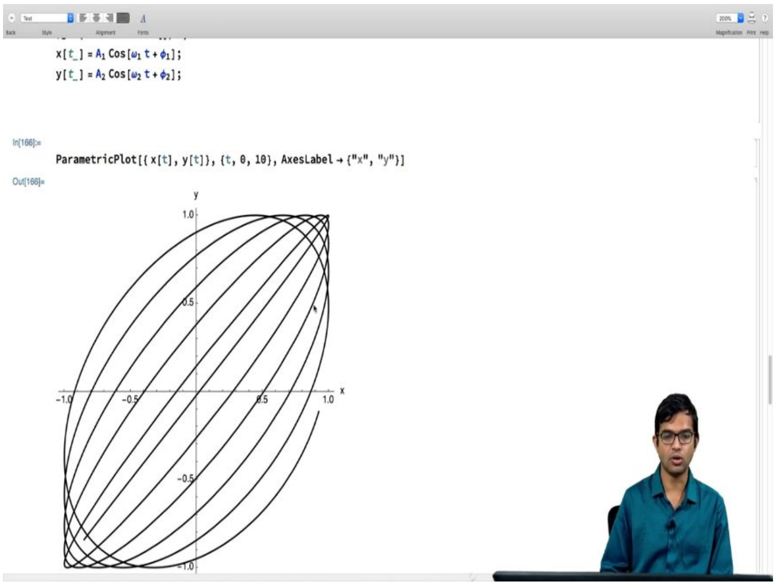

So, if I, suppose I take a different set of parameters, let me run this so then I see, it looks like this.

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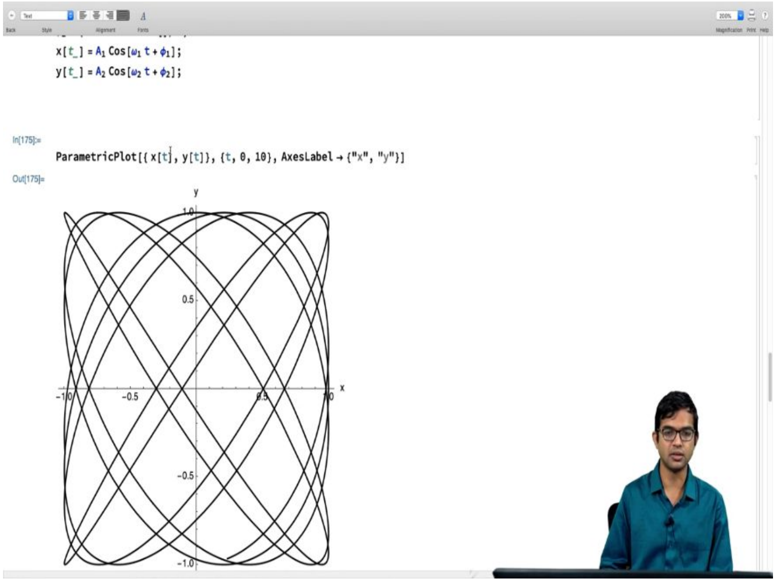

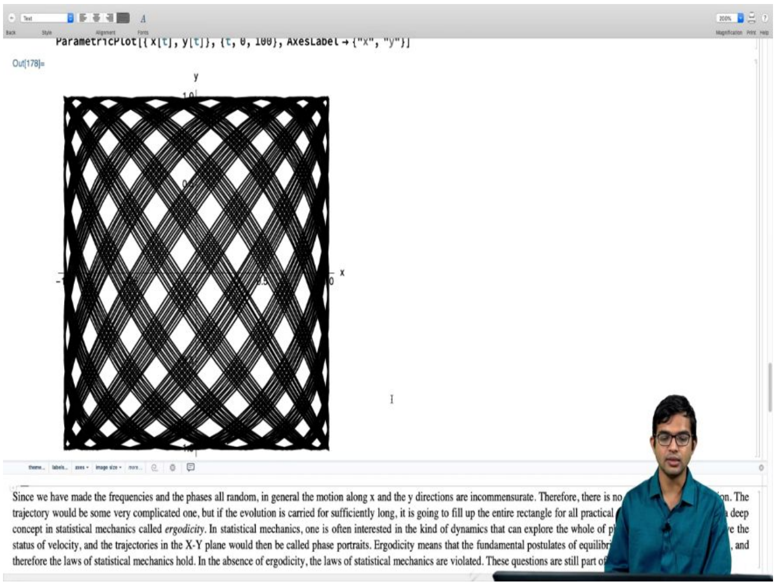

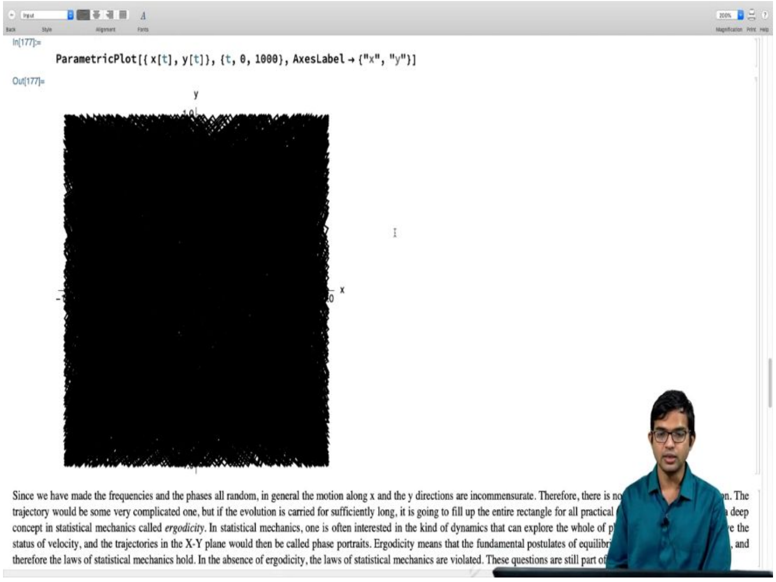

If I take t to be a 100, it is like this. A 1000, see it is covering up the whole of this region. So in fact, this is a very important question and it could have fundamental implications when one is looking at statistical mechanics. So, in statistical mechanics, if you think of this as some kind of a phase space, you are often interested in the question of whether your system is able to explore all of phase space.

If a certain dynamics can totally cover the entire phase space and such systems are called ergodic in nature. And so this ergodicity is a prerequisite for statistical mechanics work, it is taken as one of the postulates but there are systems which do not satisfy ergodicity.

And then, the dynamics confines systems to certain, some manifold which is, which does not cover the whole of phase space. And so then equilibrium statistical mechanics as usually you know studied, breaks down in such cases you know and it is, it continues to be a field of great activity, particularly when in quantum mechanics as well is involved.

There is a lot of activity even today on trying to understand these questions and how, what happens if ergodicity is broken? And what kind of statistical mechanics comes up in the quantum world? And these are all very delicate, subtle questions, which are, which continue to be questions for fundamental research.

But so yeah, so the point here is that just with some very simple games we are playing at this point, we are only just exploring things pottering around with Mathematica. And already we see that it can actually open up questions of you know fairly fundamental interest.

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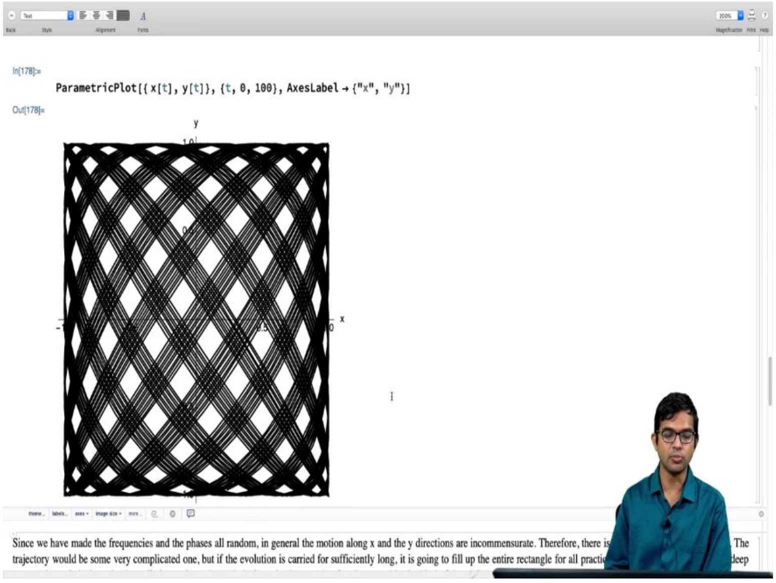

Okay, so now, the reason why this system is able to cover all of this region is because we have taken it to be truly random. And so this is by you know, you are making them incommensurate in some sense. Because typically, if you take them to be completely random, it is very unlikely that there will be some nice ratio between ω_1 and ω_2 or so on. So, let us play this game. Suppose we take $\omega_1 = \omega_2$, then what happens?

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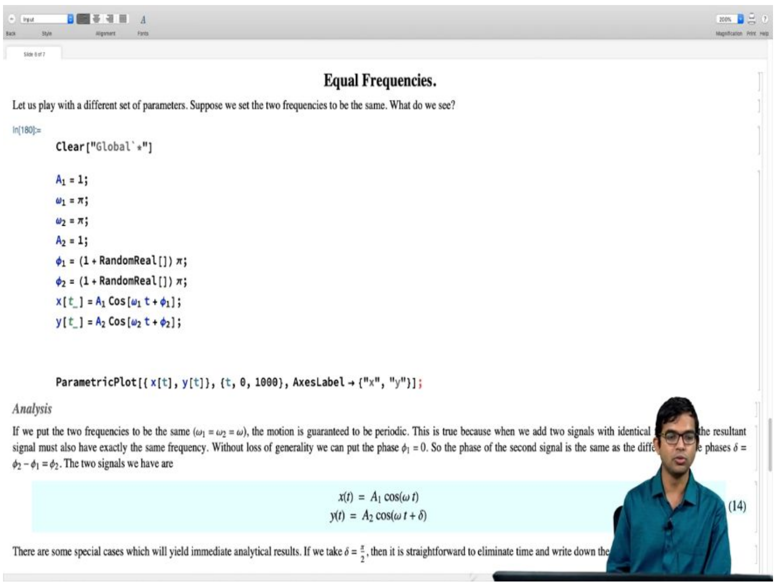

Again, I will clear this, I am taking $A_1 = 1$, $\omega_1 = \pi$, $\omega_2 = \pi$, $A_2 = 2$. Only φ_1 and φ_2, let me take them to be random. And then I am going to add them up. And then I have a parametric plot, which I will show.

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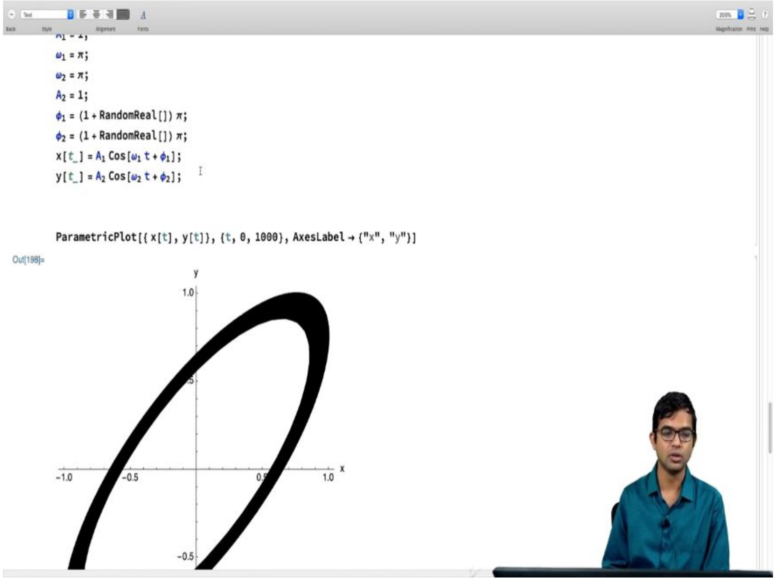

So, this is a game that you can play, right you can generate your own figures of this kind you know using your own random variables you can make all of them random, you can make a few of them random, you can try to build up the complexity. So, here you see that, in fact, it is always going to be periodic.

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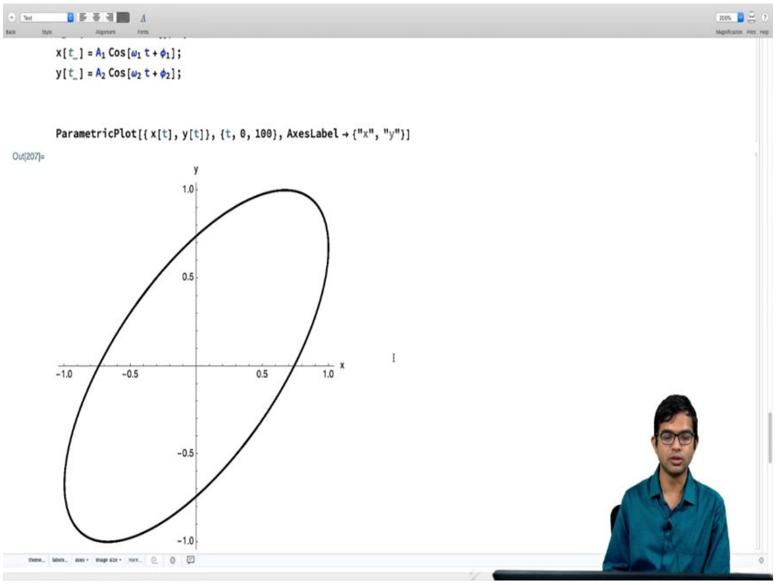

Equal to ω_2, there is no surprise that when you superpose along different directions, you are going to get an overall system.

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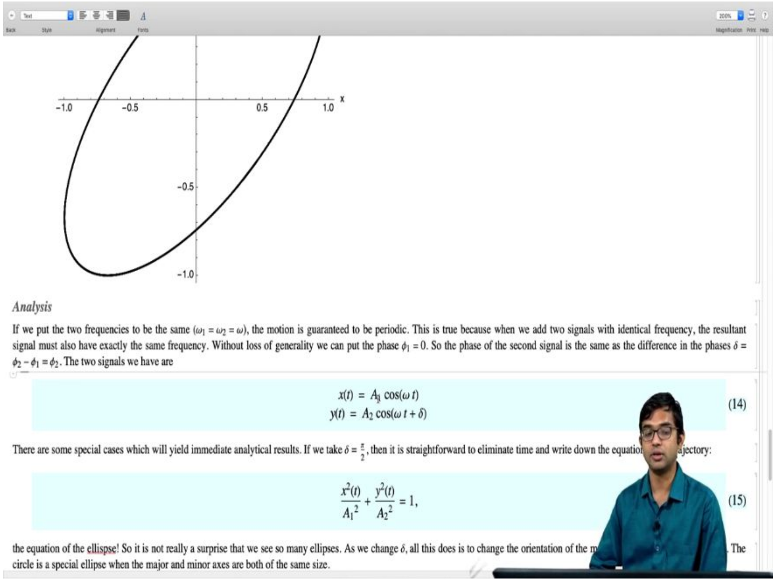

So, for certain special cases, you can actually analytically analyse, so if you put the two frequencies to be the same, then surely it is going to be periodic. So, there is going to be this phase difference. So if you take x(t) to be A_1 Cos(ωt), and $y_2 = A_2$ Cos(ωt + δ). So, then you see that for the special case of δ = π/2, you can simply eliminate ω and you can extract this curve, which just turns out to be an ellipse. So, that is why it is not a surprise if you try out some other set of parameters.

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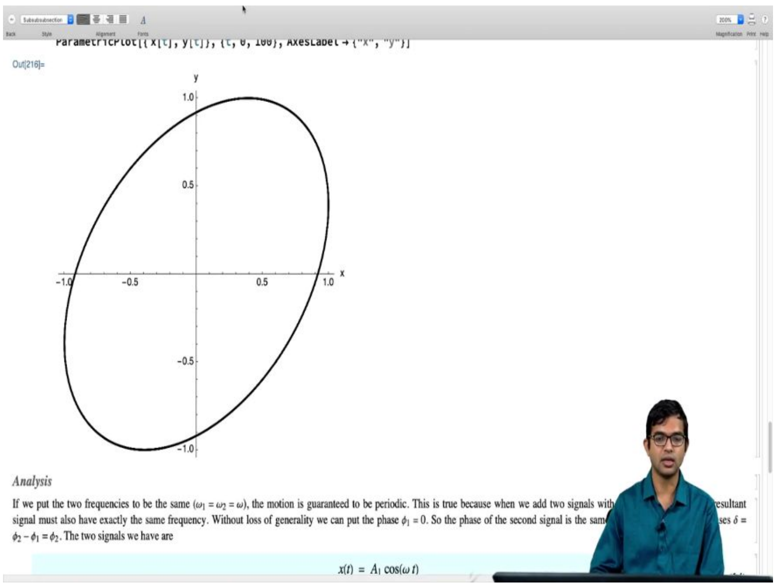

You do it again. Once again, you are going to get another ellipse and so on, you will get many ellipses.

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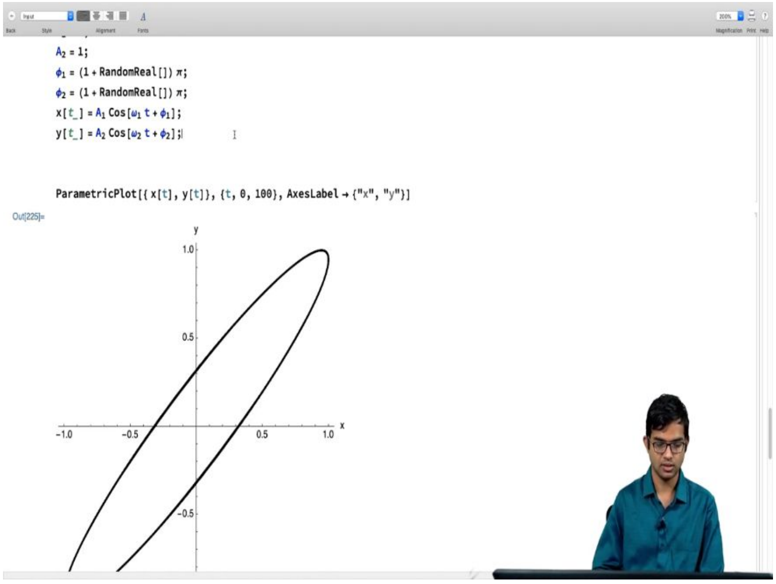

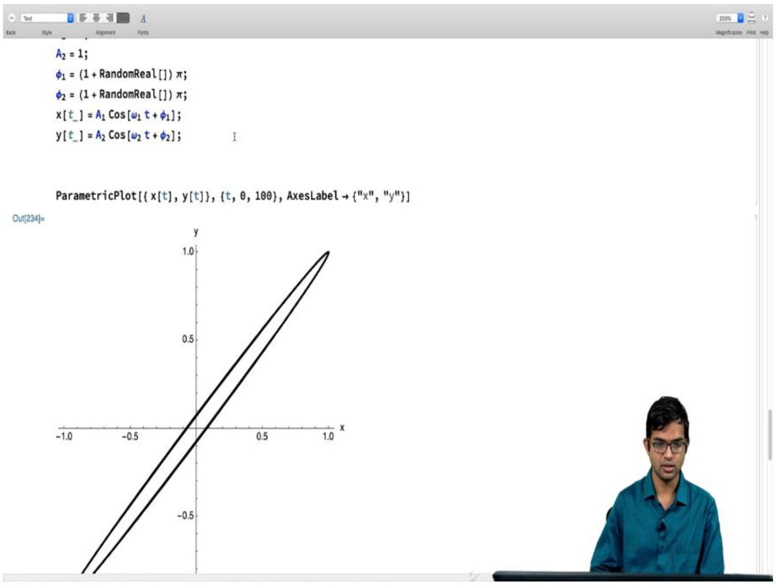

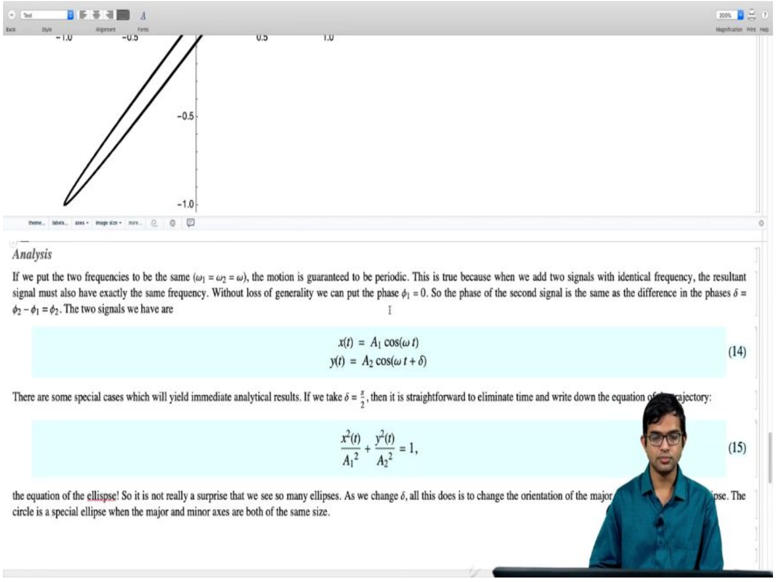

Alright. So, what happens if we change, not make the frequencies not equal, but commensurate. So if you change ω, if you make ω_1/ω_2 to be a rational fraction.

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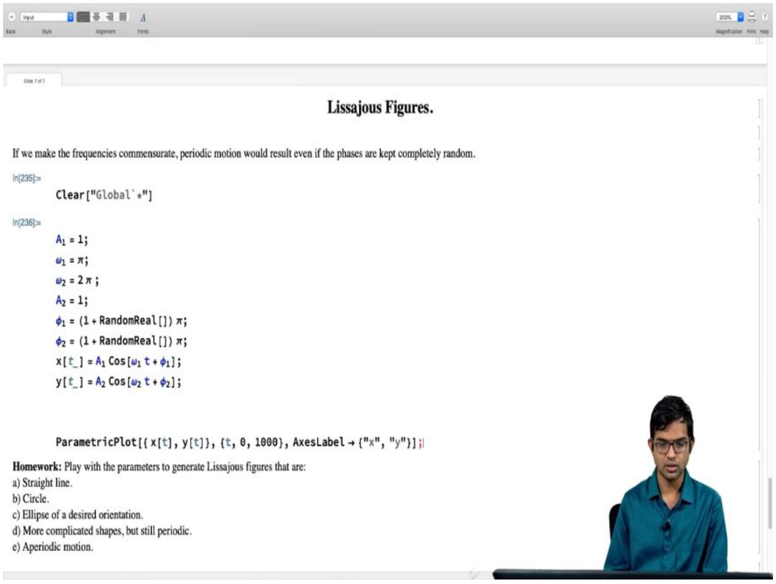

So, we will see that in fact, periodic motion would result even if the phases are kept completely random. So, let us play this. So, if I again clear this and I am choosing here for simplicity $\omega_1 = \omega_2$, but you can also take some factor let me say.

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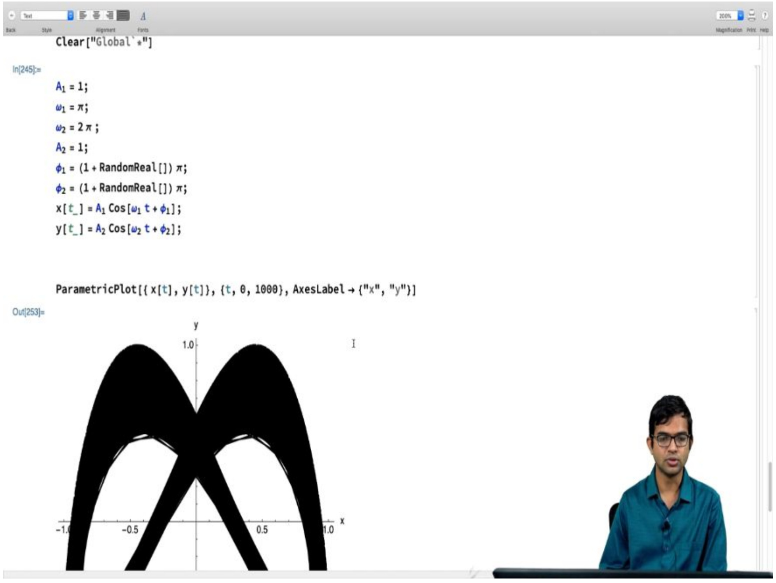

And then I make the others completely random. So, there you go. So, you get a rich set of possibilities, but these will always be periodic.

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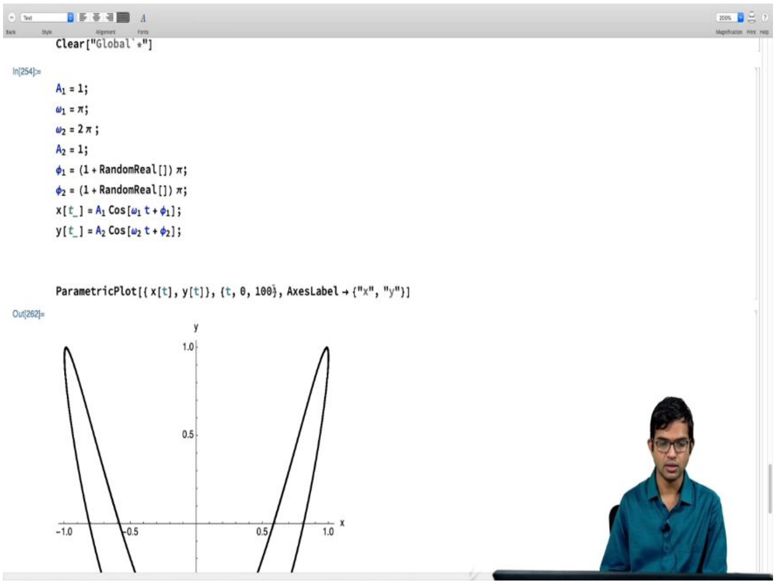

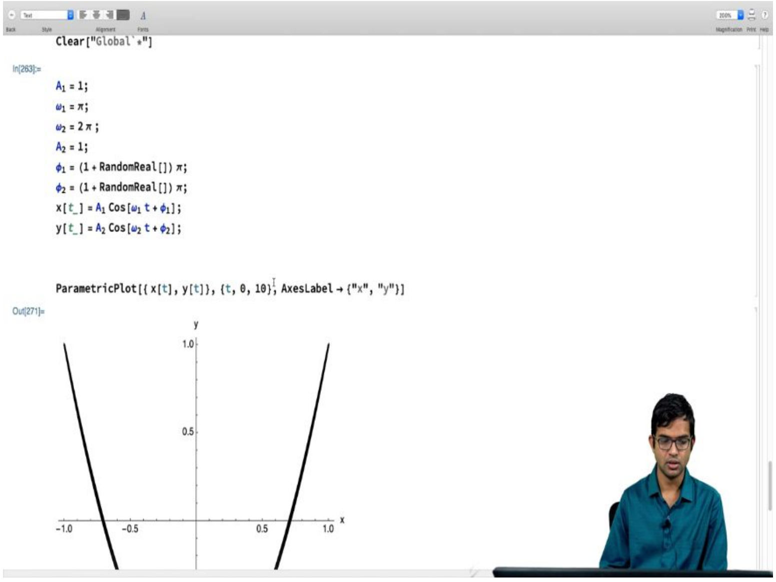

You have to you have to set the time and then the randomness and so on, but typically you will find that the system will return to where it started.

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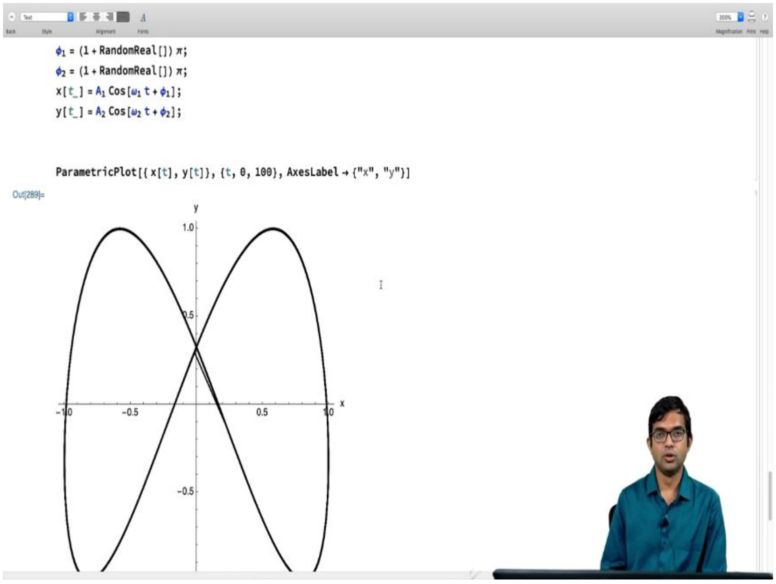

Okay, so you can go ahead and play more and see if you can.

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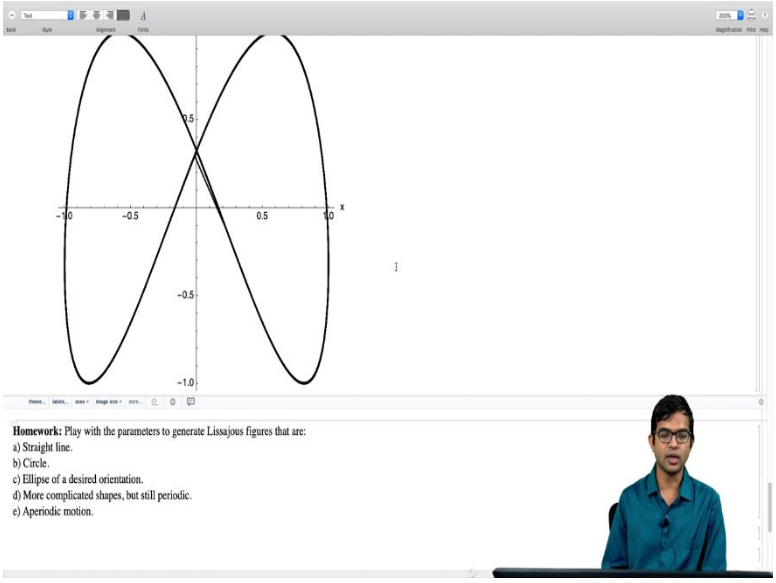

These are what are called Lissajous figures, you might have already played with this, you might have played with these kinds of figures with an oscilloscope for example.

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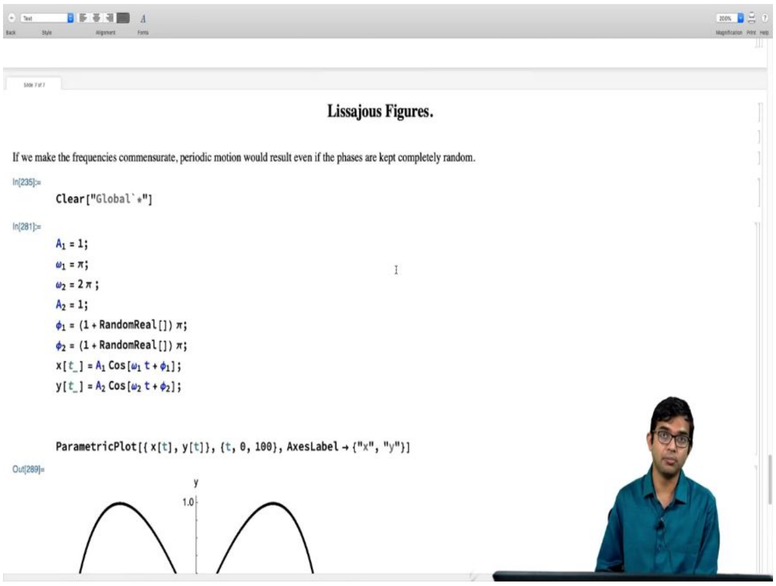

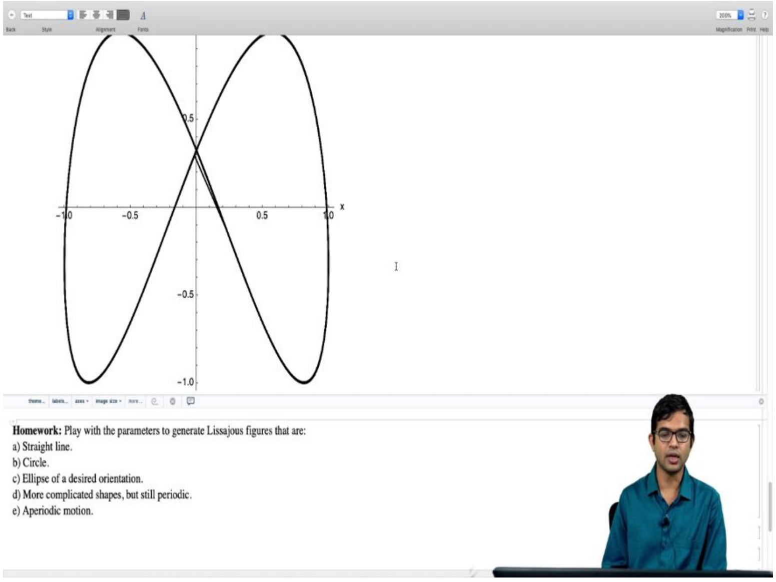

So in the lab, if you have access to an oscilloscope where you can take superpositions of waves along x direction, along y direction and you can generate all these. So, there are very specific ways of generating particular curves. So, homework for you would be to see how you can get a straight line, how you can get a circle, what kind of parameters must you do to to get ellipses of various orientation, more complicated shapes, but still periodic, and when do you get a periodic motion.

So, this is going to be homework. So, you must look at this lecture or this module mostly in the spirit of playing. So, the more you play, the more you see patterns and there is some understanding to be derived from looking at these patterns. So, that is what this lecture is about. Thank you.