Physics through Computational Thinking
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Lecture 15
Periodic Motion and Dynamics

Welcome back to Physics through Computational Thinking, today we will talk about Periodic Motion and Dynamics. We will review oscillatory motion and periodic motion, some of which you are already familiar with. So, this is going to be a quick review about the same. And we will slowly build up onto Anharmonic oscillators. Let us get started.

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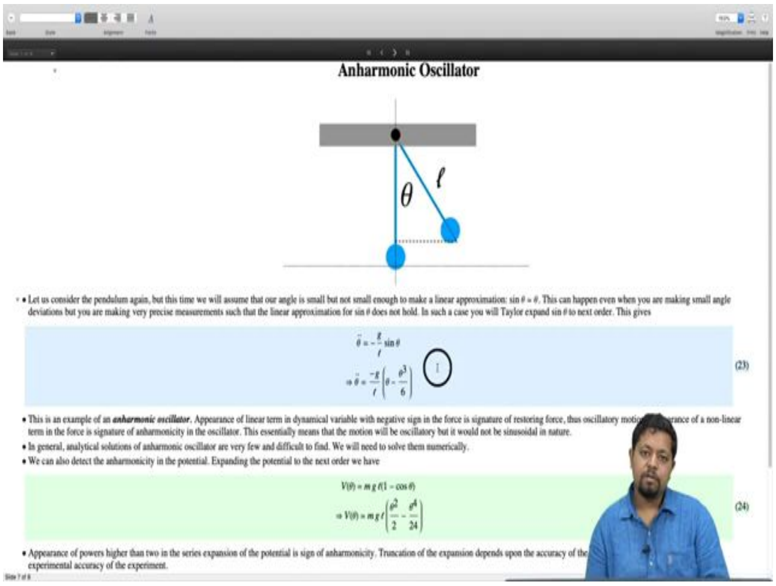

Let's go ahead and look at an example of Anharmonic oscillator. Again, building up from a simple case into a more complex case, we look back at this simple pendulum again. Last time, we showed you that the equation for the simple pendulum was $\ddot{\theta}$ = -g/l Sin(θ). Now, clearly this is not equation of a simple harmonic oscillator but we did an expansion for small θ and we have showed that for the small θ approximation, this was a good approximation, to good approximation reduced to equation for simple harmonic oscillator.

Now, suppose the angles are large and we want to consider, what is the effect of those large angles? That is small angles approximation is not valid, so we would like to include one more term in the small angle expansion and that term will become $\theta^3/6$. Now, we see that this is not a equation for simple harmonic oscillator any more, and the first correction that we see over here, this is the first Anharmonic correction, this equation is an equation of an Anharmonic oscillator, the Anharmonicity is given by θ^3 term over here.

And, the two particular features that are notable in this equation, one is, that the the dominant term is the linear restoring force that is, the first term in the force is proportional to to the linear terms θ with a negative sign, so it is a linear restoring force as a first approximation and the correction is given by $\theta^3/6$, which is not a linear term, therefore this is Anharmonic.

So, we expect an oscillation to go back and forth but not like a sinusoidal approximate not like a sinusoidal oscillation but something more general in nature. Such a equation we may not be able to solve analytically, very few Anharmonic oscillators we can solve analytically, very few solutions we know analytically. So, in this particular case we may have to resort to numerical techniques, for which we should wait for a little while and as we progress the course, we will catch upon to solving these equations by numerical techniques.

Let us also go ahead and analyze this particular Anharmonic oscillator from a point of view of a potential. As I said before that analyzing things from potential can be a lot easier than thinking in terms of force, so in this case, we will also analyze from the point of view of potential. The potential of course is $mgl(1 - \cos(\theta))$, as we have talked about it before, let us expand $\cos(\theta)$ in as a small angle approximation.

The first term is 1 the first is 1, second is $-\theta^2/2$ and the third is $+\theta^4/24$. So when I substitute that, this is what I what I get for the potential, $mgl(\theta^2/2 - \theta^4/24)$, and that potential is an Anharmonic potential because it is not quadratic any more, it is $\theta^4/24$ correction and therefore this is Anharmonic.

Now, why did we not include the other terms over here or other terms in this particular case, that will depend on at what accuracy you want to calculate? Suppose, you want to calculate perturbatively then and and really your angles are reasonably small but not small enough to consider them as harmonic oscillators, then you can truncate this expansion up to this term or over $\theta^4/24$ in the potential. So that truncation will be good truncation when the angles are small but not small enough to be considered as a simple harmonic oscillator.

But really depends on your accuracy of an experiment that you are doing or accuracy of a calculation that you are doing, you can decide where you want to truncate. If you are numerically solving, it might be just good enough to take this entire potential or this entire force, -g/l Sin(θ) into consideration for solving. So, really the approximation where you should truncate the series in theta depends on what is the accuracy of your calculation. So, this was an example of an Anharmonic oscillator and we will continue next time with more examples.