Physics through Computational Thinking
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Simple Harmonic Oscillator with a spring mass system

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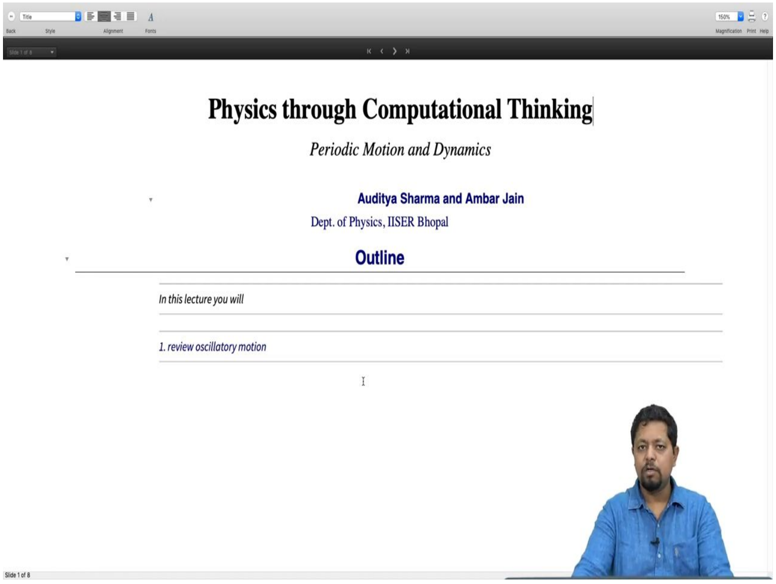

Welcome back to Physics through Computational Thinking. Today we will talk about periodic motion and dynamics, we will review oscillatory motion and periodic motion some of which you are already familiar with so this is going to be a quick review about the same and we will slowly build upon to Anharmonic Oscillators. Let us get started.

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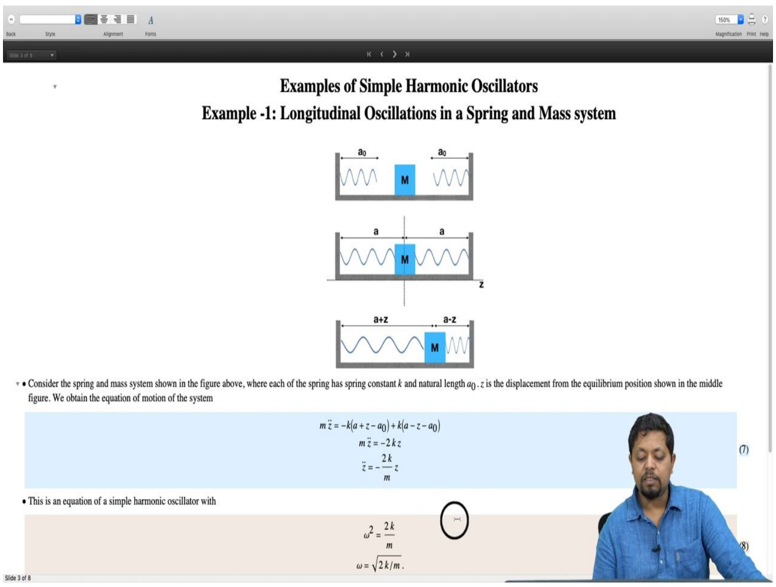

Let us take an example of a Simple Harmonic Oscillators. You are all familiar with spring and mass system. We will take an example of Longitudinal Oscillations in the spring and mass system, so consider a system given by this setup.

So, we have got a wall over here and another wall on the right side, we have got spring attached to this wall as well spring attached to this wall. Let us take these springs as ideal springs and let us take the natural length of these springs to be equal to A0. Let us say there is a mass M, a block of mass M sitting in the centre and let us assume there is no friction so this mass can slide back and forth on this floor smoothly.

Next what we do is, we connect this spring with the mass. So we stretch the spring and connect it with the mass and when we do so the spring's length becomes A. A0 was its natural length and I have to extend a little bit so that I can connect it to mass M. At this point the length of the spring becomes A.

Now, as I move the mass to one side let us say I move it to the right as shown in this picture over here. Let us say it shifts by a distance Z, then the length of the spring becomes A - Z and length of this spring becomes A + Z.

Now, the question is when I release it what happens and as you are all familiar this block is going to execute a Oscillation. Question is whether this oscillation is a Harmonic Oscillation? In order to find it out we need to work out the equations of motion.

So, we know that as we release the block this spring is going to push, this spring is going to pull. It will come to the mean position but at that time mass has already gained the speed, it will continue to move to the left and this spring will get compressed so it will start pushing to the right and this spring will get stretched, so it will start pulling to the right and eventually the mass will stop and come back and will continue to execute that motion.

In order to find out whether that motion is actually a Simple Harmonic Oscillation. We need to work out the equations of motion. In order to get the equations of motion which I have worked out over here. I want to find the acceleration of the mass, I call that $\ddot{Z}$. So, mass times $\ddot{Z}$ is a force. I need to find what is the total force acting on this mass.

The force acting on this mass, let us take this snapshot for example in this particular picture, the force acting on this mass is because of this spring to the left and because of this spring also to the left, because of this spring the force that is that is being executed to the left is equal to K times the change in the length of the spring that is its current length minus natural length.

So, current length of the spring is (A + Z) – A0, the natural length, forces to the left so K * ((A + Z) – A0), K is the spring constant.

For this other spring the force is to the left so that is K * ((A - Z) - A0). A – Z, is its current length, - A naught will be its natural length so taking a difference of the two gives me the force act because of this spring onto this mass.

Now, adding the two terms I get the total force and when I simplify I simply get, M $\ddot{Z}$ = -2KZ, A and A0 cancel out, I simply get M $\ddot{Z}$ = -2KZ. I rewrite that equation by dividing M on both sides I get $\ddot{Z}$ = -2KZ/M. Now, this equation is in the form of a Simple Harmonic Oscillator if ω^2 = 2K/M.

So, we realize that this system does satisfy equation for Simple Harmonic Oscillator and its frequency ω^2 = 2K/M or $\omega = \sqrt{2K/M}$.

And you know when a mass is attached to a perfect spring and ideal spring the oscillation frequency $\sqrt{2K/M}$. In this case the two springs so it ended up being $\sqrt{2K/M}$.

We can also analyse this from the point of view of a potential which is usually easier. If I think about this system from the point of view of Newton's laws, thinking about writing down all the forces acting on the masses in the directions, you can get confused about the directions and the signs and so on. But when you work out the potential it is usually always easier.

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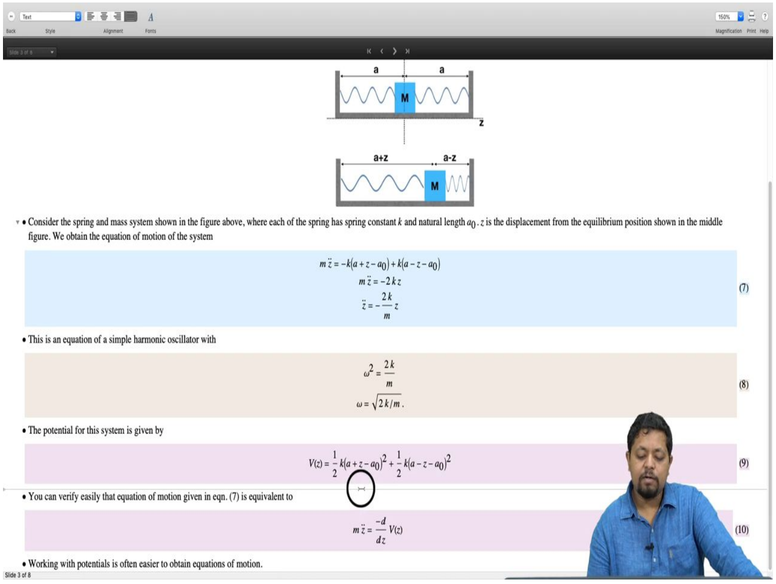

In this case the potential is given by ½ K ΔX^2 where ΔX is the elongation in the spring or the contraction in the spring. So, in this case A + Z is the length of the spring so A + Z - A0 is the elongation in the spring, squaring that multiplying with ½ K gives the potential because of this spring and similarly potential because of this spring is ½ $K((A - Z) - A0)^2$.

So, my net potential turns out to be the sum of the two terms and my equation of motion is simply given by,M $\ddot{Z}$ = -d/dz (of the potential) because force is minus the gradient of the potential. Therefore, M $\ddot{Z}$ = -d/dz (potential).

You calculate the derivative of the potential and you will find that you will get the same equation as before. So, when you simplify this equation you will see that the resultant equation will be that and that is another way of confirming that the system follows Simple Harmonic Oscillation.

The alternate way by looking at the potential also you can directly determine whether the system follows Simple Harmonic Oscillation or not. If the potential is quadratic then the system will follow Simple Harmonic Oscillation. And in this case if you want you can go ahead and test check that this potential is quadratic, if you expand this out the highest power on Z will be Z^2.

So, therefore, we see from this that for this potential that since the potential is quadratic with the coefficient of Z^2 being positive we see that the system is going to follow a Simple Harmonic Oscillation.