Physics through Computational Thinking

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Lecture 12

Introduction to Simple Harmonic Oscillator

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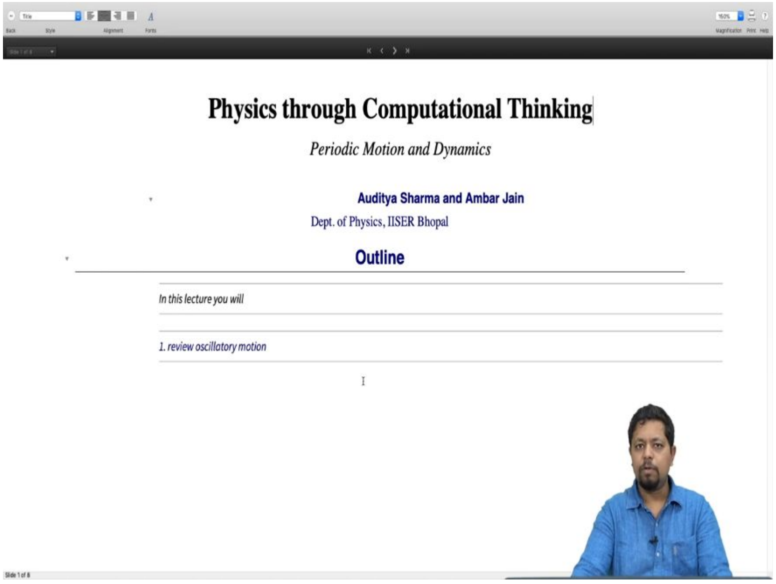

Welcome back to Physics through Computational Thinking. Today we will talk about periodic motion and dynamics. We will review oscillatory motion and periodic motion, some of which you are already familiar with. So, this is going to be a quick review of the same and we will slowly build up onto an harmonic oscillators. Let us get started.

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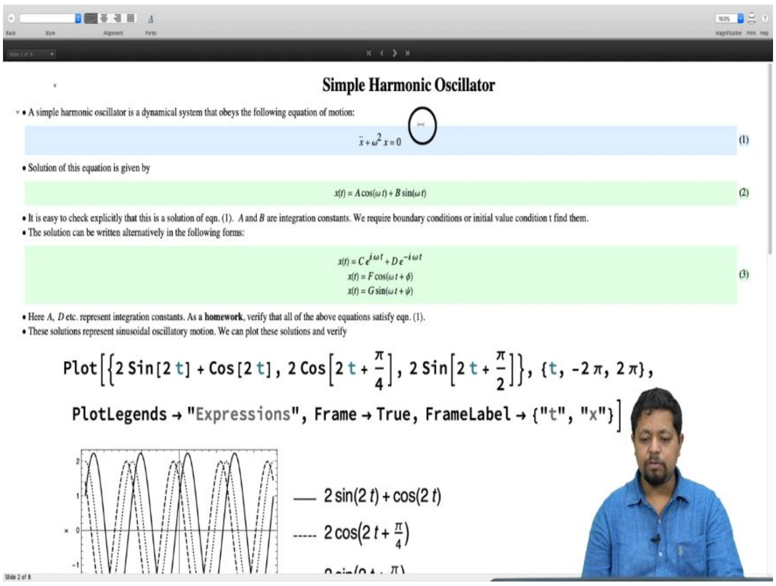

So, just do a quick recap of simple harmonic oscillators. Remember we talked about a simple pendulum which oscillates back and forth. And we said this for small angles, this executes a simple harmonic motion. So, any oscillator that oscillates like, which follows the equation of motion given by $\ddot{x} + \omega^2 x = 0$ is a simple harmonic oscillator.

This equation we refer to as the equation of simple harmonic oscillator. x is a dynamical quantity, x is something that is changing in time. It could be the position of a particle, it could be something else. In that sense, it is a generalized coordinate, it is some coordinate that you are representing by x. It could be an angle, it could be the position of a particle.

So, whenever a dynamical quantity, let us just call it x. Whenever you have $\ddot{x} + \omega^2 x = 0$ then then we say x executes a simple harmonic oscillation. The solution of this equation is given by the second line over here. $x(t) = \mathrm{A}\ \cos(\omega\, t) + \mathrm{B}\ \sin(\omega\, t)$. Now it is very easy to verify that. So, let us go ahead and quickly check that out.

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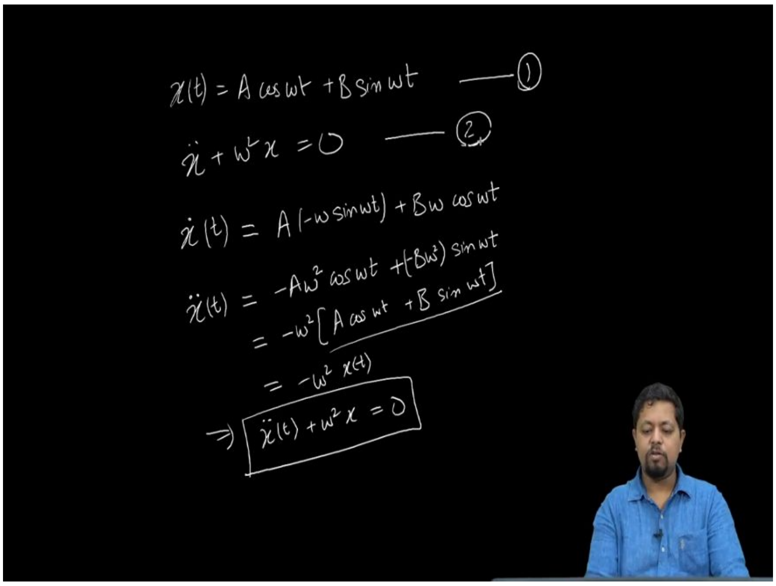

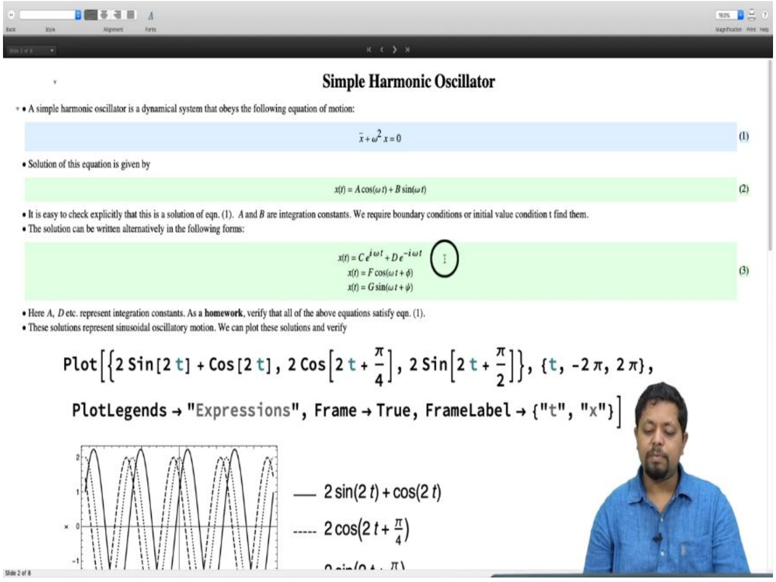

So, we are claiming that $x(t) = \mathrm{A}\,\cos(\omega\, t) + \mathrm{B}\,\sin(\omega\, t)$. One way to solve differential equations is to guess a solution and then just check it-, check it out whether that is a solution or not. So, in this case, we are doing a guess. We are claiming that $x(t) = \mathrm{A}\,\cos(\omega\, t) + \mathrm{B}\,\sin(\omega\, t)$ is the solution of the equation $\ddot{x} + \omega^2 x = 0$.

So, let us go ahead and verify. So, first let us calculate what is $\dot{x}$ of t. If $x(t)$ is given by this equation 1, I want to calculate what is $\dot{x}$ of t. $\dot{x}(t) = \mathrm{A}\, d/dt(\cos(\omega\, t))$, which is $-\omega\,\sin(\omega\, t) + \mathrm{B}\,\omega\,\cos(\omega\, t)$ for the second term. So, that is our $\dot{x}$.

Let us go ahead and calculate also $\ddot{x}(t)$. $\ddot{x}(t) = -\mathrm{A}\,\omega$, these are the constants from the first term, and derivative of $\sin(\omega\, t)$ is $\cos(\omega\, t)$. So, this becomes $\omega^2\,\cos(\omega\, t)$ plus we have got a B and ω, we will get a minus sign. So, this will become -B ω^2, and the derivative of cosine is minus $\sin(\omega\, t)$.

Okay, so now we see that this is nothing but -A, rather, we can write this as $-\omega^2\,\mathrm{A}\cos(\omega\, t) + \mathrm{B}\sin(\omega\, t)$. And whatever we have got here in the brackets, this is simply $x(t)$, so we get $-\omega^2\, x(t)$. And that gives me back my differential equation $\ddot{x} + \omega^2 x = 0$. Therefore, we say that this solution given by equation 1 is a solution for equation 2, therefore is the solution of a simple harmonic oscillator.

Let us come back to this page. Now, this is one way of writing a solution, we can also write this same solution in multiple different ways depending on what you are doing, different solutions can become useful. For example, I can also write $x(t) = \mathrm{C}\, e^{i\,\omega t} + \mathrm{D}\, e^{-i\,\omega t}$.

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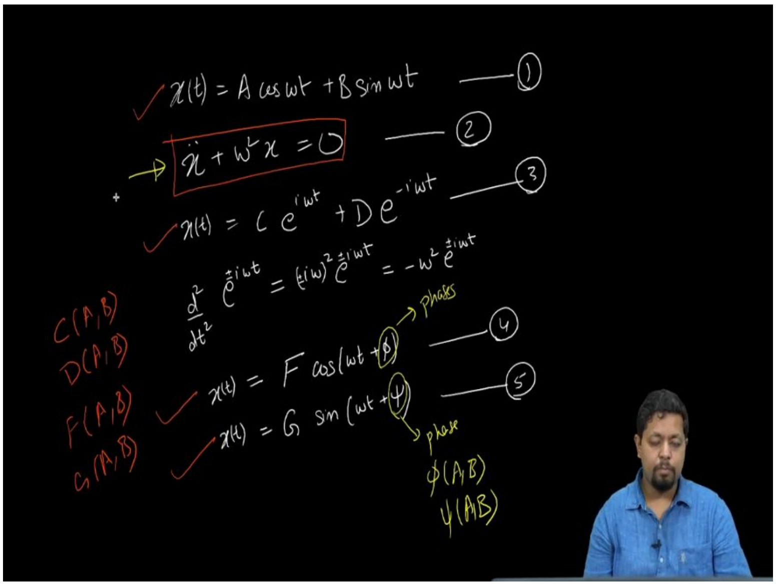

That is, now I am claiming that not only 1 gives me a solution for 2, but also the alternate solutions are $x(t) = \mathrm{C}\, e^{i\,\omega t} + \mathrm{D}\, e^{-i\,\omega t}$. Now, these solutions we can easily verify. Well, the reason this works out is because $e^{i\,\omega\,t}$ has a very nice property that you take two derivatives with respect to time and each derivative pulls out a factor of an $i\,\omega$, so two derivatives pull out a factor of $(i\,\omega)^2$, you get $e^{i\,\omega\,t}$ back and as a consequence, you get $-\omega^2\, e^{i\,\omega\,t}$.

If you put a minus sign over here, whether you take a plus or minus sign over here, it does not matter if you basically pull out a minus omega square. Therefore, this will also satisfy the first equation. So, this three is also a solution of one. Similarly, you can also write $x(t)$ equal to, let us choose a different constant. $\mathrm{F}\,\cos(\omega\,t + \phi)$ or you can write $x(t) = \mathrm{G}\,\sin(\omega\,t + \psi)$. These are two.

So, these three equations that I have got this one, that one, this one and this one, these four equations represent the same solution for this equation. These are just different ways of writing the same solution. So, as a homework exercise, I suggest that you go back and work out the relationship between the coefficients A, B, C, D, etc. So, you have to find C in terms of

A and B, D in terms of A and B. Similarly, F in terms of A and B, G in terms of A and B, and the phases, ϕ and ψ. These are called phases, these can also, ϕ and ψ are also functions of A and B.

So, as a homework exercise, I invite you to work it out what is the relationship between these various constants. Now, to remind you, we have, we have found the solution of this differential equation as any one of these four equations. In order to find the constants, you need more information, you need to know the boundary conditions or the initial conditions. If you know the initial conditions and the boundary conditions, you can solve and find out what these constants are.

So, for a differential equation, this is the most general solution, but in order to find an exact solution, you have to look at the boundary value, boundary conditions or the initial conditions given to you. Let us go back and try to understand the relationship between these four different styles of writing a solution.

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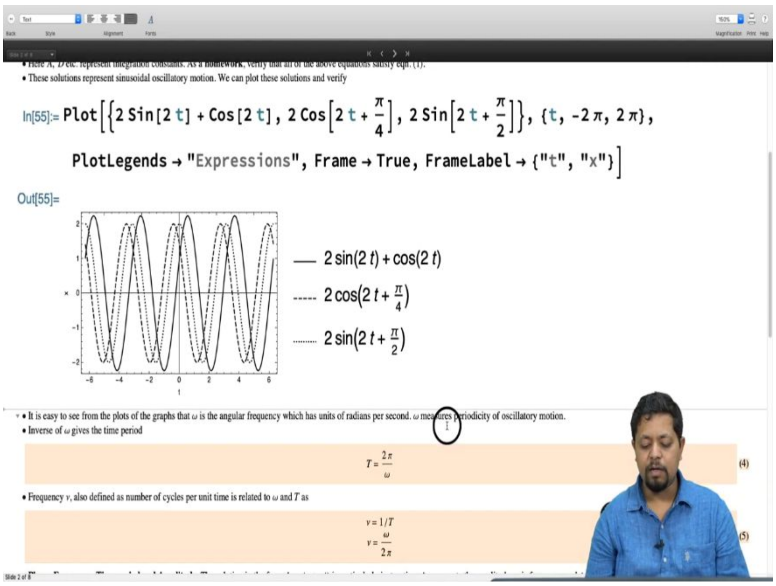

As an example over here, we will just consider this, this this style and $F\ \cos(\omega t + \phi)$ and $G\ \sin(\omega t + \psi)$. We will leave this out for a moment, I will leave this as a homework exercise for you to work it out and plot it. But these three will, we will plot it over here. And this is what I have done.

So, I have plotted $2\sin(2t)+\cos(2t)$, $2\cos(2t+\pi/4)$, and $2\sin(2t+\pi/2)$. These are three different solutions written in the styles. The first solution is written in the style of equation two and the second and the third solutions are written in the styles of last two equations or equation three.

And we plot this we are going to t equal to -2π to $+2\pi$. When I plot this, this is what I get. Here is my plot. And you see that all the three solutions represent oscillatory solutions sinusoidal oscillations, essentially. These are sinusoidal oscillations. The only difference that you see is this one is slightly higher than these two, the solid curve is slightly than the dotted and the dashed and they are shifted with respect to each other in the phase. Apart from that these solutions are essentially the same.

So, if I tune my parameters $\pi/4$ and $\pi/2$ over here, I can actually go ahead and overlap these solutions exactly, I will leave that as a homework exercise for you to try it out. Now, for simple harmonic oscillation oscillation, you can ask what is the time period?

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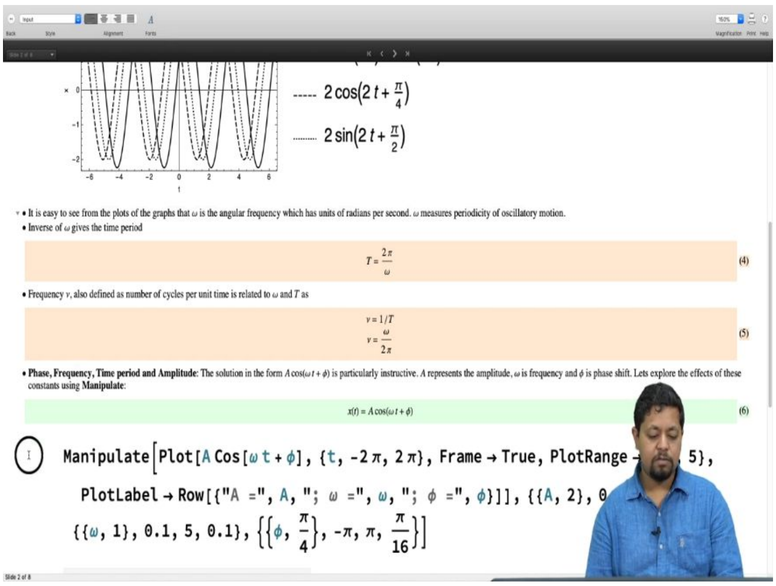

Time period is given by $2\pi/\omega$. Time period means when does your oscillator completes one oscillation, omega is the speed, angular speed given in radians per second. So, the oscillator comes back to its initial position if it has covered $+2\pi$ radians, therefore time period is

$2\pi/\omega$. This is a familiar relation. You would have seen it many times before. And we can also ask what is the frequency, or how many cycles does your oscillator do in one second? And that is given by the inverse of the time period. So, frequency ν is given by $1/t$ or $\omega/2\pi$ shown in the relations over here.

Now, it is really interesting to write the solution for the harmonic oscillator. For many physical problems, it is very, very useful to write the solution in the form of $\mathrm{A}\,\cos(\omega t + \phi)$ as shown in this equation over here.

In this particular solution, $x(t) = \mathrm{A}\,\cos(\omega\, t + \phi)$, A is the amplitude, ω is the frequency, ϕ is the phase shift. Let us go ahead and plot this solution like over here and put it inside a manipulate and see what is meant by changing A, ω and ϕ. What does A, ω and ϕ do? So, let me go ahead and execute this.

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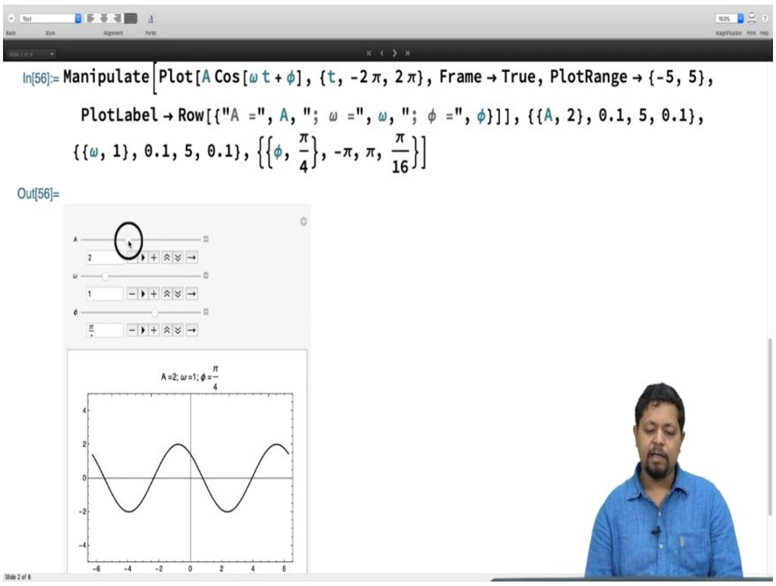

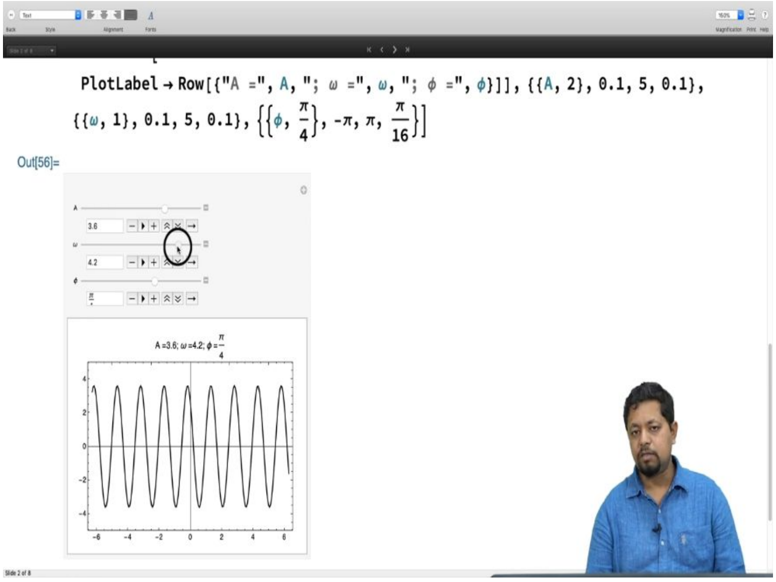

If you want you can pause the video and try it out yourself. You can copy the command from the screen over here. This is the plot I get. I have created a bounding box over here using the frame equal to true option. And I have put given three parameters in the manipulate A, ω and ϕ, where I have set the value of A between 0.1 to 5, value of omega between 0.1 to 5 and value of ϕ between $-\pi$ to π. So, let me go ahead and vary these three parameters, let me expand it out and I will vary these three parameters to show you what happens as I move the sliders.

When we move A, A is A is for amplitude. As you expect, this is going to change the amplitude of the oscillation that is going to make the oscillation bigger and bigger.

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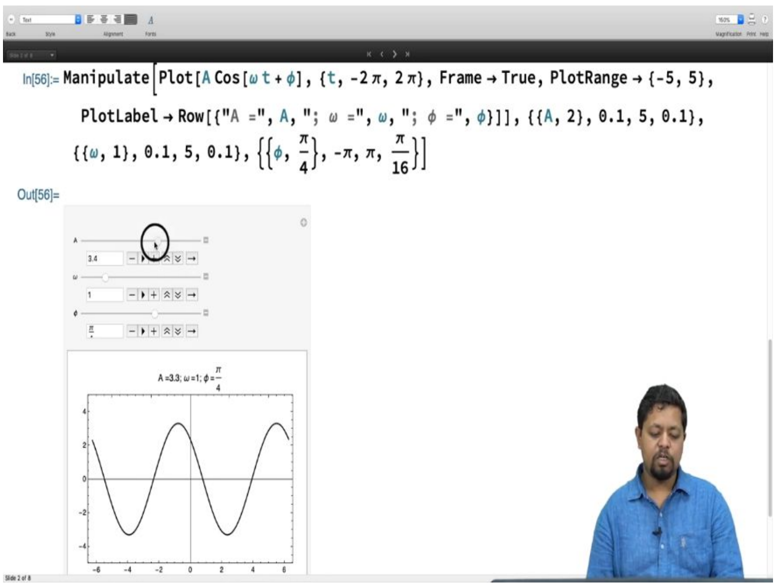

The peak will be at a higher value, the trough will be at a lower value but as we do that, notice that the 0's do not change. As I change the amplitude, increase or decrease the amplitude, notice that the 0's that is this point remains fixed. So, this time fixate your eye at this point and as I move the slider notice that that point is not changing. It remains where it is. So, the 0's do not change as I change A, it simply changes the amplitude, because A is the entire multiplicative constant to $\cos(\omega t + \phi)$. So, changing A is only going to change the whole amplitude, it is not going to change where the 0's are.

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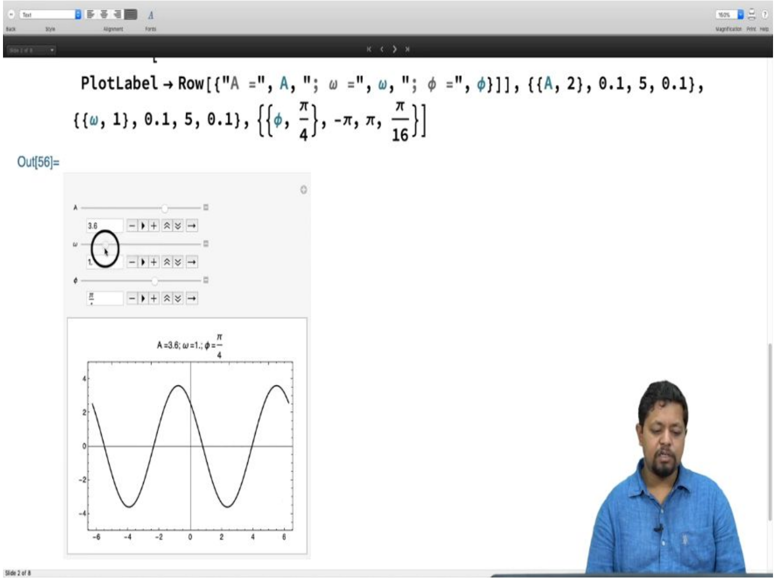

When we change ω, as I increase ω, I should see more oscillations per unit time. So, as I change ω, increase ω, you see more and more oscillations per unit time. And we say that the pendulum is oscillating or the or the harmonic oscillator is oscillating with the increased frequency. As I decrease ω the number of oscillations decreases. And similarly, over here as I change the ϕ, notice what happens.

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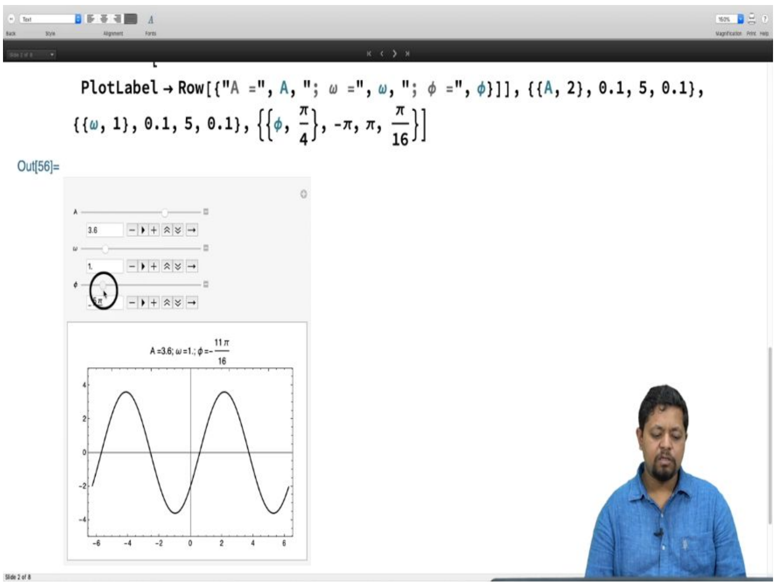

As I change ϕ, the entire oscillation, entire plot simply shifts to the left or to the right depending on what the value of ϕ is. That is why we call the ϕ as phase shift. It simply says if the maxima is happening at the origin or whether a minima is at origin, or whether there is a 0 at the origin.

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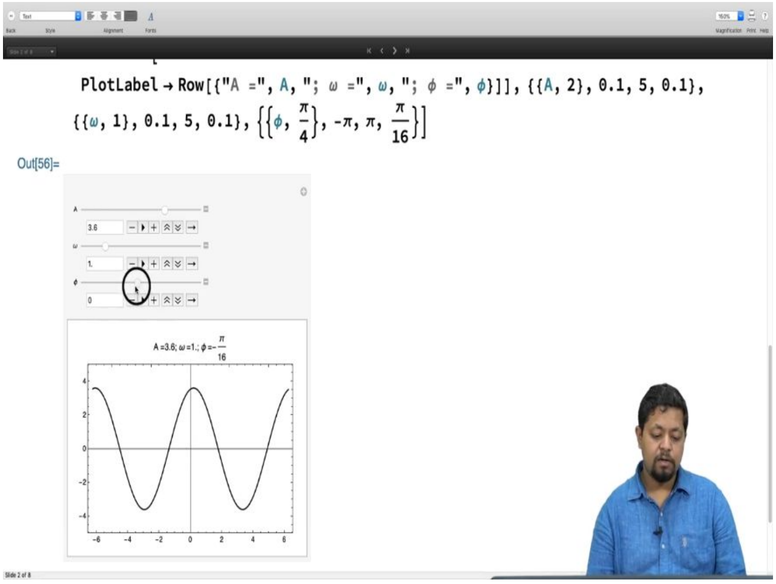

So by changing the phase phi, I can simply shift the entire plot and ask what is happening at the origin? Is it a 0? Is it a minima, or is it a maxima? Or it is some other value. So, A is called the amplitude, ω is called the frequency or the angular frequency, and ϕ is called the phase shift. It is important to understand what these three things are doing in order to understand the solution.

Notice the use of row function over here. In order to create a nice plot label as over here so that you can read the amplitude, the frequency and ϕ I made use of the row option over here, row function over here for the plot label option. It appears to be a complex construct over here, but if you play around with it you will figure out what exactly is going on over here. So, I leave that as an exercise for you, it has been explained over here.

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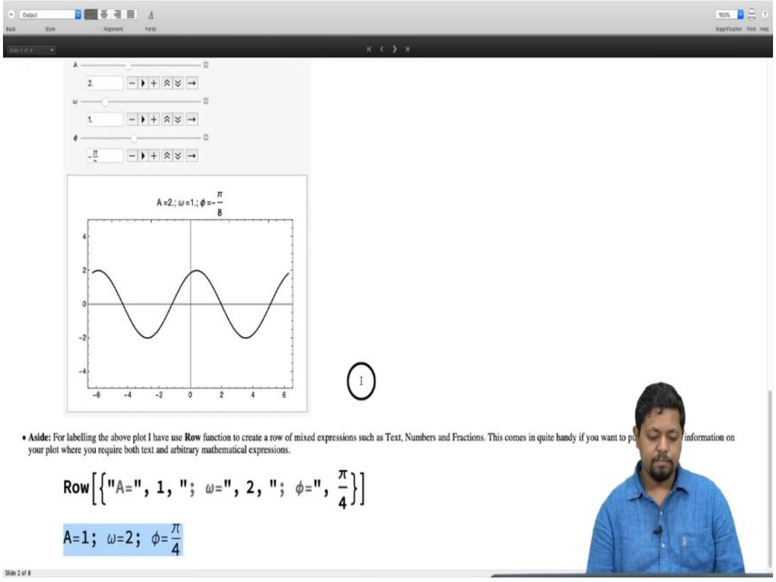

That when I use the row function like this, it simply gives me a print of the values like that and that is exactly has been used over here. From this, you can find out that, let us say, let us set A equal to 2, you see that the maxima is exactly at 2. Okay, so I I I request you to type in this command and play around more with it so that you can get a grasp of the solution.