Waves and Oscillations
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Module - 12
Lecture - 58
Waves in Quantum Mechanics and Summary

Welcome to the 5th lecture. We are in the 12th week and this is almost the last lecture of this entire course. And in this particular week we have been generally looking at waves beyond the linear region; in fact, oscillations and waves beyond the linear region.

And now we will take a small digression, in the sense that today in this last I have two things lined up, one is to give a really brief introduction to waves in quantum mechanics. And then I will give a quick pictorial summary of whatever we have sort of learnt in the last 12 weeks.

I know that is there is some possibility that most of the people following this course have not had a proper introduction to quantum mechanics. So, it is a very very top level attempt to give some basic idea, especially to focus on how wave comes into the picture in the quantum mechanics no more than that.

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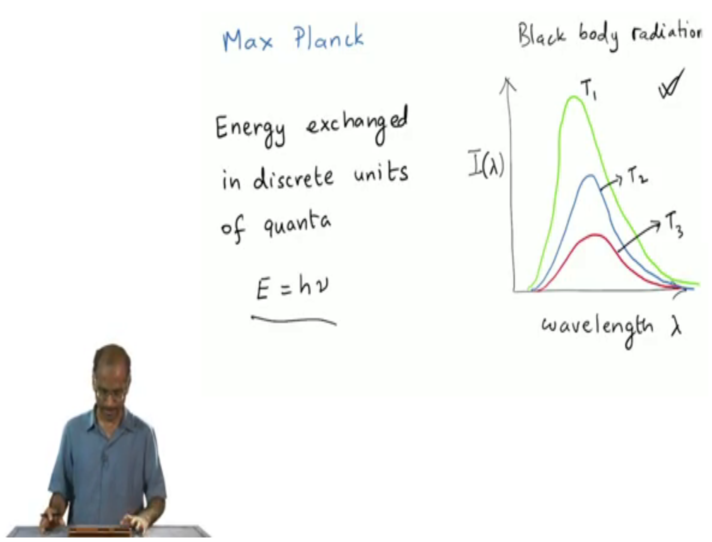

The idea of quantum can be traced back to work of Max Planck; he did this sometime in 1897 to 1900. So, he was trying to explain what is called the black body radiation. So, the black body radiation curves are shown here. So, what is plotted here is the intensity as a function of wavelength. So, this is the intensity of the electromagnetic radiation as a function of its wavelength and it has these kinds of forms.

So, each one is for a particular value of temperature. So, you could say for example, the green curve is for some temperature T_1 and a blue curve is for some temperature T_2 and red one is for some other temperature T_3. So, temperature is the constant throughout the curve. And in general T_1 would be larger than T_2, which would be larger than T_3.

In any case what is important for us is, in any case you must have studied black body radiation in some other context. But what is important here is how and explanation of black body radiation was given by Planck. So, his hypothesis was that energy is exchanged in discrete units of quanta, why is this something different? Because before Planck gave his idea, it was thought that energy is a form of wave and typically it was expected that wave is continuous. So, energy should be a continuous entity.

Max Planck tried with all these explanation and did not did not work, and by his own admission just initial frustration he made the assumption that energy can be exchanged in discrete quanta. So, like shown here energy is $h\nu$; h is what today we call as the Planck's constant and ν is the frequency.

So, if this assumption was taken in and of course one has to do a lot more work; so, you do everything then you are able to explain this intensity versus wavelength curve. So, this is what is called the black body radiation curve. So, an important assumption of physics which explains the black body radiation is that energy is exchanged in discrete units of quanta. So, this is really the beginnings of quanta as an idea.

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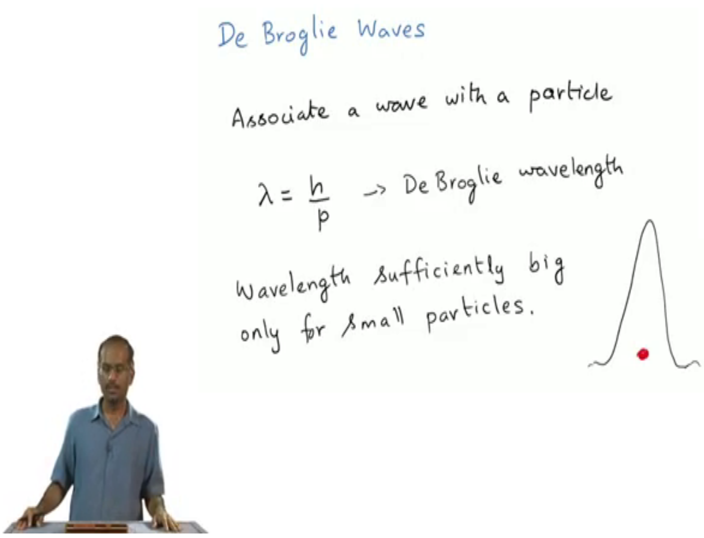

So, much later almost 20, 25 years later De Broglie said that, there has to be symmetry in nature for the following reason. Here when you say that energy is exchanged in discrete units it is almost like ascribing particle nature to wave. It is almost like saying that energy will come in small units almost like a particle. So, De Broglie's argument is that if wave can behave like particle then particles probably can also behave like wave.

So, he went ahead and made this idea quantitative by associating a wave at the particle. And the way it is done are De Broglie's postulate is that the wave can be associated with the particle for instance here I have shown a typical way of looking at it. So, this red blob that you see you can imagine as your particle and on top of it I have drawn one wave.

So, maybe it is possible to represent the particle using this wave whatever it means. But to make it quantitative in that you will have to be a little more definite about it and the De Broglie he introduced his idea of wavelength. That it is not that you can associate any wave with any particle but if you associate a wave it has a wavelength which is related to some property of the particle that is where the connection has to come from. So, this λ which is called the de Broglie wave is given by $\frac{h}{p}$, where p is the momenta. So, p is the momenta of the particle; so, that is how you connect the wave that you ascribe to the particle to some property of the particle.

Even though you are able to ascribe a wave nature to a particle, because of this particular formula the De Broglie introduced λ as equal to $\frac{h}{p}$; it is only when p is sufficiently small thus λ become reasonably observable. And the reason is that h are the Planck's constant is really a small number and if you want wavelength to be sufficiently big enough. So, that it can be observed in experiment then p has to be small. So, the momentum is related to the mass of the particle. So, you need to look for particles whose mass is really small.

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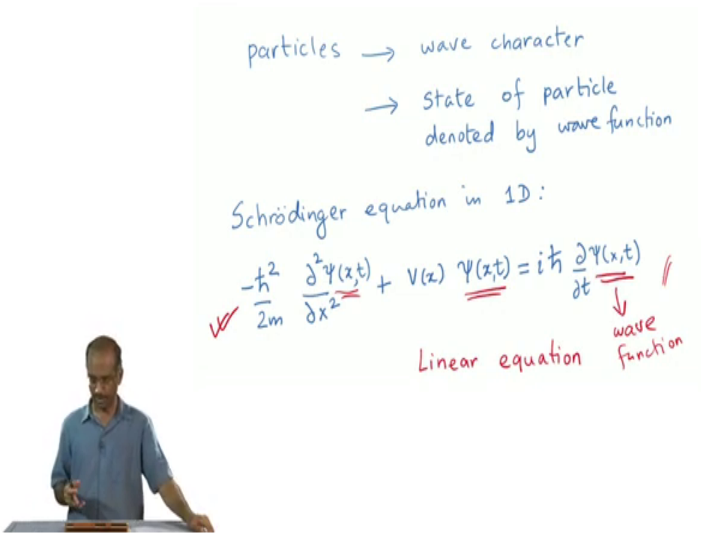

So, now if particles do have this wave character like the way I kind of picturized it, then every time I have to describe a particle I need to state some waveform associated with the particle. So, in quantum mechanics it is called a wave function. So, the state of a particle is denoted by stating its wave function. In fact, you cannot state it you need to actually calculate the wave function.

So, most of the problems associated with quantum mechanics is to find out what is the state of the particle; in other words, the question is tell me the wave function associated with the particle. In one-dimension, the dynamical equation associated with that is the Schrodinger equation which is given here.

So, I am not going to as I said describe in detail how all this came about our what are the very many ingredients of it. But, I want you to realize one thing that it is a partial differential equation, basically a linear differential equation there is no non-linearity the equation.

So, which means that, pretty much most of the things that we learnt about waves in our last 11 weeks, most of that can be used to study this equation. So, for everything that you have studied there is an analogy here with the Schrodinger equation or with the wave function which is indicated by this quantity ψ. So, the central problem at some level so, every time you want to study quantum mechanics of some problem, it boils down to finding what is the wave function.

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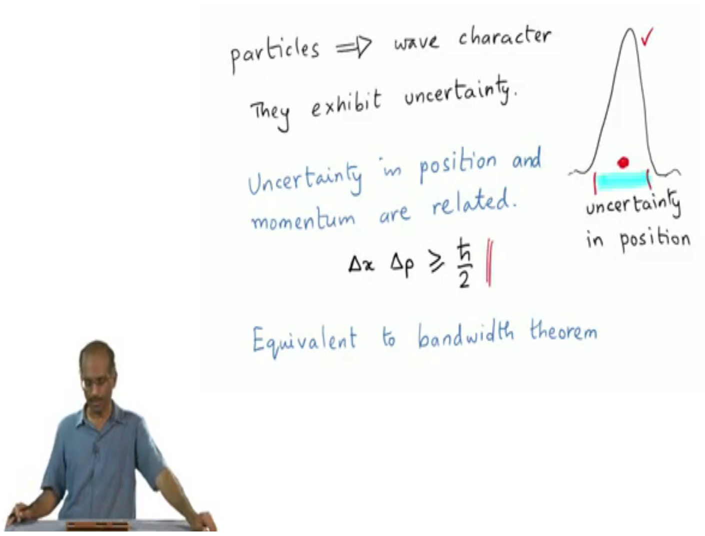

So, if the particles have wave character then there is something that follows some important implication. For instance go back to the picture that I do, when I said that I am going to associate a wave with the particle. So, here is the same picture once again.

But now, you can ask the question, if I am going to associate this wave with this particle which is shown in red colour what does it tell me about where the particle is? So, clearly it is not giving me one position unlike in the case of classical mechanics which will tell you here is the value of position of the particle, here is the value of momentum of the particle are the velocity of the particle.

So, here what I have is an uncertainty in the position like the way I have drawn here. So, given a wave function which describes this particle; the only inference I can make is maybe there is some average position for the particle but it also has some spread around it. So, to that extent the position of the particle is uncertain. And again here is an example of how useful what we learnt in wave phenomena is.

So, if I have a wave form like this, which is which has some inherent uncertainty you appeal to this bandwidth theorem that we study. So, it tells you that in two different spaces in position and in time domain the uncertainties are related. So, an equivalent of that appears quantum mechanics as well and that is called the Heisenberg's uncertainty principle. So, it simply tells you that the uncertainty in position and uncertainty in momentum are related by this relation, where this $\hbar$ is simply Planck's constant divided by 2π.

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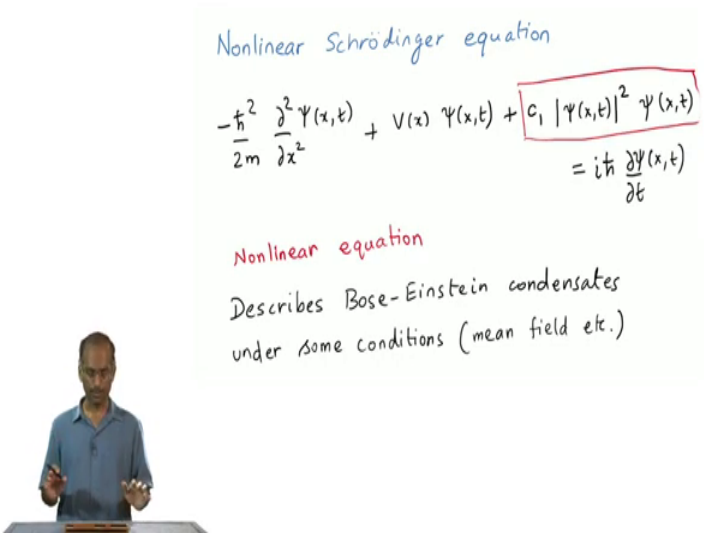

But there is also a beast so, to speak call the non-linear Schrodinger equation. So, if you look at this equation, it looks like the standard linear like Schrodinger equation except for this term here. If this were not there cc_1 is some constant, if c_1 were 0; this would be the same Schrodinger equation that I wrote down earlier on to describe a particle in quantum mechanics.

So, there are certain situation where that kind of description does not work principally for an instance when you want to study Bose Einstein condensates. Under certain conditions there are these interacting phenomena where you need to appeal to a different kind of Schrodinger equation and it is called the non-linear is Schrodinger equation. So, this term here which is now called marked in that red box introduces non-linearity which makes this a non-linear equation. Again solving this is much more harder than solving the standard linear Schrodinger equation. But the nice thing is that many of the things that one studies in the case of a non-linear wave equation can also be used here.

Again, that is a part that we did not study we just touched upon it by looking at solitones and similar solutions for certain class of non-linear wave equation, but again it is really a huge field. So, there are similarities at some level between this non-linear Schrodinger equation and the non-linear certain classes of non-linear wave equations.

So, really I have tried to convey or even motivate why understanding wave phenomena is useful in a non-classical setting; even in something like understanding behavior in the quantum domain. Simply because every particle is ascribed a wave description, which means that everything that I studied in waves and oscillations hopefully should be useful in the new setting as well. So, that is the bottom line that I would like to leave you with.

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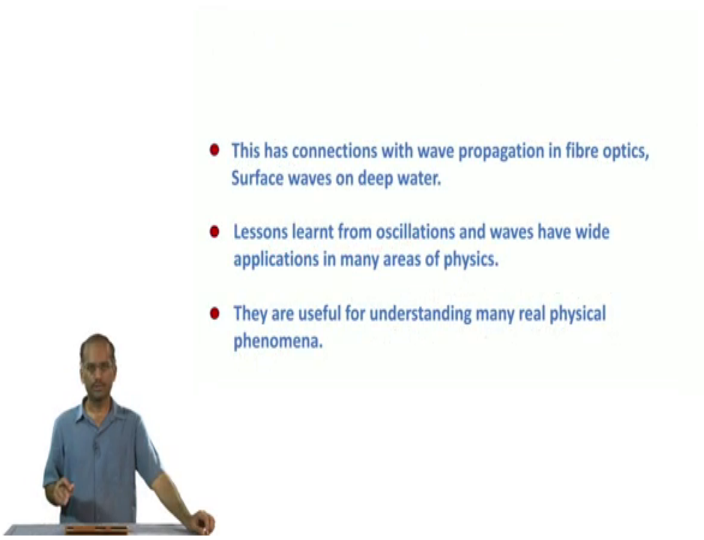

So, just to summarize this part about the non-linear Schrodinger equation it is very useful and it is again a really a huge subject with lots of emerging research work. The idea of non-linear waves of which non-linear Schrodinger equation is a part, this has connection with many things, for instance in wave propagation and fiber optics and related to surface waves on deep water.

So, the idea is that the lessons that we have learnt from oscillations and waves have wide applications in many areas of physics not just quantum mechanics, but in really many areas of physics. And more importantly they are useful for understanding many real physical phenomena.

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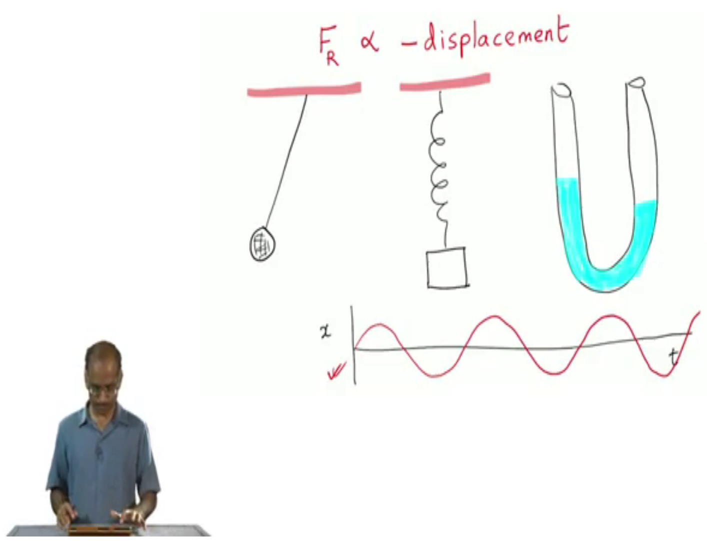

So, initially we have started with studying simple oscillations of objects like these. Simple oscillations of a pendulum for example, I have the picture here on the left hand side or the simple oscillation of a block which is connected to as a spring. So, you pull it and leave it is going to start oscillating.

The another example is I have a tube which is shaped like a *U* and there is some liquid in it. And if I depress the liquid in one end it is going to start oscillating. So, all these can be together described using one important piece of physics assumption. Namely that the restoring force is proportional to negative of displacement, it is this negative sign which gives rise to oscillations.

So, this is one thing you need to remember there may be many different systems like these. There could be many oscillatory phenomena, but at the heart of it is this idea that the restoring force is proportional to displacement with the negative sign. And if you work the math you will see that the solutions are basically sine and cosine function like the one that is shown here.

So, you could say that my displacement as a function of time is a sine function. And then you can go ahead and ask for how does two oscillators work two independent oscillators which do not interact with one another. So, all that we did; so, you can put two oscillators either parallel to one another or perpendicular to one another. And it really does not take much work to write out results for these cases.

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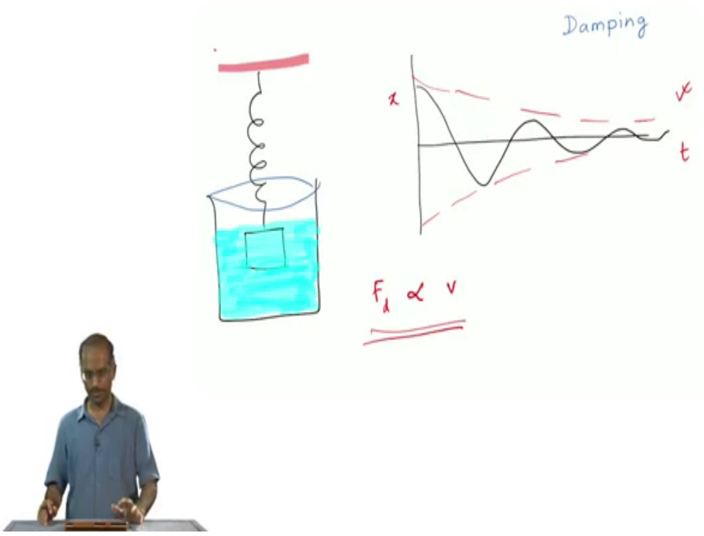

Then the next important thing that we studied was the case of oscillations with damping. So, we took specific case of viscous damping, where you could write the damping force as being proportional to the velocity. Suppose you had this system of spring with the block and you put this block and have it dipped inside let us say in water and then allow it to oscillate.

So, this system can be described using equation of motion, which will include this damping term, where the dumping them is proportional to velocity. So, in terms of phenomena what you will see is that, there will still be oscillation, but the amplitude of oscillation keeps reducing as we see here as a function of time; so, ultimately it will come to a stop. So, we described all these quantitatively using equation.

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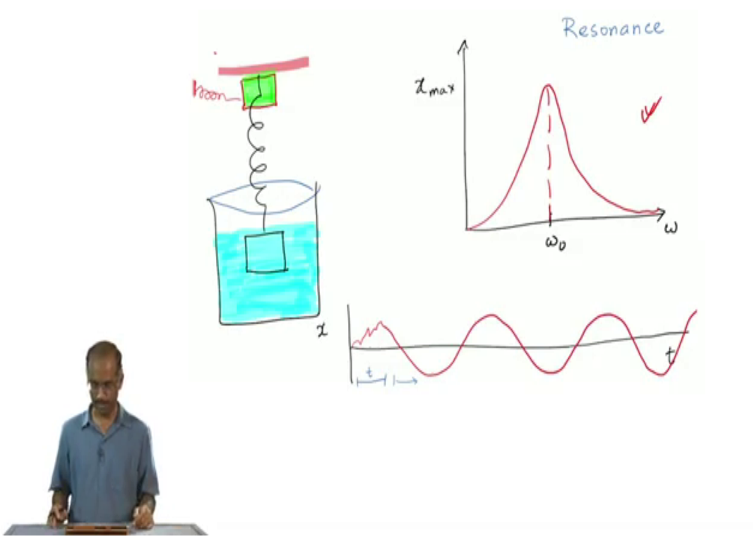

Beyond this we went to look at what happens if I forced the oscillator. So, I give it external forcing because if there is damping it is going to stop after a while, but to keep it running let us say I connected to some power source like this. And I also have damping which takes the energy away and there is a power source maybe, which gives energy to compensate for the loss.

So, in that case we saw that there is an initial transient time, but after that transient dies out what you see is that, if your external forcing has frequency of oscillation equal to ω; then the particle that you have also oscillates with frequency ω. So, in other words you apply an external frequency ω to the system, then the system also response by oscillating at the same frequency. And here what we saw is that maximum amplitude is obtained when the natural frequency of the system coincides with the frequency at which you drive the system. So, this is called the resonance phenomena.

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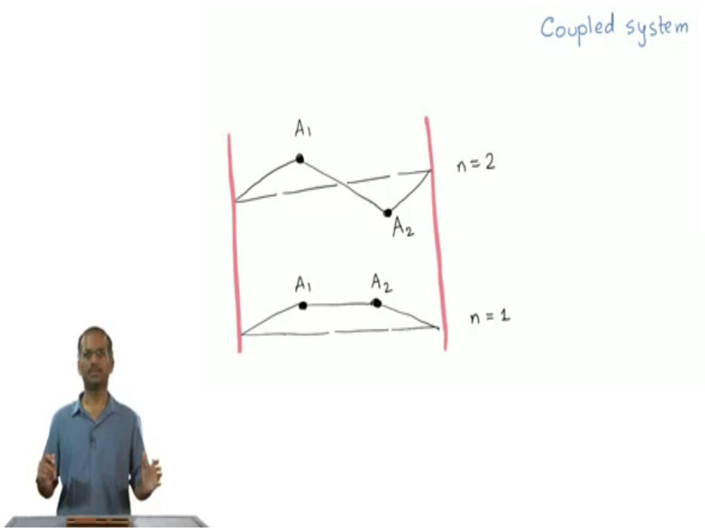

Next we asked the question what if I couple particle; now they interact with one another ok. So, you cannot anymore assume that if I have two particles like I have shown in this slide here. The two particles are coupled by a string, maybe if I move one particle the other one also would start moving so, in that sense they are coupled. And if they are coupled what is it that I am going to get. So, also the nature of question now changes ok. So, I want to know what is the pattern of oscillation of both of them, rather than asking what is particle one doing, I want to know what are they doing collectively since, it is now a collective phenomena.

So, then we saw that there were normal modes so, if you had two particles you had two normal modes; if you had three particles you had three normal modes and each of the normal mode has its own normal mode frequency. So, this is an example of a coupled system, but what we really liked was to be able to continuum.

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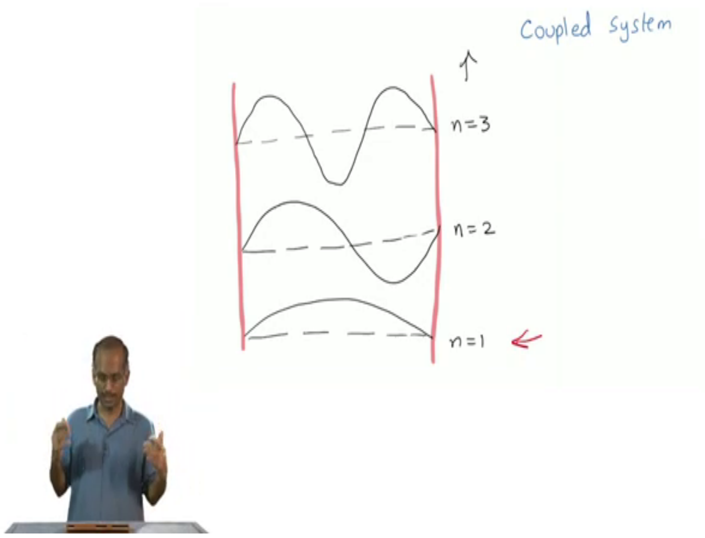

So, that now I can think of string itself as being connected through beads, but each one so, close that I can think of it as a continuum. So, a typical problem we did was suppose I had say two rigid walls and a string that is held tout between these two rigid walls, what are the patterns of oscillations it will suit. So, it is a question of finding normal modes of this system. So, here again you can find the normal modes. So, there is an upper limit to the normal mode frequency in such a system.

So, we saw that if ν_0 is the fundamental frequency corresponding to this $2\nu_0$ would be the largest frequency possible in this system. And we also saw that you could also think of it as two waves propagating in opposite direction and they interfere and produce a standing wave pattern.

So, in principle these are the only possible unique templates or patterns you can get. So, any arbitrary pattern you will see in such a system can be written as linear combination of these, we wanted something more.

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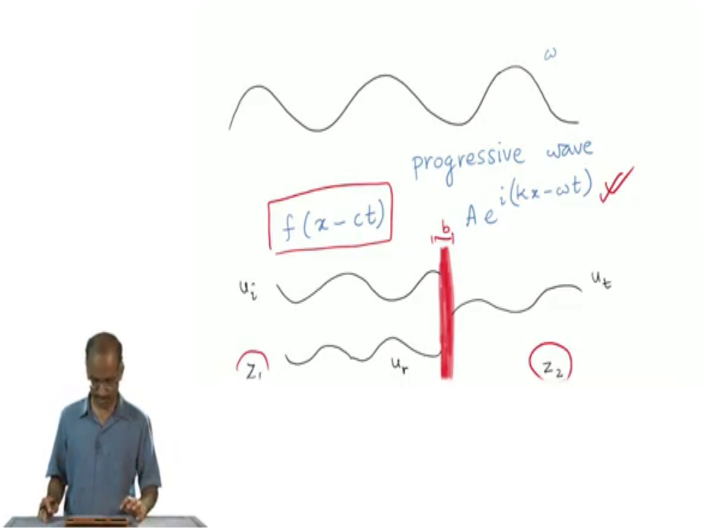

In the sense that in the previous case we had put in this boundary condition that there is a wall and string was tied at the wall. But suppose I want to be freed from the boundary condition, let me worry about boundary conditions later. But, now just let me describe the oscillation of a string without worrying about what the boundary condition is, which I will apply later depending on what the boundary condition is. Maybe I have a free end at one end why should I always consider a string tied between two walls.

So, in that case we actually went ahead and applied obtain the wave equation. And we saw that you can solve the wave equation and the solutions are plane waves of this type. So, you call it a progressive wave with some frequency or angular frequency ω. So, in general we saw that any function which has this form any function of $x - ct$ will be a solution of the wave equation.

So, once you have the wave equation you can write down the solutions straight away. So, every time you see a wave, you can straight away write this solution; it does not matter that it was generated by a physical system which obeyed a wave equation. So, now, in some sense the life is easier you see a wave I can write this equation. Of course, you should keep in mind that the wave equation itself has been derived under certain approximations. Especially keep in mind that we had strongly applied the approximation setting it to linearity. So, in other words every time we had a $\sin\theta$ we said it is equal to θ.

So, those kinds of approximations have been put in obtaining the wave equations itself and.

So, now, you could handle problems like I have a incoming wave there is a barrier what happens in such a situation. So, I have an incoming wave that hits the barrier. So, part of that is reflected and part of it is transmitted across into the other side of the barrier. And in fact, you can add more complexity to the problem by saying that on the left hand side I have this medium which is characterized by Z_1 the impedance. And on the right hand side, I have a different medium which is characterized by Z_2, ok. So, how do I relate these incoming reflected and a transmitted component.

So, we looked at all these we obtained what is reflection co efficient transmission co efficient. And suppose if your barrier itself is another medium here with width b; we also found condition under which the entire incoming wave can be transmitted without a reflected component. So, this is called impedance matching condition we derived all these cases.

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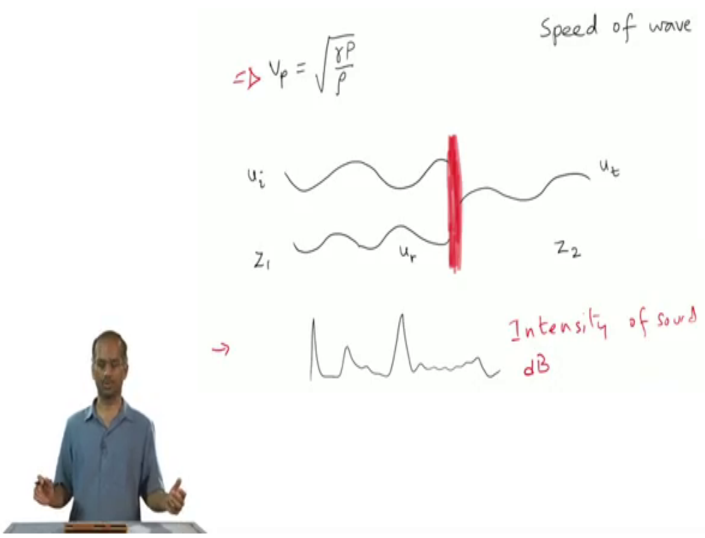

And we spent sometime almost a week in looking at speed of waves in particular we studied the special case of sound waves. So, sound waves are longitudinal waves the oscillations are in are in the same direction as the direction of propagation. So, there we obtain the speed of sound, the phase velocity of sound or the velocity of sound is simply $\sqrt{\frac{\gamma P}{\rho}}$, P is the pressure and ρ is the density of the medium.

So, here again we could handle problems like you have a incoming sound wave and there is a change of medium, how much of it is transmitted how much of it is reflected? And if it were going from one medium with impedance, Z_1 to another medium with impedance Z_2 what happens. So, all these things we studied we also looked at measures like intensity of sound how it is expressed in decibels and so on.

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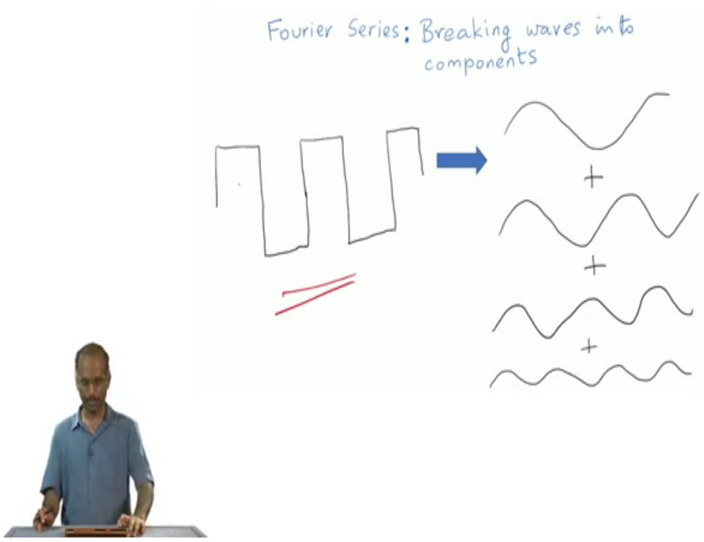

We spent almost a week in looking at Fourier series. So, if you remember initially we said that, if I have two wave forms I can add them together thanks for super position principal for linear wave equations. I can add two of them and produce a new wave form.

Now, if I have a wave form which is a sum of these can I again reconstruct the once or the components that were added together that is done by Fourier series. So, it essentially breaks the waves into its component parts. What are the component parts? The component parts are themselves sine and cosines of different frequencies we also saw Fourier transform we also saw how it can be used to obtain energy of the waves.

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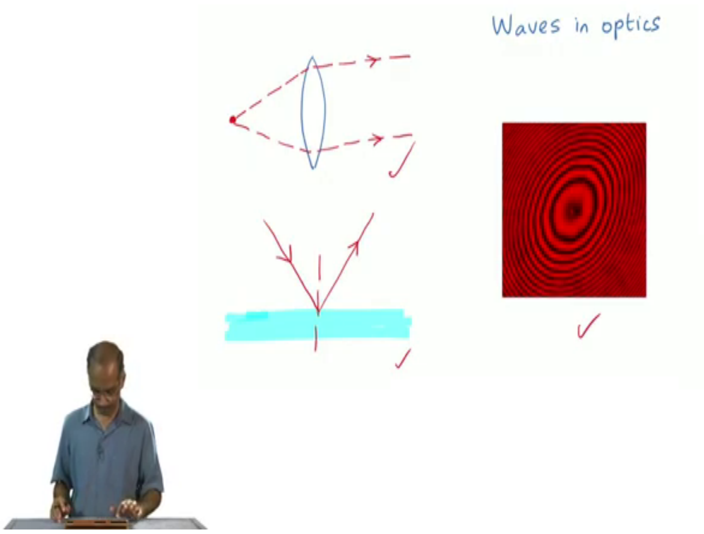

Then we spent almost two weeks in looking at waves in optics really too shorter time to recall all the results. But in particular, if you remember we looked at two limits one where you could think of your wave form the wave length of your wave to be tending to be 0; in such case you can describe it by a ray. And reflection and refraction are nicely accounted for by the ray; ray phenomena like these. On the other hand if you want to look at interference like these, this is the Newton rings as you might remember.

So, if you want to explain Newton's ring you need to go beyond this ray form you need to consider the oscillation or the oscillatory parts as well. So, you need to consider the entire wave form. So, by looking at two wave forms which are reflected from two different parts of the same system and essentially the idea there was to look at the path difference converted into phase difference. And see how many wave lengths you can accommodate in that path difference. And depending on how many wave lengths you can accommodate you can either get a destructive or a constructive interference.

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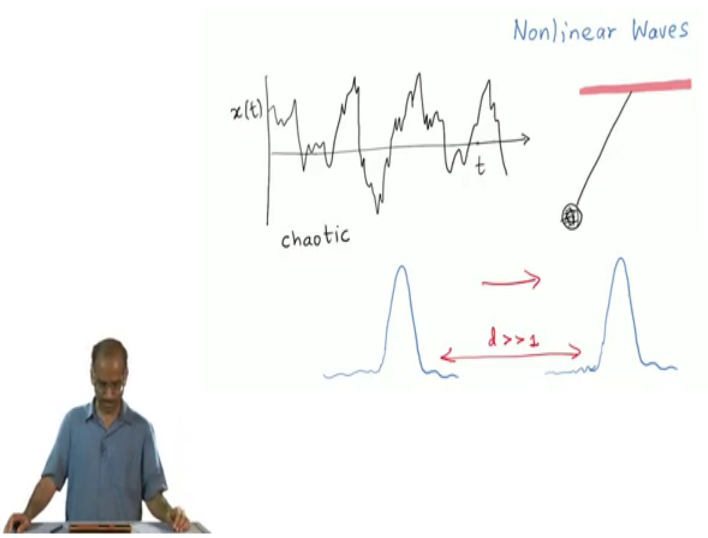

Finally, we spent this week looking at what happens beyond all these linear assumptions. So, the first eleven weeks everything we studied has to do with had to do with linearity assumptions.

In other words if you remember the very first week we said that the restoring force is proportional to displacement or x in principal; it works for many simple systems. But, there are full lot of phenomena where the restoring force is not simply proportional to x its in fact, some non-linear function. So, those are more complicated functions. So, once you put in even something like a pendulum, if you have arbitrary amplitude you need to it is non-linear system and solving it is not as easy as solving a harmonic oscillator problem.

So, we looked at those kinds of non-linear oscillations and one of the important new and novel phenomena we said that we came across was that of chaos. System can show chaotic behavior meaning that it is irregular, but has a keep emphasizing chaos is much more than this visual irregularity in the displacement as a function of time, there is lot more in it ok, again it can be a separate course in itself.

And finally, we also studied a little bit about solitons, which are essentially describing non-linear partial differential equations. So, these are special because these waves do not easily disperse. So, they can travel for long distances and stay around for long times without dispersing. In other words the shape of the wave like I have shown here it does not distort itself does not die away when it travels over long distances and for long times.

This is very different from standard wave forms, you see as I said you throw a stone into water, it produces waves, but very soon the waves die out. So, it is really remarkable that the shape of waves can be maintained over a long distance and this really has important application as well.

So, this essentially is the summary of everything that we did in the last 12 weeks; of course, I gave a short, really really short intro to quantum mechanics where wave function is just another wave form. So, with this I would like to conclude this course and I hope you enjoyed and benefited from this course as much as I enjoyed this course.

Thank you very much.