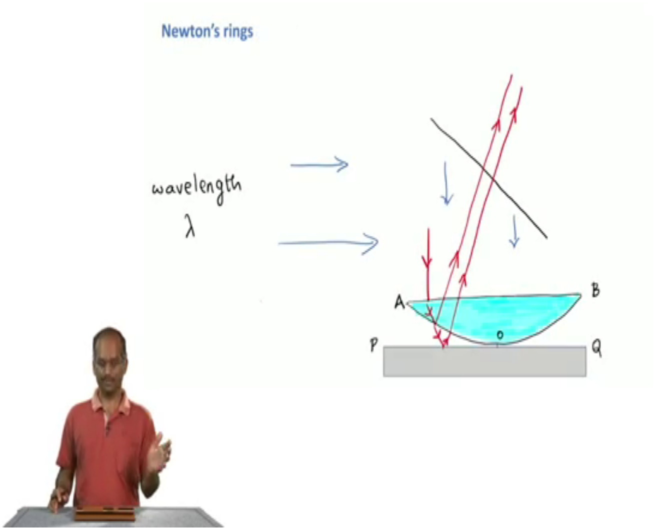
Waves and Oscillations
Prof. M S Santhanam
Department of Physics
Indian Institute of Science Education and Research, Pune

Lecture – 52
Young's Double Slit Experiment

Welcome to the fourth module of eleventh week. This week we have been seeing interference; interference phenomena and in today's lecture we will look at Young's Double Slit Experiment, one of the classic experiments and I must tell you first an important distinction between this and whatever we studied earlier. So, in all the cases that we looked at earlier Michelson and Fabry-Perot and Newton's rings and so on, in all those cases they are a class of interference where you divide the amplitude and look for interference phenomena or interference through division of amplitude.

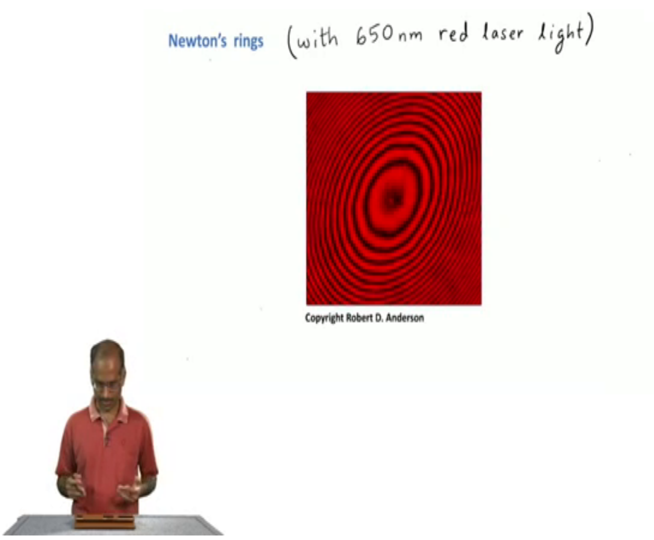
So, what you had was you had one beam which was used to generate two beams by a process of reflection or even transmission or a combination of reflection and transmission. On the other hand, in what we are going to see today will actually divide a wave front. So, there will be an incoming wave front and the path of the wave front you put in a screen with maybe some slits. So, the same wave front is now divided into two and generates for us two beams which would possibly interfere. So, Young's double slit experiment belongs to this class.

(Refer Slide Time: 01:46)



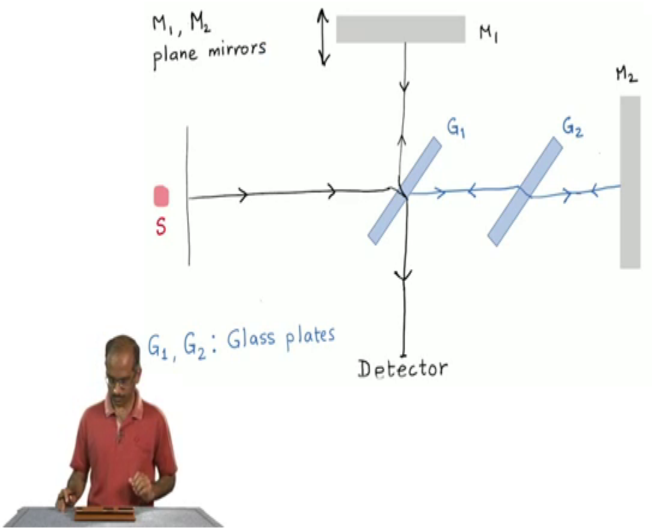
So, very quick review of what we did we looked at interference through division by amplitude; Newton's ring is a good example of that. So, in this case one beam becomes two beams through a process of reflection. So, in this case the reflection takes place from the bottom surface of this plano convex lens and the top surface of the glass plate P and then of course, it satisfies the usual conditions for destructive or constructive interference and you can of course, observe it.

(Refer Slide Time: 02:17)



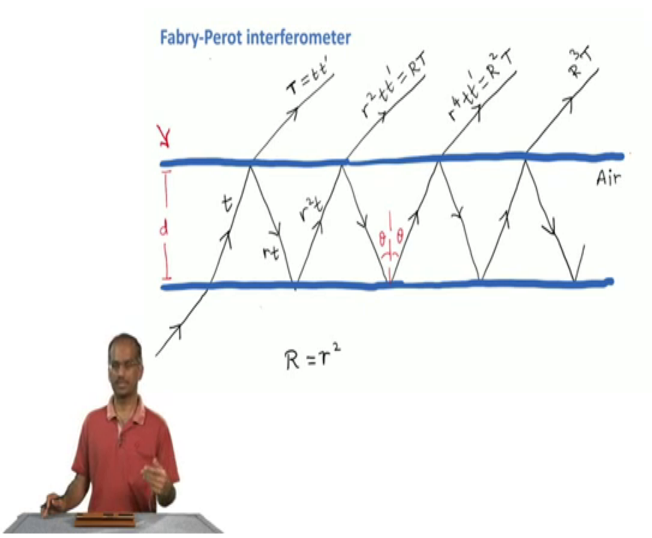
This of course is one of those observed interference pattern.

(Refer Slide Time: 02:23)



And in the case of Michelson's interferometer again what was involved was reflections are two mirrors M_1 and M_2. You had one source of beam and two mirrors and the path length to these two mirrors can be varied which provides the path difference and hence the phase difference and, again at the detector depending on the settings you could either see a constructive or a destructive interference.

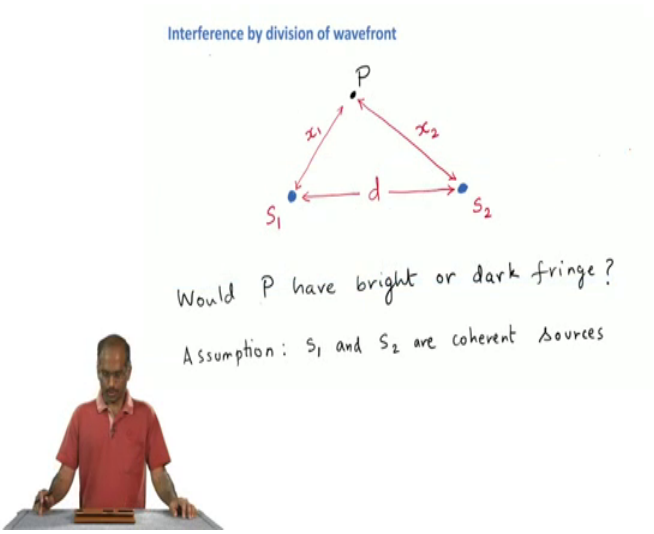
(Refer Slide Time: 02:59)



The other hand Fabry-Perot interferometer was the case of interference in a setup with multiple beams because you had one incoming beam instead of just producing 2 which normally interfered now you had many which interfere. At the top end of this Fabry-Perot interferometer this one it is partially reflecting and partially transmitting. So, there is a transmitted component which is finally detected and there is a reflected component. The reflected component gets transmitted partially transmitted later on and so on.

So, you had a large number of these beam which come out and at some point external to the etalon you could ask whether they would all constructively interfere or there would be destructive interference. So, all that depends on the phase difference between the successive beams that are coming out and depending on conditions on these again you could work out when you will see a constructive and the destructive interference. So, the point to remember is that this is a case of many beams interfering together as opposed to two that we saw for Newton's rings and Michelson's interferometer.

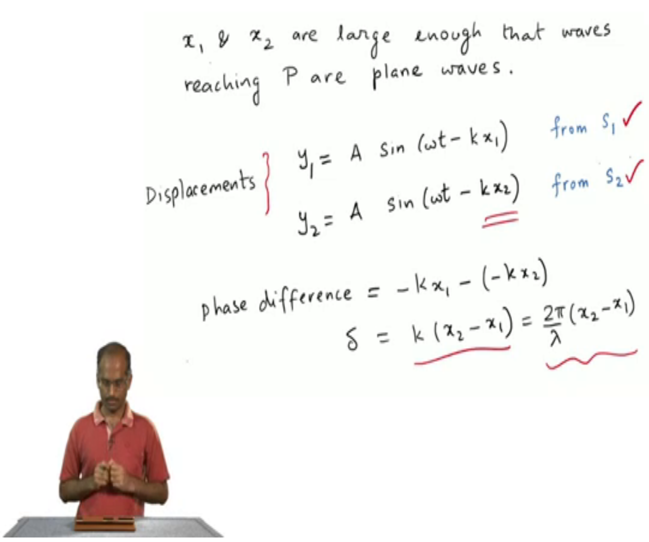
(Refer Slide Time: 04:21)



So, here I have two sources S_1 and S_2 you can take them to be point sources and you can also assume that they are going to emit light in all directions if you like. Now, the question is the following. So, if I have these two sources S_1 and S_2 they are monochromatic they are coherent sources and they are separated by a distance d between them. Now, I take some arbitrary point P which is shown here what would I see at P? That is the question.

So, this point P is located at a distance x_1 from the source S_1 and it is located at a distance x_2 from S_2. Would I see a dark fringe or a bright fringe at P that is a question that we need to answer and here we are assuming that these two light sources S_1 and S_2 are coherent. They have the same frequency and more importantly they will maintain a constant phase difference between them.

(Refer Slide Time: 05:35)



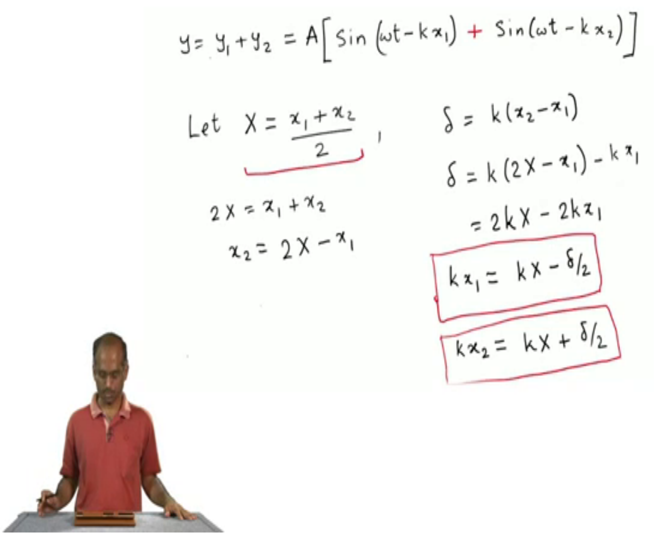
And, another piece of assumption that I need to take to make the analysis easier is that these distances x_1 and X_2 that I have defined here are sufficiently large enough in the sense that the point P is sufficiently far from these two sources S_1 and S_2, so that when the waves wave fronts reach point P you can assume that it is nearly plane waves. So, in that case I can write expression for displacement of each of these waves.

So, y_1 is the displacement of the wave that is reaching point P from source S_1 and y_2 is the displacement of the wave front that is reaching point x_2 and this comes from source S_2. So, I have this these two expressions and capital A is of course, the amplitude and again for simplicity I have taken amplitude to be the same in general they need not be, but makes analysis easier if I do so and here calculating the phase difference is straightforward because the equation that we have written down is already in a form where we can just take the phases of each one of them.

So, the phase difference is simply the phase of the first one which is this is $-kx_1$ and phase of the second one is $-kx_2$. So, I just need to take the difference between the two. So, phase difference is $-kx_1 - (-kx_2)$. So, that gives me $k(x_2 - x_1)$ and remember k is the wave number and we assumed that both the sources have exactly the same frequency and same wavelength λ. Hence k will be equal to $\frac{2\pi}{\lambda}$. So, I have substituted that as well here. So, the phase difference is $\frac{2\pi}{\lambda}(x_2 - x_1)$.

Now, what I need to see is the net effect of both these waves reaching the point P. So, just add both of them.

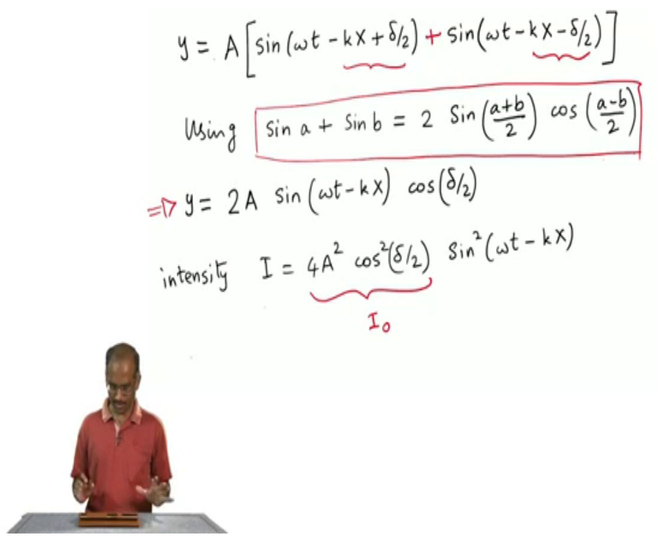
(Refer Slide Time: 07:52)



This is a kind of problem that we have done many times over when we looked at combinations of simple harmonic oscillations. It is pretty much a similar kind of exercise. So, I have this y_1 and y_2, now I need to find the resultant which is simply $y_1 + y_2$ and just add both the waveforms.

Now, what I am going to do is for ease of analysis I am going to introduce two new coordinates; my existing coordinates are x_1 and x_2 in some sense and I want to introduce this capital X which is related to x_1 and x_2 like this it is simply the average of x_1 and x_2 and also I have already defined this δ in terms of k. So, that is another relation that I have here; so, which means that I have two relations which connect x_1 and x_2 in terms of some new variable capital X and δ.

(Refer Slide Time: 09:05)

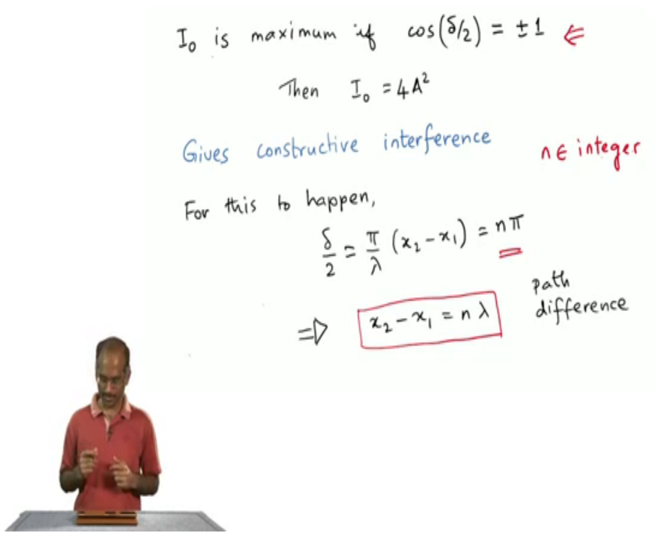


Now, I substitute for kx_1 and kx_2 I have already done that you can see that kx_1 here has been replaced in terms of x and delta and similarly kx_2 has also been replaced. And, now what I have is like $\sin a + \sin b$ relation. So, all I need to do is to use this standard trigonometric identity and write it in terms of product of sine and cosine. So, it turns out that in this case if you apply this formula; so, a is this $\omega t - kX + \frac{\delta}{2}$ and b is this quantity $\omega t - kX - \frac{\delta}{2}$.

So, you do that and you will get this relation simple application of a trigonometric identity. And now this is of course, the resultant and now, I can write what is the intensity associated with this resultant form y which is as we know simply square of this quantity. And, the square of this quantity is $4A^2\cos^2\frac{\delta}{2}$ and the usual $\sin^2\omega t$ function. So, we have been calling this as I_0 which is what is important for the interference phenomena.

This is time dependent and not quite important for our purposes immediately. So, we will look at what will happen to I_0 under various conditions of path difference or phase difference.

(Refer Slide Time: 11:02)

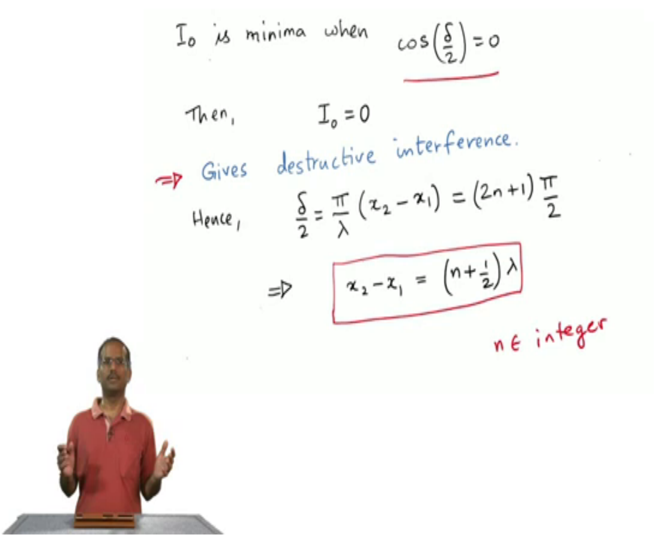
\


So, from here from this formula it is very easy to see what we should expect. So, I_0 will be maximum. In fact, you will notice that I_0 is a function of a $\cos^2$ term. So, it is going to be an oscillatory function and I_0 will be maximum if $\cos\frac{\delta}{2}$ is plus or minus 1 that is a maximum value that a cosine function can take. The maximum value a cos function can take is $+1$, minimum value it can take is -1 and since we are considering the square of cos function the maximum value it can take is $+1$.

And, this will happen if $\cos\frac{\delta}{2}$ is plus or minus 1; in either case $\cos^2$ will be plus 1 and in that case I_0 will simply be equal to $4A^2$ and this will give us constructive interference because this is the largest intensity possible, you cannot get better than this. You notice that the other term is sin square term which also will lie between plus and minus 1. So, you cannot get intensity larger than I_0 which is equal to $4A^2$ and that will happen if $\frac{\delta}{2} = n\pi$, where n is an integer.

So, this tells me that $x_2 - x_1$ should be simply integral multiple of wavelength λ or in other words the path difference should be equal to integral multiple of wavelength λ and this gives constructive interference. You notice that with respect to whatever we had seen earlier, in this case there are no additional phase factors that we need to consider due to reflection and so on. There are no additional contributions to the phase. So, directly if the path difference is equal to integral multiple of a wavelength you will get constructive interference.

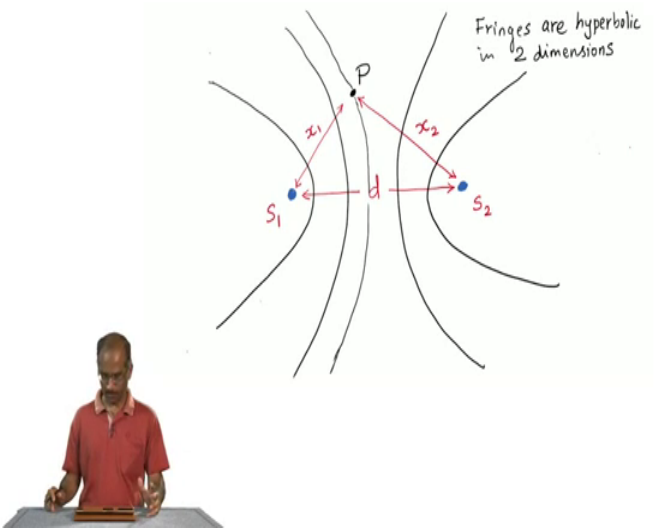
(Refer Slide Time: 13:14)



So, I_0 would be minimum and $\cos\frac{\delta}{2} = 0$ of course, in that case I itself would be equal to 0. So, I_0 will be 0 if $\cos\frac{\delta}{2} = 0$ and this will give destructive interference and the condition for $\cos\frac{\delta}{2}$ to be equal to 0 is that $\frac{\delta}{2}$ should be equal to $(2n+1)\frac{\pi}{2}$ where n is a integer. $x_2 - x_1$ which is simply the path difference is equal to $\left(n + \frac{1}{2}\right)\lambda$. So, this is the condition for destructive interference will give you 0 intensity.

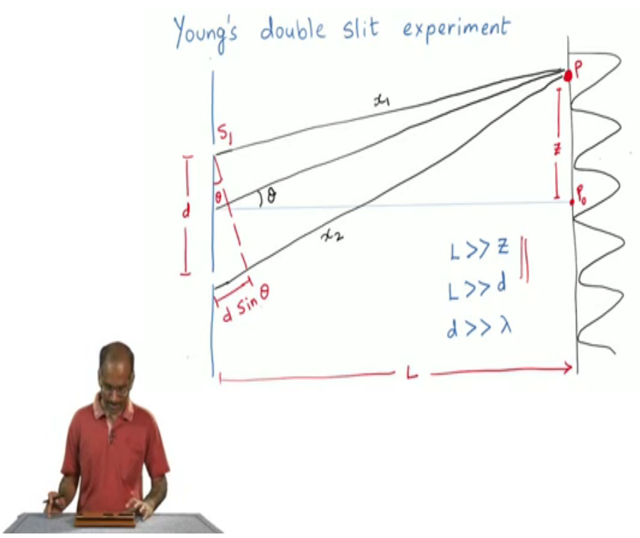
So, now, you can ask now that we have worked out the condition for constructive and destructive interference, now you can ask the question what would be the pattern that I will see.

(Refer Slide Time: 14:20)



So, we have this S_1 and S_2 which are source of monochromatic coherent source of light. They are separated by a distance d and point P which we are monitoring is distant x_1 from S_1 and x_2 from S_2. The locus of points which will give us the bright or dark fringes will form a hyperbola in 2-dimensions. So, in 3-dimensions it will be this 2-dimensional hyperbola that is rotated. So, you will see patterns like this. So, there would be alternating dark and bright fringes which have the shape of a hyperbola.

(Refer Slide Time: 15:10)



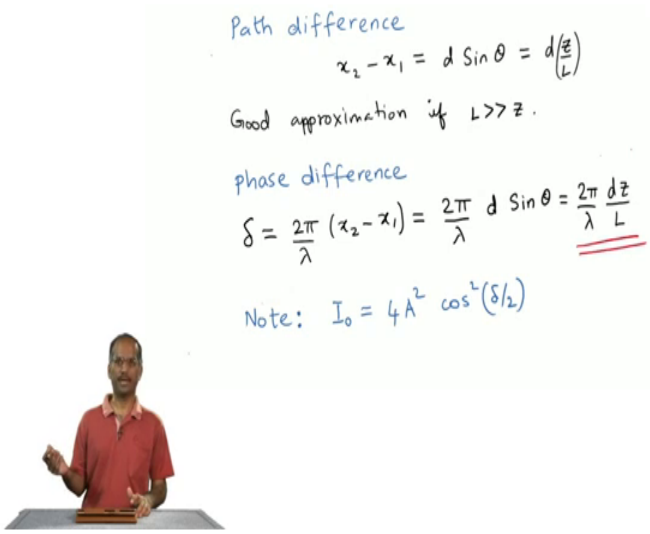
Let us move to the Young's slit experiment or in this case the double slit experiment. So, here the setup is as follows. So, I have this source of light which is at the left side and the light wave front from this source hits this screen which has two slits S_1 and S_2 to begin with it is easier to assume that S_1 and S_2 would produce point would be point sources of light.

And, S_1 and S_2 are separated by a distance d and at the far end on the right side I have this screen and the light beam coming from S_1 and S_2 would of course, make its contribution at any arbitrary point on this screen. Suppose, I choose a point P here which is indicated by this red dot, the question is what would I see there, would it be a bright fringe or a dark fringe? The distance between the double slit and the screen at the far end is capital L and of course, S_1 and S_2 are separated by distance d.

And, from this point here which I will call P_0 to point P is a distance of is Z. So, P_0 is the point which is precisely perpendicular to the midpoint between S_1 and S_2 and with respect to that line that is drawn there at the ray that is emanating from there a notional ray of course, there would not be any ray from that, but if I draw a line from there to point P it will make an angle θ.

This distance capital L is much larger than this distance is that between P_0 and P and capital L is also much larger than this distance d between the two slits and also I want the wavelength of light to be much smaller than the distance between the slits. These assumptions will guarantee for me that the two rays which are starting from S_1 and S_2 and reach point P they are nearly parallel when they begin with.

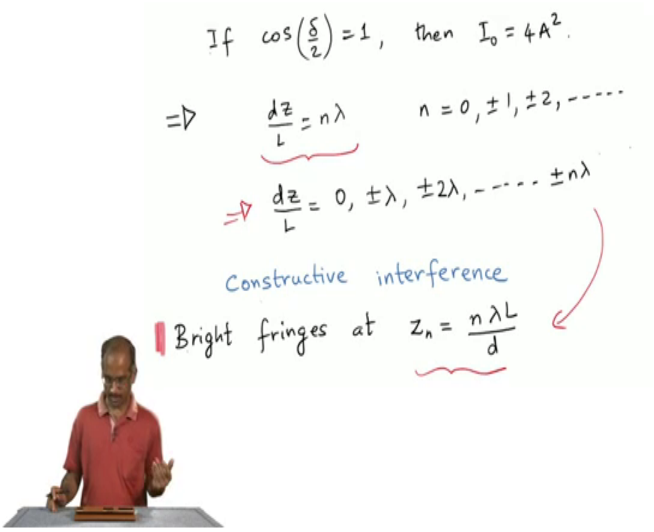
(Refer Slide Time: 18:01)



And, because they are nearly parallel we can approximately write the path difference simply $d \sin\theta$. So, you can see that the path difference that I have drawn here is $d \sin\theta$. So, that is the path difference between the two beams that reach P from S_1 and S_2 you take this triangle between P_0, P and the midpoint here. So, from that you can write $\sin\theta$ to be z by capital L.

So, once we know the path difference straightforward to write the phase difference so, simply substitute $x_2 - x_1$ to be $\frac{dZ}{L}$ and I have this phase difference and as usual our intensity is $4A^2 \cos^2 \frac{\delta}{2}$. So, we just need to substitute $\frac{\delta}{2}$ here and see whether we are going to get a bright fringe or a dark fringe.

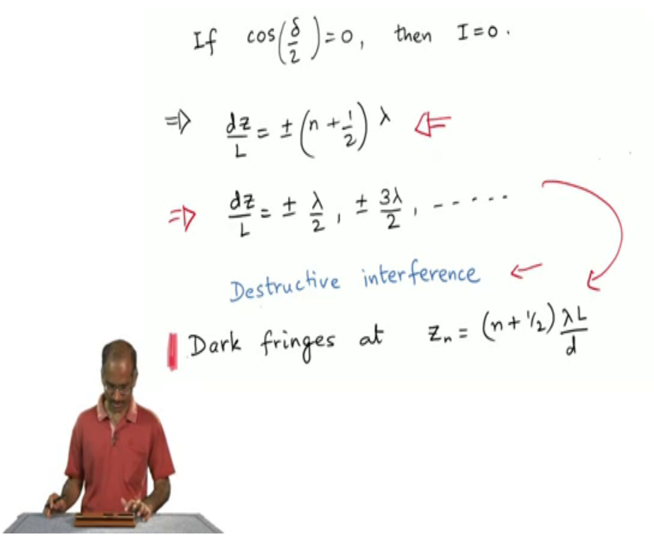
(Refer Slide Time: 19:06)



The fringe will be of course, bright if $\cos\frac{\delta}{2}$ is 1 or more correctly plus or minus 1. So, in that case the intensity would be $4A^2$ just like we saw the last time and then the condition is that this would happen if this $\frac{\delta}{2}$ is equal to integral multiple of wavelength which is given by this condition and that is $\frac{dZ}{L} = n\lambda$ where n is 0 plus or minus 1, plus or minus 2 and so on.

So, here is my condition $\frac{dZ}{L}$ is 0 plus or minus 1 and so on. So, which means that this is the condition for constructive interference, using this I can write this relation. So, it tells me that Z_n where the n would be the n-th order fringe you will get bright fringes at Z_n equal to $n\lambda L/d$; d is the distance between the 2 sources S_1 and S_2. Now, you can do a similar exercise for minima or destructive interference.

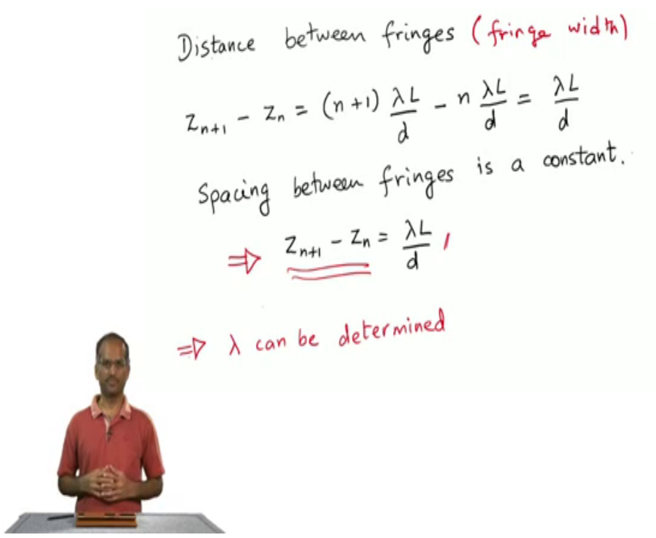
(Refer Slide Time: 20:33)



So, the condition is $\frac{dZ}{L}$ is equal to plus or minus $\left(n+\frac{1}{2}\right)\lambda$. From here I can extract the condition for destructive interference or a minima or a dark fringe $\frac{dZ}{L}$ should be equal to $\pm\frac{\lambda}{2}, \pm\frac{3\lambda}{2}$ and so on.

And of course, this is a case of destructive interference and like we did earlier on you can now write out an expression for where I can observe a dark fringe. So, Z_n then from this you could write down this; Z_n will be observed at $\left(n+\frac{1}{2}\right)\frac{\lambda L}{d}$; n, n is all integers. So, you are going to see a series of dark and bright fringes.

(Refer Slide Time: 21:30)



What is the distance between two bright points, two consecutive bright points and or two consecutive dark points? So, that is simply given by $Z_{n+1} - Z_n$. So, just write out the expressions that we just derived and you will notice that the difference this is equal to $\frac{\lambda L}{d}$. So, the spacing between the fringes as you can see is a constant simply because the three factors which determinate are all constants this also offers a way of calculating λ provided I know other things.

I know what is capital L because in an experiment if you are doing it you set up the distance between the source of radiation and the screen, so, you know what is capital L. You know the value of d because again it is under our control. Once I know all this I also I can measure the fringe width experimentally. So, if I can measure this difference experimentally then the only unknown is λ.

So, just like we saw with the Fabry-Perot or Michelson interferometer the Young's double slit experiment can also be used to determine the wavelength of unknown source of light.