Waves and Oscillations

Prof. M S Santhanam
Department of Physics
Indian Institute of Science Education and Research, Pune

Lecture - 51
Michelson and Fabry-Perot Interferometers

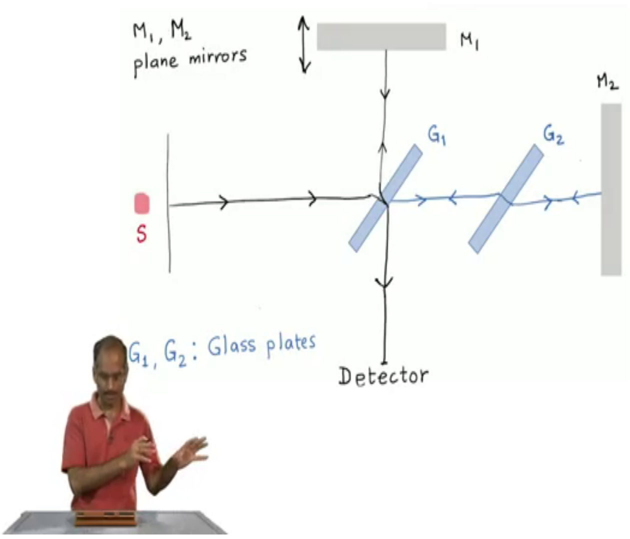
Welcome to this third lecture, we are in the tenth week; this entire week we are going to look at interference phenomena and diffraction phenomenon. Today in particular we will look at interference again with two really important contributions towards the end of last, towards the end of 19th century, the Michelson and Fabry Perot Interferometers.

But before we get into that, let me quickly remind you that interference is a general class of beautiful wave phenomena. So, in particular you need waves to be able to explain interference; in the sense that we cannot escape by taking the limit lambda tending to 0 and working with race, which we did, when we worked for, when we try to explain reflection and refraction.

So, here we need to keep the wave forms and to see any kind of interference effect, you need two coherent beams. Generally you will produce them from a single source. The main reason being that, if you take two independent sources, they are unlikely to be coherent; the phase relation is not going to be constant which is one reason and the main reason why you need to draw your two light sources from a single beam.

Today we will start with the Michelson interferometer; historically it was one of the most important contributions towards the end of a 19th century, which allowed experimental determination of large number of spectral lines.

(Refer Slide Time: 01:58)



So what I have is a source of light which is shown in red color, on the left side you can see that here, this one. So, that is a source of light and I have some kind of a glass plate here, which makes it an extended source. So, from what is essentially a, what might look like a point source of light, we get an extended source of light. And this light goes through to the right side and there are these two glass plates and the glass plate let us say G_1 is such that it is partially, it is partially mirrored. So, it reflects the light that is coming from left side upwards and partially transmits it.

So, you can for instance assume that 50 percent of the light is transmitted through it and another 50 percent is reflected. So, the reflected component goes up and hits the mirror M_1 and comes back that is reflected 100 percent and then passes through G_1 and directly comes to the detector here. And the beam of light which got transmitted through G_1, it goes straight hits another glass plate G_2; it is often called the compensatory glass plate, which is located extreme right.

And then gets reflected from there, and again comes and hits the passes through G_2 again comes and hits glass plate G_1 and is reflected towards the detector. So, starting from one beam of light, we have now two beams of light; one which comes from M_1 passing through G_1 and comes and hits the detector, the other one which comes from M_2 passes through G_2 gets reflected by G_1 and finally, hits the detector.

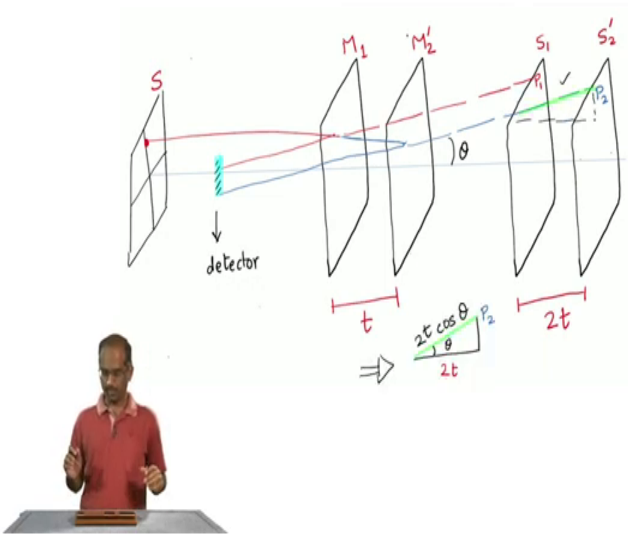
So, there are two beams of light and hopefully under good set of conditions they will be able to interfere and produce interference pattern for us. So, in particular we take these two mirrors M_1 and M_2 to be plane mirrors fully reflecting and M_1 itself is movable whereas, M_2 is fixed. So, the way to create path differences by making this M_1 moveable, this glass plate G_2 is often called compensatory glass plate and it is kept mainly because you want to equalize the path difference in the two arms.

And the reason why we need to put that is, if you carefully look at the diagram here; you will notice that the beam that is starting from the source going through G_1 hitting M_1 and finally, coming to the detector passes through G_1 three times. On the other hand the beam of light which passes straight through G_1 and hits the mirror M_2 and then finally, comes back to detector would have passed through a glass plate only once.

Whereas, the first one passed through it 3 times and this one passed through the glass plate basically G_1 glass plate; it passed through only once and to compensate for this difference you introduced this G_2. So, that would equalize the path difference or the optical path difference. Of course, in principle you need not really require the second glass plate, you can equally achieve that by adjusting the length of M_2.

Imagine that you are actually putting yourself, putting your eye at the place where the detector is. So, what you would see is that, you will see M_1; because you are looking straight through the detector and through glass plate G_1. So, you should see M_1; but you will also see the mirror M_2. And in addition you will also see the source S, which is shown in the red color on the left side, you will see the reflection of source S in M_1 and M_2.

(Refer Slide Time: 06:45)



So, here I have the source on the left side. So, that is a screen and I have just taken one point which is denoted by that red dot that you see there. So, S is the screen from where you have, you are getting that extended source and red dot is the one particular point we are monitoring. And my detector is at the position that is shown here. So, as we said, the first thing you will see is this light from the source reaches M_1 and comes back and hits your eye at the detector.

So, that is marked here, through this red colored line. So, M_1 is of course the, this mirror M_1 here. Now as I said you will also see M_2 and we shall call this M_2 prime in this picture, we see M_2 as a reflection; which is denoted by M_2' here. So, the same point will be seen in M_2' as well. So, the light coming from M_2' will also reach the detector, in other words it reaches my eye if I am seeing it there.

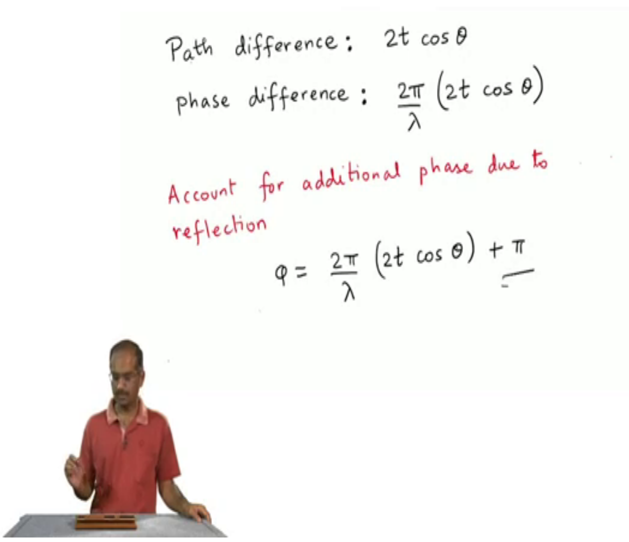
And that is also shown here as the blue line. Now if there is any path difference between this M_1 and M_2 that would be the distance between this M_1 and M_2'. So, that is the optical path difference between these two paths. So, it is essentially these two arms of the interferometer. Now as I said, we will also see the reflection of the source or in other words we will also see the reflection of the screen itself.

So, here what I have shown you is the reflection of the screen which I call S_1 coming from M_1 and the light coming from there reaches the detector through this red line. And then we also see the reflection of the screen from M_2 and I should have indicated that this red dot is this point P; this point P has a position in screen S_1 and S_2', which is called P_1 and P_2 as you see in this diagram.

Now the distance between them S_1 and S_2' will be $2t$, provided that your optical path difference is t. So, now, what I have in front of me is the complete picture of our analog of this interferometer, as a sort of linear arrangement; the path difference is simply this region which is marked in green color. So, here is the standard way of looking at the Michelson interferometer with two arms and here is the same Michelson interferometer drawn out linearly.

So, M_1 and M_2' are the position of mirrors as seen by the detector and S_1 and S_2' are the reflection of the screens as seen by the detector.

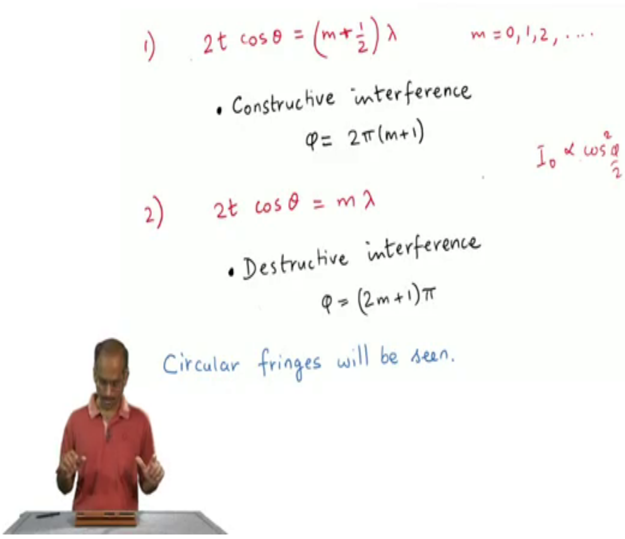
(Refer Slide Time: 10:23)



From this we obtain this path difference. So the path difference is $2t\cos\theta$. So, phase difference is simple to calculate, $\frac{2\pi}{\lambda} \times 2t\cos\theta$. Now we need to account for any additional phases that get accumulated due to reflection. As far as the path difference is concerned one important reflection we need to take into account is this; horizontal beam when it comes and hits G_1 and gets reflected downwards, it gets an abrupt change in phase by a quantity π that needs to be added.

So, in principle when reflection takes place at the mirrors M_1 and M_2, there will be a phase change of π; but we can ignore that, because that is going to add π to both the arms. So, we can sort of cancel out both and additional phase change of π that comes from the reflection that happens here. That is the contribution that we need to add. So, I have accounted for that additional phase by adding this π here.

(Refer Slide Time: 11:39)



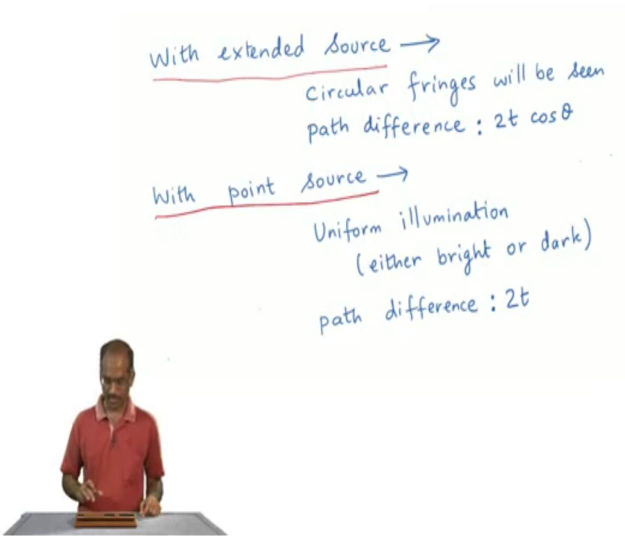
Condition for constructive interference is that, this $2t\cos\theta$ should be $\left(m+\frac{1}{2}\right)\lambda$, this is after taking into account that additional phase difference. So, I simply have to substitute $2t\cos\theta = \left(m+\frac{1}{2}\right)\lambda$ in this equation here or more correctly in this equation here. And if I do that, you will see that, I will get phase ϕ to be equal to $2\pi(m+1)$, m is integer. And similarly the other condition is I can take $2t\cos\theta$ to be equal to $m \times \lambda$.

So, once again you go back to this and substitute $m \times \lambda$ here and that is going to give you phase which is equal to $(2m+1)\pi$ and this is of course, the condition for destructive interference. There is the situation that we had seen for Newton's rings as well as for the case of difference coming from a glass slab of thickness t.

A constant θ value will have a locus, which will look like a circle. Hence all the points that have same θ value would have either a bright fringe or a dark fringe, depending on which of the condition is being satisfied. Hence you should see circular fringes. So, remember that this θ is defined for a particular point on the screen. So, different points on the screen are going to define for you different values of θ.

So, hence the whole pattern of interference that you will see, we will take contributions from all this range of θ values. Hence whenever the θ value is right in the sense of satisfying one of these conditions, you are going to get either bright or a dark fringe; but in general the result is that it is going to be a circular fringe.

(Refer Slide Time: 13:52)

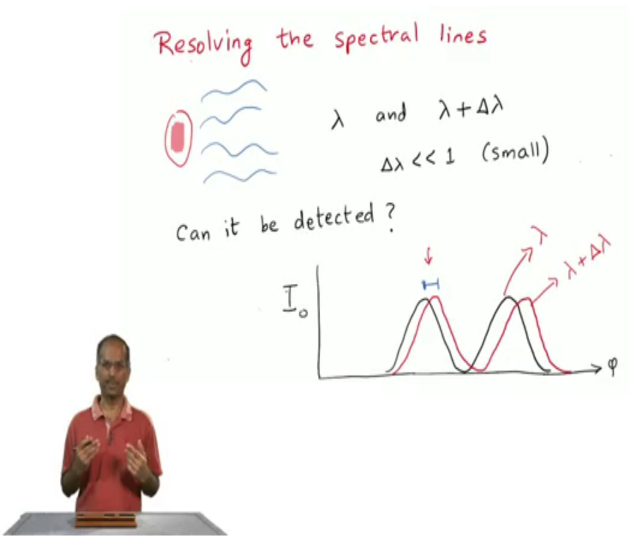


Here I can summarize these results. So, with an extended source like the one which we analyzed, we actually started with the point source and made it into an extended source by putting in some glass or something like that. With an extended source of light, what you will see is circular fringes and the path difference in this case is $2t \cos\theta$. So, this is the case that we analyzed. On the other hand you can consider a somewhat simpler case by simply using a point source of light.

If you use point source of light, it is like saying that there is only one point here; there is only one point on the screen, which lights up, everything else is dark. So, you are looking at an interference that is happening due to one ray of light coming from the source. And in that case $\theta = 0$, so the path difference will simply be $2t$; it is equivalent to saying that there is one ray of light which passes through two arms of the interferometer.

And the path difference is simply the difference in the travel distances and that is simply equal to $2t$. And what you will see is not circular fringes, but since now every parameter is fixed there is no θ at all in this problem, t is the only value and that is also fixed; hence you will either see a, you will basically see uniform illumination, it would either be dark or bright depending on which of these conditions is satisfied.

(Refer Slide Time: 15:42)



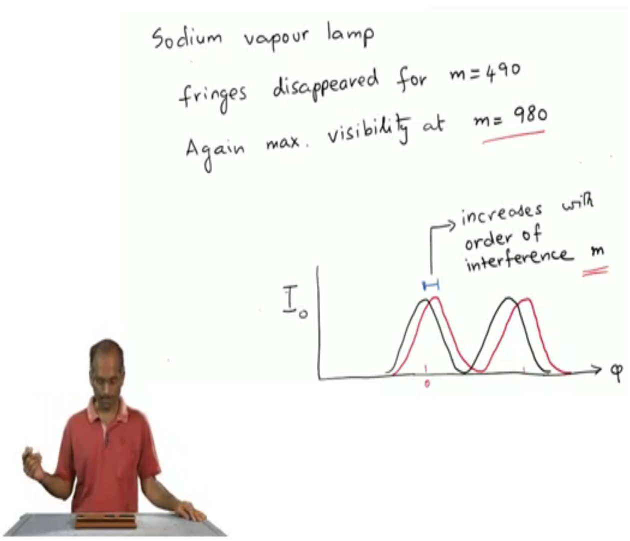
There is an important application of this Michelson interferometer. As I said in determining the wavelength of spectra; but before that there is one important question one needs to answer. Suppose there are, your source of light has two wavelengths which are very close to one another; for instance the sodium line, $D2$ line is actually made up of two lines which are very close to each other in wavelength or frequency.

So, can this interferometer tell us that it is actually made up of two lines; that is the question of resolution of spectral lines. So, here I have assumed that I have this source which is marked as marked in red and it is giving out light. So, typically when we do this analysis with interferometers, we assume that it is a coherent source of light and it is monochromatic; meaning that it has precisely one wavelength.

But now I assume that it has actually two wavelengths; one is λ, other is $\lambda + \Delta\lambda$. And I use $\Delta\lambda$ to indicate that the second one is very close to the first one. So, here is the statement that $\Delta\lambda$ is much less than 1 so, it is very small. Now what would I expect; if suppose I put this light through my Michelson interferometer and I plot the intensity, I expect something like this.

So, I have shown the expected intensity as a function of the phase difference ϕ. So, there is this black line corresponding to the wavelength λ and then there is this red line that you could see; where the intensity is plotted as a function of ϕ. So, this red line is for $\lambda + \Delta\lambda$. So, the question is in practice, can you differentiate these two peaks, can you resolve these two peaks?

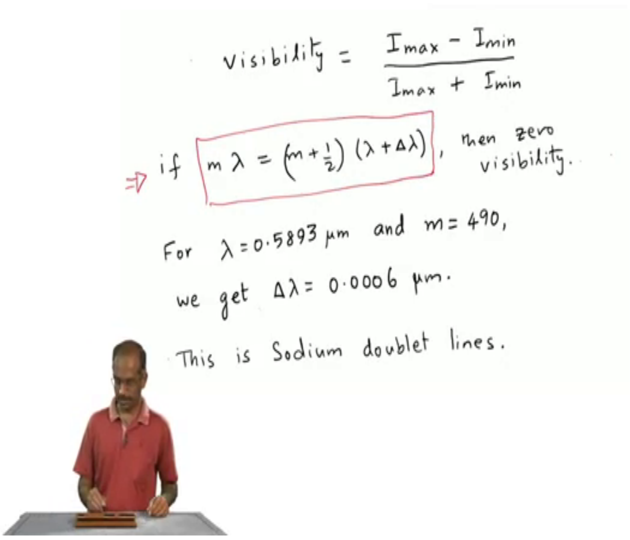
(Refer Slide Time: 17:57)



The sodium vapour lamp is made up of two sodium lines $D1$ and $D2$ as it is called. What was noticed was that, these fringes which are drawn here; when m is 490 that is the order of the interference is 490, it is really very large order. So, when m is 490, these fringes disappeared and again they appeared when m became 980, this was something that was observed. So, typically this distance between the two peaks increases with the order of interference m.

So, remember that m is what we defined as this integer here. So, $m = 0$ would mean 0 th order fringe, $m = 1$ is the first order fringe and so on. This is simply because this distance between the peaks that is indicated here changes with the order and at some order the peaks coincide when the interference pattern is clearly visible that is such a thing happens at 980; whereas, at 480 it disappears simply because peak of one coincides with the minimum of the other.

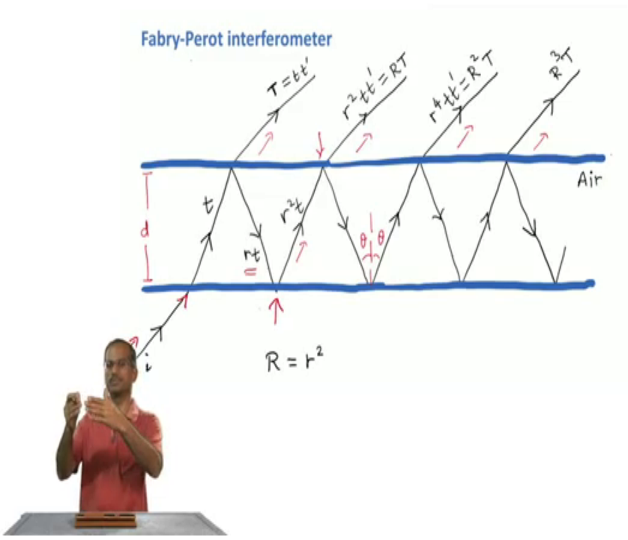
(Refer Slide Time: 19:24)



So based on this a quantity called visibility was defined by Michelson himself. So, it is $\frac{I_{max} - I_{min}}{I_{max} + I_{min}}$. You can convince yourself that, if you are using monochromatic light; then I_{min} is 0, I_{max} is some number you can scale it and take it to be 1. So, the visibility will be 1, the condition for 0 visibility is when as I said the peak of one falls on the 0 of the other; which means that, this condition is being satisfied.

If I know one of the wavelengths, let us say λ is known and if I know that experimentally at $m = 490$ the visibility basically becomes small; in that case from this equation you can find out what is $\Delta\lambda$. And for the case of sodium lines, which come from sodium vapour lamp; the $\Delta\lambda$ is of the order of 0.0006 micro meters. So, this tells you the two different lines that make up the sodium $D2$ lines, $D1$ and $D2$ lines ok. So, it is called the sodium doublet.

(Refer Slide Time: 20:54)



Now, let us look at the Fabry Perot interferometer. So, I have this incident beam which is coming from the lower left end. So, the core part of the Fabry Perot interferometer is this etalon, which allows transmission. So, the incoming beam gets transmitted into the etalon and we assume that the material inside is Air. So, the fraction that is got transmitted is t and at some point.

So, it goes and hits the top surface and at that point a part of it gets transmitted outside and the part gets reflected back inside. And the part that gets reflected back inside, again gets reflected from the lower end of the etalon and again it gets reflected from the top end of the etalon and so on. So, adjusted that, transmission is possible only from the top end and not from the lower end.

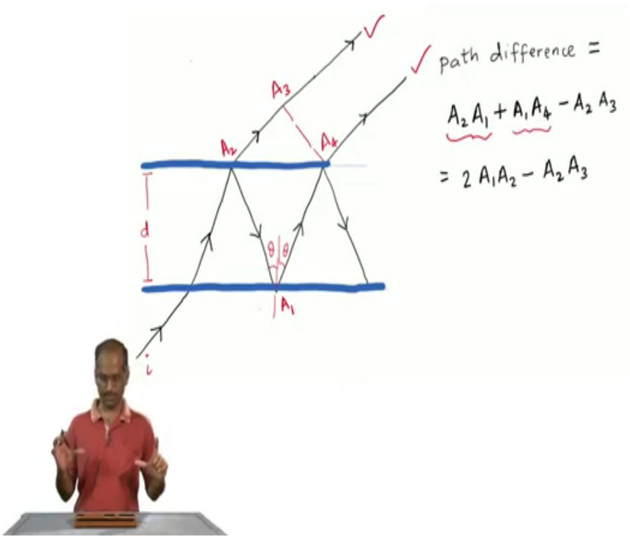
Then what you will see at the output end or from the top end is, multiple beams that are coming out; each one is transmitted partially, partially transmitted beam. Of the initial beam, a fraction t is transmitted into the etalon and of that t' is transmitted out. So, I have tt' that goes out and of the t which has entered a part r is reflected back. So, I have this

rt fraction which is reflected back. And now it is again reflected back from the lower end at this point.

And hence the component here would have $r \times r \times t$ two, it has suffered two reflections and one transmission. So, that is r^2t and when it reaches this point here there is going to be one more transmission. So, it gets transmitted by a factor t'. So, that is going to be r^2tt'. There are these beams that come out from the top end and these are going to interfere. The crucial point to note here is that, each of these outgoing beams here, they maintain a constant phase difference with respect to the other, with respect to the neighboring ones.

Now if I experimentally want to see this interference pattern, all I need to do is to put in some converging lens, a convex lens maybe and then put in a detector, then you can converge all these beams to a point and see the net result.

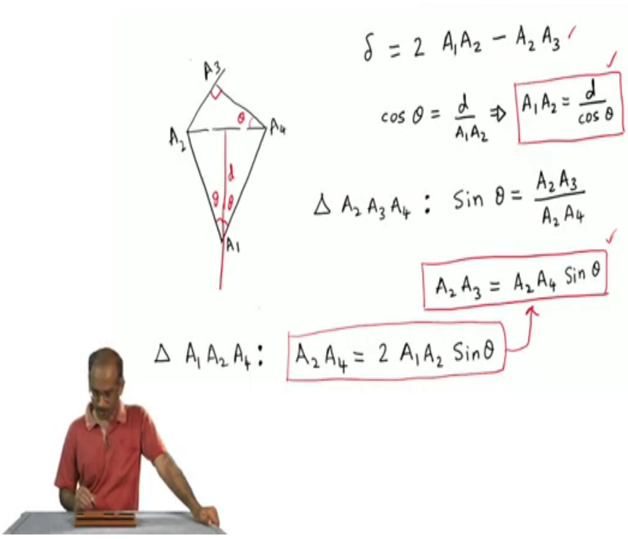
(Refer Slide Time: 23:54)



So, I have just taken the first two of them here. So, this is the incoming beam and of course, it goes through, gets reflected at point A_2 and there is a part that gets transmitted and goes through point A_3 and so on. And the reflected part again gets reflected at A_1 and then there is a part that gets transmitted at A_4 and so on; two neighboring beams which are coming out of the etalon.

Now what is the path difference between these two neighboring beams? So, you can easily see that the path difference will be this part $A_1A_2 + A_1A_4$ and you need to subtract this A_2A_3 and it is also easy to realize that these two quantities are exactly equal A_1A_2 and A_2A_4.

(Refer Slide Time: 25:02)

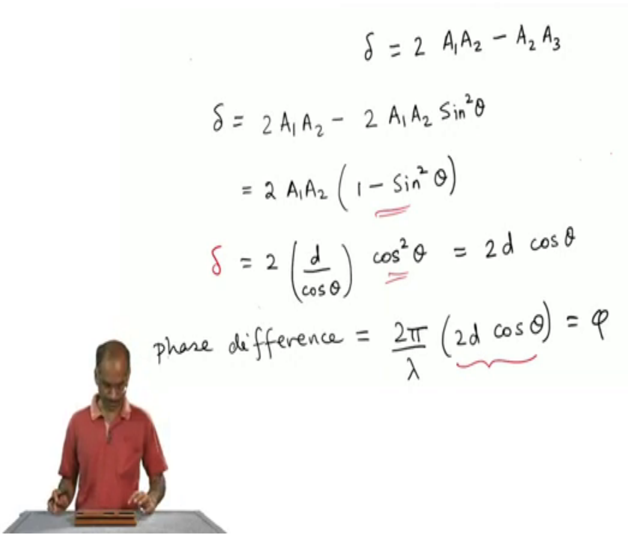


So let me indicate my path difference as δ again $2A_1A_2 - A_2A_3$. So, I have drawn this figure once again here, this triangle here A_1A_2 and the line read line d together form a right triangle. And from there we can define what is $\cos\theta$ and that is simply d; remember d is the distance between the top and lower portion of the etalon. Hence $\cos\theta$ is simply $\frac{d}{A_1A_2}$.

And so, from here I can write out an expression for A_1A_2 that will be $\frac{d}{\cos\theta}$. Now let us look at this triangle $A_2A_3A_4$, the triangle at the top. From here I can write $\sin\theta$. So, here notice that the angle at point A_4 is actually theta same as the angle at A_1. So, A_1 has two θ in which case this will be θ. So, $\sin\theta$ will then be equal to A_2A_3, that is opposite divided by opposite side by hypotenuse and hypotenuse is A_2A_4.

And A_2A_4 as you see is this horizontal line. And to write it in terms of known parameters I can look at this triangle once again $A_1A_2A_4$. And from here you will see that A_2A_4 is simply equal to $2 \times A_1A_2 \sin\theta$; once you substitute for A_2A_3 that is $2A_1A_2 \sin^2\theta$.

(Refer Slide Time: 26:48)

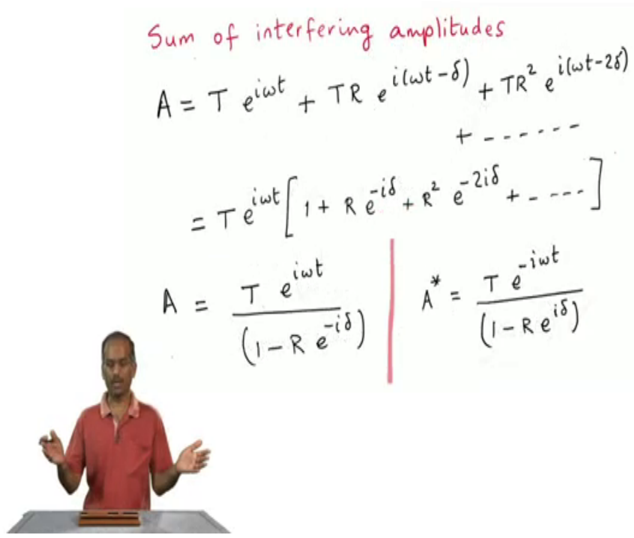


Finally the expression for δ is equal to $2d \times \cos\theta$; this is the path difference or the optical path difference. If I had taken the material inside the etalon to be of refractive index μ, this would have been $2\mu d \cos\theta$. With this the phase difference is $\frac{2\pi}{\lambda} \times 2d\cos\theta$; but there is a catch here. So, this is a phase difference only between two of the beams, I mean what we have obtained is just the phase difference between two beams.

So, there are other beams, if you assume there are n number of beams, you will have n phase differences with respect to the first one. So, we need to take all that into account. But fortunately you do not have to work out the phase difference between the first beam and every other beam. The successive beams, you take any two beams; they will always have the same phase difference which we calculated.

So, which means that all we need to do, to decide if a particular point is going to have a bright or a dark fringe is to accumulate these phases. And look at what is the net effect of all the beams interfering with the phases going like with respect to first one, you have a phase ϕ, the next one would be twice that, thrice that and so on.

(Refer Slide Time: 28:16)

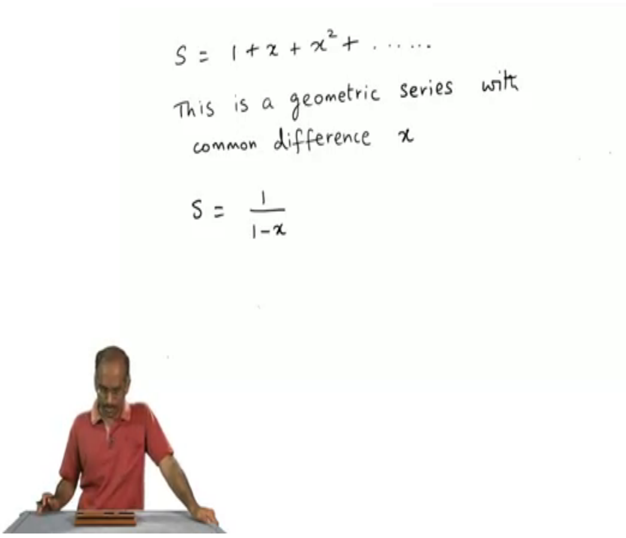


So, all I am going to do is to sum the interfering amplitudes. So, the amplitudes are $e^{i\omega t}$; that is like taking oscillatory solution and T as we had taken. So, T is tt', so this quantity. So, that is the amplitude. And the second one as you can see is $T \times R$ and the third one is $R^2 \times T$; next one is R^3T and so on; where R is r^2.

So, remember the T is the fraction of beam that is transmitted and R is the fraction that is reflected. So, what we need to do is to sum all these amplitudes, sum all these waves; keeping in mind that, each one of these waves which you take the first one with assume that has zero phase and the next one will have a phase ϕ, the one after that will have phase 2ϕ and so on. And phase ϕ is what we just now calculated.

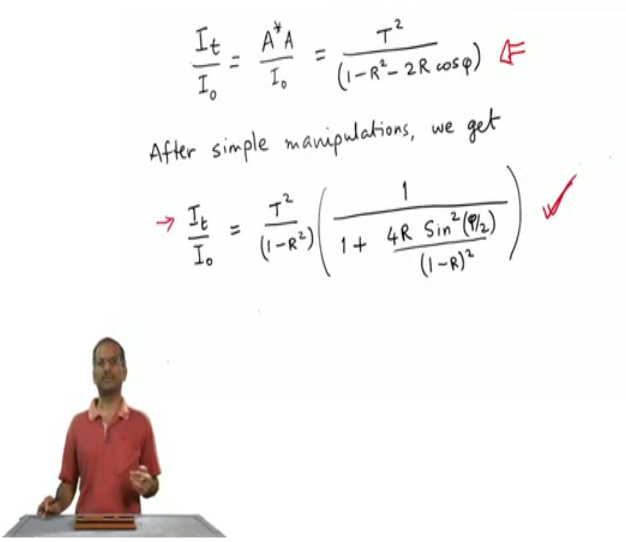
So, we can say that this is equal to phase difference ϕ. So, in that case here is the framework. So, I want to add all the amplitudes. So, it will be $Te^{i\omega t}$ that is the first term; second one will maintain a ϕ phase difference with respect to the first. So, you see a phase difference of ϕ. Third one will maintain a phase difference of 2ϕ with respect to the first one. So, you see this 2ϕ here and so on, you will immediately realize that this is simply a geometric series.

(Refer Slide Time: 30:18)



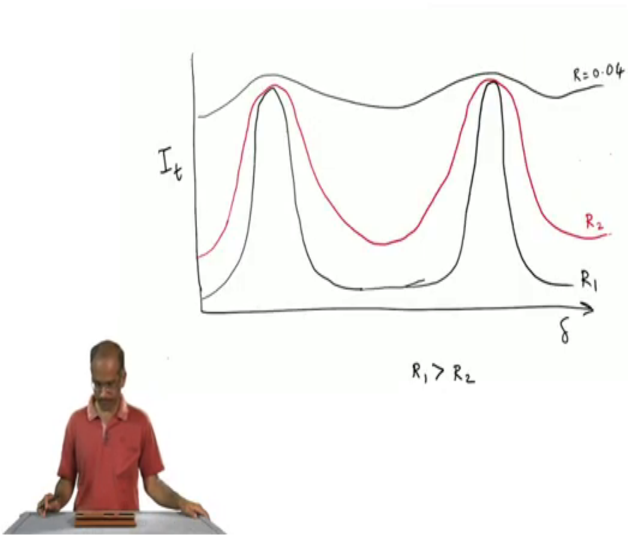
And if I have a geometric series of this type; $1 + x + x^2 + \cdots$ and so on; so I want to sum to infinity. In that case and of course, assuming x is much less than 1; otherwise it may not even converge. In that case, the sum, the infinite sum of this geometric series is simply $\frac{1}{1-x}$; you will get this sum for the sum of the interfering amplitudes.

(Refer Slide Time: 30:46)



The transmitted intensity will then be equal to $|A|^2$ or it is equal to A^*A. So, calculate that, that is going to give you this relation. I_0 you can take to be the intensity of the incoming beam, it is a constant and with simple manipulation you will get this result. You will notice that there is the $\sin^2\frac{\phi}{2}$, crucially that is where all the phase information is located. You can look at how I_t that is a transmitted intensity, behaves as a function of ϕ.

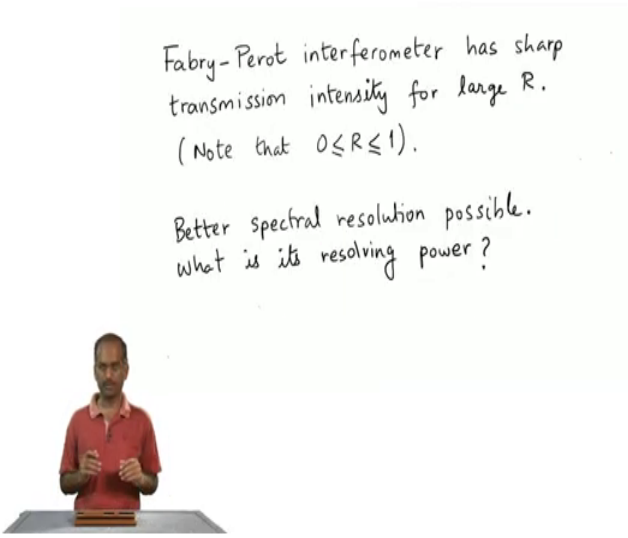
(Refer Slide Time: 31:22)



And this is how it is. It is not easy to realize from this, but it is very useful if you plot this function. So, if you plot this function what you will notice is that, for very small values of R, most of it is transmitted; so which means that the reflection is very less. But what is very crucial for our purpose is that, if you increase R to very large number something like R becomes like R_1 for instance and R_1 could be something like 0.9 and so on.

Then the curve becomes very sharp. So, as R increases, you get this very sharp peaks and remember that R is a number between 0 and 1. So, as R tends closer and closer to 1 this peaks become very sharp.

(Refer Slide Time: 32:16)



So, the transmission intensity for larger shows very sharp peaks. Now it is useful because, if I can resolve transmission intensity peaks here; I can find out the values of ϕ. And if I know the value of ϕ, I can actually plug it in this equation for phase difference and use it to determine the wavelength of an unknown light source, to a very good accuracy.

When we say that I can resolve these peaks very well; that means that, I assume that there is some measure of spectral resolution that is possible. So, if I have to measure the value of wavelength of an incoming light, I should be able to measure these, differentiate these peaks quite precisely. And how well I can do that is given by what is called the resolving power of the Fabry Perot interferometer. You can use both these Michelson and Fabry Perot interferometers as a means for determining wavelength of an unknown source of light.