Waves and Oscillations
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Lecture - 50
Interference: Newton's Rings

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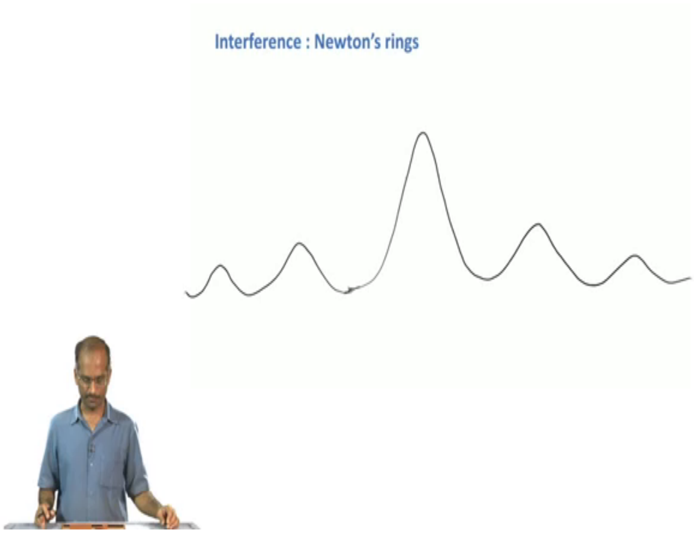

Welcome back, we are looking at Interference yet and this is the second lecture of 11th week. In fact, if you look around for interference patterns you will see that, there are quite many very visually striking patterns that you get and alternating patterns of darkness and bright fringes, ok. That is characteristic of interference in general; like for example, some intensity pattern that I have drawn here, this could be indicative of that.

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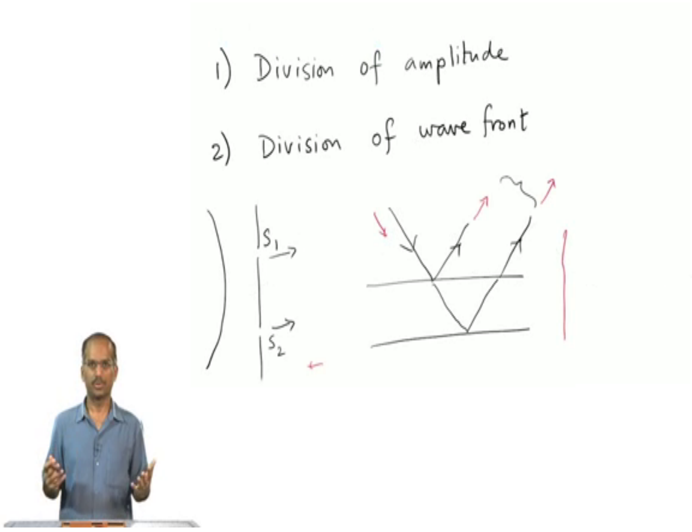

Just to recall once again we said that, for interference to happen you need two beams of light. Typically you should split a single beam of light into two. So, depending on how you do that splitting, you could either have a division of amplitude or a division of wave front. In the division of amplitude, the way it works is like in the figure that is shown here on the right side; what you have is one beam that is let us say coming and hitting the a glass slab or some such thing and there are reflections taking place.

So, typically from two different points of that material, and because the reflection takes place from two different points in that material, there is a path difference that is introduced between these two beams. So, like in the picture that is shown here on the right side, you will see that one beam which is coming from the left side is reflected from two successive planes; and what comes out is essentially two beams, except that one has taken a longer path and other one has taken a shorter path. So, there is a path difference and corresponding to the path difference there is always a phase difference. So, what we have is two beams of same frequency. So, we have assumed that they have same frequencies.

So, these two can interfere and produce an interference pattern. So, this is division of amplitude. On the other hand division of wave front is something that we are rare to see, but it should work like this. So, you have one wave front like in the figure that shown on the left side here, you have one wave front that comes and hits let us say some screen and there are two slits. So, now, we are able to produce from one wave front, two waves; one that would come out of slit S_1 and the other one that would come out of slit S_2, and these in principle can again interfere and produce interference fringes.

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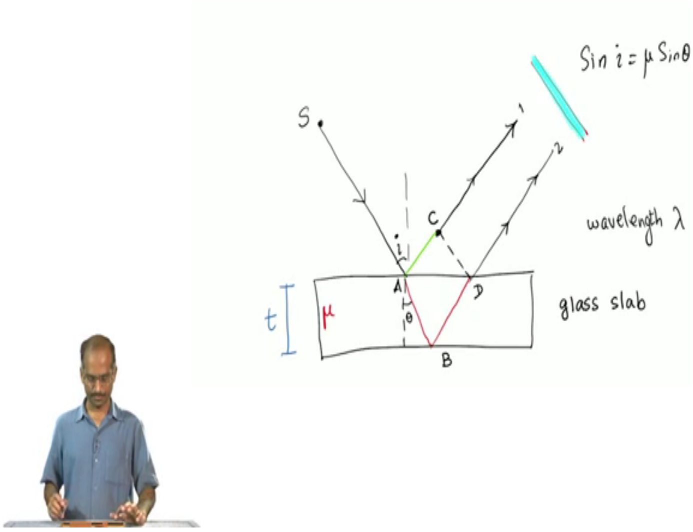

In particular what we saw in the last class was this setup. So, you had this glass slab, the material of glass slab has a refractive index μ and you had this point source of light, which is generating light with wavelength λ. As I just described, the; this light when it hits the top surface of this glass slab, it gets reflected. So, part of the beam gets reflected and there is other part which gets transmitted inside the glass slab and that in turn gets reflected from the bottom surface of the glass slab. So, that is what is shown here in red color. So, the one that gets reflected at the top is shown in green color ok.

So, until the point C and D they take different paths, which is giving rise to that path difference; and what we did in the last class was to actually calculate this path difference.

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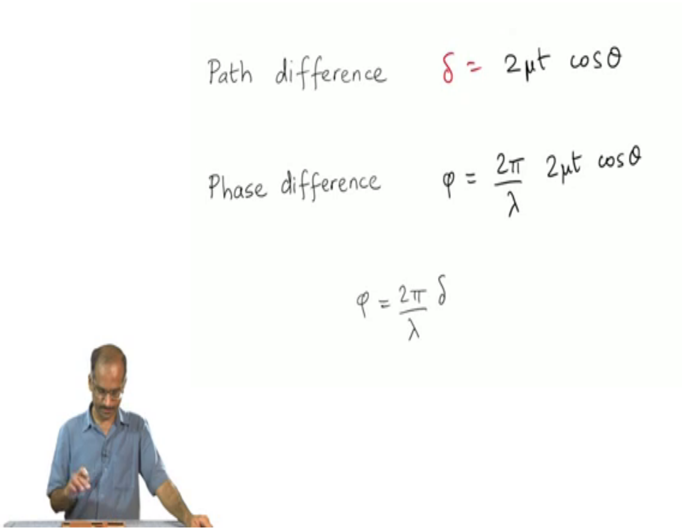

So, we showed that in general the path difference is $2\mu t \cos\theta$, t is the thickness of the glass slab and θ is the angle of refraction. And corresponding to this path difference, the phase difference is $\frac{2\pi}{\lambda}$ multiplied by path difference. In general it is true that, if you had a path difference δ, the corresponding phase difference would be $\frac{2\pi}{\lambda} \times \delta$; this is the relation between the phase difference and the path difference.

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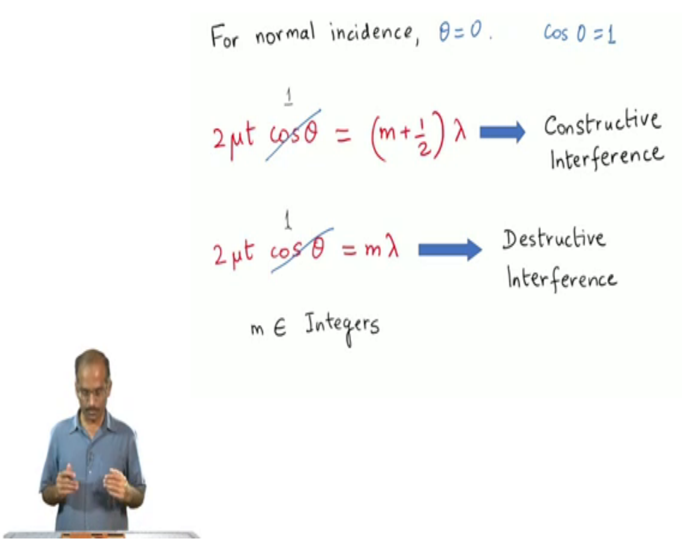

And we also saw that, if this condition is satisfied, that $2\mu t \cos\theta$ which is the path difference. If the path difference is equal to $m + 1/2$ times the wavelength, then you get constructive interference. So, you should see bright fringes and if $2\mu t \cos\theta$ is equal to integral times λ; that is an integer multiplied to the wavelength, then you will see destructive interference. So, which means that there, the two beams are precisely in anti phase and they cancel with one another; whereas, in the case of constructive interference, both the waves precisely are phase matched.

So, the first requirement is of course, you need coherent set of beams to begin with; and you generate that by possibly splitting a single beam into two, by some reflection process like the one that we did in this case. And then calculate the path difference, calculate the phase difference and take care of any other factors involved; like for instance in this case we needed to take care of an additional factor of phase ϕ, which was arising because there was an reflection that is happening at the top of the surface.

So, take care of all those set it equal to an integral and integer times wavelength or integer plus half times the wavelength. So, they generate for you the condition for constructive and destructive a difference.

Let us look at some variations on this problem itself.

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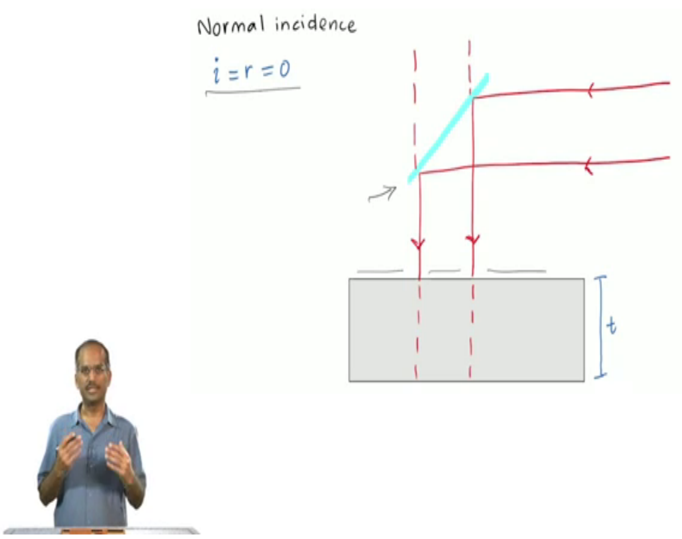

So, first of them is the case of normal incidence. Now what you have is an incoming beam that comes from this right side, as you can see, and I have put a partially reflecting mirror, roughly inclined at 45 degrees. So, what it does is, it allows partial transmission and partial reflection. So, the partial reflection is directed towards the glass slab; whereas, the partial transmission of course, we will go through straight away and nothing we do not worry about that, it just goes through.

Now, the one that travels straight below into the glass slab goes into the glass lab obviously; but there are two components here. So, one is that is part of that which would get reflected at the top surface of the glass slab here and then there is partial transmission. So, part of the light actually travels right into the glass slab and to the lower surface of the glass slab and again it gets reflected back.

And, as you can see in the figure everything is so aligned that, the incident angle, the angle at which the light ray is incident is 90 degrees with respect to the glass slab. On the other hand, the angle of incidence when we typically measure angle of incidence, we do that with respect to a normal at the point of incidence.

So, if we draw a normal at the point of incidence, ask for what is the angle of incidence that is 0. And similarly angle of refraction is also measured with respect to the normal at the point of incidence that is also 0. So, it is correct to state that in this case, i the angle of incidence, r the angle of refraction both are equal to 0. And now what we have is two sets of beams; one which is reflected from the top of the slab, other from the bottom surface of the slab, and there is a path difference which is generated by this thickness t of the slab.

So, now, we go and work out the relevant formulas for path difference and the phase difference. But then this is something that we had already done, as we had done it for the previous case; except that now the θ which was there in the earlier case which corresponds to the angle of refraction, now we should set it to 0. Now, if I put θ equal to 0 for the case of normal incidence; then $\cos 0$ is 1. So, this term here, $\cos\theta$ will just vanish and will give me 1, similarly here as well. So, the condition that is relevant for this problem is that, $2\mu t$ is equal to either integer times the wave length or $m + 1/2$, integer plus half times the wave length.

And again, you will notice that here too, when the beam gets reflected from top surface of the glass slab and additional phase difference of π is generated; and that needs to be taken into account. So, once you take that into account, the correct conditions for interference are exactly what we already flashed.

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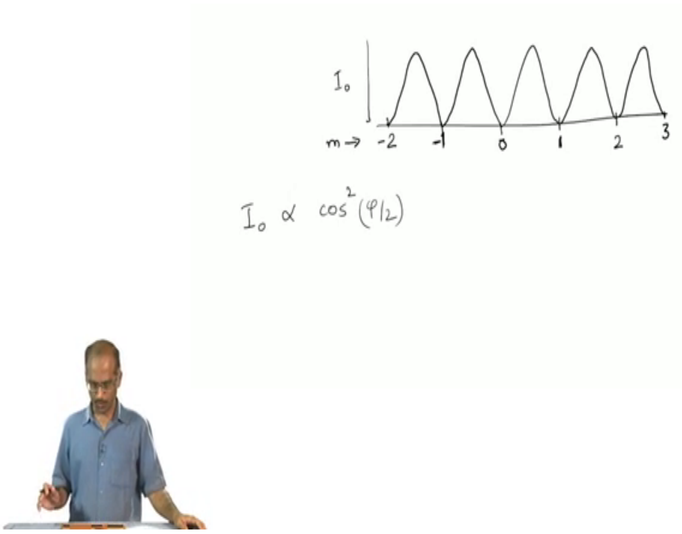

I_0 is proportional to $\cos^2 \frac{\phi}{2}$, where ϕ is the phase difference. So, now, you can substitute the phase difference; calculate the corresponding phase difference and substitute them here. And again you will get pictorial representation which would look like this. In the first version of the problem that we did, we started with source of light which is a point source; but here we are using maybe an extended source. So, one can ask what happens if I use a point source here; again the principles are the same, it just requires to thoughtfully work it out.

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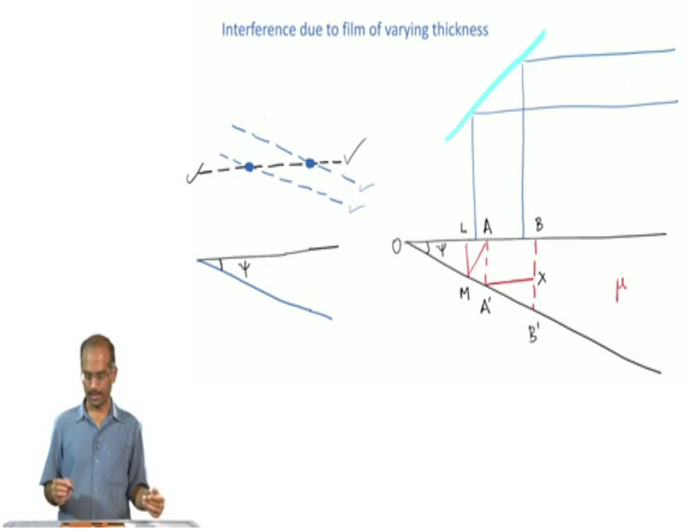

Another version of same problem which is doable and useful for applications is, when the glass slab that you have is not necessarily having a uniform thickness t. So, one thing you could think of is, what if my glass slab had the shape that is shown here. So, it is flat at the top, but the lower part is a certain angle ψ with respect to the top part. So, the whole thing looks like a wedge. So, if your glass slab, who with refractive index μ has this shape of a wedge and you do the same experiment again, what would I expect.

So, what is going to happen is, there is going to be a reflection from the top surface as usual and there is also going to be a reflection from the slanting surface, the lower one; in the left side of this figure you will see that I have drawn a black dashed line. So, imagine that is your reflected wave front coming from the reflection that is taking place on top of; from the top surface of this wedge. And then I have this blue dashed lines that you will see here, that is the wave front that has got reflected from the slanted surface; this maintains an angle ψ with respect to the top surface and there are some points where they meet one another.

So, those points are marked in dark circles here, just two of those points. So, those would be the points which would, which you can expect to have either bright or dark spots. So, here I have the setup shown on the right hand side; as usual I am shining some light which comes as a parallel beam of light from right side, and there is a partial, a mirror that partially reflects it. So, there is partial transmission which goes out and we do not care about that part. And the partial reflection basically turns it by 90 degrees and it is directed towards the wedge.

Now, if I consider let us say one beam, which enters the glass slab at point L in this figure. So, it goes straight in and it hits the lower surface at position M, and from M it gets reflected back and goes to position A and from A of course, it goes out. And the path difference is simply this that it is $LM + MA$.

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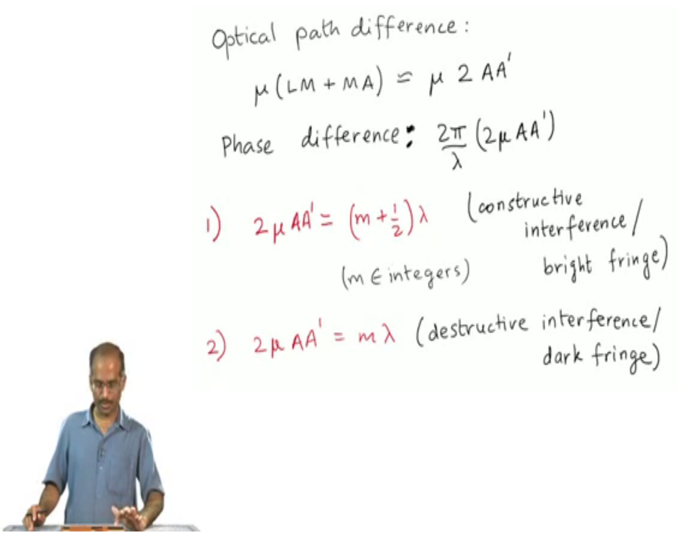

So, which means that, my optical path difference is μ multiplied to $LM + MA$ and you might imagine that well, when a beam goes out in the direction of MA, there is still some more path difference we need to account for. So, the difference here is that, we are assuming a condition that ψ is sufficiently small that this MA is approximately equal to AA'. So, we will assume that both LM and MA are approximately equal to AA'. So, this is equivalent to saying that the wedge that is formed of this material of glass with refractive index μ that angle ψ is really small.

So, typically these kind of things are formed with some thin films, where the angles are very small that this approximation holds good. In that case the optical path difference is just the refractive index times multiplied to $2AA'$. And once I have this straightforward to write down the phase difference, $\frac{2\pi}{\lambda}$ into this optical path difference. And from here we are just one step away from writing the conditions for bright and dark fringes, which I have written down here.

So, the condition for bright fringes $2\mu AA'$ and that is equal this should be equal to $m + 1/2$ times the wavelength. And similarly when this path difference is equal to integral multiple of wavelength, you get destructive interference.

So, this set of condition would look like similar to what we had seen for the previous problem when the thickness of glass was a constant. Again the reason that you get the same thing is because, from the top surface when reflection takes place, there is an additional phase of π that gets added, which needs to be accounted for, and we have taken that into account already here.

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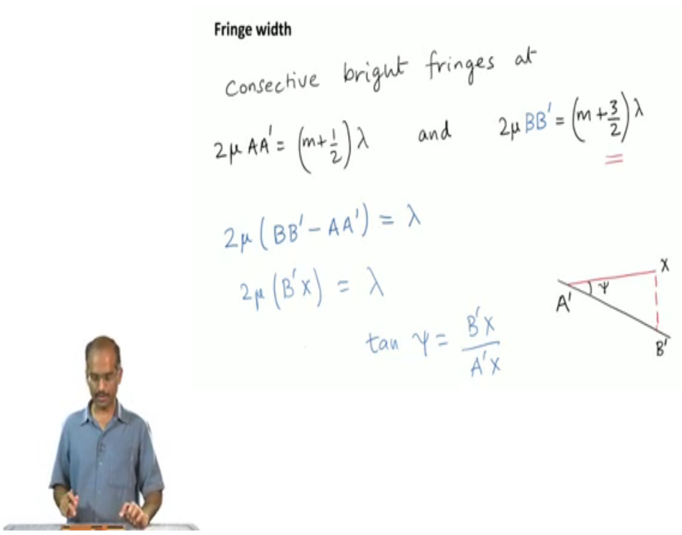

Now you can ask for what is the fringe width? In the sense that what is the distance between two consecutive bright fringes or two consecutive dark fringes; to do that, we just need the conditions for say one of them. So, let us take the condition for bright fringe ok. So, if I take the condition for bright fringe. So, again going back to this figure, let us assume that there is one bright fringe that takes place at AA' or when the distance AA' is such that it matches the condition for bright fringe, you do get a bright fringe. And the next one, when this condition gets satisfied is at the position where this BB' is marked.

So, I want to know what is this distance between these two positions AA' and BB'. So, I have written down the condition for bright fringe; since they are consecutive bright fringes, you will notice that here I have to put $3/2$.

So, one case it is $m + 1/2$; in the other case m actually increments by 1. So, it is $m + 1 + 1/2$, hence $m + 3/2$. Now, I want this distance between these two. So, I just subtract the two and you will get this relation. What is $BB' - AA'$? Let us look at this figure here, $BB' - AA'$ will be equal to $B'X$, ok. So, let me plug in that value here. So, this will be $B'X$ and that is equal to λ.

Now if you look at this figure. So, what I have done is to draw this part of the figure $A'XB'$. I have drawn it here again indicating the angles ψ as well, because the angles ψ which is at position O would be the same here as well, at the point A' as well. So, I have just blown up that part of the angle here. And now $\tan\psi$, tan of the angle ψ would be equal to $B'X$ divided by $A'X$. So, when I substitute for $B'X$ from this relation involving $\tan\psi$, I will get this relation.

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$$2\mu (A'X) \tan\psi = \lambda$$

$$A'X = \frac{\lambda}{2\mu \tan\psi}$$

$$\boxed{A'X \simeq \frac{\lambda}{2\mu\psi}}$$

Now you will notice that what really I am looking for is this distance $A'X$, and remember that we discussed this approximation that ψ is very small, in which case $\tan\psi$ would be ψ itself; hence $A'X$ can be approximately taken to be $\frac{\lambda}{2\mu\psi}$. So, in this case you are going to get fringes of equal width; simply because you will see that this fringe width that we calculated, is simply a constant; λ which is the wavelength of light is a constant, μ that is a constant and ψ is also a constant.

In relation to this experimental setup, either I can see it with my naked eye or put some photographic plate; what is it that I will actually see. So, it depending on the value of thickness at the range, at the region that we are seeing; we will either see a dark or a bright fringe, ok. And depending on the thickness again, we will see that the fringes have equal thickness.

But similarly, if you go back to the previous problem, which is the case of a glass slab with constant thickness t; what is it that we will see, if we actually view the output from the top for instance from this region here. In this case you will notice that the results that we had got, tells you that is either constructive or destructive interference and it all depends on only these parameters; wavelength, refractive index μ and thickness t all three of them are constants.

So, which means that depending on how you have set your instrument, you are going to see either a dark or a bright fringe; in other words it will be equally illuminated throughout, it will be either dark or bright. And, if you want to see something else, suppose let us say that for a particular choice of parameters you are actually seeing a bright equally illumination of bright patch. Now, if you want to see the dark patch you might, you will have to change some or the other parameter.

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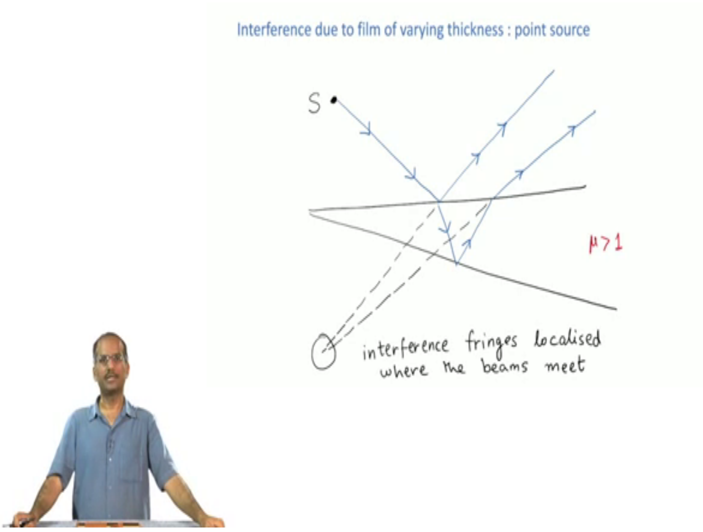

Now, finally, before I leave this subject of glass slab, will also again look at this same problem of wedge shaped glass slab with the light beam as point source, which is at an angle to the top of the glass slab. So, here is your source of light, S and you have a beam which comes and hits the top surface of this glass slab, which has the shape of a wedge and the glass slab as usual has refractive index μ assumed to be, assumed to have a value greater than 1.

So, at this point when it hits that top surface, there are going to be two components; one which is reflected and other which gets transmitted inside and gets reflected from the lower part of the glass slab and that would form the two beams that would come out. So, in this case you can go through the calculation; but you will see that interference fringes are localized near the region where the beams meet. In the sense that, so these are the two beams, if you produce them backwards, it would appear as though they are coming from here.

So, that is where your interference fringes are sort of localized, here it is virtual in this case; but if you had taken the wedge to be a mirror image of what I have here with wedge shape being on the other side, it will actually be at a real point. With this let me close this chapter on glass slabs with one application, which is that of what is called non-reflecting films.

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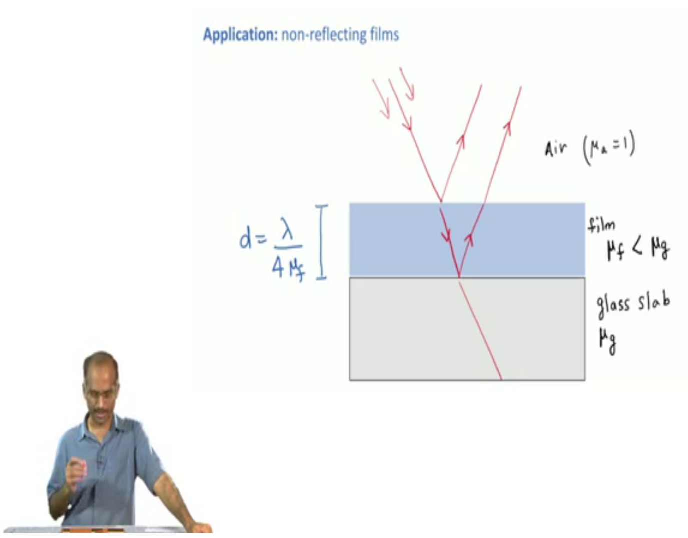

So, I have been using the term glass slabs all the time, because I imagine that we will be doing possibly experiments with glass slabs. So, you can assume at least in small patches that some thin films of possibly oil or of some other material are formed over, let us say water or in general over any other surface.

So, in all such cases, you can apply the ideas that we sort of explored in the last class and today's class until now. Let us say that I have this glass slab, which is shown in gray color the lower end and it has refractive index μ_g. On top of it I have a thin film, so the sizes here are exaggerated; but really it is a thin film and the refractive index of this thin film is assumed to be smaller than the refractive index of the glass material. And then on top of it of course, is air whose refractive index we take to be 1.

Now, in general if I have a beam of light that is coming in this direction, as usual is going to hit the top surface, get reflected from there, part of it would get transmitted and at the second surface that is on top of glass slab, there is going to be one more reflected component and part of that would also get transmitted right into the glass slab. Now, after going through quite a bit of calculations and I am not going to do this here, because I am just posting it as an application.

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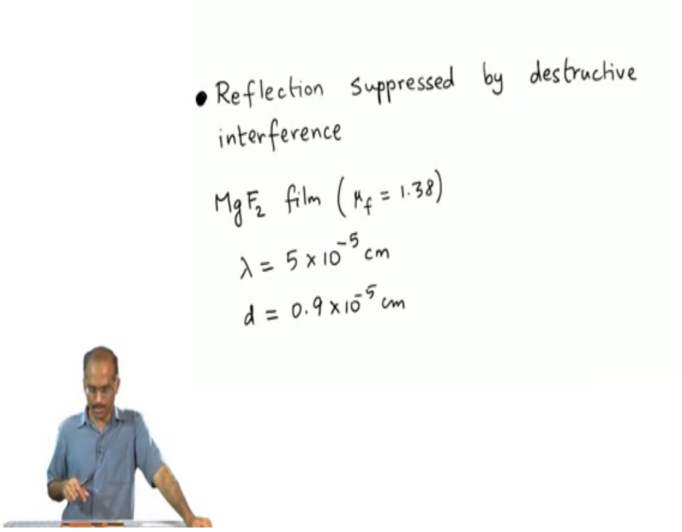

You can work out the conditions under which the reflection can be suppressed. In the sense that, you are sending in some energy in the form of light, you do not want anything to be reflected back, everything needs to go in into this. What is the condition for that to happen; if λ is the wavelength of light which is incoming, if the thickness of this glass slab is λ divided by $4 \times \mu_f$; μ_f being the refractive index of this thin film. In such a case, the reflection is strongly suppressed and it is strongly suppressed because of destructive interference that happens.

So, for instance if it is a MgF_2 film whose refractive index is about 1.38; and let us say you are using visible spectrum of light. So, if you take the wavelength of the middle portion of the visible spectrum that is about 5×10^{-5} centimeters. In that case you can use this formula and d which is the thickness of this film would turn out to be of the order of 0.9×10^{-5} centimeters.

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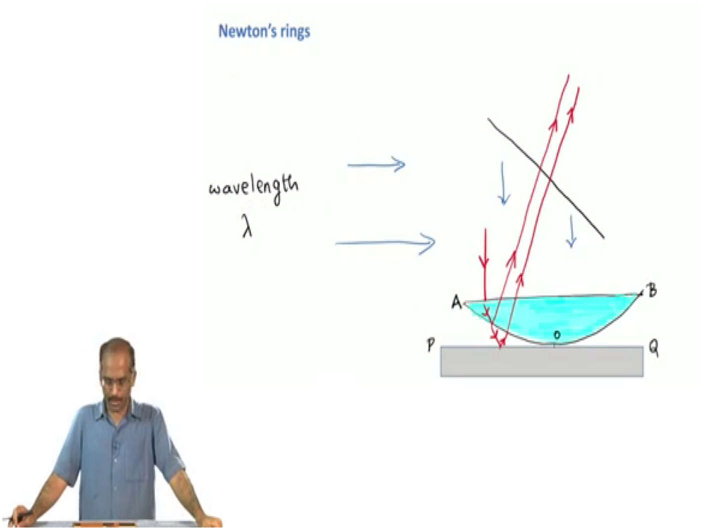

Now let us start with Newton's rings, the idea behind Newton's rings is about all this interference and constructive and destructive interferences. It was actually observed much before Newton; but Newton was the first person to analyze it in some detail. So, here I have this typical set up to observe Newton's rings; from the left side let us say I have a source of light of wavelength lambda, which comes and hits partially silvered mirror. So, it allows a partial transmission, which just goes through and we do not worry about it and part of it is reflected into this Plano convex lens that I have.

So, this AOB is a Plano convex lens and the Plano convex lens itself is placed on a slab PQ; there is this rays which are shown in blue, which will go and hit the top part of the Plano convex lens. So, this corresponds to the case of normal incidence. So, the angle of incidence is 0 and when the ray hits the top of this surface, there is a refracted ray inside the glass, inside the Plano convex lens which hits the curved portion of the Plano convex lens AOB, the curved portion.

And there is one component which gets reflected and goes straight out. So, that is this line which is shown here, and at that point part of it is transmitted into air again. So, part of it is transmitted into the air again and it hits this glass slab PQ and then there is a reflection again from that. So, these two beams which are reflected from the lower portion of this Plano convex lens and top surface of this PQ, they interfere and produce the interference fringes.

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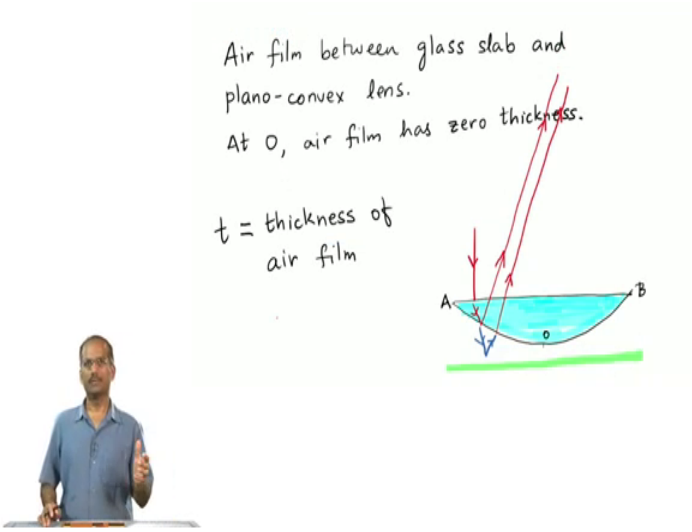

So, what we shall assume is that there is a air film between the glass slab and the Plano convex lens. And at the point O, which is this point here, at the center; the Plano convex lens touches the glass slab and at that point the thickness of air film is 0. And in general at any point we take t to be a variable, which indicates thickness of the air film.

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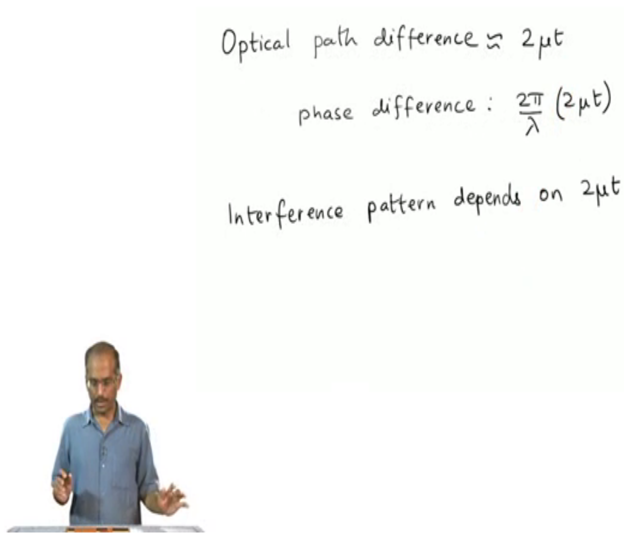

Now as usual the quantity of interest for us would be what is the optical path difference? So, we know which are the two beams which are interfering; to repeat it again, the one that comes or gets reflected from the bottom portion of the Plano convex lens and the top portion of the glass, so these two beams. And I want to find out first, what is the path difference.

And, if I assume that t is the thickness of the air column at that point, then the optical path difference is $2\mu t$. And phase difference as usual is $2\pi/\lambda$ multiplied by $2\mu t$. So, now, the interference pattern will depend on what happens to this quantity $2\mu t$.

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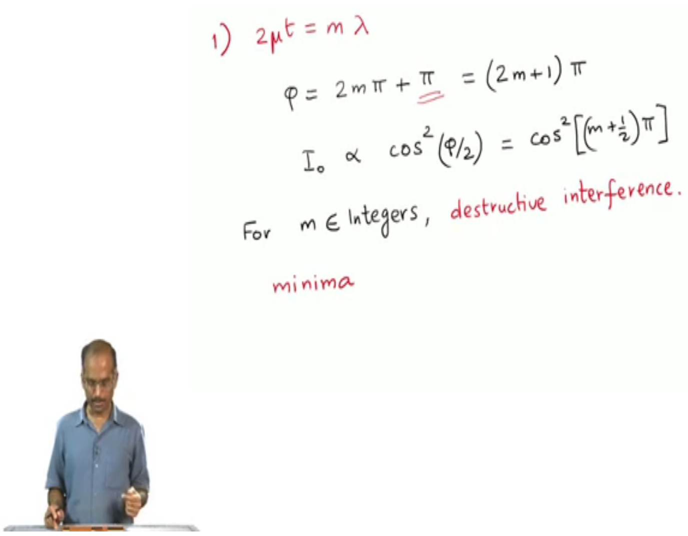

So I will cut the story short, and the first case is when $2\mu t$, the path difference is equal to some integer time's λ. So, in that case like we did before you compute the total phase. So, it will be now substitute this $m\lambda$ here in this equation and also the effective phase will be, the phase generated through path difference plus that additional phase of π which got generated because there was this reflection at the lower surface. So, this π is coming from that reflection from the lower surface the glass slab, you will notice that this is precisely the condition for destructive interference or minima; intensity is going to be minimum at these values.

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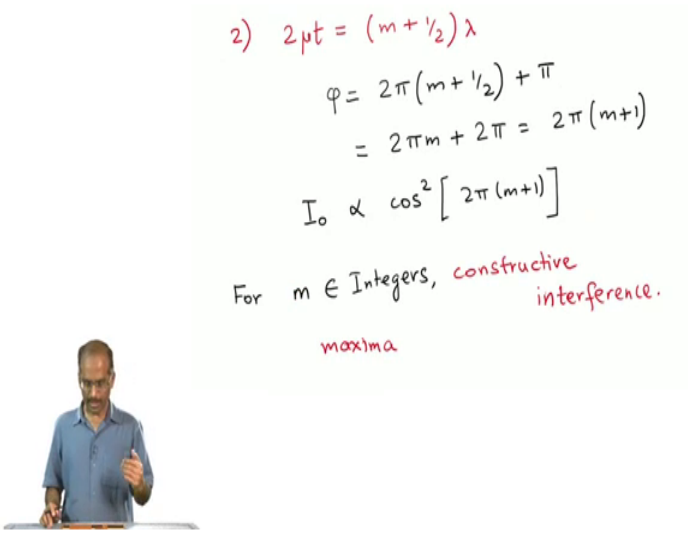

Similarly, one can analyze the other case; you take the path difference to be equal to $m+\frac{1}{2}$ times the wavelength. And as usual the effective phase differences, the usual phase difference that comes from this path difference plus an additional phase difference of π that got generated due to reflection. So, simplify that, your effective phase difference is $2\pi(m + 1)$; this is the case for constructive interference and we expect to see maxima.

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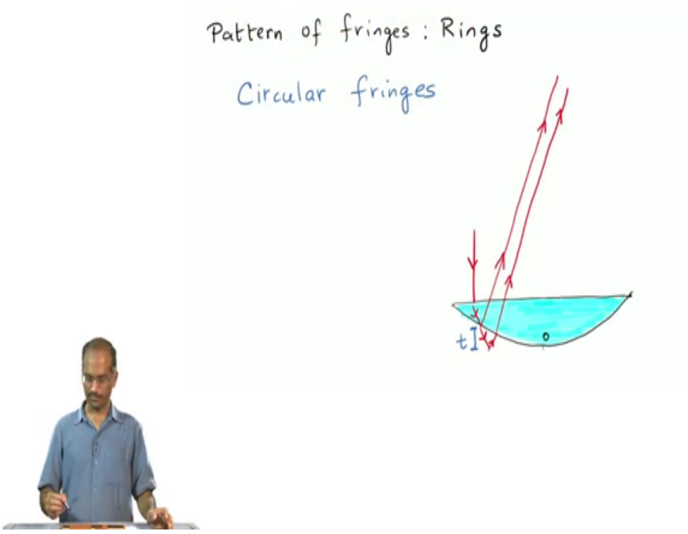

So, if I; let us say put in, some maybe I can put in a lens there and converge these rays then what do I expect to see. In that case, we should expect to see circular fringes; which is why it is called rings. These conditions for bright and dark fringes depends on t, μ is a constant; once you chosen your Plano convex lens of given refractive index, then that is fixed.

So, the only thing that is a variable is t, which is thickness of that the air film that is trapped between the lower glass slab and the Plano convex lens. So, t is 0 at the center, and the locus of t is a circle; in other words the points at which the value of t is the same, if you mark that out that will form a circle, all the points having the same value of t would be either bright or dark depending on which condition they satisfy. So, you should see fringes that have a circular pattern; which is why as I said it is called Newton's rings.

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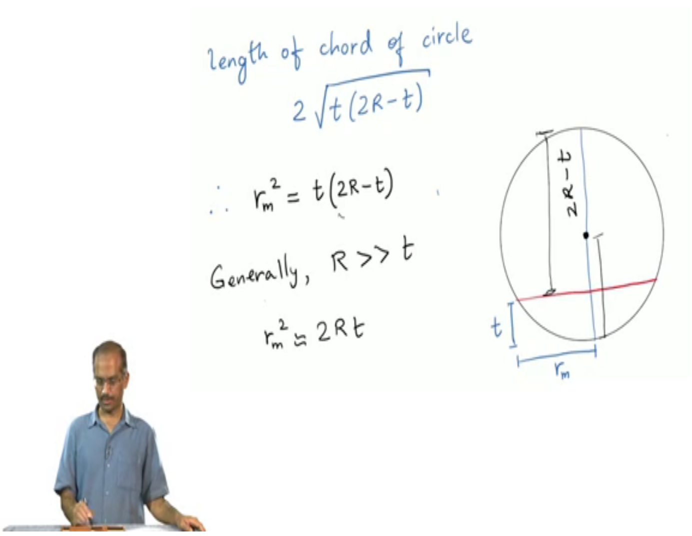

Now, if this is my setup in abstractions. So, the red color is top of your Plano convex lens and if I draw a circle around it, so this distance would be the radius of curvature of the lens; and this height here t is the thickness of the air film and of course, it is a variable depending on where I am. So, if I am looking at m th fringe, r_m would denote the distance of that fringe from 0 from the center.

Now, I would like to calculate what is the distance of the m th fringe from the center? And that is very easy to calculate, if you realize that since this is this distance is t here, this thickness $2R - t$ would indicate this distance. And this red line that I have drawn is of course, the top portion of your Plano convex lens; but that is like a chord of a circle in this case. And length of a chord of a circle in terms of parameters shown here is given by this formula is $2\sqrt{t(2R-t)}$.

And therefore, this distance r_m which is half the chord of the circle, would simply be equal to $t(2R-t)$; capital R is the radius of curvature. And general in these things we often use thin lens. So, this thickness of air column would be very small and radius of curvature will be much larger than this t, in which case I can ignore this t in this factor $2R-t$; which means that for dark fringes; we already saw this relation $2\mu t = m\lambda$ is what would happen for dark fringes.

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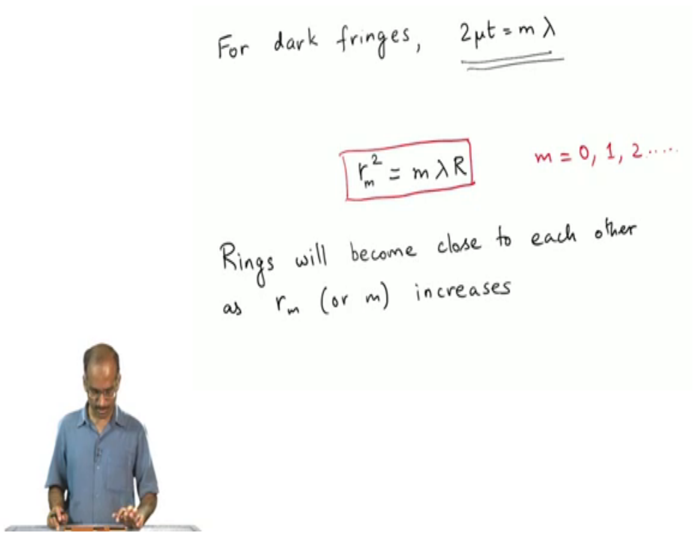

Now, what I am going to do is to simply use this relation that I got here. So, substitute for $2t$ from this relation that would be r^2 by R; and if I do that, it tells me that r_m^2, the distance from the center for the m th fringe is equal to $m \times \lambda$ multiplied by R and m is of course, integer.

So, you will see that as m increases, the distance from the center also increases and what we can infer also from this is that, the rings will become close to each other as r_m increases. In other words the fringe width gets smaller and smaller.

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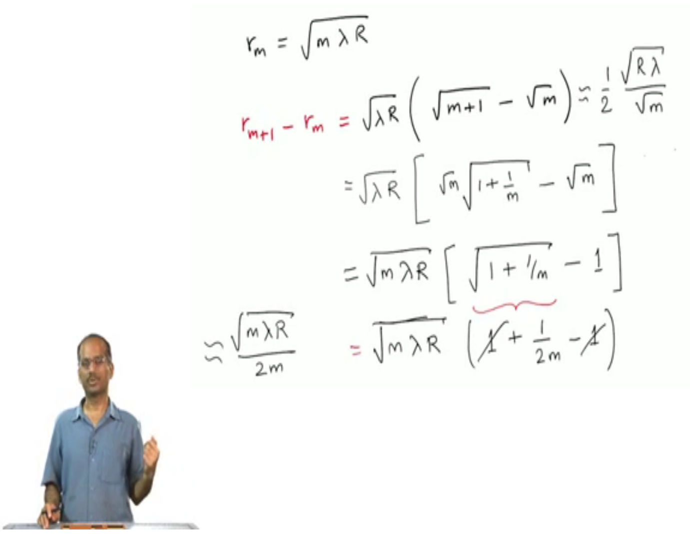

Now I want to find out what is $r_{m+1} - r_m$; which is simply the fringe width and that is given by this relation. For this part here, if m is very large, m is a large order in that case $\frac{1}{m}$ will be very small, I can apply the usual binomial approximation. So, this would be approximately equal to and the final result would approximately turn out to be. So, the fringe width is going to decrease as $\sqrt{m}$.

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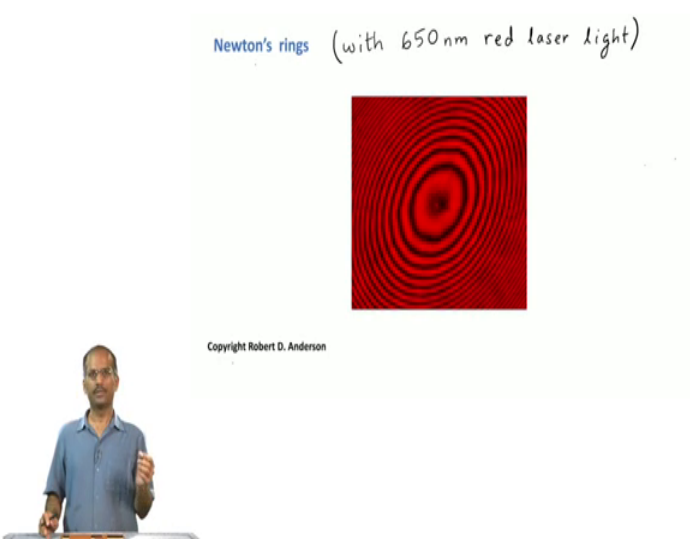

So, here is a result of actual Newton's rings, it was generated with 650 nanometer red laser light and you will see the first thing is that at the center there is a dark spot. So, clearly the center, the condition satisfy destructive interference. And as you can see, so large higher order fringes have been generated in this case; successively the fringes towards the edges of this picture are having fringe with which are smaller. So, as m increases or the order increases, the fringe width decays as a $\frac{1}{\sqrt{m}}$.