Waves and Oscillations

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Module – 01
Simple Harmonic Motion: Problems

Welcome to the 5th module in this 1st week. So, if we have to summarize everything that we did in the last four modules we initially started by looking at a simple experiment of the pendulum and we sort of deduced that at least for the case of small displacements about the equilibrium position the restoring force is proportional to displacement and there was the negative sign and from there we went on to get an equation of motion and once we know the equation of motion we were able to deduce the time period of oscillation.

And then we also saw that we can solve the equation for displacement as a function of time and once we know the displacement we can also find the velocity and acceleration, and we also saw that there were systemic relations between displacement and velocity and acceleration. And the last part of it was we computed the potential energy and kinetic energy of the oscillating system while each of these components the potential and the kinetic energy were individually time dependent, but the total energy was independent of time.

So, again that is not surprising given the fact that when we initially model the system we did not consider dissipation as a possible avenue for energy to be dissipated. So, we have a system which is ideal in the sense that it does not dissipate energy. So, we will look at adding realistic effects like dissipation in the subsequent modules. So, to conclude this week we will go through a set of illustrative problems.

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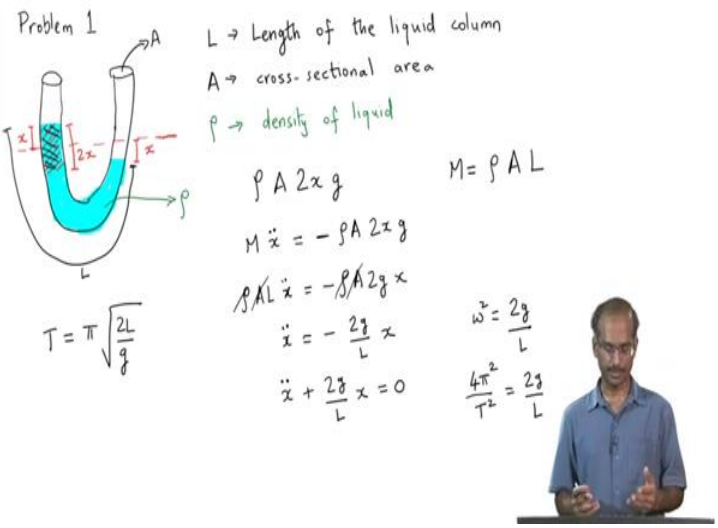

So, let me begin with the first problem which is a tube, which is in the shape of U as you can see in the figure here and originally in this tube the water let us say was there at this level the height of the water is same in both the arms of this U tube. But when let us say I depress the water in one of its arms like this by an amount equal tox, the water in the other arm rises by an amount as well.

So, that the net displacement here would be $2x$ the difference between the level of liquid in both the arms after I depress it. And once I leave it, it's going to oscillate up and down and of course, because of dissipation finally, it will come back and settle at the equilibrium position which is this. So, now, the question is what is the period of time period of oscillation for this case? To do this we will need to we will need few other parameters let us assume that L is length of the liquid column, that is this entire length starting from here to here. So, this is length if you sort of make it horizontal and let us also assume that area the cross-sectional area of the tube is A.

Let us assume that this liquid has density ρ. So, this is density of liquid in U tube. So, I would like to know what is the time period of oscillation in the system if I set it to oscillate by depressing the liquid in one of the arms. So, the question that you will have to answer is where is the restoring force coming from. So, clearly when the liquid is at the same height in both the arms of this U tube there is no net restoring force, but when I depress it in one of the arms the net restoring force comes from the liquid which is excess in one of the arms basically this. So, the weight of that liquid essentially provides the restoring force.

So, what I would first like to know is what is the weight of this column of liquid. So, that would be ρ which is the density. So, weight would be ρ which is the density so, density is mass divided by volume. So, I can get mass if I actually multiply this by volume. So, volume would be cross-sectional area multiplied by the length of the excess length of the column which is $2x$g.

So, this is the weight of this region of liquid, now this will have to be equated to mass of the oscillating part of the liquid. So, I would write $M\ddot{x} = -\rho A 2xg$ So, what is the mass of the entire liquid that we have here let me write it out here. So, that would be; so the mass of the entire liquid that we have here would be again ρ which is the density multiplied by the area of cross section multiplied by the total length of the liquid column.

So, if I substitute here I have $\rho \times A \times L \times \ddot{x}$ = - ρ A $2g \times x$ and of course, I can cancel off several things here. So, ρ and A would be cancelled off I would get $\ddot{x}$ = - $(2g/L) \times x$ and by a small rearrangement this could be written as $\ddot{x} + (2g/L) \times x = 0$.

So, clearly the equation that you have obtained to represent the oscillations of this liquid column looks like are familiar simple harmonic motion equation except that I need to make the identification that ω^2 which is the of the angular frequency is equal to $2g/L$. And if ω^2 is $2g/L$ its straightforward from here onwards to find out the time period that is $4\pi^2/T^2 = 2g/L$.

So, now, you take this equation and rearrange and get the value of time period. So, if you do that time period will be equal to $\pi \times \sqrt{(2L/g)}$

and I leave it as an exercise for you to check if the right hand side of this equation has dimensions of time. So, let us now go the next problem.

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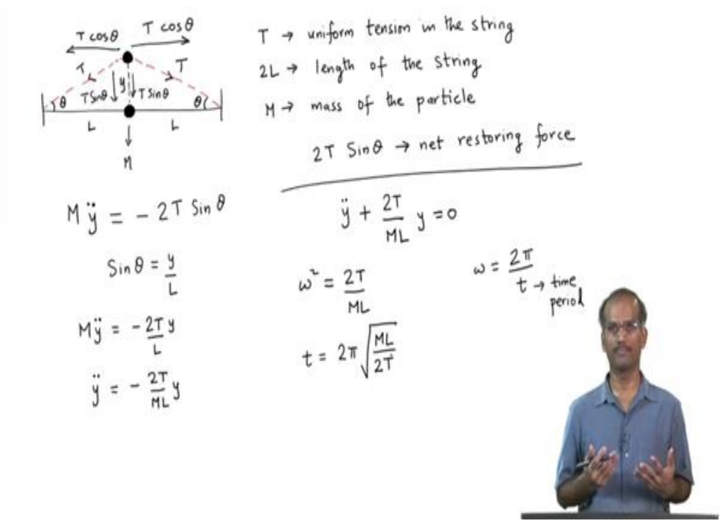

So, this is a problem of a mass in the form of a let us say a ball solid sphere of mass M which is tied between 2 rigid supports by use of a string. And we shall; so, if you pull the mass M up and leave it; it would start oscillating this is something that you can easily do with the string and a mass tied to it. So, you will indeed see oscillations happening. So, the question is what is the time period of oscillations? And to do this we will have to make a few simplifying assumptions. So, we will one of the assumptions we will make is that the oscillation precisely takes place only in the y direction.

So, there's no component of oscillation or dynamics in the direction perpendicular to the vertical so, entire oscillation is in 1 dimension in that sense. And of course, we have already taken to be, we have already taken M to be mass of this block and let us say that I am going to pull it up and leave it and θ is the angle at which I am going to leave it and so this is our equilibrium position the point at which nothing happens you do not do anything to it nothing happens to this block. So, that shall; that will define our equilibrium position and with respect to the equilibrium position this will be the variable y.

So, that would be our displacement and important thing for us is there is tension T in the string. So, T is the uniform tension in the string and we shall also assume that $2L$ is the entire length of the string and M of course, is mass of the particle ok. So, now, when I pull it up let us say by an angle theta. So, there is this tension which is directed in these in this direction and we will we are going to resolve this tension in the perpendicular and horizontal component. So, if this is theta and this is also theta; so there is going to be the horizontal component of tension which will be $T\ cos\ \theta$ in this direction and $T\ cos\ \theta$ in this direction as well.

So, because the magnitude of the tension that is opposing one another in the horizontal direction is equal and opposite, the implication of this is that there is no net motion of the particle in this direction along the horizontal. So, the only degree of freedom or the only direction in which the particle is going to move is up and down. Now, what about the component of tension along the vertical. So, there will be due to this side there will be a $\mathrm{T} \sin \theta$ here and another $\mathrm{T} \sin \theta$ from this side. So, the net downward force due to tension is $2\mathrm{T} \sin \theta$. So, this is the net restoring force.

So, in this system the restoring force is provided by the tension in the string, if there is no tension in the string there is no restoring force and there will be no oscillations. So, what we have done is to calculate the magnitude of this restoring force which is $2T \sin \theta$ So, now, I can write the equation of motion. So, the equation of motion will be $M\ddot{y}$ that is equal to $-2T \sin \theta$ and now I need to replace $\sin \theta$ by y/L which you can figure out from basic definition of sine function. So, given all these now I put it back in this equation. So, I will have $\mathrm{M}\ddot{y} = -2\mathrm{T}/L$

And I can rearrange this equation, so that will give me $\ddot{y} + (2T/ML)y = 0$. So, once again what we have obtained is an equation of motion that looks very similar to the identical to the equation; generic equation for simple harmonic motion. So, from here I can straightaway deduce what is the time period.

So, here the ω^2 the of the angular frequency will be $2\, T/\, ML$ and since we know that $\omega = \ 2\, \pi/\, T$ we can substitute this here in the equation for ω and rearrange the terms. We know that ω is the angular frequency which is defined as $2\, \pi/t$ because I am using T for tension. So, t is the time period and this is the time period that I want to find out. So, if you substitute ω in this equation and do a simple rearrangement we should be able to get this t which is the time period is equal to $2\pi\sqrt{ML/2T}$ And again I will leave it as an exercise for you to verify that the right hand side of this equation does indeed have the dimensions of time.

So, with this; so we have done two problems where we computed the time period of the system. So, if you look at the sort of algorithm that we have been following you need to identify where the restoring force is coming from in the case of liquid in a U tube it comes from the excess length of liquid which is in the case of liquid in a U tube it comes from this excess column of liquid which provides the restoring force, and in the case of a mass tied through a string to rigid supports it comes from the tension in the string.

Once you identify the restoring force you write the equation of motion and once the equation of motion is in the standard form corresponding to the standard simple harmonic motion directly you can identify the angular frequency from where we can write down the time period. So, this recipe can be followed for pretty much most of the simple problems of this type.

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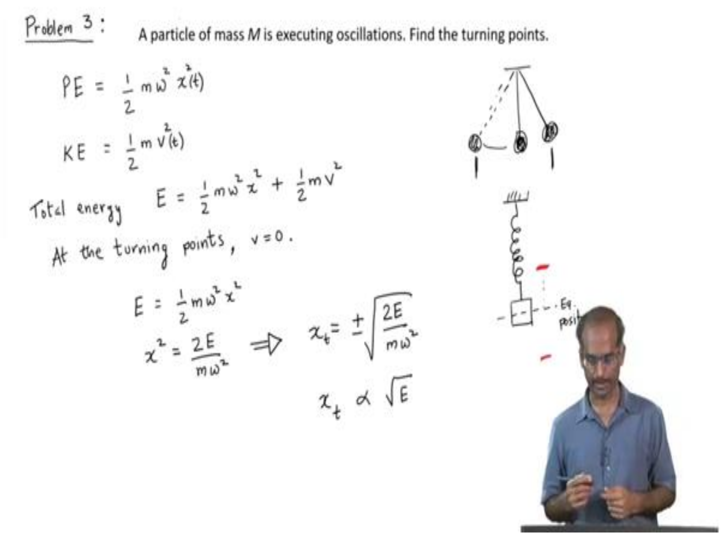

So, in this problem we will deal with the case of particle of mass M that is executing oscillations find the turning points. So, let me first explain this problem through an example, suppose consider the example of a pendulum. So, this is its let us say the normal equilibrium position and when you try and give it a push you are oscillating it and so it starts from the equilibrium position goes to one end and turns back. So, this position here where it turns back is called the turning point. So, it goes in one direction and turns back and again it goes in the other direction and turns back. So, again somewhere here maybe so, this is; these two are the turning points.

So, for an oscillating system like this there are two turning points; one on either side of the equilibrium position. So, equilibrium position is somewhere in between the two turning points. Irrespective of which system which oscillating system you consider there would always be two turning points for instance if you think of this as a mass that is oscillating from a spring which is tied to a rigid support like this.

And let us say that this is its equilibrium position and once you give it some oscillation you are giving it some energy and consistent with the total energy that you have given it would go up and down and let us say that this is the point where it goes up and this is where it goes down. So, these are the two turning points it goes up turns back and comes down and similarly it goes below the equilibrium position again rebounds back.

So, these are the two turning points. So, for every such oscillating system you can even go back and look at the two problems that we just now did both the case of liquid in a U tube and a mass tied to a string and given an oscillation in all these cases there are two turning points. So, I would like to find the turning points.

So, to do that we can do that in general without referring to any one of these problems and that is because we know that for any oscillating system potential energy is given by $(1/2)m\omega^2x^2$ So, just to remind you that x which is displacement as a function of position I have indicated that x is a function of T Similarly, kinetic energy is equal to $(1/2)mv^2$ again v is the velocity and velocity is also a function of time. So, in general both kinetic and potential energies are functions of time.

So, the total energy E would be equal to $(1/2)m\omega^2x^2 + (1/2)mv^2$ Now, from this how do I find out the turning points? So, you should carefully note what happens at the turning points. So, at the turning points the particle let us say if it is a pendulum it goes in one direction stops momentarily and turns back. So, precisely at that point the velocity is 0. So, it is reasonable and correct to say that at the turning points velocity is 0.

So, all I need to do then is simply plug in that velocity is 0 and find the value of position x. So, if I put in that velocity is 0 in the equation for energy that I had written above I will get $E = (1/2)m\omega^2x^2$ and since v is 0 that term does not come here and now I need to find the value of x. So, $x^2 = 2E/m\omega^2$, this implies that $x = \pm\sqrt{2E/m\omega^2}$

and to indicate that these are turning points let me call It x_T here.

So, x_T is the position of turning points as you can see there are two turning points $x = \pm\sqrt{2E/m\omega^2}$

on either side of your equilibrium position if your equilibrium position corresponds to 0 in your scale there are two turning points one on the positive and other on the negative side.

So, you should note the feature that the turning points depend on energy. So, turning points is proportional to $\sqrt{E}$. So, more the energy you give the turning points would be farther away. So, clearly if you give a strong push to the pendulum it is going to go far before it turns back which is what is the lesson that we learn from this equation as well. And also it tallies with the physical intuition that more energy the particle would go further would oscillate with larger amplitude before turning back.

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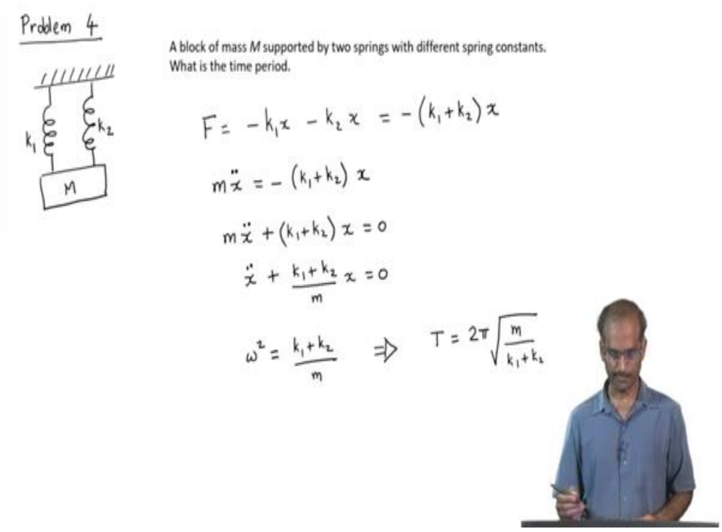

Let us look at a slightly different problem. So, here I have a mass M or block of mass M which is supported by two springs and the springs have different spring constants k_1 and k_2. Now, if I give it a little bit of a push what would be the time period of oscillations that it would execute. So, how do we solve it? The principle here is that the extensions produced by both the springs will be equal.

Imagine suppose if they were not equal if the one spring produces a different extension from the other then the block would not stay horizontal, but would have some tilt to it. So, we avoid that situation in which case the extensions produced by both the springs will be same in which case the total restoring force would simply be the sum of the restoring forces. So, I could write it as $F = -k_1 x - k_2 x$, this will be equal to $-(k_1 + k_2)x$.

So, from here on its very simple we have identified the restoring force, from here I can straight away identify what is ω^2 which is my angular frequency is $(k_1 + k_2)/m$ from which it is straight forward to write an expression for time period the time period will be $2\pi\sqrt{m/(k_1 + k_2)}$. So, this gives us an expression for time period of this system.

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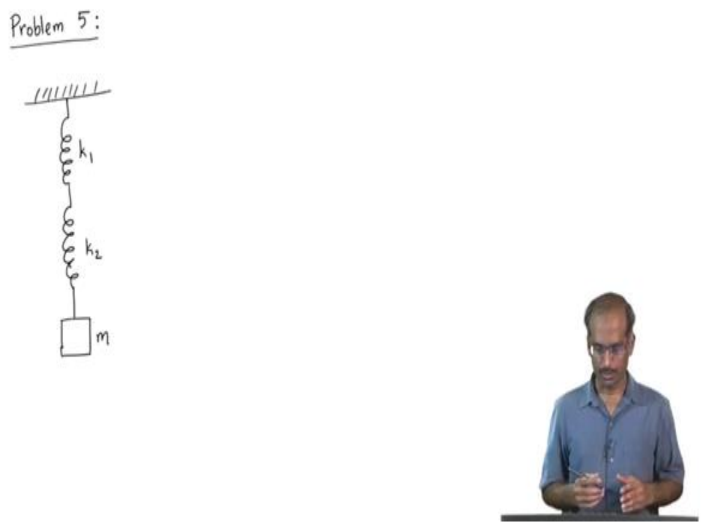

The next problem is a variant of the earlier problem we had seen. So, earlier the two springs were sort of connected in parallel, but now they are connected in series one below the other, and again you have mass m which is being supported by these two pendula from a rigid support. So, I will not do this problem in full, but I encourage you to try it yourself and the key idea here is that the restoring forces on the two springs would be the same. So, if you use this principle you should be able to get the get the time period of this system. So, I am going to leave it for you to do this problem.

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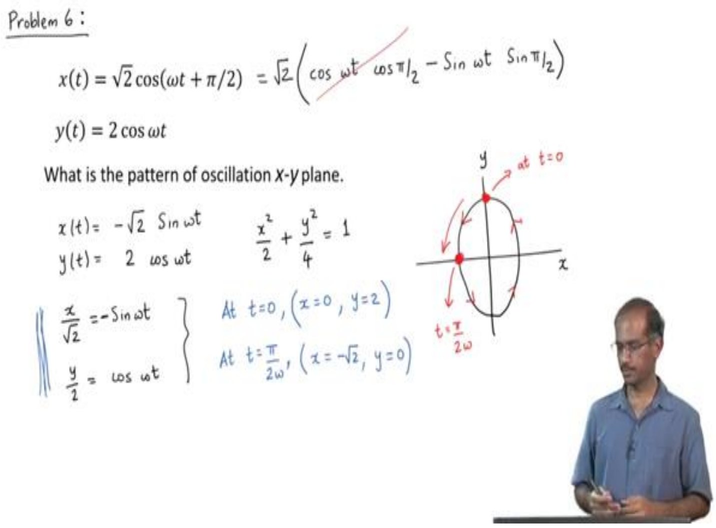

So, in this problem we are looking at two oscillations which are perpendicular to one another and the solutions are given here the question is what is the superposition of these two oscillations? And I would like to know the pattern of oscillation in x, y plane. So, the way to do it is; so as usual we want to eliminate time from these equations and plot the oscillation and as usual let us expand the first equation if I do that I will get $\sqrt{2}\ \cos\omega\,t\cos\pi/2 - \sin\omega\,t\sin\pi/2$ and here $\cos\pi/2 = 0$. So, the first term would be cancelled.

So, what I have is $x(t)$ is simply equal to $-\sqrt{2}\sin\omega\,t$and $\sin\pi/2 = 1$ and of course, $y(t) = 2\cos\omega\,t$ from here it is very easy to do rest of the job $x/\sqrt{2} = \sin\omega\,t$and $y/2 = \cos\omega\,t$and if I square and add I will get (I missed a - sign here) $x^2/2 + y^2/4 = 1$.

So, clearly this is an ellipse as what we have got is a equation of; now, that we have obtained the sketch the dynamics in the $x\ y$ plane we can also ask the question what is the direction of motion is it going in this direction or is it going in this direction. So, that can be determined as well and to do that let us say at some particular time the easiest to pick is $t = 0$ where is the position of the oscillating particle. So, at time $t = 0$ all you need to do is to substitute $t = 0$ in these two equations if you do that it tells you that at $t = 0$, $x = 0$ and the value of $y = 2$ because $\cos 0 = 1$, so y is equal to 2. So, this is the position of particle at $t = 0$. So, I have identified one position at $t = 0$, so I can put that. So, $x = 0$, $y = 2$ will be this point. So, this is the point at $t = 0$.

Now, I need one more point as well let me ask the question what where is the particle at $t = \pi/2\omega$. So, you could take any other time and do it, but I have just chosen $\pi/2\omega$ and if you do that plug in this value of t in these two equations here and that would tell you that the value of x is equal to $-\sqrt{2}$ and if I put in t equal to $\pi/2\omega$ here in the second one it tells me that $y = 0$ and where is this point this will be a point that is located here.

So, this is the position at t equal to $\pi/2\omega$. So, we are going from 0 to some positive value of time which means in that intervening time period the particle has moved from this point here to this point. So, the direction of motion is actually this. So, I need to erase this. So, now, I can put in the direction of motion. So, by choosing various values of time you can not only plot the pattern of oscillation in xy plane, but also obtain the direction of motion.