Waves and Oscillations
Prof. M S Santhanam
Department of Physics
Indian Institute of Science Education and Research, Pune

Lecture – 49
Interference: Part 1

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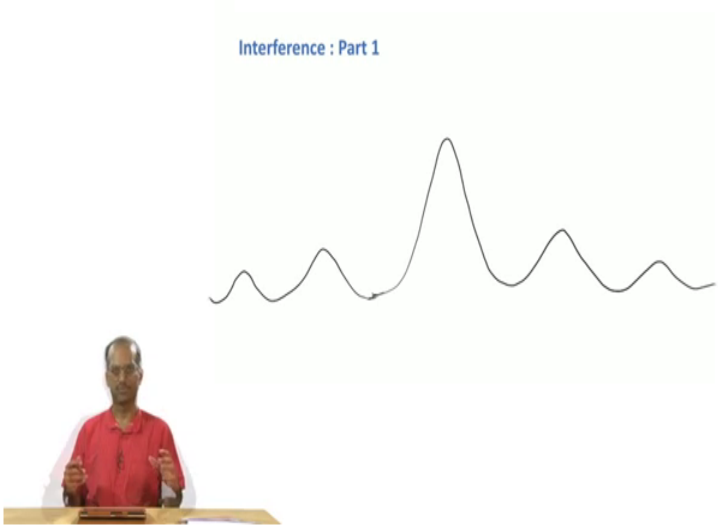

Welcome to the eleventh week of this course. We are still studying waves in optical systems. In the last week, we looked at refraction and reflection the important point to be said about them is that; we looked at optical waves in the limit when the wavelength tends to 0, or this is the limit in which the diffraction effects are sort of suppressed. So, you could treat a wave almost like as though it is a pencil of ray.

So, we were drawing ray diagrams for light rays to be going from some point a to some point b in a homogeneous medium or in a inhomogeneous medium. And we were even able to handle cases, where there were many different optical devices. However, this week we are going to look at interference and diffraction. First we will study interference for the first two lectures also. Here we cannot really ignore the diffraction effects. We cannot say that ok, let us work in the limit where λ tends to 0, simply because these effects arise because λ is not 0.

Typically, when I speak of interference; I really mean that somehow, there are two beams or let say two waves which combined together and produce some interesting visuals; like for instance something that I have drawn here. You could imagine this intensity of light as a function of may be position and there are places where intensity is more and there are places where intensity is less and so on. So, this could be thought of as an interference effect.

So, any time that I see or that I talk of interference the basic requirement is at least I need two ways. I need at least two of them for them to interfere. So, underlying all these phenomena; both interference and diffraction is idea that light is a waveform. So, we had already spoken about that, we made statements saying that light is a form of electromagnetic radiation and so on, but we really did not write out the wave equation. So, for instance you might remember that, when we studied sound waves. We obtain the equation for propagation of sound and that was a wave equation.

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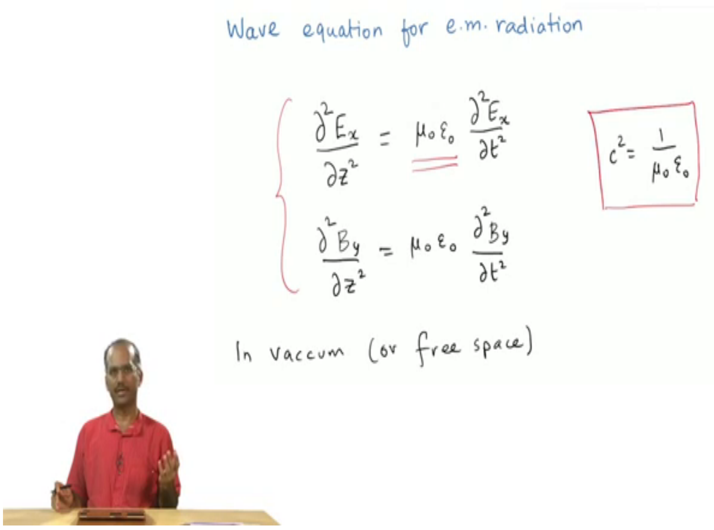

So, likewise light are more generally electromagnetic radiation is indeed a disturbance. It is a electromagnetic disturbance that, propagates in space. When I mean light at least in the sense in which it has to be applied for this course; I mean electromagnetic radiation in the visible region, in the visible region of the electromagnetic spectrum.

We can start from Maxwell's equations and derive wave equation, corresponding to this electromagnetic radiation. However, this is going to involve fairly, sufficiently sophisticated vector calculus. You need to note Δ^2 and how these operators work and. And I am not entirely sure that, everyone listening and following this course might be aware of these mathematical techniques.

So, hence what I will do is directly show you the wave equation corresponding to EM radiation or electromagnetic radiation. So, what I have written here is the wave equation for components of electric and magnetic field. So, E_x represents electric field in x direction and B_y represents magnetic field in the y direction. And their corresponding wave equations are; what is shown here.

You will immediately recognize that this is very similar to the general wave equation that we derived several weeks back. And going by what we already know, this quantity here should be related to the speed of wave propagation or in this case the speed of light itself. μ_0 being the permeability of free space and ϵ_0 is the permittivity of free space.

So, these are the equations that describe electromagnetic disturbance and it is propagation in free space or vacuum. The direction of propagation is actually z. So, if you look at the equations themselves there is a $\frac{\partial^2 E_x}{\partial z^2}$. So, the position is z and the x component of electric field depends on position which is z and time t.

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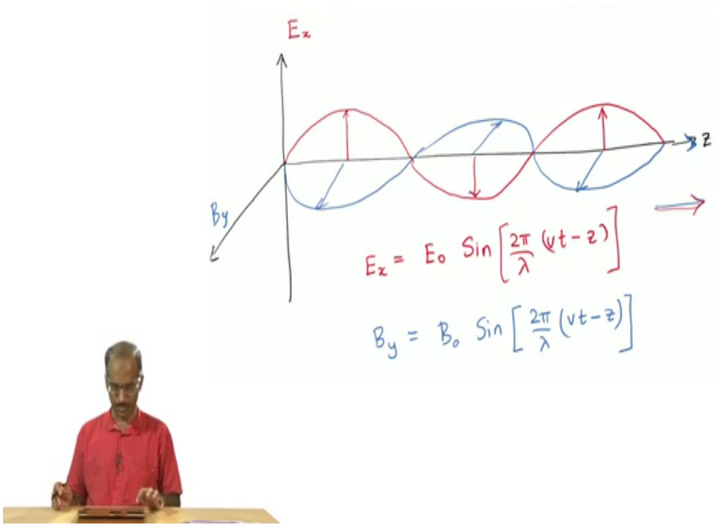

So, here I have plotted E_x; the x component of the electric field as a function of z and that is the red curve that you see here. On the other hand; the magnetic field component in y direction is shown as blue curve. You will notice that they are always perpendicular to one another and the direction of propagation is indicated by this arrow shown here. So for us, when I talk of light wave, I actually mean electromagnetic wave forms of this type or as visualized here. So, these are the waveforms that could combined together to form interference and diffraction patterns.

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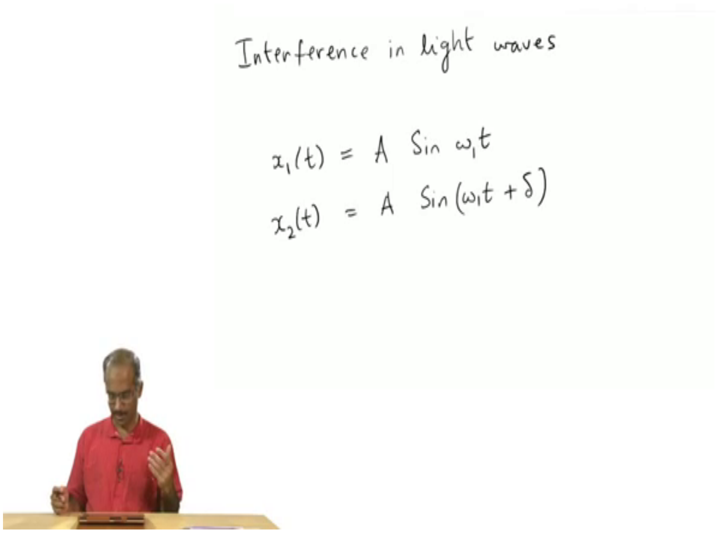

So, in general I could choose any suitable solution for these waveforms. So, $x_1(t)$ is some $A \sin \omega_1 t$ and $x_2(t)$ is $A \sin \omega_1 t$ plus some phase δ. Now, if I combined these together, what I will see is some the resultant wave will show me peaks and troughs or the intensity would show some maxima and minima and of course, all these would depend on the value of δ itself.

The δ here is independent of time a constant. So, here I have chosen the frequency to be same for both the oscillating forms; which is ω_1 here. So, you would say that it is a monochromatic wave. If you have many frequencies mathematical analysis becomes a little more difficult.

Hence we pretty much most of the time stick to cases where; the two interfering waveforms will have same frequency or wavelength.

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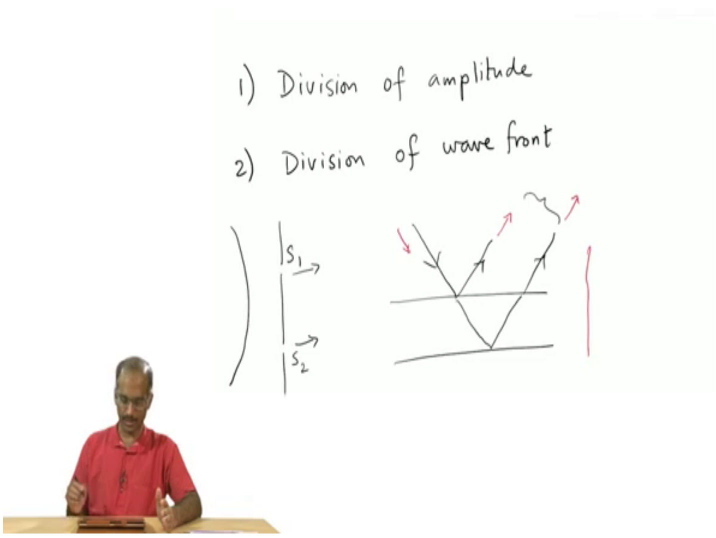

So, with this introduction; there are two broad classes of interference possible. One is what would be called division of amplitude. In this case what happens is that you start with let us say one waveform, may be its generated by a point source of light. And let us say it strikes glass slab as shown here, like this.

So, at the top of the at the top surface of the glass slab, part of the radiation is immediately reflected and the other part actually gets transmitted inside. Now the one part which got transmitted inside would go down and would finally get reflected again at the lower surface. And these two beams can in principle combine interfere and produce an interference pattern.

So, from one point source which is coming in this direction. We have finally, ended up with two beams. And there is a constant phase difference between these two beams. On the other hand you can do it by slightly different means; which is called the division of wavefront.

So, in this case if you look at the left figure here; what we have is an incoming maybe a plane wave and incoming plane wave from left side. And it hits a screen with two small slits. The slits are named S_1 and S_2 here and of course, then they light which hits the slit will pass through, and again under certain conditions on the width of the slit the distance between slits and many other factors taken together it is possible that these two beams which are coming out of the slits S_1 and S_1. Might interfere and produce interference pattern. So, I will start with the simple case.

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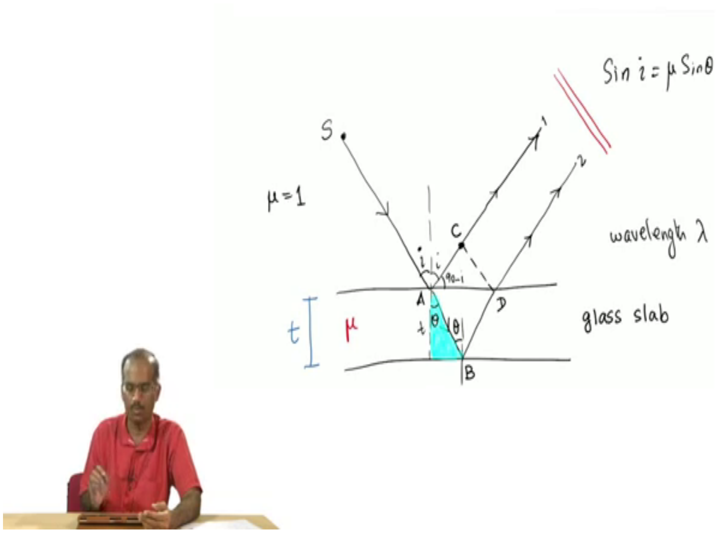

So, the setup is as follows. So, I have let us say a point source of light which is indicated by S here in this figure and this point source of light shines light at an oblique angle to the glass slab the glass slab has thickness t and the refractive index of the glass slab is μ, which written here now part of this beam is refracted goes inside the glass and this beam goes all the way down to the lower surface of this glass lab and again gets reflected that.

So, there is one incoming being starting from S and out comes two beams. So, indicated by C D or one and two. So, here we have assume that there is one beam of light starting from S and wavelength is λ. So, it has all the ingredients necessary for interference. So, there are two beams; which are sort of coherent, in the sense that they have same frequency and they have a constant phase difference between these two beams one and two.

And where is this phase difference coming from? It comes on the fact that, the path taken by the two beams are different. The first beam takes the bath SAC, $SAC1$ if you like. Whereas, the second beam takes a longer path it goes as $SABD2$. Hence, depending on the value of phase difference; you will either see a bright or a dark patch at the output, in the place where there are these two red lines drawn here.

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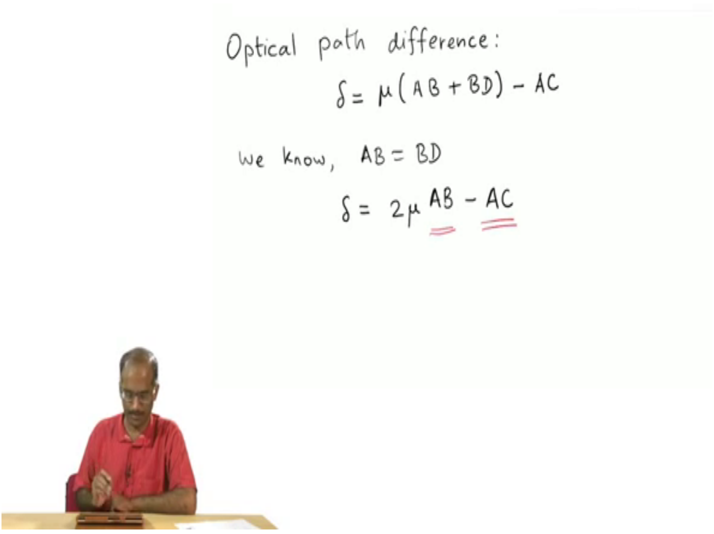

Referring back to this figure; the optical path difference is given by $\mu(AB + BD) - AC$. So, just go back and see this figure. So, $AB + BD$ is this length of beam traversed inside the medium of glass, whose refractive index is μ, between S and A; both the beams in principle travel the same distance. So, there is no path difference.

The path difference begins to happen only at point A. Because sum one part of light goes into the glass slab and the other immediately gets reflected. And from C and D onwards; again the path is the same. And if you look at the geometry of this figure you can see that $AB = BD$. So, if $AB = BD$; I can simply right my path difference as $2\mu \times (AB - AC)$. The quantity is I need to find our AB and AC, let us do that. To find ABI invite your attention to this triangle which is given some color.

So, you can see that it is a right angled triangle, and angle θ is marked. And the vertical distance is simply the thickness of the slab and that is equal to t that is also marked here. So, I know the, I know one angle and one distance which is the thickness t then I can find AB using trigonometric relations; by our law of reflection the incident angle should be equal to the angle of reflection, so this should be equal to i, as well which means that this angle here would be $90 - i$.

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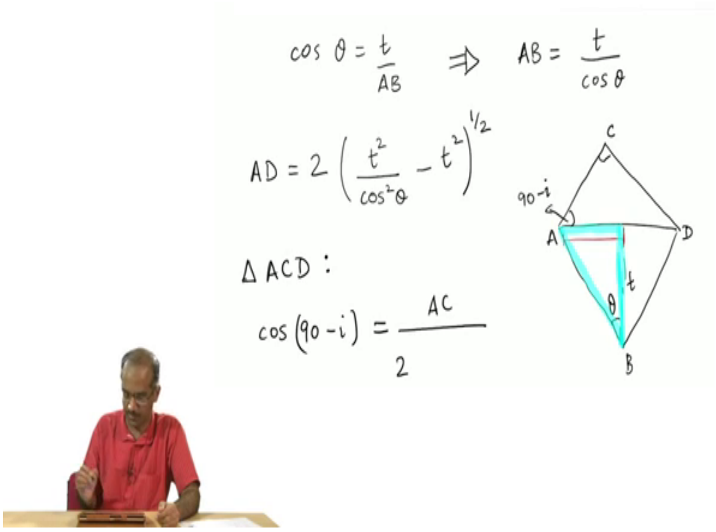

Keep in mind that, we want to find AC ultimately, and this vertical line here is thickness of the glass t and of course, angle θ is also marked here, but we just found AB that is a distance we know. So, we know AB, we know this t which means that I can calculate this distance. So, AD is twice this distance.

Now, that I know AD, I want to calculate AC ultimately. So, AD is known and one angle at A is known. So, calculating AC should be straightforward. I should point out that AD is actually 2 times, but square root of this quantity. So, I is the incident angle, angle of incidence from point source S. And if I apply the law of refraction at point A, you will see that $\sin i = \mu \sin\theta$; because, we have implicitly assume that μ here is 1.

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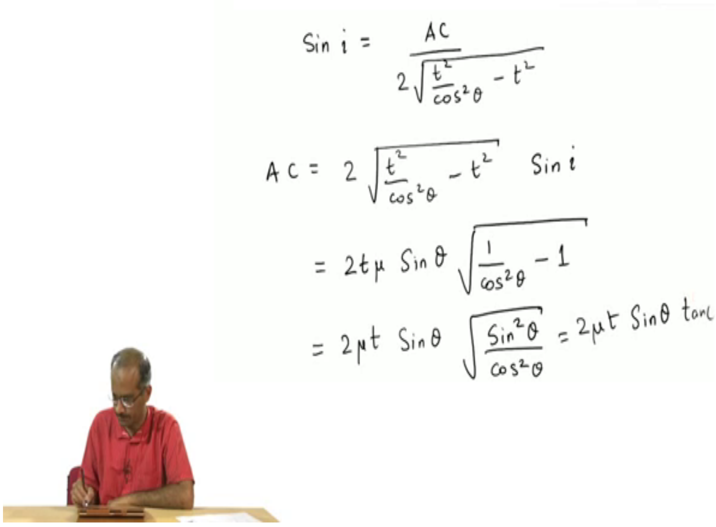

Now, let me substitute this expression for $\sin i$, here in which case I can on the way also simplify this, so it will be $2t \times$; $\sin i$ is $\mu \sin\theta\sqrt{\frac{1}{\cos^2\theta}-1}$. So, this will be $2\mu t \sin\theta$. This is going to be $1 - \cos^2\theta$; which is going to be $\sin^2\theta$ divided by $\cos^2\theta$ and a square root. And this will finally give me $2\mu t \sin\theta \tan\theta$.

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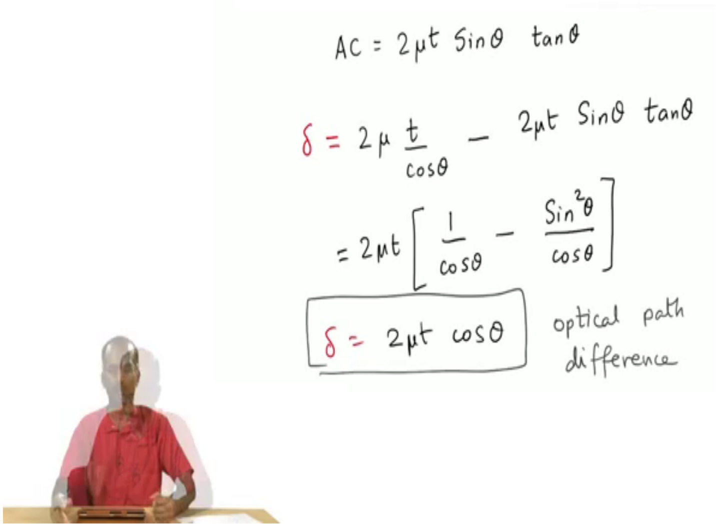

So, the optical path difference $\delta = 2\mu t \cos\theta$. The reason that, I did this in sufficient detail is because more or less pretty much in this entire class of interference and even diffraction problems; the crucial part and the major part of the work is actually to calculate the optical path difference. Now I have calculated the optical path difference. What should I do from here?

So, when would I expect to see, may be bright fringes and when should I expect to see dark fringes. You have this beam starting from S and gets reflected at point A and goes towards C and then to point 1. So, whenever you have this kind of a reflection taking place at a boundary like this. So, the light is trying to go from a medium with refractive index 1 to some other medium whose refractive index is greater than 1.

So, in all such cases an additional phase of π gets added at the point of reflection. So, it is equivalent to like one of the problems that we did in the case of a waves travelling on a string. So, you have this string tied between two walls and you excite a wave there; the wave travels let us say to one end of your string and then it meets a wall; whose impedance is infinity, ok. It cannot penetrate into the wall, so it turns back.

So, there is a phase change of π upon reflection. So, when we now calculate the phase difference corresponding to this path difference; we also need to add this additional phase, that comes because one of the beam actually got reflected.

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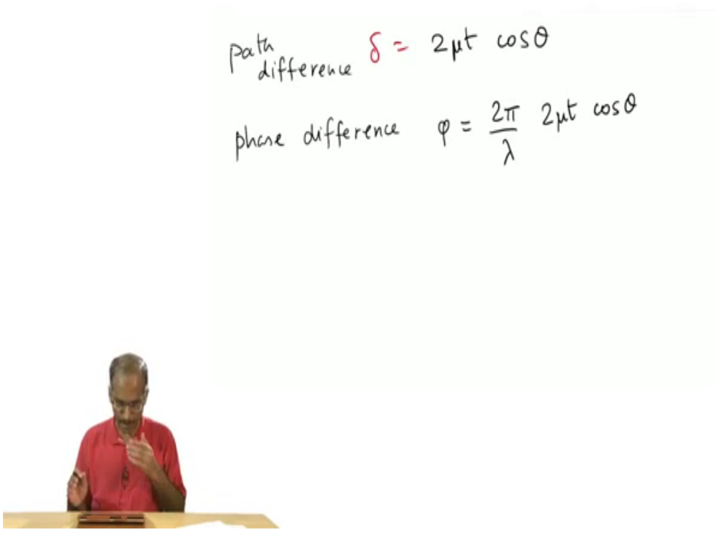

Here, I have the path difference. Now let me calculate the phase difference corresponding to this and let me call that quantity ϕ. How do you obtain this? This is simply by the argument that a path difference of λ or path difference of one wavelength would equal a phase difference of 2π.

So, then you ask the question; if I have path difference δ, what is the phase difference? So, from that consideration, you will get this relation for the phase difference given path difference. Now that I have the phase difference, let us see what are the possibilities.

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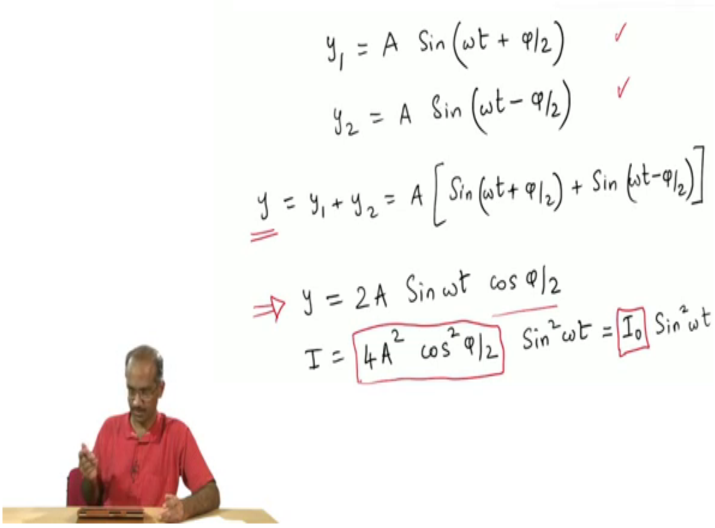

But before I really do that so I want to go back to a simple waveforms and see what is a result we get; if phase difference is ϕ.

So, you take two simple waveforms; $A\ \sin(\omega t+\frac{\phi}{2})$, like y_1 that is shown here. And the other one is y_2; which is $A\ \sin(\omega t-\frac{\phi}{2})$. So, you can note one thing that this is coherent set of waves two of them coherent because frequencies are same, and there is a constant phase difference between y_1 and y_2.

So, the resultant waveform in this case is; as we had done earlier on, simply add the two. $y_1 + y_2$; this $y_1 + y_2$ which you indicated by y. Now can be written in the following way: so, this is the result for y which is the net displacement, when I sum these two waves with a phase difference equaling ϕ so, you notice that there is a term of $\cos\frac{\phi}{2}$, that is appearing.

And I can ask for, what is the intensity? We know that, intensity is square of this wave. So, essentially what I have done is to simply square the wave. So, intensity is this quantity for instance. $\sin^2\omega t$ gives you the time variation, right now I am not, so much interested in the time variation. So, I am more interested in the square of the amplitude; which would be $4A^2\cos^2\frac{\phi}{2}$.

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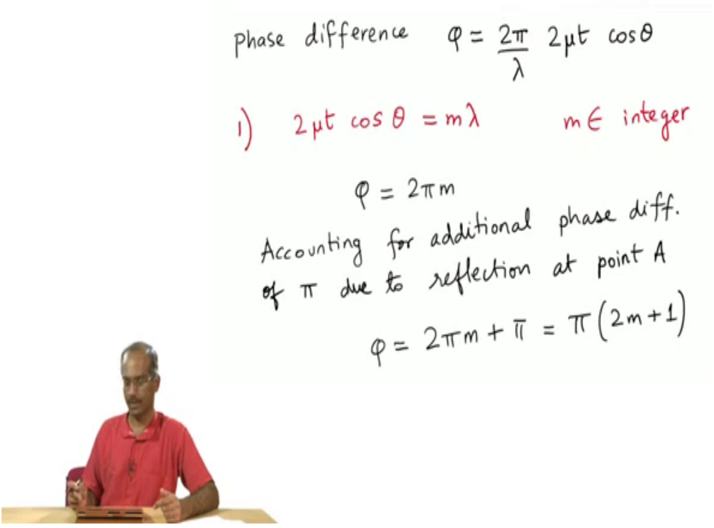

Phase difference is of course, something I calculated here; I have just rewritten it again for you to see it and $2\mu t \cos\theta$ is the path difference, that we calculated; the optical path difference.

So, first let us see, if this optical path difference is an integral multiple of the wavelength. So, phase difference is integral multiple of 2π. So, if I account for this additional phase difference of π, due to reflection at point A, then I just needed to add ϕ here hence, now after accounting for this phase difference; it will be $2\pi m + \pi$, which would mean that, if I take π outside it will be $2m + 1$. Going back to this intensity, I can write it as $I_0 \sin^2 \omega t$; where this I_0 is this quantity which is written in red square; namely that it is equal to $4A^2 \cos^2 \frac{\phi}{2}$. I am interested in I_0 alone.

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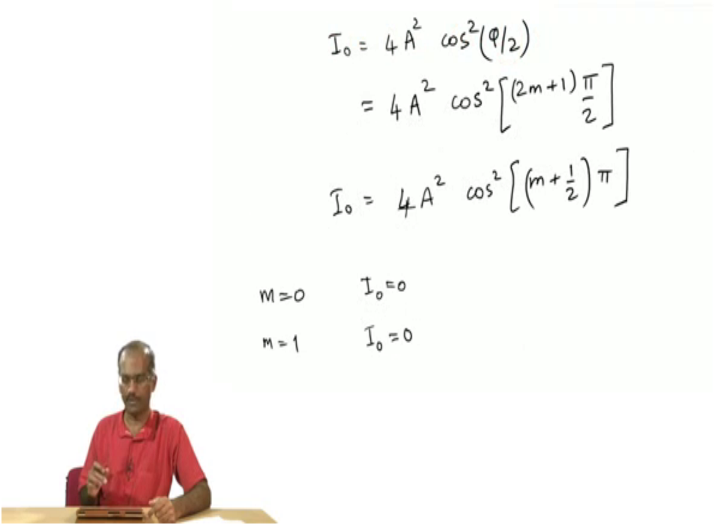

So, now let me write, I_0, which is equal to $4A^2 \cos^2 \frac{\phi}{2}$. So, let me substitute for ϕ from here as $(2m+1)\pi$. So, here I have these expression for I_0. Now you will see that if you put $m = 0$ here, this I_0 would be $4A^2 \cos^2 \frac{\pi}{2}$, $\cos \frac{\pi}{2}$ is 0. So, $I_0 = 0$. If I put $m = 1$, it is going to give me $1 + 1/2$ is $3/2$, $3\pi/2$.

So, clearly again $I_0 = 0$. So, in general for all integer values of m, I_0 is going to be equal to 0.

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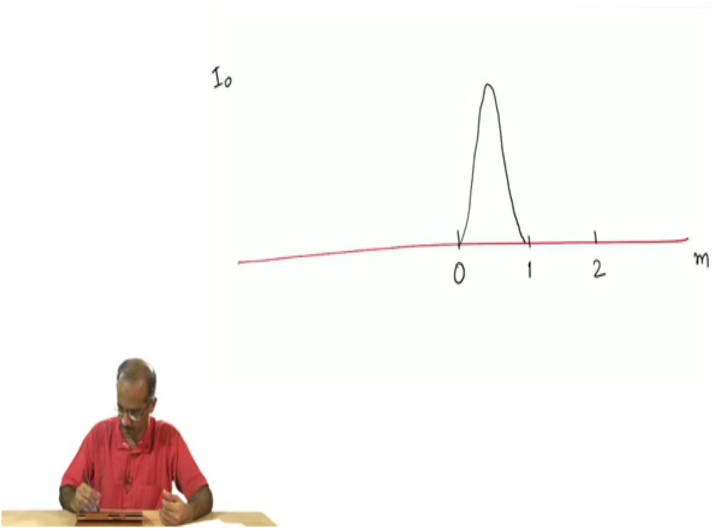

So here, if I try to plot this I_0 as a function of m, so, it is going to be 0 at integer values here, I am going to get something like this. So, that is for the case when $2\mu t \cos\theta = m\lambda$. So, when this condition is met; the intensity is 0 or we are going to see dark fringes.

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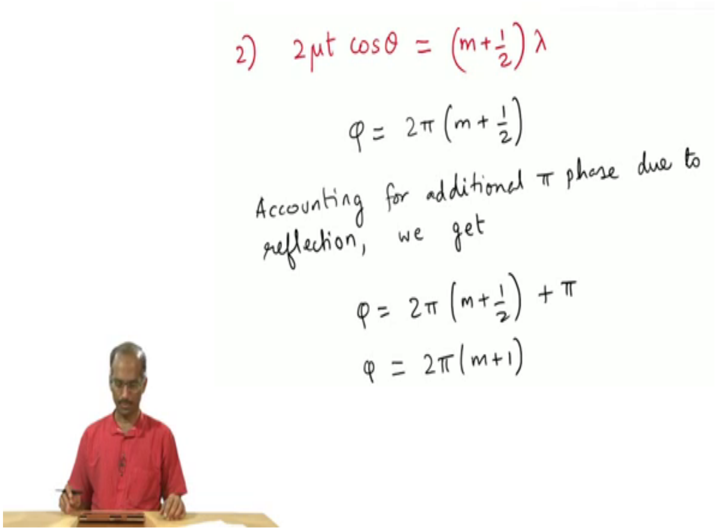

What happens if $2\mu t \cos\theta = \left(m+\frac{1}{2}\right)\lambda$. Again remember that m is a integer. Now, if I put in this value of $2\mu t \cos\theta$ in the expression for phase difference; that will give me the following. So, we need to account for this additional phase π that arises from reflection at the top surface of the slab. Hence, if I take that into account, this ϕ will become 2π plus 2π into $m + 1/2$ with another π added.

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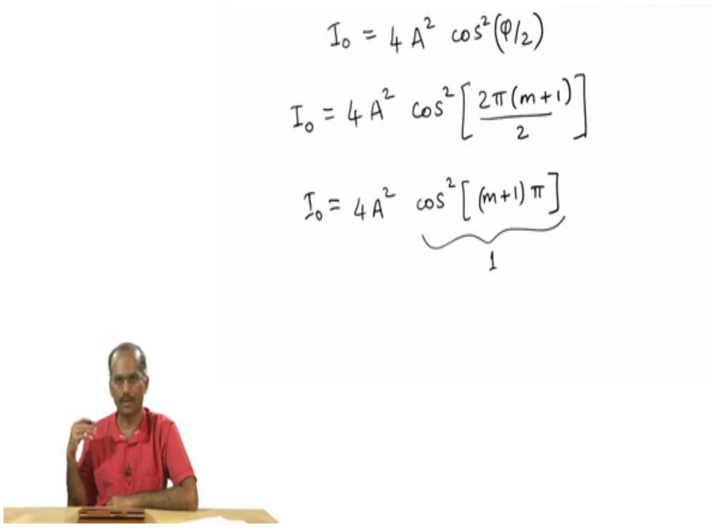

Now, let us plug this value of ϕ in these equation for intensity. You will see that I am going to get the following here; any value of integer, it does not matter what you take it is always going to be equal to 1 this factor.

So, if I put $m = 0$, I am going to get the first bright fringe. And similarly if I put $m = 1$, it will be $\cos^2 2\pi$ which will be one again. So, in other word, you will see that this interest purses with the dark spots that were obtained for the previous case.

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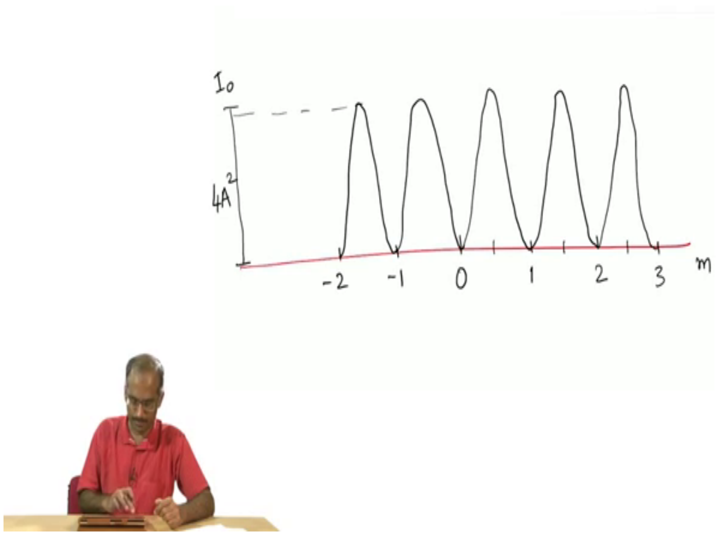

So, all these points here, this all this value which corresponds to $4A^2$, they are the positions where; this equation is satisfied and now, if I were actually going to put a photographic plate at this place here.

So, what you will see? We will depend on of course, the parameters. Because, if you notice the condition for whether you will get brighter dark band depends on this combination of parameters $2\mu t \cos\theta$ whether it is equal to $m + 1/2$ or whether its equal to $m\lambda$.

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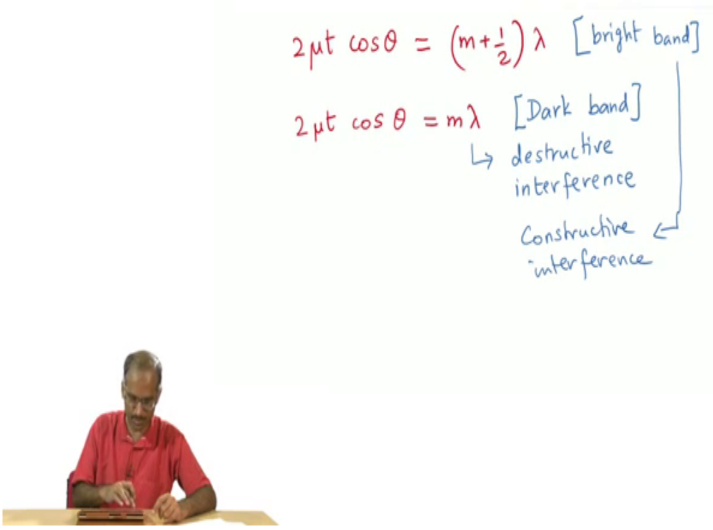

So, I had these two condition; one is $2\mu t \cos\theta = m\lambda$, in which case we get dark band let us say or dark fringe; whereas, if $2\mu t \cos\theta = (m + 1/2)\,\lambda$, you are going to get a bright band. And θ also is a constant for a given configuration and the setup, in the sense that once I have chosen how I am going to light up my source. How the source is going to be oriented with respect to this glass.

In other words it is going to fix the angle of incidence which in turn will fix the angle of refraction. So, all these are constants, and if that is the case; what we will actually see when we view this view at the output end is a constant illumination. That constant illumination could either be bright or dark depending on whether it satisfies which of these two conditions. So, if it satisfies the first condition, it will the illumination will be very bright, you should see a bright spot everywhere, or if it satisfies the second condition you will see dark spot, dark band everywhere.

Now, if you really want to see fringes, you need to do something more to it. You need an extended source of light. When you have extended source of light; then of course, you are going to have a range of θ values, because each of the beam starting from different points on that source. We will give rise to different angle of incidence and hence different angle of refraction.

So, there is going to be a range of θ values. And what you will see? Will be the locus of, will be a kind of pattern; which will follow the locus of constant θ, in that case. So, you are going to have several θ values and constant θ would be a circle. So, you are going to see, circular fringes provided of course, you replace this source of light by an extended source of light.