Waves and Oscillations
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Lecture - 46
Waves in Optical Systems: Thin Lens

Welcome to the third lecture this week. This is not really a comprehensive course on optics, neither is it a comprehensive module on optics. So, you should think of this as a barely superficial introduction to how waves come into play in optical systems. In particular we are looking at the limit when wavelength is very small compared to some typical say aperture sizes, such that diffraction effects are suppressed. In this limit we can treat waves like as though they are rays and the whole idea about what we are doing is to trace the path, to trace the path of a ray from some point *a* to point *b*.

So, in the process of going from point *a* to point *b*, this ray could be traveling from one medium which is characterized by refractive index μ_1 to another medium, which is characterized by refracts refractive index μ_2. So, it could get reflected it could get refracted. So, all these effects are possible.

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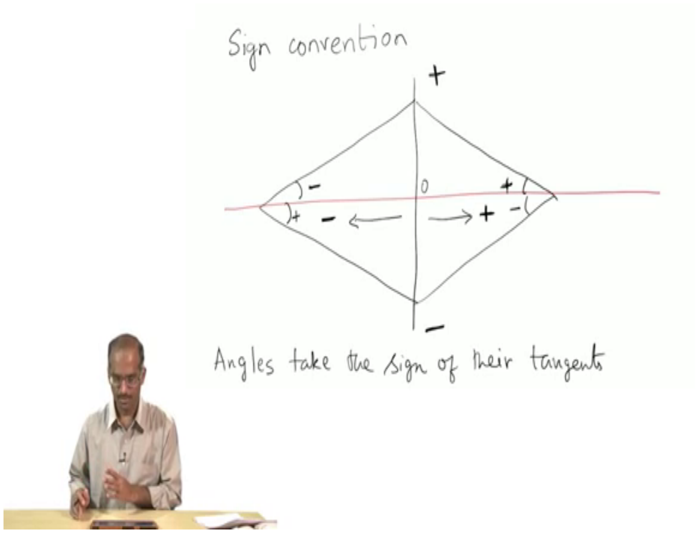

We did use the sign convention last time, but let me specifically talk about it for a minute. So, in general anything to the positive of origin here; so here origin is O maybe you have some reflecting surface there or in any case anything to the positive side would be taken as positive length and anything to the negative side is taken as to be taken as negative length.

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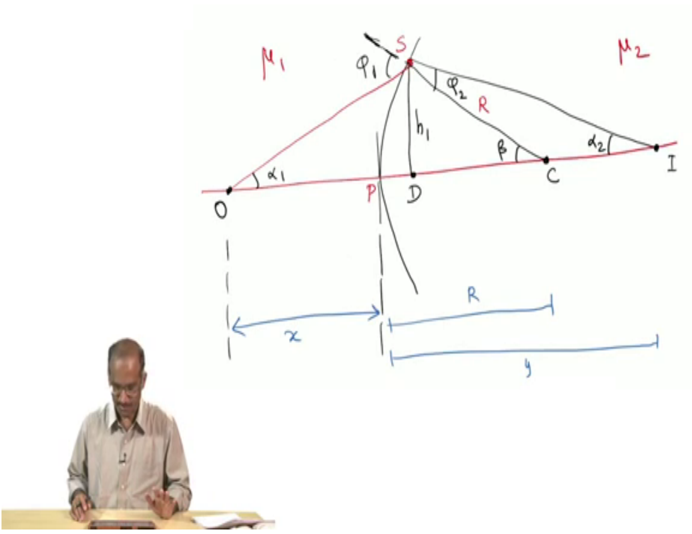

For instance in the figure that is shown here, P is that cutoff position. P at the center determines that. So, anything to the left of P is going to be negative which means, that x would take negative sign, on the other hand y which is on the right side to P will take positive sign. And, similarly when we look at the angles, so here 4 angles are marked and the signs are also marked there. So, typically angles take the signs of their tangents. We discussed how to obtain the power of a refracting surface; we also looked at how to obtain the power of a plano convex lens.

So, we dealt with something like this. In fact, this figure is same as what I showed you in the last class. So, we wanted to know if O is the position of the object on the left side; where should I see the image of O on the right side, given that I have a refracting surface. The most important thing here is that this ray goes from a medium whose refractive index is μ_1 to the medium on the right side, whose refractive index is μ_2. So, there is change in medium when it traverses from one side to the other.

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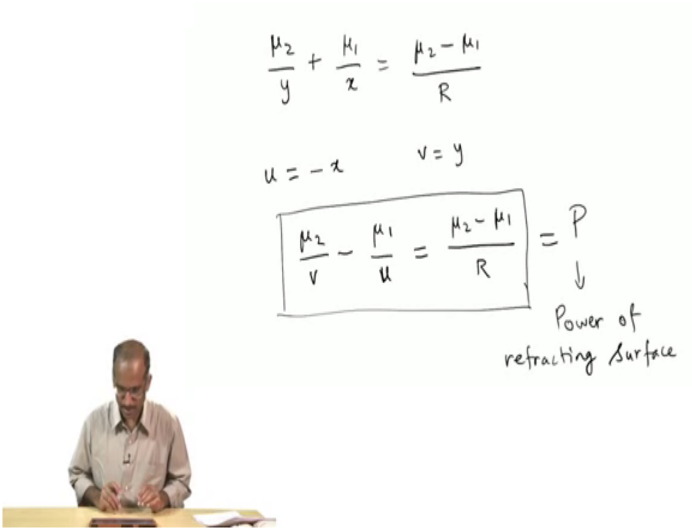

Before I go ahead let me flash for you the slide which I had shown in the last class, so we did obtain this result in the last class which is the formula for power of a refracting surface. This u and v are distances notice here that in saying that u carries a negative sign we have used this sign convention.

Now, if I apply the law of refraction to this system, the ray changes goes from one medium to another medium at point S. And, I know that incident angle is ϕ_1 refracting angle is ϕ_2. So, I can use the law of refraction and write the following equation. So, this is simply the law of refraction.

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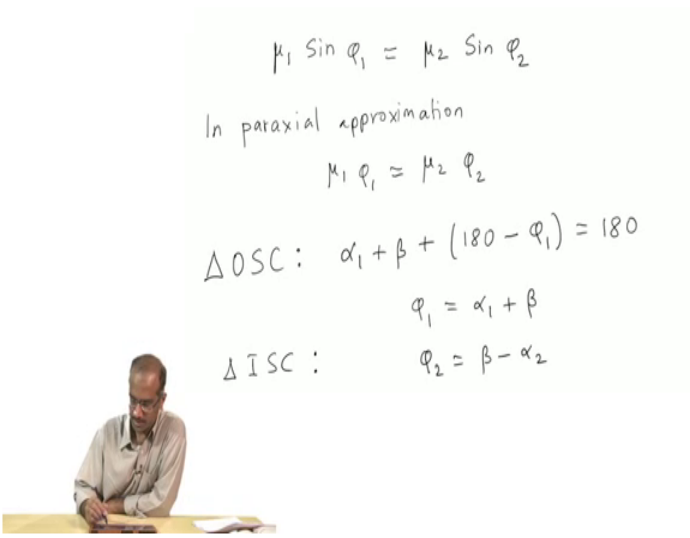

In paraxial approximation which means that I assume that all these angles are small enough, ϕ_1 is small; ϕ_2 is small in which case μ_1 times ϕ_1 will be equal to μ_2 times ϕ_2. To be able to do anything with this formula I need to know what is ϕ_1 and ϕ_2.

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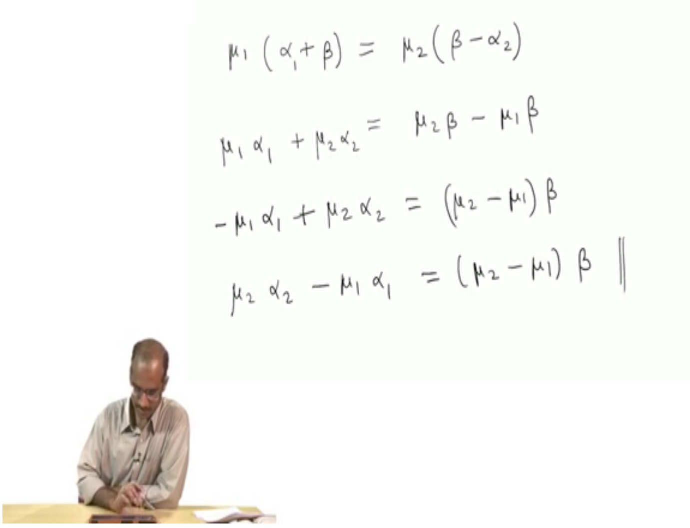

So, now let me substitute these in the law of refraction it is going to give me the following. Now, once again if I apply my sign convention that we just discussed here α_1 would take the negative sign hence I would have $\mu_1\alpha_1 + \mu_2\alpha_2 = (\mu_2 - \mu_1)\beta$.

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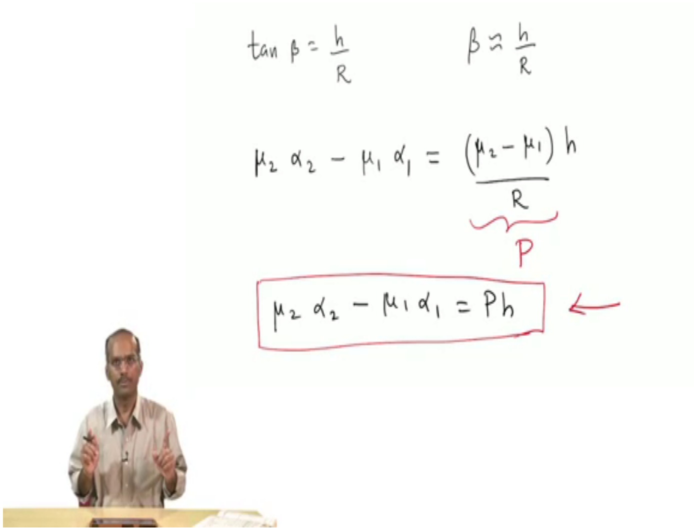

In the spirit of paraxial approximation this would imply that $\beta = \frac{h}{R}$. Because we are stating that $\tan\beta$ is approximately equal to β in the limit of small angles. Then if I substitute this in my former equation this one, I am going to get the following.

Now, this quantity $\frac{\mu_2 - \mu_1}{R}$ simply the power 1 by focal length; so, let me call this P. So, now I would have so it is a useful way of rewriting what we already knew.

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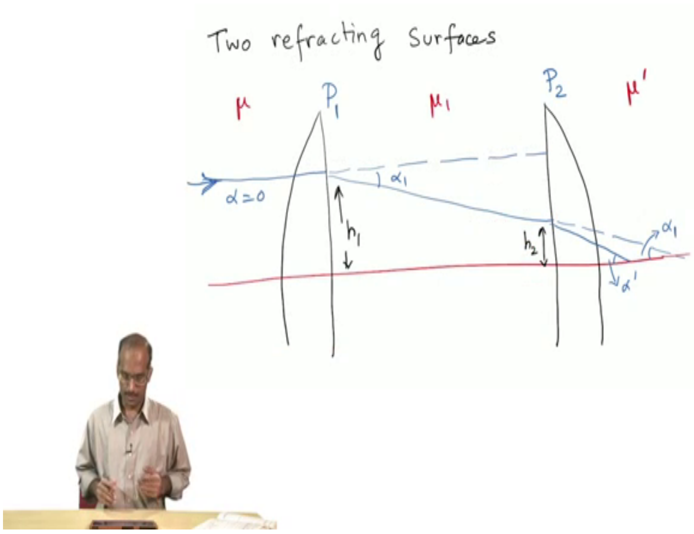

So, here is a problem of two refracting surfaces. So, earlier we dealt with one, now we have two of them and their powers are specified P_1 and P_2. And, also we have three different media here: one characterized by μ refractive index μ other one which lies in between these two refracting surfaces and that is μ_1 and there is a third one which we shall call μ'. And, as you can see I have assumed that there is a ray which is coming really from a far. So, it comes as plain wave front parallel to the axis which is drawn in red color.

What we shall do is actually apply the this formula that we have got this one, at both the points meaning at the position of both the refracting surfaces. And, ultimately when I want a formula for thin lens, what I am going to do is to bring together these two pieces such that the medium μ_1 would not even exist.

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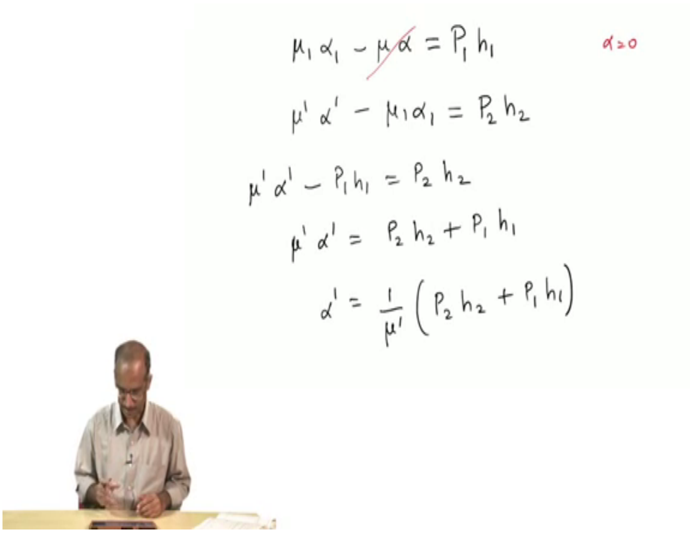

So, I have these two sets of relations which we can now manipulate. An obvious thing to do is to first set $\alpha = 0$, because as you can see we are working with light ray coming from in principle infinite distance which means, that $P_1 h_1 = \mu_1 \alpha_1$. So, I can substitute here and that is going to give me the; from here I can write an expression for α'. Now, to get the power of this combined system all I need to do is to simply do a small manipulation.

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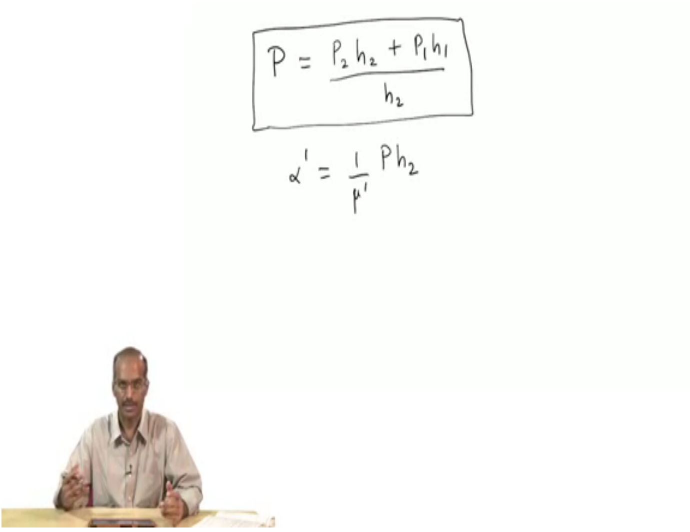

So, now let me call this effective power as simply P. So, if I use the effective power which is P and rewrite this relation for α', α' would simply become α' is equal to; so this gives me the effective power of the system of two refracting surfaces. So, this will be the equation that we will now use as I explained earlier to find the power for a thin lens. As we begin to calculate the power of thin lens, here I have the relevant diagram for you.

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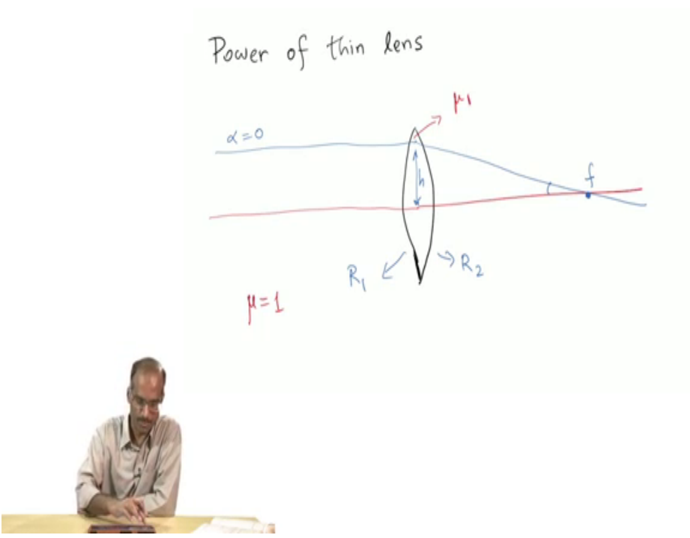

So, I have the lens at the centre and I am assuming that a ray of light is coming from in principle infinite distance. So, the angle of incidence $\alpha = 0$, it is getting refracted and as we know it finally, converges at a point which we call the focus of the lens and the entire region or you might call the atmosphere we assume that has that it has refractive index equal to 1 and the material of the lens has refractive index equal to μ_1. And, then I have also assumed a general situation where the radius of curvature of the two sides that make up the two halves that make up the lens are different what we know is the power of each half of the lens so let me, indicate by P_1 this particular half of this lens which has radius of curvature R_1.

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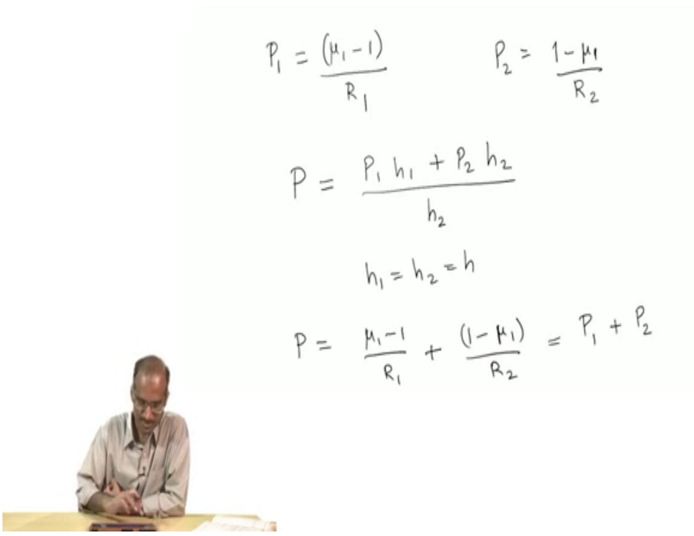

So, in that case it would be, so that would be the power for one half. And, similarly I can write an expression for power for the second half. Now, the effective power we already have this formula for effective power let me rewrite that we know what is P_1, we know what is P_2, formulas are written there they are the power corresponding to Plano convex lens. Now, we should note that as far as this lens is concerned there is only one h. So, remember that this is put together by squashing these two halves.

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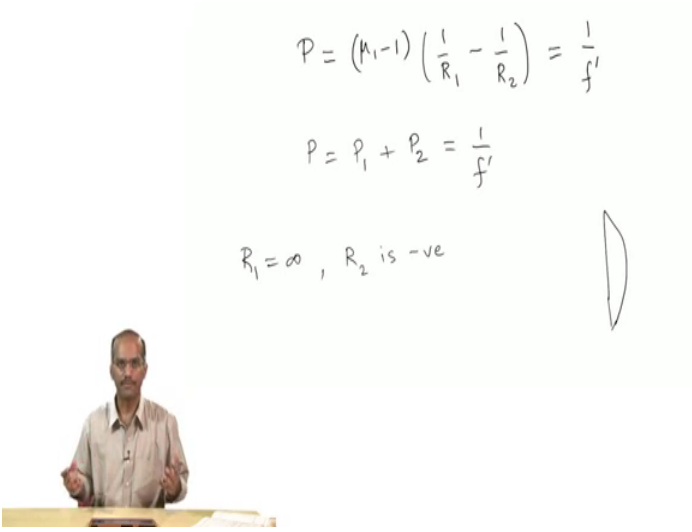

So, h_1 and h_2 become equal and it is equal to h. And, of course, h would cancel out completely this formula for power can be written in a simpler form. So, just by simple manipulation I get this final form for the power of thin lens and we know that this power is inversely, is inversely related to the focal length. So, I could call this as some $\frac{1}{f'}$. And, as you can see just going one step backward this expression for power is simply nothing, but $P_1 + P_2$ itself.

So, ultimately we have come back to the point where we could say that $P = P_1 + P_2$, the power of the combination; here combination being two halves of the refracting surfaces and that is equal to $\frac{1}{f'}$ you can go back to ask so if I had a Plano convex lens would this formula help me give the correct limiting result indeed for a Plano convex lens of this type R_1 is simply equal to infinity and R_1 is negative by our sign convention, hence this gives a positive value for power. Which means, that it is able to converge a light that is coming really from a infinite distance so to speak.

In the next module we will try and generalize this to a method where we can apply to several such elements.