Waves and Oscillations

Prof. M S Santhanam
Department of Physics
Indian Institute of Science Education and Research, Pune

Lecture – 44
Waves in Optical Systems

Welcome to the 10th week, we look at Waves in Optical System or in other words the light waves or electromagnetic radiation. Thanks to Maxwell and many others we know that light is a form of a wave. Let us see if we can describe propagation of light and maybe more common properties of light for example, reflection refraction by just not worrying about the wave form, but simply looking at it as ray that travels from point *a* to point *b*.

So, if you take this view point we need to decide first in what limit of wave would this be valid and how are we going to go about working with this limit. This sort of description works well in what is called the limit of ray optics.

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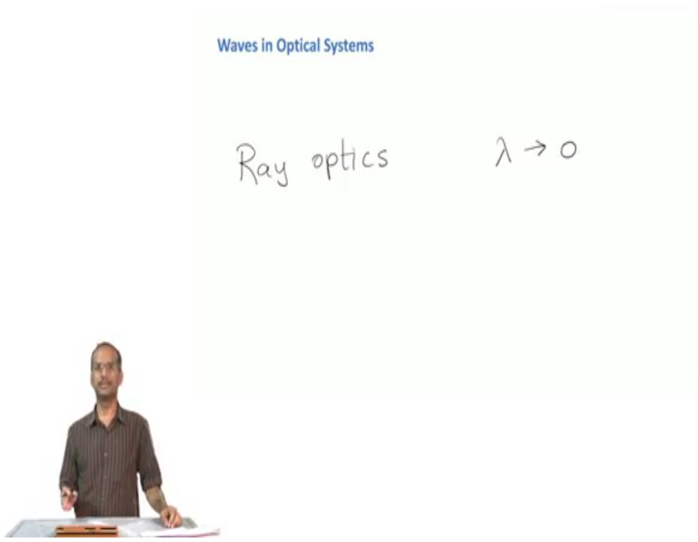

And this corresponds to taking the limit that wavelength of light or wavelength of the waveform is small or it tends to 0.

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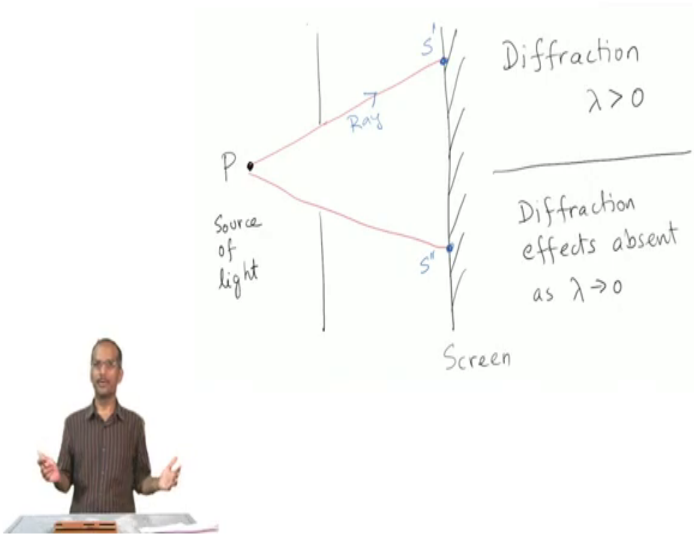

Let us consider a source of light here from point P and let us say that I have a circular aperture given by this and at the other end I have a screen here. If P is any source of light now question is what is it that we will see on the screen? And in this case it will depend on dimension of this aperture. Suppose this aperture is small enough in such a situation the light from P will of course, go like this, but we are unlikely to see a very clear boundaries.

So, this effect that we see where you see that there is some light even in a region which is expected to be dark happens due to what is called diffraction effects. And, this diffraction effect is essentially an effect that arises because of the wave nature of light. So, we need to go to a limit where we would not be able to see the effects of diffraction.

In other words the conclusion is that you will see effects of diffraction whenever λ is larger than 0. So, in the limit of $\lambda \rightarrow 0$ even if you bring in make the aperture sufficiently small you would not see diffraction effects. In that case where we can very clearly see the sharp distinction between the region that is lit up and the region that that falls in the shadow below this point S' on the screen you do see light and above S' on the screen it is dark.

So, in this case it goes from P to S' and of course, on the other side it would also go from P to let us call this point S''. So, between S and S' and S'' you do see light and above S' below S'', it is very dark. In that case we can describe our light wave using ray diagrams basically lines like the ones that I have drawn here. So, this is the regime of what is called the ray optics or geometric optics. Every time a wave of light goes from point a to point b we can just represent it as a line going from point a to point b.

So, the question is how do I know that light that starting from point P takes this particular path and this question is answered by what is call the Fermat's principle.

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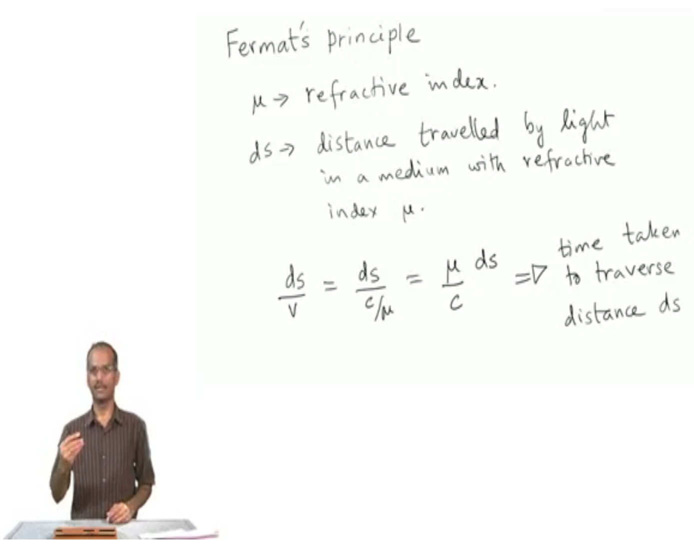

We need to state the refractive index let me assume that light travels in a medium with refractive index μ. Let me denote by ds the distance travelled by light in a medium with refractive index μ if ds is the distance and v is let us say the speed of light, then of course, speed of light in the medium with refractive index μ in that case I can write the time taken to travel this distance ds as. So, I will simply use the definition of refractive index to rewrite the velocity of light in the medium that is characterized by refractive index μ.

Now, I have assume that light travels in a medium that is characterized by refractive index μ and there is no change in the medium. We can generalize this idea further by saying that light probably travels in a series of different media each one characterized by a different refractive index.

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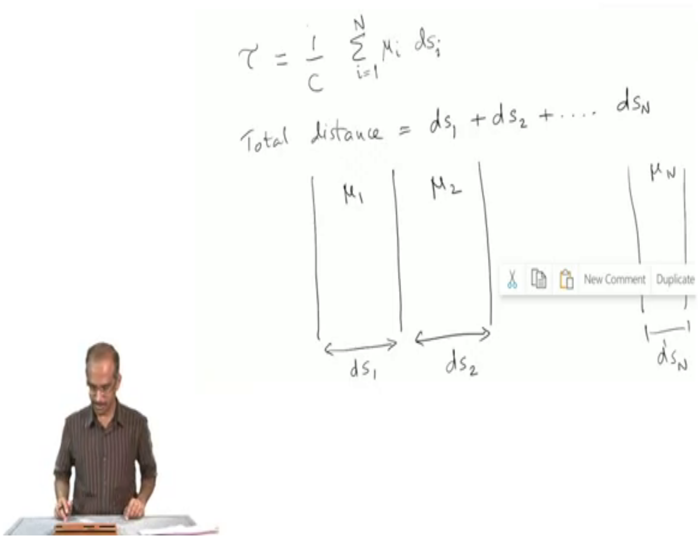

So, let me denote by τ the time taken to reverse distance ds. So, generalizing this formula I can write it as. So, each one of these indices represent a different medium and there is a change in refractive index. So, that is characterized by μ_i.

So, ds_1 is the distance traveled in a medium characterized by refractive index μ_1, ds_2 is the distance traveled in a medium that is characterized by μ_2 and so on. If there is a continuously changing media we can replace the summation by an integration. In which case the distance traveled which is denoted by τ will be given by.

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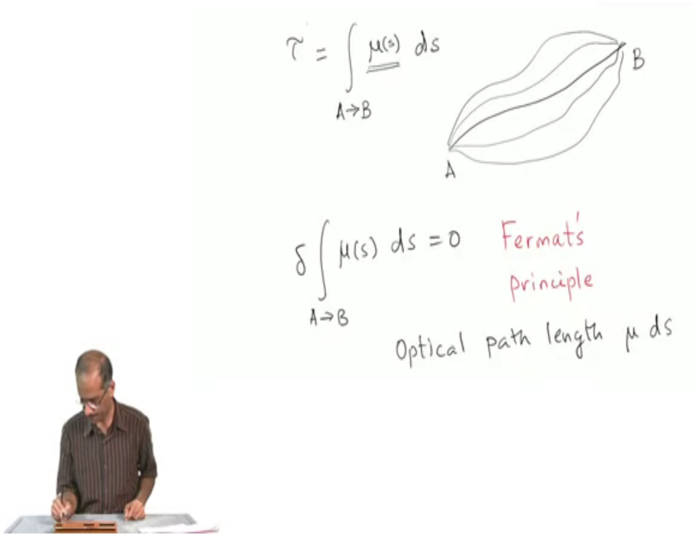

So, I have this sort of formal expression for the time taken by light to travel from point A to point B in this picture of ray approximation and between point A and point B depending on where you are the refractive index does change. So, that is taken into account by this μ which depends on s. Fermat's principle can be mathematically written in the following way I have written it here for you. So, the variation of this integral is 0.

So, the physical content of Fermat's principle is that if you have chosen two points and light travels from A to B you could consider many possible paths here through which the light might possibly travel from A to B. And, the one it actually takes is the one for which the variation of this quantity equals 0. Very loosely speaking its equivalent to saying that the time taken to go from A to B is the least, this quantity $\mu \times ds$ is called the optical path length.

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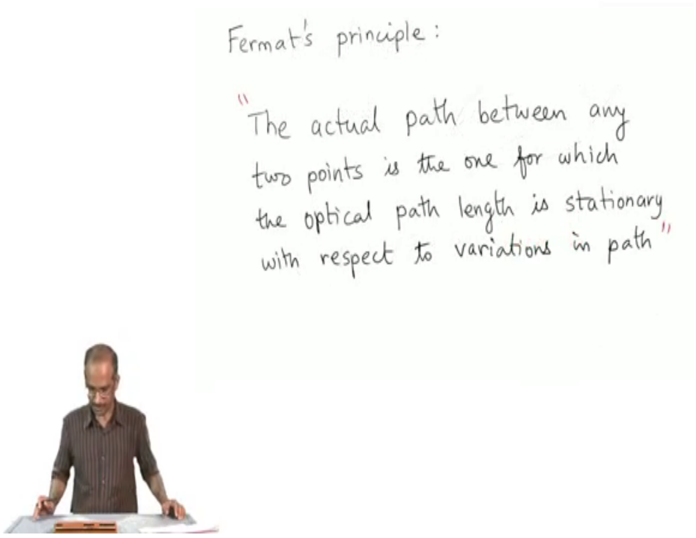

So, here is Fermat's principal written out for you. The actual path between any two points like A and B that we had drawn in the previous slide is the one for which the optical path length is stationary. So, you would see that μds represents the optical path length. So, this quantity is stationary with respect to variations in the path. So, the light actually takes that path for which the optical path length is an extremum.

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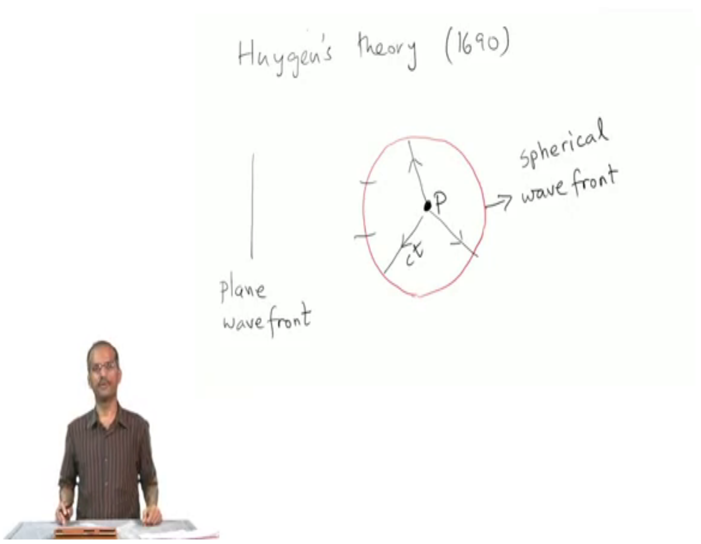

So, Huygen's theory sort of provides this bridge between light as a wave and light as a ray. So, his idea is that light can be represented as a wave front; I have this point P which is a source of light and if that is a source of light. So, light is going to travel in all the directions outward from this point P.

Now, you can ask the question what is the locus of those points which have the same face after all whichever direction light travels around P its traveling with same speed. This locus of these points with same face have travelled a distance equal to $c \times t$.

Now, this is a wave front for us and specifically in this case it would be called a spherical wave front the light coming out of point sources of light would define a spherical wave front.

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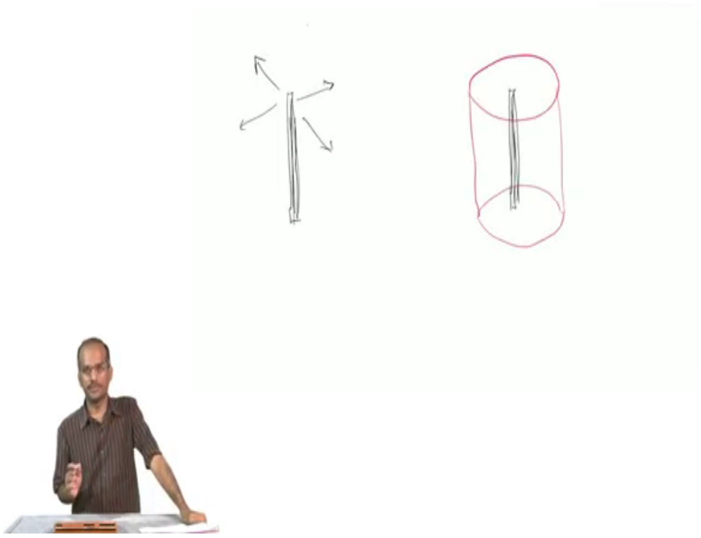

On the other hand if I had a source of light which is linear. So, all the points with the same face here have travelled the same distance and the locus of such points they form a cylinder. So, this is called a cylindrical wave front and if you are looking at say a point source of light which is really very far away, but if it has traveled sufficiently long distance and you are looking at a small portion of that.

And that for example, something like this would look like a plane wave. So, that would be called a plane wave front for instance the sunlight which is reaching us from really long distance. So, in such cases you could assume that you are actually observing a plane wave front.

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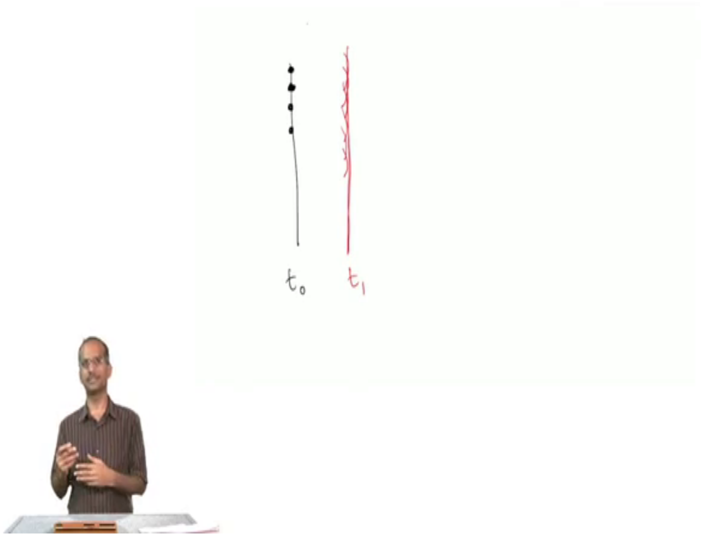

So, let us say that I have this plane wave front at some time *t* maybe it originally started out as a spherical wave front, but at the point where I am observing it what I see is a plane wave front. Huygens idea is that each point on this wave front acts as a source of secondary wavelets and let us say you take one point here its a source of secondary wavelet which is going to be spherical.

Now, you take the point which is very close to this point draw a similar wave front which came out from the second point and so on. So, if you put all of them together what you get is a new position of the wave front. So, essentially you transported a wave front at time t_0 to let us say at some time *t* one.

The central and core idea is that every point on a given wave front is a source of new wavelets and if you join the points with same face for this secondary wavelets you are going to produce a new wave front and you moved your wave front from old position to new position and you can continue this process. It requires a little more involved exercise including wave phenomena to explain why it moves forward rather than backward.

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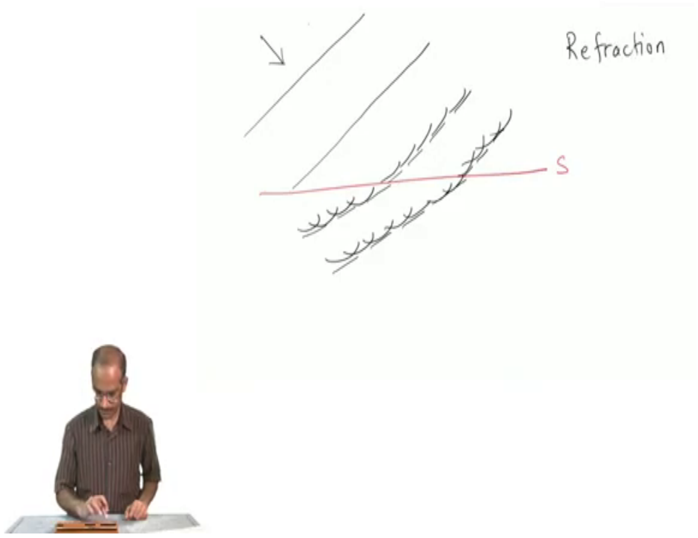

Let us say that I have some surface here which I call S and there is an incoming wave front like this. So, I consider secondary wavelets and I can move it let us say to this point which is essentially the locus of all the secondary wavelets with the same face and here I assume that the media are different at the top and bottom. So, at the surface a reflected component of light emerges and that would go something like this.

So, in other words you have a light wave that is coming let us say in this direction I have drawn the arrows here, it gets reflected at surface S that is a change of media there and finally, the reflected component goes in this direction that I have shown here. And by the same token one can also explain refraction, so this is for the case of refraction.

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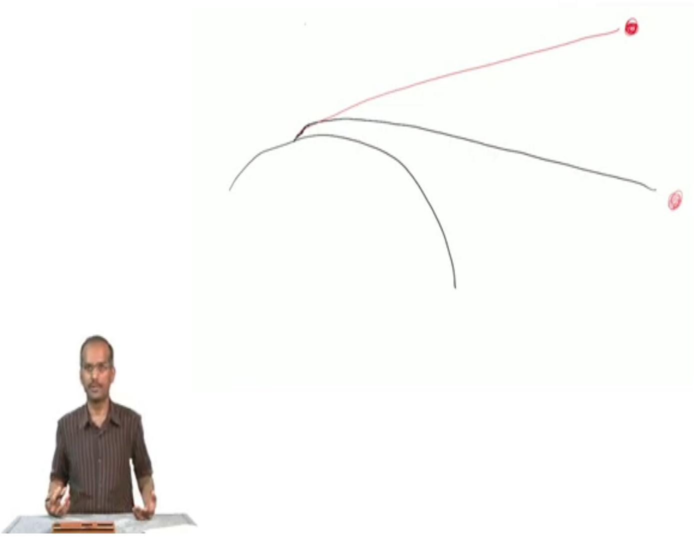

So, in the evenings the light travels longer distance to the atmosphere hence if this is the ray that is coming from the sun it actually bends when it encounters the atmosphere and reaches our eyes. And, from our perspective it would look like the sun is somewhere here whereas, the more correct position of the sun is somewhere way below this point and this is because of refractive effects of the atmosphere.

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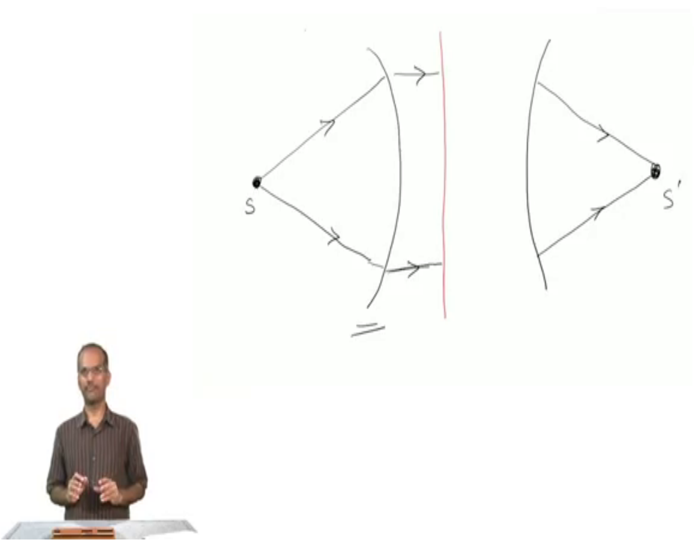

Let us say that I have a source of light S and from here I have a diverging beams which go out as spherical waves and after let us say it travels over sufficient distances it becomes plane wave. So, if you put in optical devices you can make it finally, converge at some other point here and of course, the converging wave front might look like this. The point you note is that all the points along the wave front let us take for example, this wave front all the points along the wave front have traveled the same optical distance.

So, in other words they are at the same optical distance from the source. So, the path that light takes from let us say point S to S' can be accounted for by Fermat's principle and to map this wave front we basically use Huygens idea of secondary wavelets. So, you can move your wave front forward using secondary wavelets. Now, let us use all this machinery to account for reflection and refraction.

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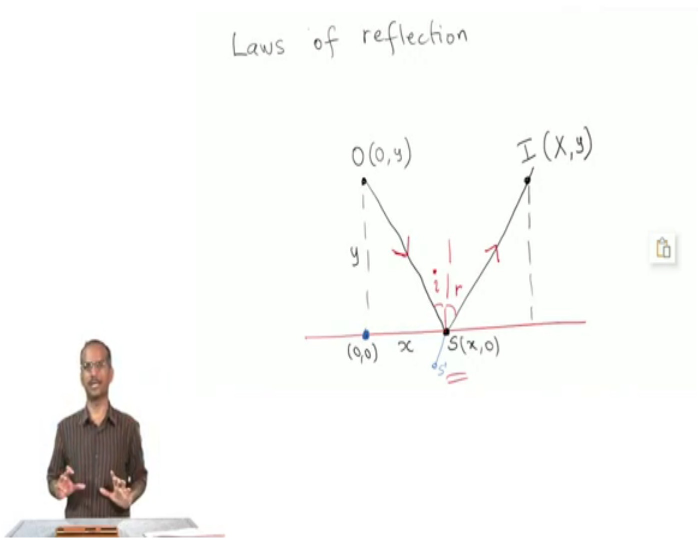

So, what we have is this surface S there is an incoming incident wave which comes in a sense from the left of this point S that I have and gets reflected at this surface S and you have this reflected component. And, at this point S where the reflection takes place I have a normal drawn and with respect to that perpendicular or normal there are two angles defined; one is angle i which is call the angle of incidence which is the angle at which the incident ray hits the surface S. And, there is another angle defined which is r this is the angle at which there is an outgoing ray the reflected ray.

So, this entire action is taking place in a plane that is perpendicular to S because suppose if I take some other point let us say S' and I connect these points object O to S' and S' to I. So, these distances will actually be larger than OS and SI purely from the geometry of what we are looking at and the only time this travel distance I mean the optical distance will be smallest is when OSI is in a plane that is perpendicular to this surface S. So, let us apply Fermat's principle to this problem I need to know what is the time taken for light to travel from O to S and from S to I.

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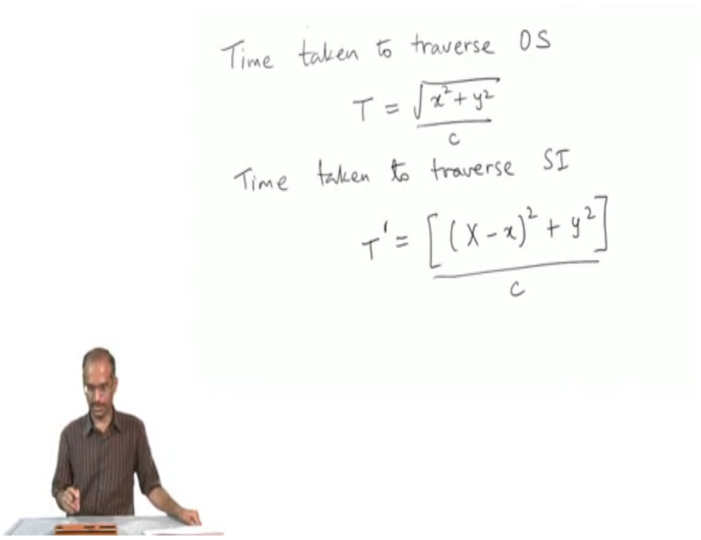

So, to begin with I need to know first the distances OS and SI and from the geometry of figure I can calculate both of them that would be $\frac{\sqrt{x^2+y^2}}{c}$, where c is the speed of light. T is equal to $\frac{\sqrt{(X-x)^2+y^2}}{c}$; let us give it a different name T' instead of T.

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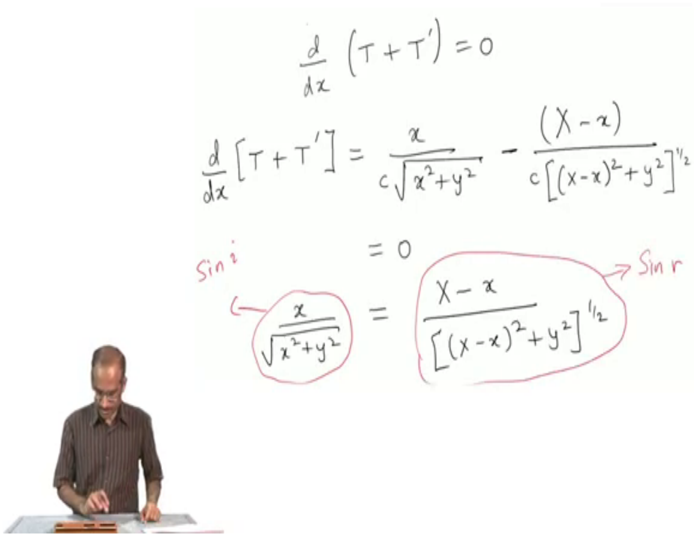

Now, Fermat's theorem tells us that $\frac{d}{dx}(T+T')=0$. So, substituting expressions for T and T' that we just got in here I will get and this is to be say $T=0$ and of course, there is a minus sign here when we differentiate and this is going to give me the following equation that. Now, if you look at these expressions on the left hand side and right hand side of this equation you will notice that this quantity here is simply equal to $\sin i$ and similarly what would be $\sin r$? That is precisely what I have here on the right hand side of this equation.

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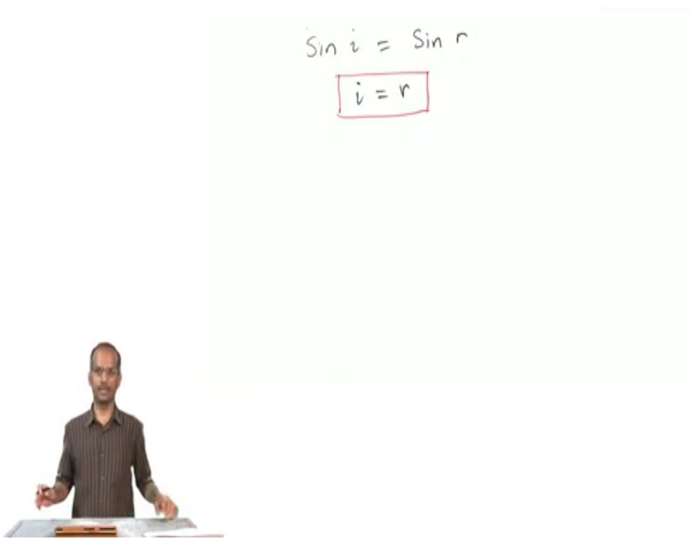

So, Fermat's principle in this problem tells us that $\sin i = \sin r$ of course, this is a problem with multiple possible solutions, but if we take the simplest possible solution this tells us that i is equal to r angle of incidence is equal to the angle of reflection. So, this is one of the commonly stated laws of reflection.

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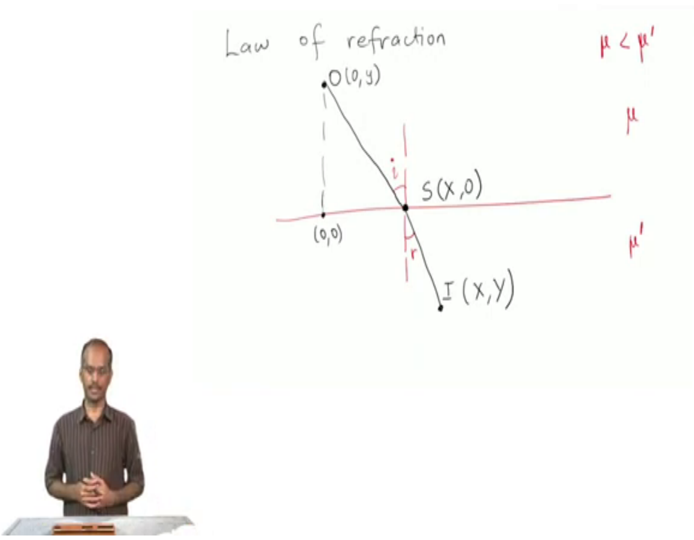

Now, let us do a similar kind of calculation for refraction here again I have this surface and presumably the media above and below the surface are different characterized by two different index of refraction.

So, here I have drawn my setting the point s is where refraction takes place is given by $(X,0)$ and at that point we draw this normal and define angle of incidence and an angle of refraction. Now, the question is what does Fermat's principle tell us in this scenario.

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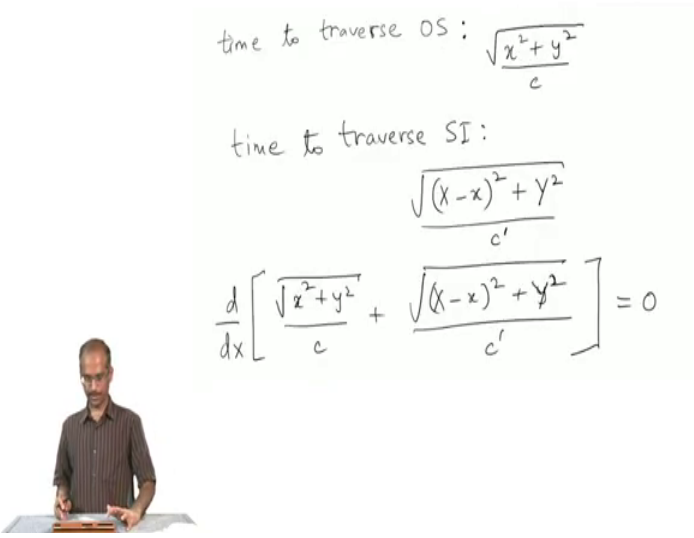

So, time to traverse OS that would be given by I need to calculate the time to traverse SI let me write down that expression in the expression I have divided it by c'. So, c is the velocity of or speed of light in the first media which is characterized by refractive index μ and c' is the speed of light in the second media which is characterized by refractive index μ'. So, now, I need to take the sum of these two times differentiate them with respect to x.

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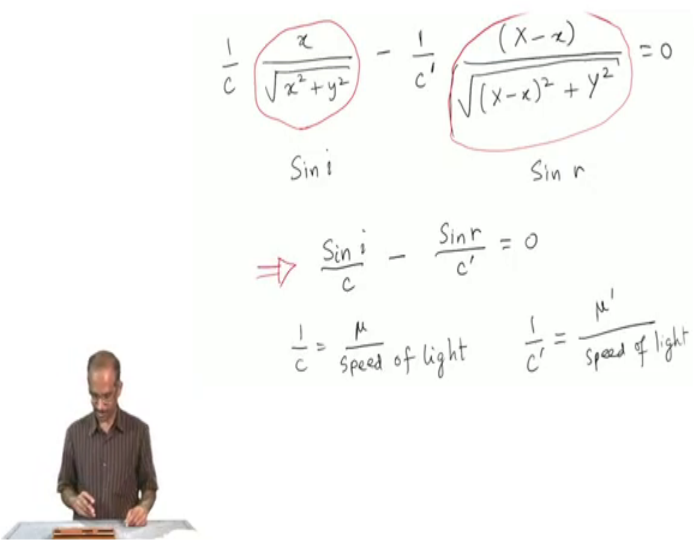

So, apart differentiating this expression with respect to x this is what I am going to get this term here and correlated with this figure that I have it will tell you that what I have circled here in red is simply sin i sin of the angle of incidence this quantity and again you correlated with this figure you will notice that that is simply equal to sin r sin of angle of refraction.

So, $\frac{1}{c}$ would be μ divided by speed of light and similarly $\frac{1}{c'}$ would be μ' divided by speed of light. Now, if I substitute these two quantities in this equation this quantity speed of light will cancel out and I would get the final form that.

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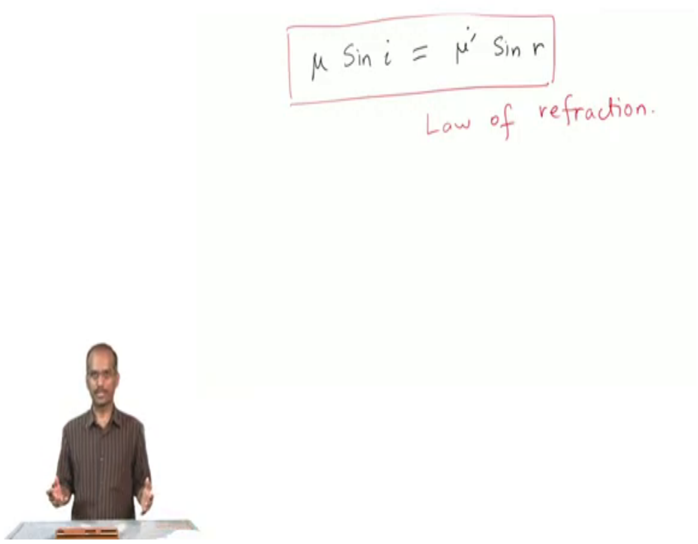

So, this is the law of refraction. In the next lesson we will try and look at some more examples of this, but involving optical elements, we will see how we can map out path of array in the presence of say lenses mirrors etcetera.