Waves and Oscillations
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Lecture – 38
Longitudnal Waves: Problems

So, in this lecture I am going to basically do a few problems, but as I do some of these problems I will try and point to, point out to you some interesting facets of the results.

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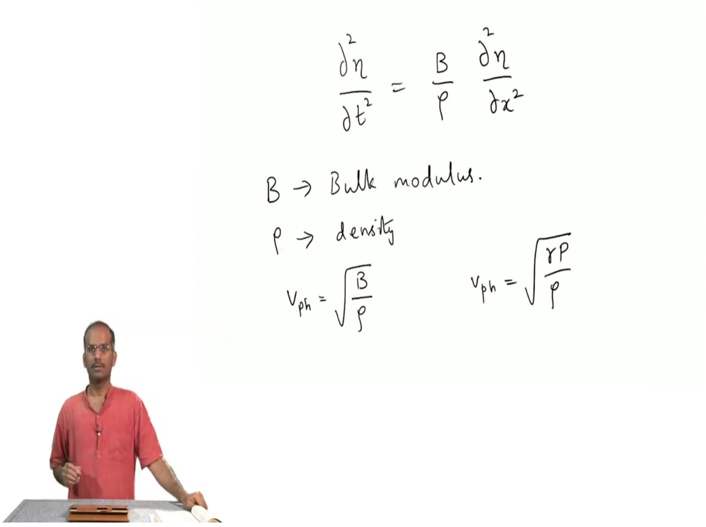

So, to begin with let me quickly point out that the basic and the fundamental equation that we are going to again and again use is this wave equation. η, here is the displacement. In the lecture today, we are going to confine ourselves to longitudinal Waves. And, the phase velocity here can be easily identified from this equation. $\sqrt{\dfrac{B}{\rho}}$, B is the bulk modulus and ρ is the density of the medium.

And when we specialize to the case of sound waves in air, as we had seen before this formula for the speed of sound becomes γ P by ρ; where P is the pressure, γ is the ratio of specific heat. This wave equation that we have obtained is also valid for longitudinal waves in a solid. Typically the additional complication in a solid is that we knew let us say, try and excite a wave in a solid. You are actually stressing the solid. In the sense that you applied some pressure, maybe you hit it with a hammer for example, and that is how you created a propagating wave inside a solid.

So, the situation is a lot more complicated in general. So, if we had a huge solid body and if such waves are traveling, so it is going to create pressure variations inside the solid. There is going to be dilation of the material in some places, there is going to be a compression of the material in some other places. What happens is that if you try to stress let us say in one particular direction, it is going to strain in directions that are perpendicular to it, so which means; that you have to worry about stresses in different directions and the resultant strains in all other directions.

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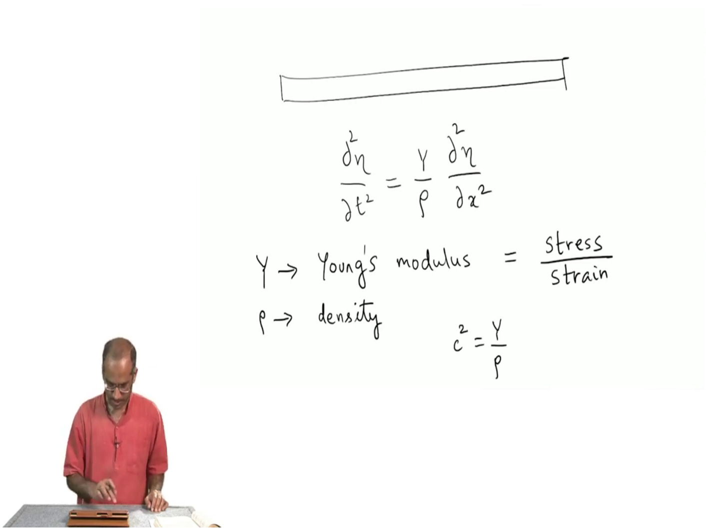

If I confine myself to thin object like this, something like a long rod where one of the dimensions is much smaller than the others.

In such a case, you could ignore these effects, these effects of strains being produced in perpendicular directions. In that case, it so happens that you could use the same wave equation as the governing equation for propagation of longitudinal waves in solid. Hence, we can still be fine with $\frac{\partial^2\eta}{\partial t^2}$ is proportional to $\frac{\partial^2\eta}{\partial x^2}$. So, here the proportionality constant will be bulk modulus by ρ, but bulk modulus should be replaced by Young's modulus here, by ρ.

So, Young's modulus is of course, stress divided by strain. So, that is the correct elastic property that should be used in the case of a solid. And here, it must be used more correctly in the case of a solid which are in the form of a rod or something similar, where one of the dimensions is much smaller than the others. So, in this case of course, once we have made this modification, immediately we can write out the velocity of sound in a solid as c^2 is $\frac{Y}{\rho}$.

So, once we sort of understand this generalization, before we start doing some of these problems. Let us look at the standard results.

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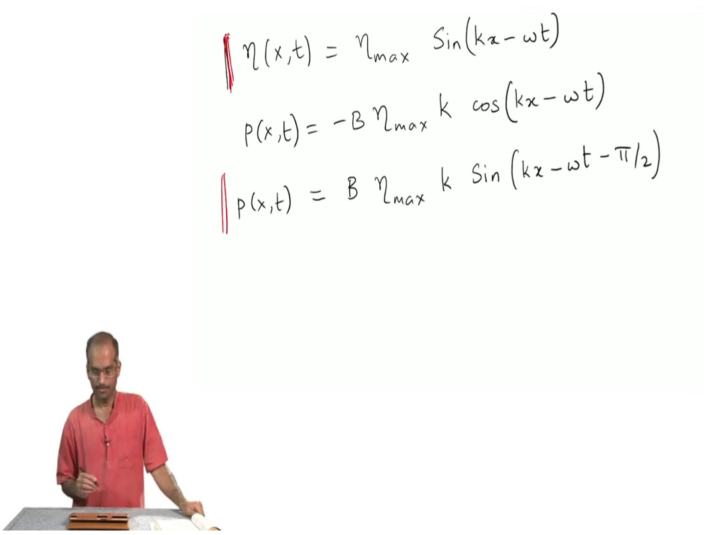

So, we know that the solution for the wave equation is η, which is the displacement as a function of position and time. So, it is either a sine or a cosine function. You can choose any of that until it is a function of $kx - \omega t$ finally. And the solution can also be written in terms of pressure variations. So, here the solution is written in terms of pressure variations, indicated by P; which is a function of both position x and time t. And note crucially that between displacement and pressure, there is always a phase lag, a phase lag of $\frac{\pi}{2}$.

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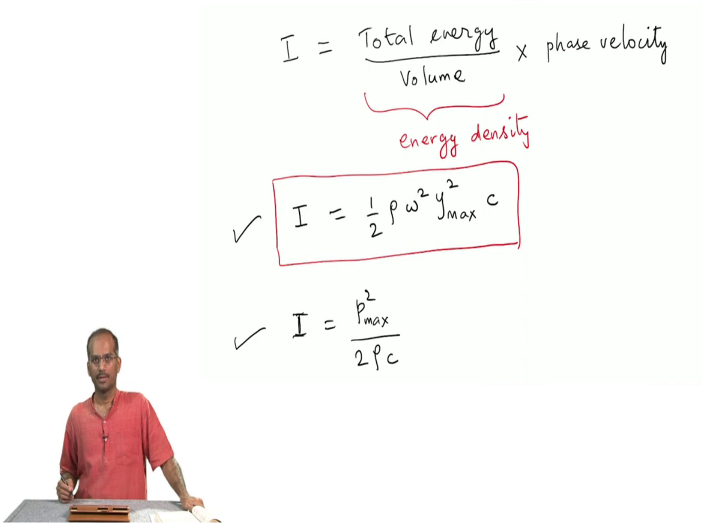

And we also saw that, we can define intensity of these waves. So, we went through this whole lecture on deriving the intensity as the total energy which is transferred per unit area.

So, when you go through all the derivation finally you get this result that, intensity is the expression that is given here. Importantly you notice that intensity depends linearly on ρ and on c. And the first of these expressions that I have written here is intensity which is in terms of the maximum amplitude. So, it is proportional to the square of the maximum amplitude. But as I explained in the last lecture, the intensity can also be written in terms of pressure variations or velocity variations or acceleration variations.

So, here I have written another of those expressions which is in terms of pressure variations, so the maximum amplitude of pressure which is P_{max}.

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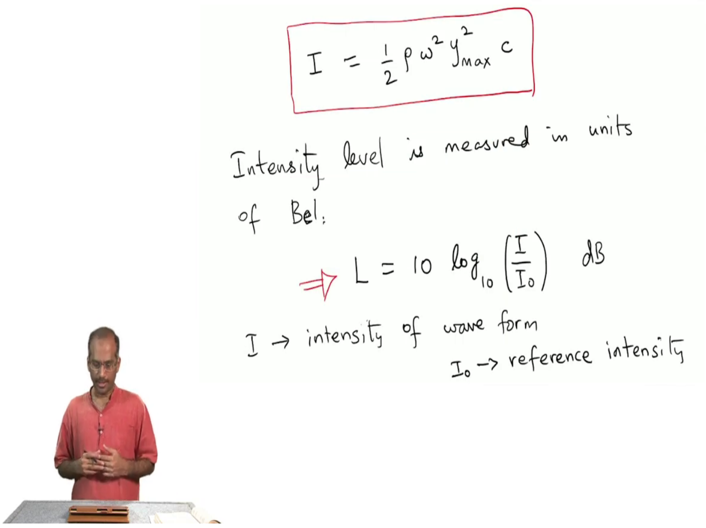

And then we went ahead and in order to standardize the intensity, we assume that there is a reference intensity level I_0; which is identified with the just about audible voice for a human being and that is I_0. And any other intensity is measured, as a ratio with respect to I_0. So, the intensity level is measured in units of Bel or Bel is a very large unit. Hence often the standard unit that is used as decibel, so that is given by this formula here, L is $10 \log_{10} \frac{I}{I_0}$; where I is the intensity of our waveform, I_0 is the reference intensity.

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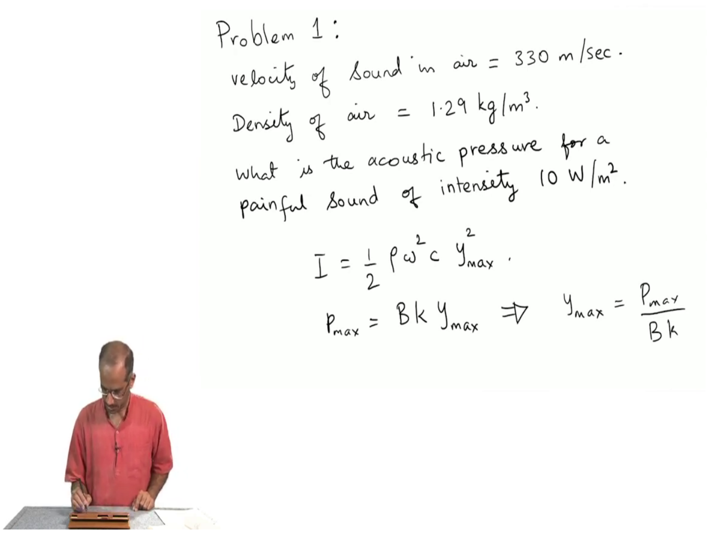

So, the question is as follows, the velocity of sound in air is given it is 330 meters/second and density of air is 1.29 kilogram/meter^3. The question is what is the acoustic pressure for painful sound of intensity, which is about 10 watt/meter^2 ? So, the sound intensity is 10 watt/meter^2. What is the corresponding acoustic pressure? So, we need to find the maximum amplitude of pressure variations, for this kind of intensity of sound.

So, let us collect two of the formulas that we know. So, one is I is equal to half $\rho\ \omega^2\ c$; c is the velocity of sound; $y_{\max}^2$. So, this form relates intensity to the maximum of displacement, which is $y_{\max}$. And we also know that, $P_{\max}$ is related to $y_{\max}$, $B\ k$ into $y_{\max}$. So, you will notice one thing that, I have been using this η and y interchangeably, for me both are displacements, both are longitudinal displacements. If you want to fix your ideas, you can think of them as longitudinal displacements of a parcel of gas, for example.

So, most of the time I seem to have used η in the equation of motion, in the wave equation and I have quoted the solution here in terms of η; but many times when I was doing problems, I have used y. So, you should keep in mind that both η and y represent displacement for us. And $y_{\max}$ or $\eta_{\max}$ is the amplitude. Problem is sort of simple. All I need to do is to substitute, eliminate $y_{\max}$ because I do not know $y_{\max}$. What I know is intensity of sound and what I want to find out is $P_{\max}$.

So, using these two equations, I can simply eliminate $y_{\max}$ and directly relate intensity to $P_{\max}$. So this, for example will give me that, $y_{\max}$ is equal to $P_{\max}$ divided by, B is the bulk modulus and k. So, if I substitute this, in this equation I will get the following.

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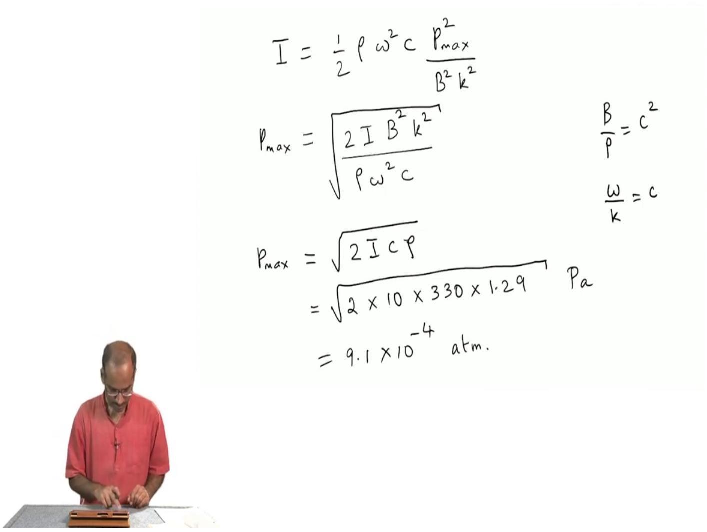

And from this relation, I can extract what is P_{max}. So, that would be $\frac{2IB^2k^2}{\rho\omega^2 c}$ and of course, I need to take a whole square root here. But you will notice that, we do not have all the information that is needed for this formula. For instance, we are not told what is the bulk modulus and k the wave number. So, we do not have all this, but clearly there has to be a way out.

The point is to recognize that $\frac{B}{\rho} = c^2$, hence we can eliminate B using this relation and also that $\frac{\omega}{k} = c$. So, if I substitute all this in this equation, you will notice that. So, this tells me that, to calculate P_{max}, I need to know intensity, velocity of sound, and the density. That is exactly the information that we have. So, we just need to put in all those numbers. And that will be 2 multiplied by intensity which is 10 watt/meter^2 and c is given as 330 multiplied by 1.29 which is the density. And this, you can use your calculator to calculate.

So, notice that this one will give you the pressure in Pascals and I have converted it into atmospheres and the reason is that, the standard atmospheric pressure is of the order of 10^{-6} atmospheres.

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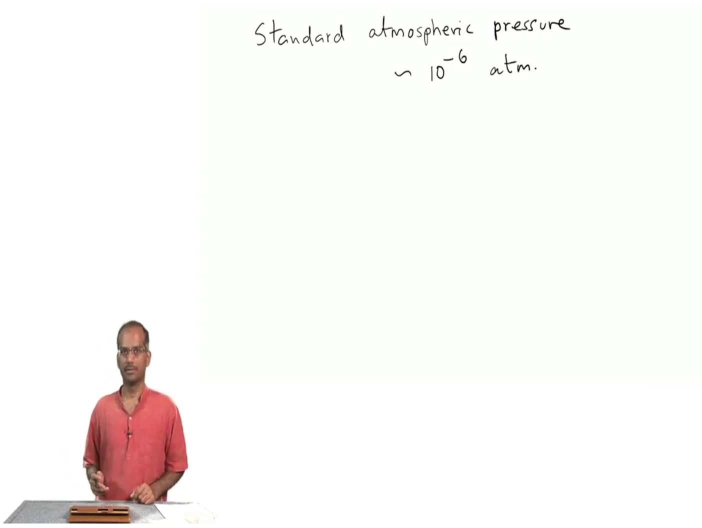

So, clearly the value that we have obtained for a painful sound of intensity 10 watt /meter^2 is 100 times larger than the standard atmospheric pressure. So, which means that if you are exposed to such a sound whose intensity is like 10 watts/meter^2, we are talking of 100 times more pressure on your, let us say the eardrum than what is imparted by the standard atmospheric pressure.

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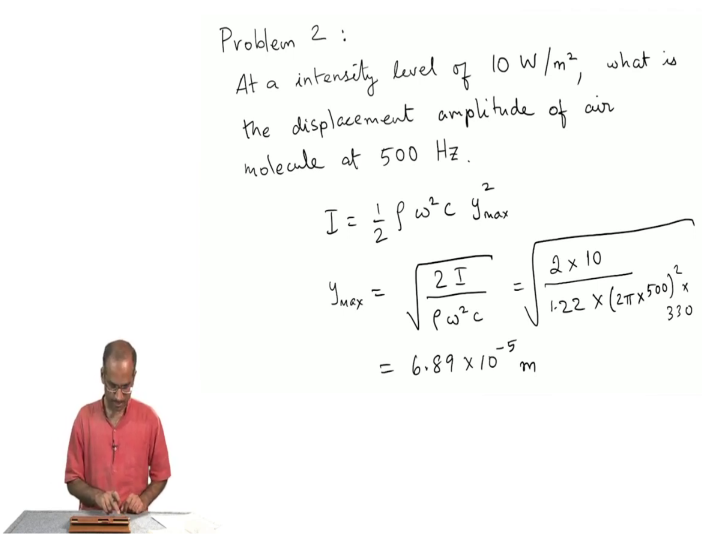

The next problem is as follows. So, there is sound at an intensity level of about 10 watts /meter^2. The question is what is the displacement amplitude of air? So, which means find the maximum of the displacement of your air parcels, when the sound is of about 500 Hertz frequency? So, y_{max} will be $\frac{2I}{\rho\omega^2 c}$ square root. And now we just need to put in the numbers, keeping in mind that ω is 2π times the frequency; where the frequency is given as 500 Hertz. So, this will be 2 multiplied by intensity which is $\frac{10}{\rho}$.

Of course, we know the value of ρ, which is about 1.22 multiplied by, so this is 2π times the frequency whole square, 2π times; frequency is 500, whole square multiplied by c, c is of course the velocity of sound, which I will take as 330. So, this is the displacement amplitude for an intensity level of 10 watt/meter^2.

Now, you should compare with what we had done before.

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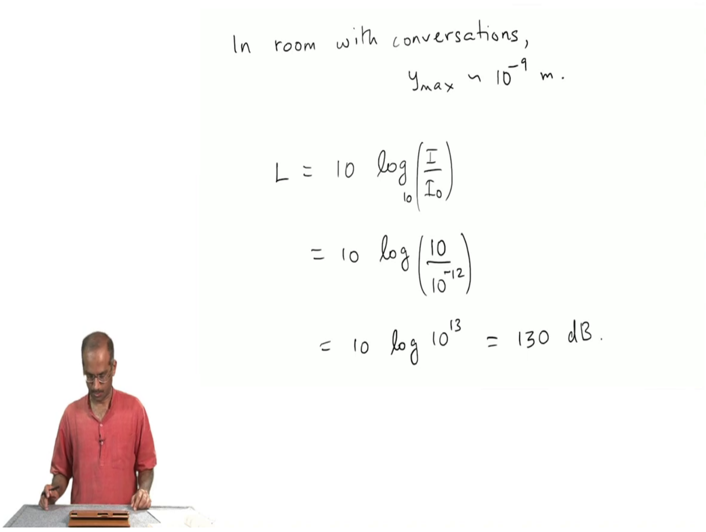

Earlier on towards the end of the last module we said that, in quiet room or room with conversations between just one or two people, we saw that the displacement amplitude y_{max} was of the order of 10^{-9} meters. So, clearly in this case, the amplitudes are 10^4 times larger compared to the amplitude of sound in a quiet room. We can also calculate what would be the sound level in decibels and that would be $10\log\frac{I}{I_0}$; log taken in with respect to base 10, so this would be 10 times log of 10 divided by, 10^{-12}.

So, this gives us $10\log 10^{13}$. So, that is 130 decibels, which is very high. Continuous exposure to such kind of sound levels can be quite damaging to the ears.

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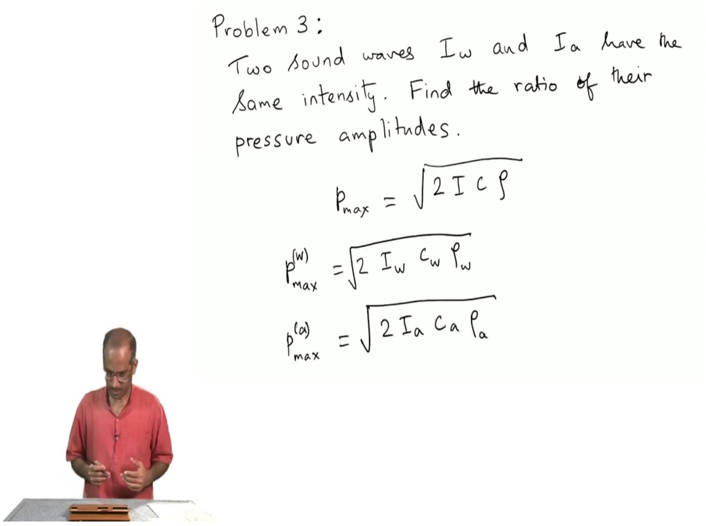

The third problem, we have two sound waves. One of them, a progressive wave in water and other in air and we are told that both have the same intensities. So, ideally this I_w and I_a that I have written down are actually equal to one another. So, the question is, find the ratio of their pressure amplitudes or the maximum of the pressure displacements.

So, I had already introduced to you this formula, which relates the pressure amplitude or amplitude of pressure variations to the intensity I. So, we are going to use this one and from here it is a very straightforward exercise. $P_{\max}$ is $\sqrt{2Ic\rho}$; c is of course the speed of sound. So, now, we have two media, water and air, so we just need to consider both of them. So, let me say that P_w, so the w tells you that it is water; maybe I will put it in brackets. So, this is $2I_w$ to indicate that intensity of sound in water, c velocity of sound in water and ρ density of water.

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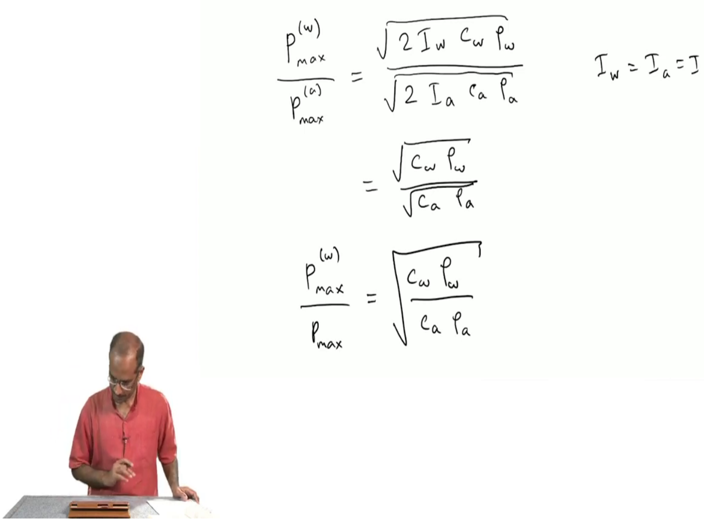

So, we just need to divide one by the other, to get the ratio; the ratio is equal to $\frac{c_w \rho_w}{c_a \rho_a}$.

So, the information that I need is, the speed of sound in water, density of water speed of sound in air and density of air.

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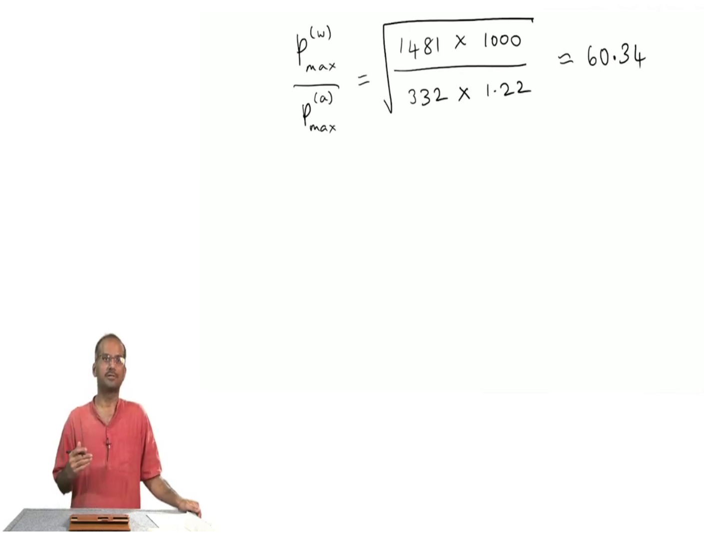

So, speed of sound in water is roughly 1481 meters/second, multiplied by the density of water; which is about 1000 kg/meter^3 divided by speed of sound in air which is 330 or approximately 330 or 332 multiplied by density of air which is 1.22 and we need to take the square root.

So, if you do this, we will get something like 60.34. The amplitude of pressure variations in water when say, a sound wave travels through it, 60 times larger than the corresponding pressure variations in air for a fixed intensity of sound waves.

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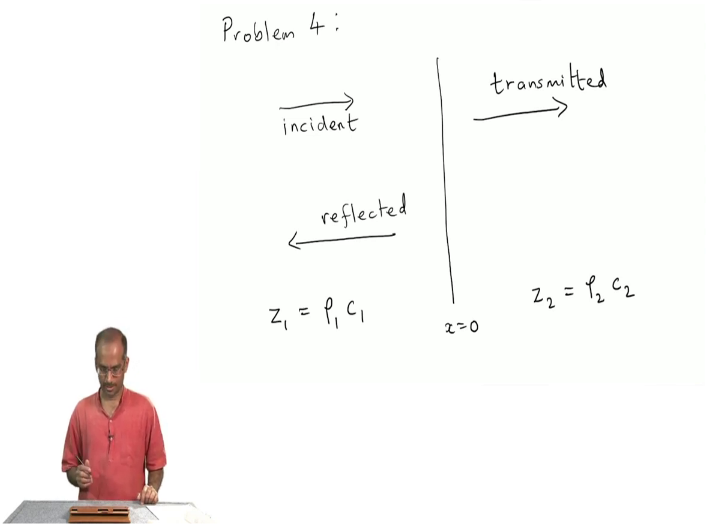

So, the last problem I am not going to do in full. It is a version of the problem that we had done before.

So, it is a same question that keeps occurring again and again. Suppose sound waves traveled from one medium to another medium, what happens? So, you have an incoming wave form and then reflected wave form. So, this is reflected. This is of course, the incident wave and then there is a transmitted wave in the second medium. So, here, so you could assume that $x = 0$ is the point where the interface between two media is; and the first medium corresponds to acoustic impedance Z_1, which will be $\rho_1 c_1$; and the media on the right hand side corresponds to acoustic impedance of Z_2 which is $\rho_2 c_2$. So, ρ_2 is a density and c_2 is the velocity of sound in that medium.

So, the question is standard. Like, if I had an incoming wave; what fraction of it is reflected back into the same medium, what fraction of it is transmitted into the second medium. The way to do it is precisely the same as what we had done with the transverse waves. The only difference and the crucial difference comes in the treatment of the boundary conditions. So, here we will have two boundary conditions like we had in that case there.

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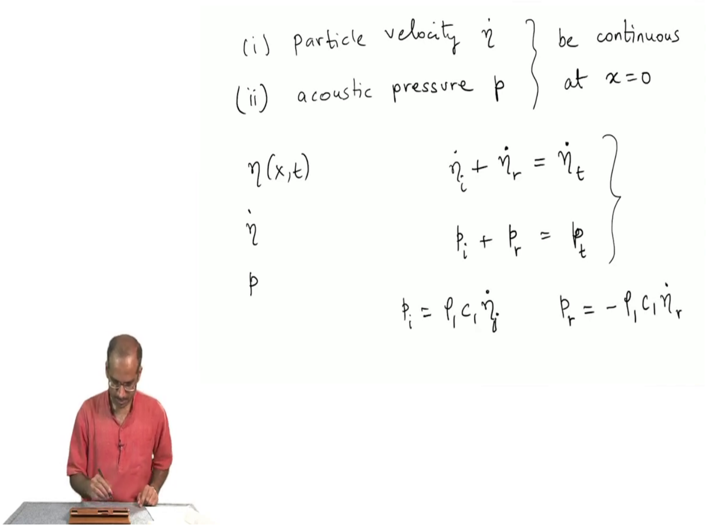

So, one is, that the particle velocity which is $\dot{\eta}$ has to be continuous at $x = 0$. And, secondly, the acoustic pressure given by P; that this has to be continuous at the interface between the two boundaries.

Clearly you can see that the boundary conditions are physically motivated. So, you do not want particle velocities to be suddenly changing across the boundary. Does not make physical sense, it does not happen; neither does pressure suddenly change in the absence of any other external mechanism to make it happen. Hence, we want that both these quantities be continuous at $x = 0$, which is the interface between the two media. So, this is one set of relations that you will get from implementing these boundary conditions.

So, now putting all these together, we will have the following equation which is.

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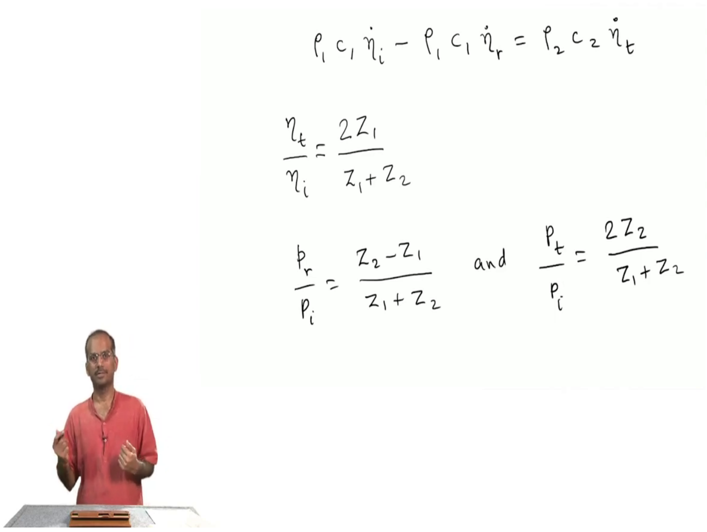

So, now rest of the exercise is to simply write these equations in terms of Z_1 and Z_2. So, eliminate everything else. So, that your final result for the ratio of $\frac{\eta_t}{\eta_i}$ and $\frac{P_t}{P_i}$ comes out in terms of Z_1 and Z_2. So, I will leave it as an exercise for you to do that. So, that $\frac{\eta_t}{\eta_i}$ should finally, turn out to be $\frac{2Z_1}{Z_1 + Z_2}$. $\frac{P_r}{P_i}$ will be $\frac{Z_2 - Z_1}{Z_1 + Z_2}$ and you can also calculate $\frac{P_t}{P_i}$, which will be $\frac{2Z_2}{Z_1 + Z_2}$.

So, again you will come back to the familiar result that the, intensities of waves that are transmitted and the intensities of the waves that are reflected. The sum of these is equal to the intensity of the incoming wave. So, it is just a restatement of the principle of energy conservation. It is a simple question of manipulating these three equations to obtain these results. And further, I would urge you to rewrite all these in terms of the intensities and you will see that you can finally make a statement that the incoming intensity has been distributed between the transmitted intensity and the reflected intensity.