Waves and Oscillations
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Lecture - 37
Sound Intensity

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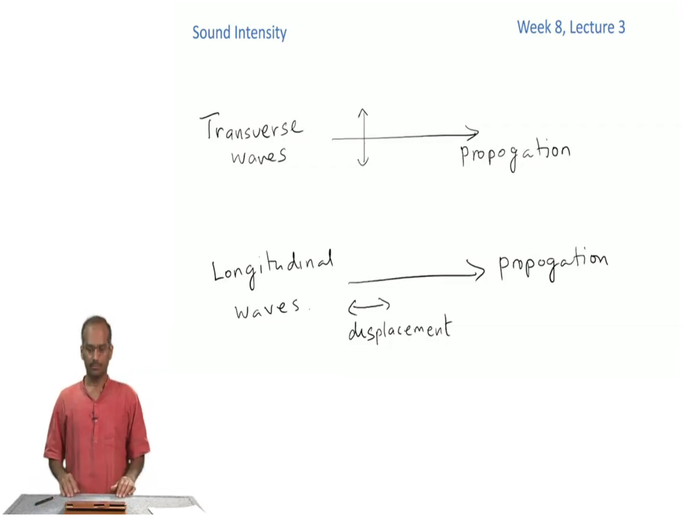

So, welcome to the third lecture, we are in the 8th week. So, we were looking at the equation that governs the propagation of sound waves and we also were able to identify the speed of sound. We also realized that sound wave is a form of longitudinal wave. So, as I shown here in this slide, longitudinal waves, just to remind you again, are the ones where the displacement and the direction of propagation are in the same direction or parallel to one another more correctly.

As opposed to transverse waves where the direction of propagation and the direction in which the individual oscillations take place they are perpendicular to one another.

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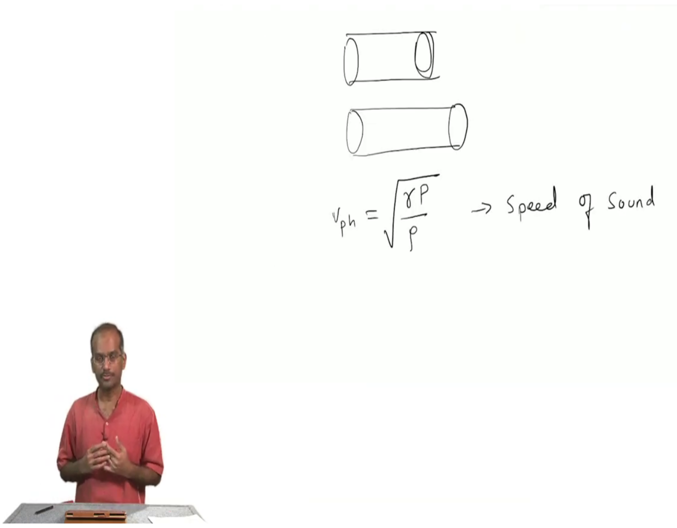

You think of sound waves as something like a parcel of air which one gets disturbed, disturbs the next parcel of air and so on and so forth. So, you create a disturbance here. I can do that just by moving my hand across the air here and that is going to convey the disturbance to the next parcel and to the next parcel and so on.

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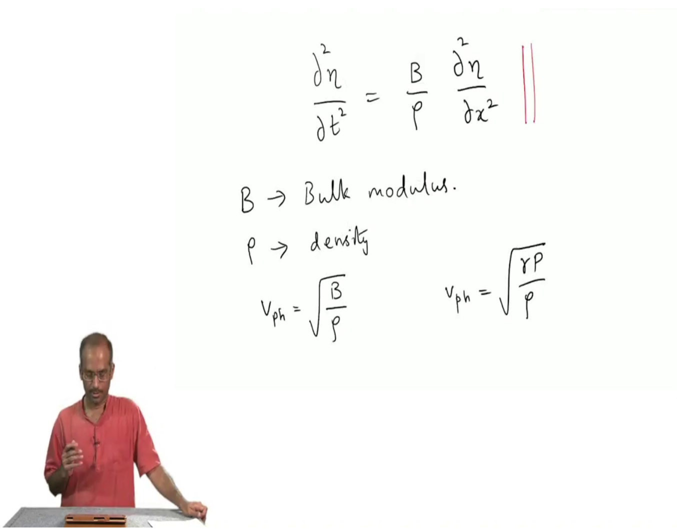

The equation got that governs the propagation of the wave is given the way it is shown here and η is the displacement. Displacement let us say of your parcel of air. And it is a function of both position and time. B is the bulk modulus and ρ is the density of the medium. And from this you can immediately write down the an expression for velocity the phase velocity which is $\sqrt{\frac{B}{\rho}}$ and for the case of sound waves go back to the ideal gas equations and you can replace bulk modulus and finally, write an expression in terms of pressure. So, the speed of sound is $\sqrt{\frac{\gamma P}{\rho}}$.

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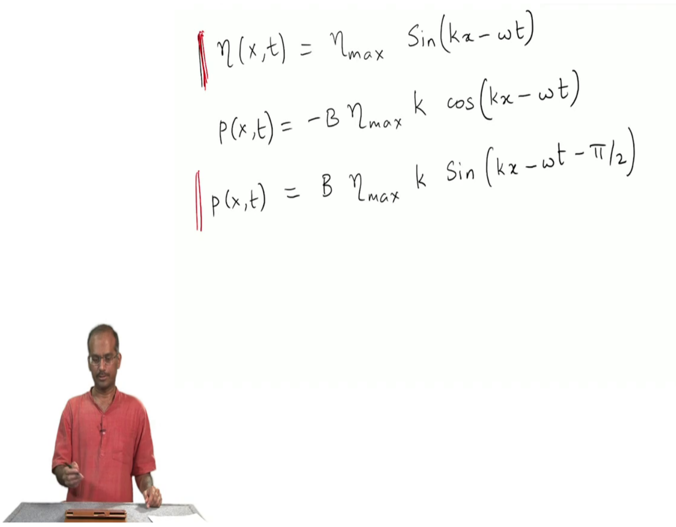

To summarize all this together we have η which is displacement as a function of time and its given by this solution of the wave equation and remember that η and the excess pressure P are related through a gradient $\frac{\partial y}{\partial x}$ and hence you can also write an expression for the excess pressure. So, when I mean excess pressure it is the pressure above or below the atmospheric pressure. And again that has this expression which is in terms of this cosine function.

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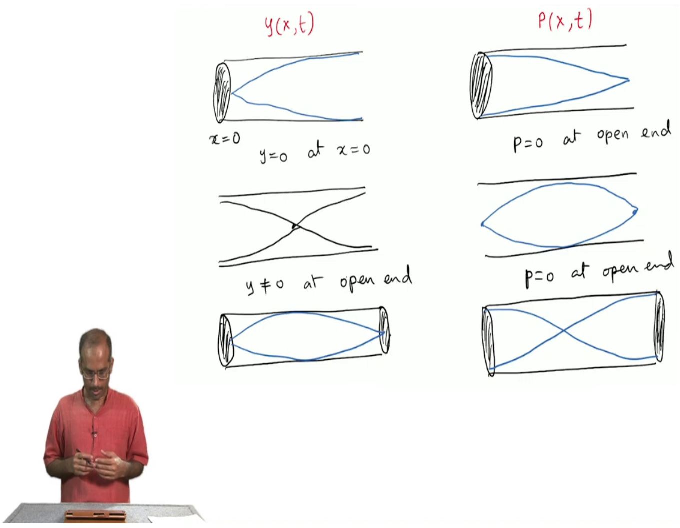

And we discussed about the boundary conditions. So, at the open end the displacement is not equal to 0, but the excess pressure is equal to 0 simply because at the open end the pressure has to be continuous with the atmospheric pressure, which is lying just outside the your cylindrical body. So, with these boundary conditions we also saw the progression or systematics of the standing modes in such a body and one can also look at velocity as a function of position and time, then there is even acceleration which is a function of position and time, and even density changes.

So, pretty much all these quantities are varying as a function of position and time and associated with each one of these quantities there is an amplitude. Going back to one of the earliest lessons that we learnt that the energy is proportional to square of the amplitude. So, if you keep that in mind you will realize that somehow the intensity has to be proportional to square of these amplitudes.

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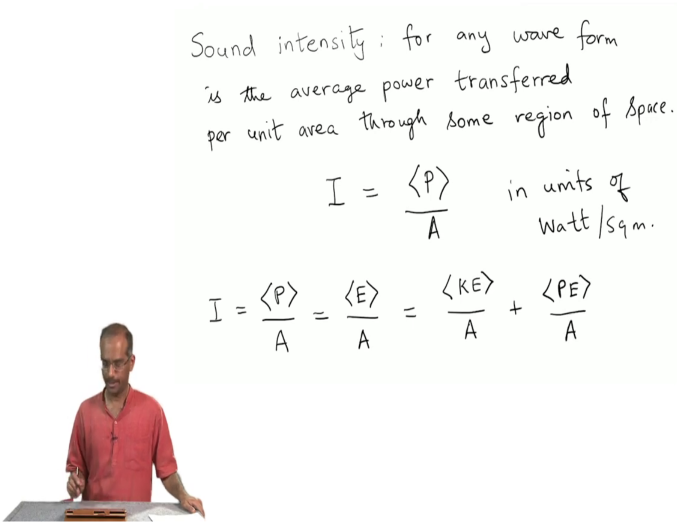

Sound intensity for any waveform is the average power that is transmitted per unit area through any region of space. If I interpret it in a sort of straightforward way I would say that intensity is simply the average power per unit area. Now, from this definition I could write I to be $\frac{\langle P \rangle}{A}$ and which I would say is the average energy transferred per unit area.

And the average total energy that is transferred per unit area you could of course, compartmentalize it into two parts one is the energy that is coming from the kinetic energy term and the other which comes from the potential energy term. So, this I could write it as the average kinetic energy per unit area plus the average potential energy per unit area.

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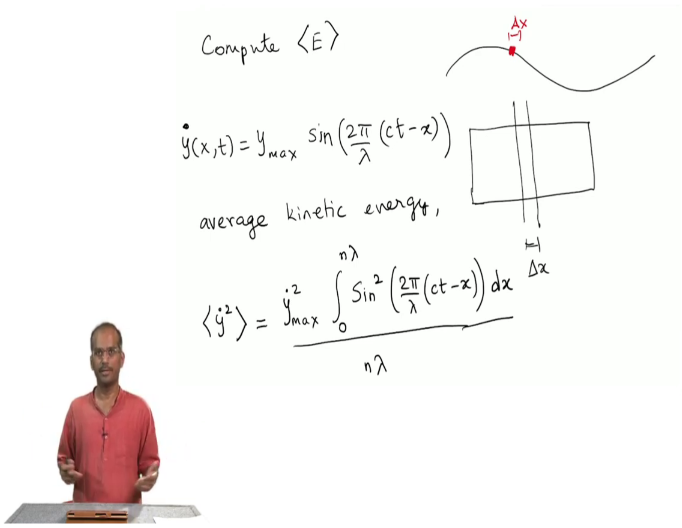

So, if you remember an exercise that we did some time back when we looked at how much energy is being transferred. So, we said that in waveforms you consider a small segment of the wave. So, in the earlier argument we were looking at the string and we would wanted to and we wanted to know how much of energy is transferred by the waveform that is propagating through a string.

So, in that case we said that we will consider a small segment of the string. You can say that the segment of the string has a length δx and if you focus your eyes only on this small segment of the string that is δx, this segment of the string is doing nothing more than oscillating up and down. So, it is just executing simple harmonic oscillations. And we know what the energy of the simple harmonic oscillator is from what we had studied several weeks back. So, all we needed to do was to simply put in the expression for that energy and compute the result.

Now, let us translate this technique for our case. So, what we have is a parcel a small parcel of gas maybe and I am going to consider an even smaller strip of this parcel which I will call it by stating that. The smaller strip has a length that is δx. So, we will consider the motion of this small strip of length δx. Since, I want to compute kinetic energy, let me directly state that $\dot{y}$ which is $\frac{\partial y}{\partial t}$ can be written in this form.

So, this implies that I have already chosen some form for the displacement as a function of position and time and that will be a cos function, only then $\dot{y}$ will give me a sine function, but the results do not depend on whether I choose my initial solution to be a cos or a sine function. So, I just have to make any one choice and go ahead with the calculations, so that that would be consistent. And since, I am going to require only $\frac{\partial y}{\partial t}$. I am directly stating that $\dot{y}$ has this form.

Now, I want to calculate the mean kinetic energy, I mean that it is averaged over one or over several wavelengths. The average of $\dot{y}^2$ will give me a following relation. So, what I have done is, so write the expression for $\dot{y}^2$ over a small strip δx. So, n is the number of wavelengths that I am adding these small strips and then divide by the total length which will be $n \times \lambda$, so λ is one wavelength here.

So, while doing this integral we need to keep in mind that we are integrating over a total length which is equal to $n \times \lambda$. And t will be n times the time period and of course, time period itself will be given by $\frac{\lambda}{c}$, where c is the phase velocity or velocity of wave. So, in this case that is the speed of sound.

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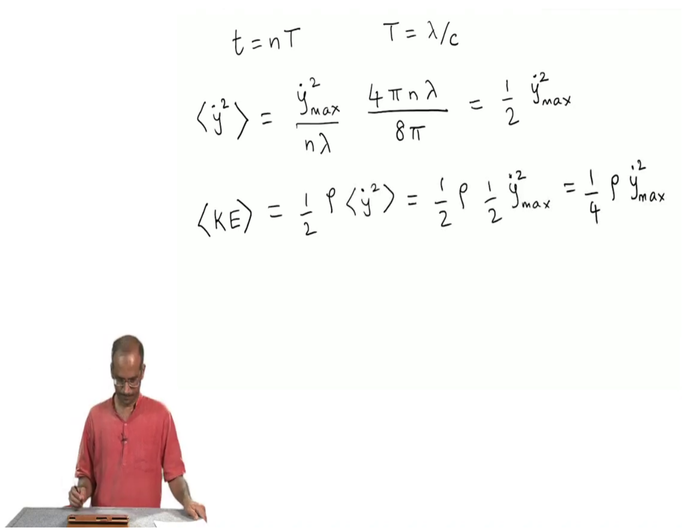

So, this is the result that I get from integrating it. The integral itself is not very hard to do it is a $\sin^2\theta d\theta$ kind of integral. I can write the average value for the kinetic energy. The quantity that I have on left hand side should more properly be called as the kinetic energy density.

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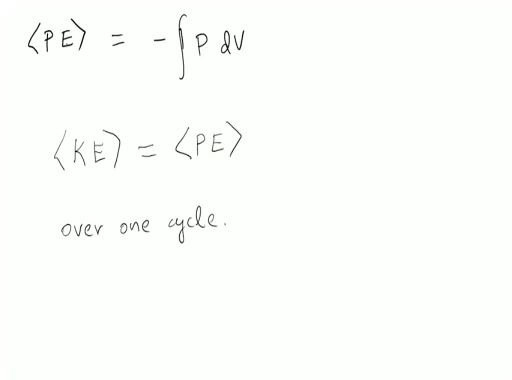

So, the next part is I want to calculate the average potential energy density. So, potential energy is the work done on a fixed mass of gas during adiabatic changes in the sound wave. So, this quantity can be written as $-\int PdV$. So, PdV is of course, the expression for work done by a gas. And the negative sign that you see here should tell you that, if pressure increases volume decreases or if volume increases pressure decreases. So, there is that inverse relation between pressure and volume. So, the negative sign basically indicates this sort of an inverse relation.

So, even though we can calculate the average potential energy by looking at the work done by an adiabatic process which is given by this integral pressure integrated over the volume it is a longer haul. So, instead it is more easier to use something that we had learnt when we were looking at the simple harmonic oscillator. So, there one of the results was that the average kinetic energy and average potential energy were exactly equal to one another.

So, where both the kinetic and potential energies were averaged over one cycle. So, if we use the fact that average potential energy or one cycle is equal to the average kinetic energy over one cycle, we have already calculated the result for the average kinetic energy and we just need to restate the same result that the average potential energy is equal to what we had already calculated. So, instead of recalculating the average potential energy, we will just use this result and proceed ahead to calculate the average energy density.

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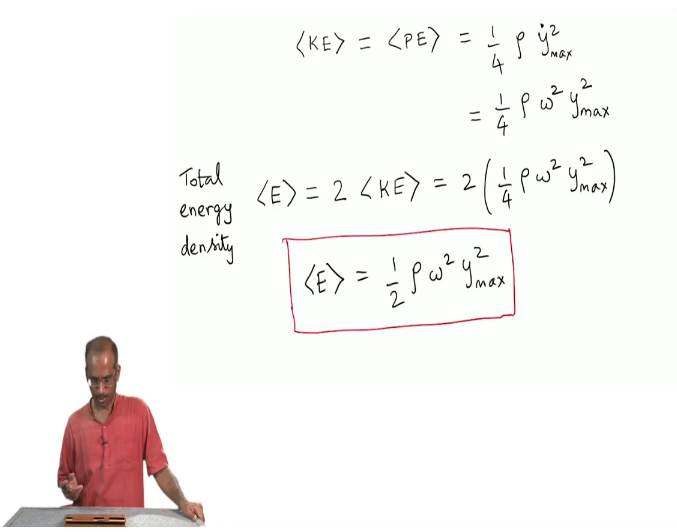

So, both are equal to $\frac{1}{4}\rho\dot{y}_{\max}^{2}$. And this expression can also be written in terms of the amplitude of the displacement itself in which case all these will simply boil down to $\frac{1}{4}\rho\omega^{2}y_{\max}^{2}$. Since, the kinetic and potential energies are exactly equal the total energy is simply 2 times any one of them.

So, let me take it as 2 times the averaged kinetic energy. Hence, the total energy density is equal to 2 times $\frac{1}{4}\rho\omega^{2}y_{\max}^{2}$ and that is equal to $\frac{1}{2}\rho\omega^{2}y_{\max}^{2}$. So, this is our expression for the total energy density.

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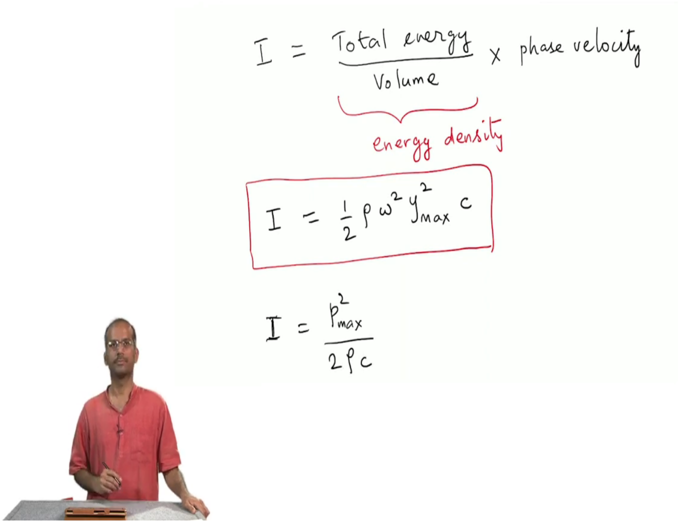

Hence, finally, intensity will be the total energy density transferred which will be total energy divided by volume and this is the energy density multiplied by the phase velocity, and this quantity let me remind you is the energy density. So, what we have already computed here is the average total energy density. So, all I need to do is to simply substitute it here. So, I would have $\frac{1}{2}\rho\omega^2 y_{\max}^2$ multiplied by c, which is the phase velocity. So, this gives me an expression for the intensity.

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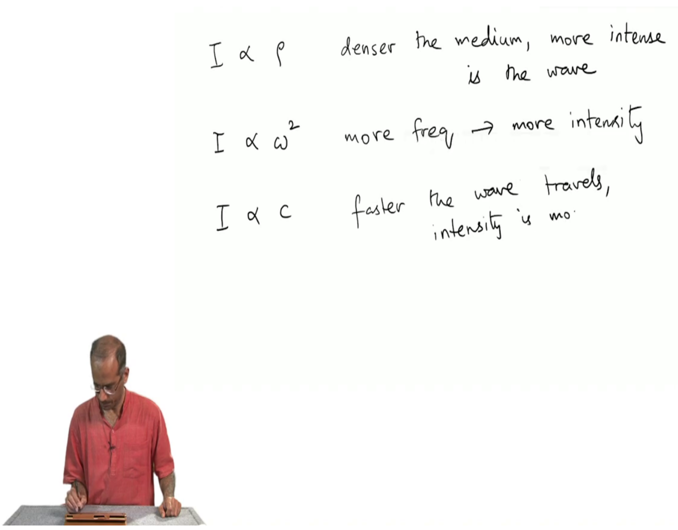

Now, let us collect these information together. So, first is that the intensity of sound is directly proportional to the density. Clearly, it tells me that denser the medium more intense is the sound or more intense is the wave. Secondly, intensity is proportional to square of the frequency. So, this is the maybe let me use the angular frequency, its proportional to the angular frequency, hence more frequency implies more intensity.

And similarly, intensity is also proportional to the speed of the wave, faster the wave travels intensity is more. Expression that we have derived for intensity explicitly is in terms of the displacement amplitude. So, the displacement amplitude here is $y_{\max}$, as I said we could have derived it in terms of velocity amplitude or acceleration amplitude or even pressure amplitude for instance if you go through a similar derivation and obtain an intensity formula for, intensity formula for a wave form in terms of the pressure amplitude we will get an expression of this form.

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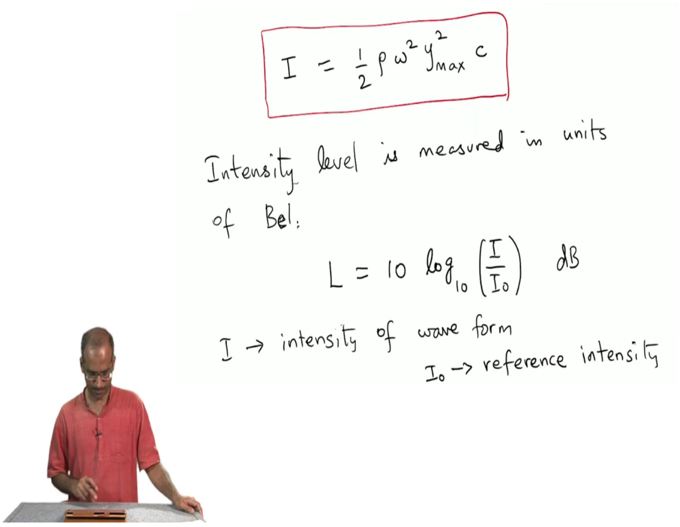

In general for any periodic signal, the intensity level is measured in terms of in units of Bel. The intensity level is defined as follows. So, let me call it L that would be $10 \log_{10} \frac{I}{I_0}$. So, I is the intensity of our waveform and I_0 is some reference intensity. So, this way of measuring intensity is equivalent to saying that I have decided some reference level and every other intensity will be measured with respect to that reference level. And since, we are dividing I by I_0 the units would cancel and the level that we get would be a dimensionless number.

The way the intensity level is defined Bel is a very large unit. So, in general this formula measures in units of dB or decibel. So, clearly 1 Bel is divided into 10 smaller units. So, by convention reference intensity is taken to be the threshold sound intensity for humans. So, what is the lowest intensity that is just about audible for a human being?

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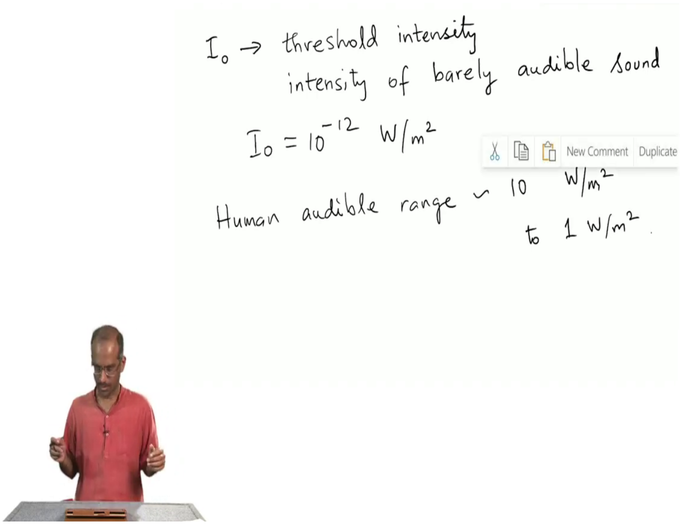

And the intensity of such a sound is of a magnitude that is of the order of 10^{-12} Watt /meter^2. So, with respect to this we can compare any other sound, and it makes sense because any intensity smaller than this is not something that humans can hear. And in general, human audible range runs from an intensity that is of the order of 10^{-12} Watts /meter^2 to 1 Watt/meter^2.

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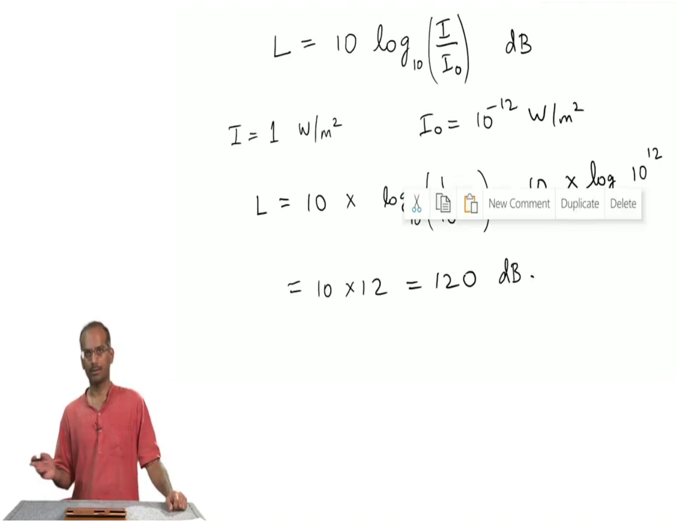

So, clearly the audible range is quite large. To get the full audible range, I need to simply set I equal to 1 Watt/meter^2 and I_0 is 10^{-12} Watt/meter^2 in which case L will be $10\log_{10}\frac{1}{10^{-12}}$ and that is just $10\log_{10}10^{12}$. So, that is going to give me 10 multiplied by 12 which is equal to 120.

So, between the lower limit which is 10^{-12} to upper limit the range is 120 decibels or 12 Bels. So, what we know is that from typical measurements, sound intensity is in a generally quiet place, maybe a quiet home or even a quiet office where there are no huge amounts of external noise; in all such cases the intensity is about 10^{-8} Watts/meter^2.

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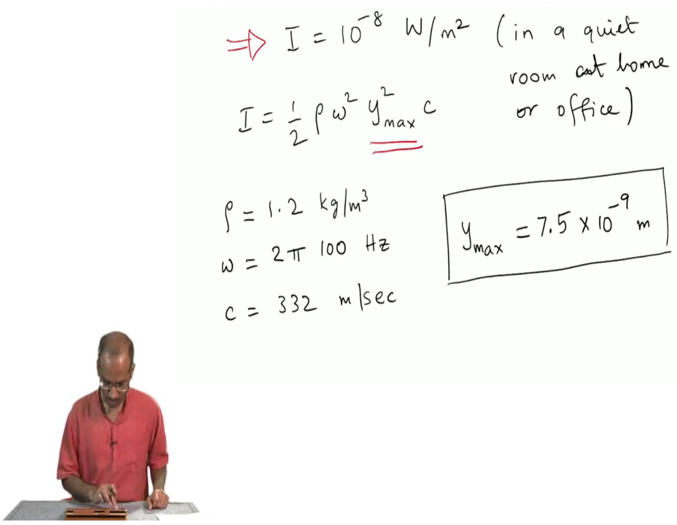

I want to find out in terms of decibels what is a sound level? And given this intensity, I can also find out what would be the amplitude of the displacements of the parcels of air which gives rise to sound intensities of this, for that purpose we need the relation that we just derived.

So, we said that intensity is equal to half $\rho\ \omega^2\ y$ mark y_{max}^2 multiplied by c. And to get this information I need a few things one is density of air which is about 1.2 kilogram /meter^3 and I need also the angular frequency which is 2π times the frequency of our normal speech. So, when few people are speaking typically the kind of frequencies involved are of the order of about 100 to 200 Hertz. So, let me take the number as about 100 Hertz and c of course, is the speed of sound roughly about 332 meters/second.

So, now I have all the information I know ρ, I know ω, I know c and I know the intensity measured intensity in a quiet room or an office like this and from this I can find out what is y_{max} it is a simple exercise to do. So, you can put in the numbers and you will realize that y_{max} that we are getting. So, in typical situations, the displacements of the air parcels are no more than a few 100 or a 1000 times the molecular diameters. And given that this is the intensity in a quiet office or a room let us calculate what is the sound level.

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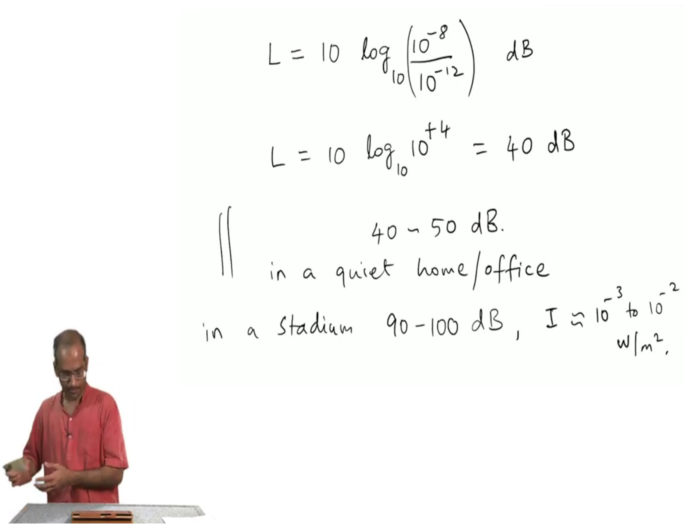

So, for which I need to know this formula $\log_{10}$ and the intensity that I have in mind is 10^{-8} divided by 10^{-12} dB. So, this will be 10 and of course, this is $\log_{10}$ 10 power minus 10^4 and this is about 40 decibels. So, in a place where there is not too much of external noise, just few people talking with one another you could expect sound levels to be in the range between about 40 to 50 decibels, I would say in a quiet home or office. The kind of noise that you get in a stadium would be of the order of about 90 to 100 decibels.

In a stadium corresponds to intensity level of about 10^{-3} to 10^{-2} watt/meter^2. Now, that we have some number for the intensity level in a quiet home or office and here I am the only person speaking, so I could quickly check whether it is right or not. For instance I have opened android app which directly measures and tells you the sound level, for instance in a place like this it has measured an average of about 49 decibels. So, you can see that the number lies somewhere between 40 and 50 in a typical place that has no external noise.

In the next module, we will do some problems related to sound waves and all that we have learnt in this week.