Waves and Oscillations
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Lecture - 35
Longitudnal Waves and Speed of Sound

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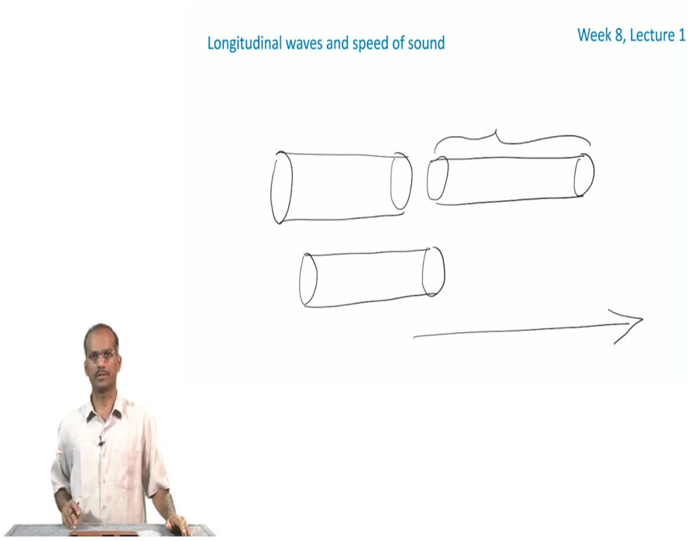

Welcome to the 8th week, this is the first lecture our canonical idea of what a wave is basically a mechanical disturbance but while the disturbance itself travels forward in one direction; the up and down motion is perpendicular to it whereas what you see as wave phase disturbance travels in a direction that is perpendicular to that up and down motion.

So, there is an oscillatory motion which is about some equilibrium position and then we do see a wave the disturbance that is propagating in a direction that is perpendicular to the direction in which the simple harmonic oscillation is taking place. So, that is what would be called as a transverse waves because the direction of propagation is transverse to the direction of up and down motion for instance.

So, on the other hand today we will start by looking at Longitudinal Waves; let us say that I have a small parcel of air in this volume and when you displace it a little bit I mean you can very easily do this displacing; a small parcel of air by very many different ways just push your hand like this in the air you would have done that or make a noise you would have done that. So, there are many ways of doing this.

So, you push this small parcel of air. So, it is going to move a little bit let us say in this sign and even as it moves; the volume is going to change, the volume of this parcel is going to change and when all this is happening it is also going to disturb the next parcel which is; which is probably somewhere here. So, as a whole this disturbance is going to propagate maybe in this direction.

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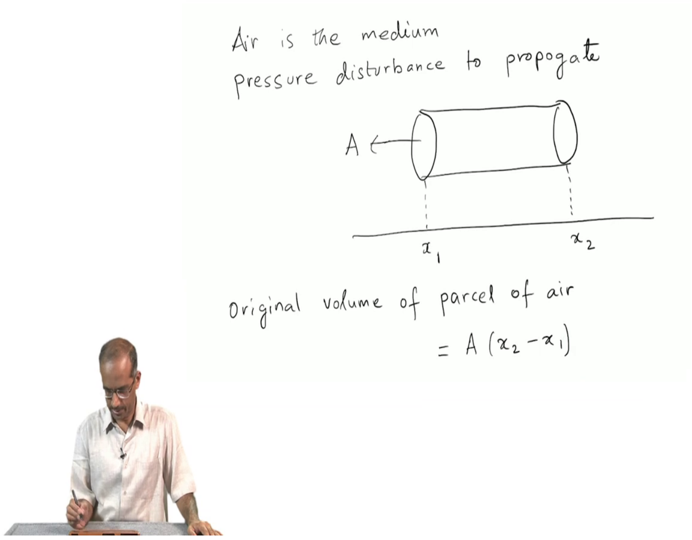

So, I have taken a small arbitrary volume, parcel of air and let me say that this point here is x_1 and this point here is x_2 and let me assume that this area of cross section is A. So, a small parcel of air is in equilibrium with rest of the air surrounding it; so that is the statement. So, if you go back to your idea of string the equivalent thing is to say that the string is held tight between two rigid support and nothing happens to it; in the sense that you have not excited it by pulling it one side and so on.

That is its equilibrium configuration. I am calling it to original volume simply because I am going to soon consider a volume which is a little bit deformed from this situation.

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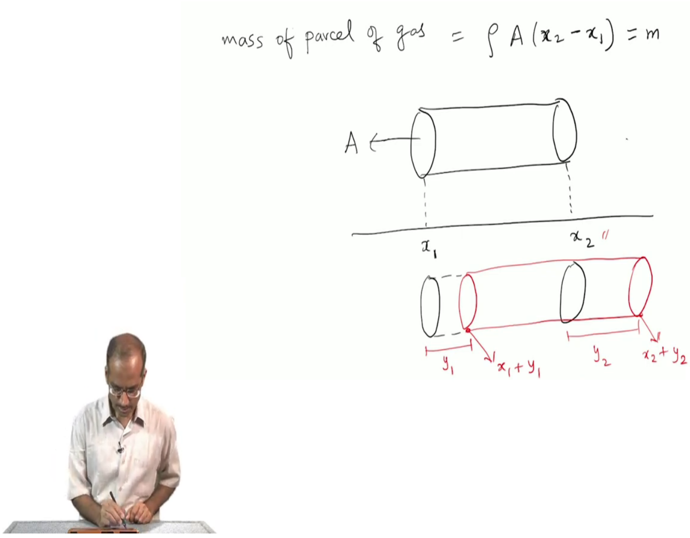

So, clearly this volume is $A(x_1 - x_2)$ and with this; it is also easier easy to write down what is the mass of this parcel of gas would the simply be equal to ρ, it will be the density of gas multiplied by the volume and the volume is $A(x_2 - x_1)$; so let me call it m.

When this parcel of gas is displaced, this is the figure that we are going to have; I have already drawn it for you here. The point which was originally at position x_1 has moved by a distance y_1. So, the new coordinate is $x_1 + y_1$; correspondingly the point originally or the phase which was at x_2 which is this, now has moved by a distance y_2. So, the new point or the new coordinate of that phase is now $x_2 + y_2$. So, we assume that there is no net displacement in any other direction other than one horizontal direction; not only that it has moved but its volume has changed as well; so we need to find the new volume.

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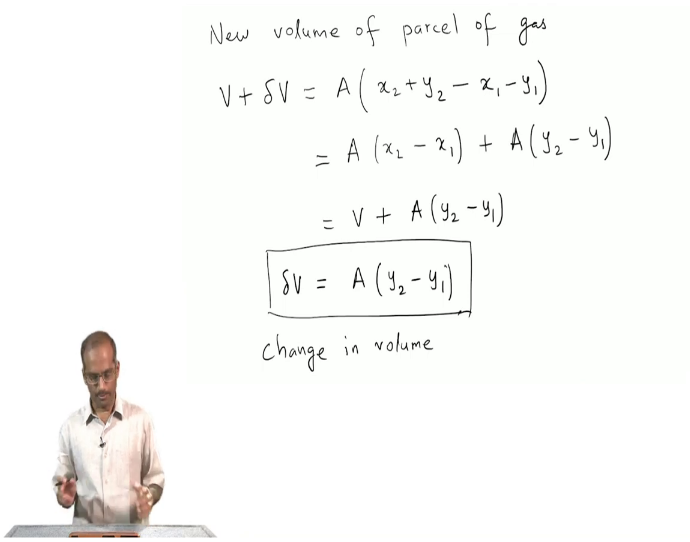

So, let me call this as $V + \delta V$ and the original volume of the parcel of air; let us give it a name let us call it V. So, the change in volume is δV.

So, now this new volume would be $A(x_2 + y_2 - x_1 - y_1)$ and this can be written as $A(x_2 - x_1) + A(y_2 - y_1)$. So, that will be $V + A(y_2 - y_1)$ and V and V will cancel on the left hand side and the right hand side. So, I will be left with an expression for change in volume; change in volume is $A(y_2 - y_1)$. So, this tells me by how much the volume has changed.

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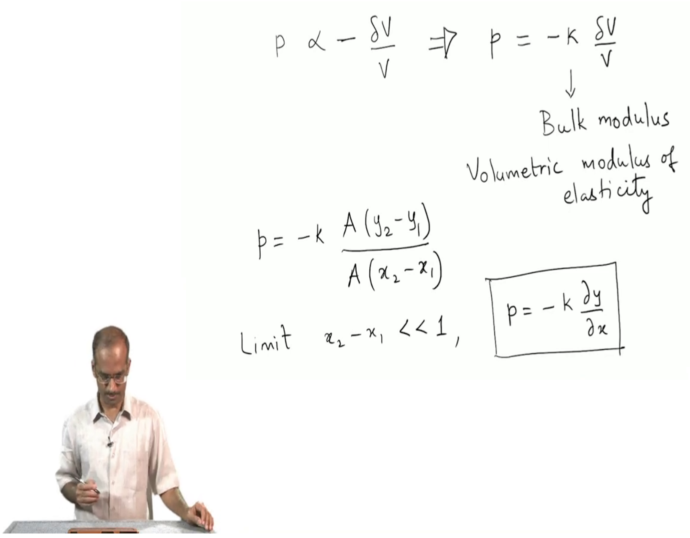

We are dealing with gas, so when volume changes; there will be change in pressure as well. So, I am going to relate this change in pressure to change in volume I would have a relation like this; change in pressure would be proportional to fractional change in volume.

And because they are inversely related in the sense that increase in volume corresponds to decrease in pressure and vice versa; you will need a minus sign here and of course, we can replace this proportionality by an equality. So, you can say that let me write this as P, change in pressure is equal to minus some $\frac{K\delta V}{V}$. This quantity, K is of course, a constant it is called the bulk modulus or in other words this is what would be called the volumetric modulus of elasticity.

Now we can estimate this P for our parcel of air because P is simply equal to $-K$ times change in volume by original volume and we know both these quantities. Change in volume is $A(y_2 - y_1)$ divided by original volume is $A(x_2 - x_1)$. So; that means, that the parcel of air that we are considering for our analysis is infinitesimally small. In that limit this P will be equal to $-K\,\frac{\partial y}{\partial x}$. So, what we have here is an equation that relates pressure changes to the gradient of the displacement.

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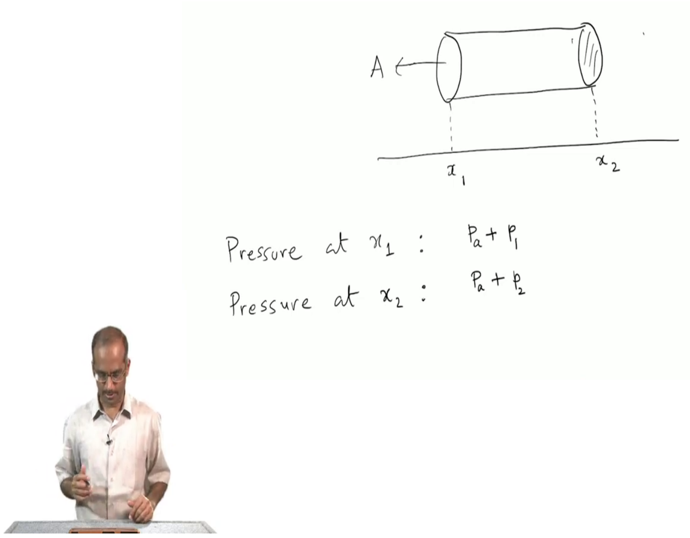

Now, if you go back to the figure that we had started with this parcel of air which was in equilibrium was displaced because; obviously, there is a net force acting on this parcel of air that pushed it one side.

So, let us assume that at point x_1; the phase of this parcel of air here this cross section here experienced as a pressure which is some average pressure plus P_1 and the average pressure here in this case would be the atmospheric pressure; simply because in the absence of any other forcing mechanism, atmospheric pressure is always there. So that the pressure on both of these phases are not equal and clearly there is going to be a; net pressure acting on this parcel of air and if you multiply that net pressure by the area of cross section which we have said is A; we will get the net force acting on this parcel of air that is what we are going to do next.

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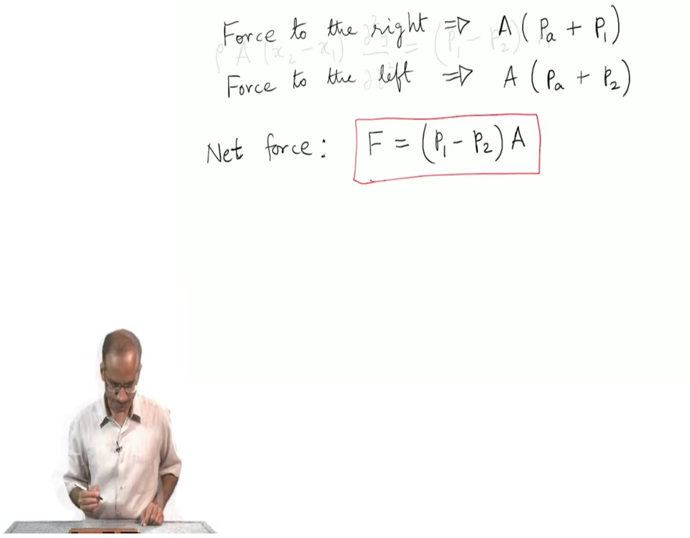

Now, the net force on the parcel is simply the difference between the forces applied from both the cross sectional areas. So, the net force experienced by this parcel of air; the expression is here it is simply the difference between these two and it turns out to be $P_1 - P_2$ multiplied by the area of the cross section.

So, now we have an expression for the force; all we need to do is to simply equate it to the mass times acceleration of this parcel of air. Of course the A and A would cancel; we are left with all the other quantities.

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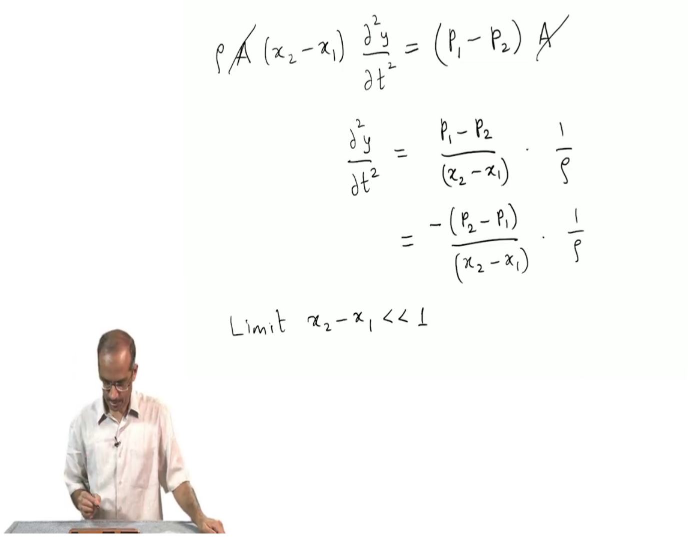

So, I should have $\frac{\partial^2 y}{\partial t^2}$ which is equal to $\frac{P_1 - P_2}{x_2 - x_1}$ and of course, there is $\frac{1}{\rho}$. Let us take the limit when $x_2 - x_1$ is much less than 1.

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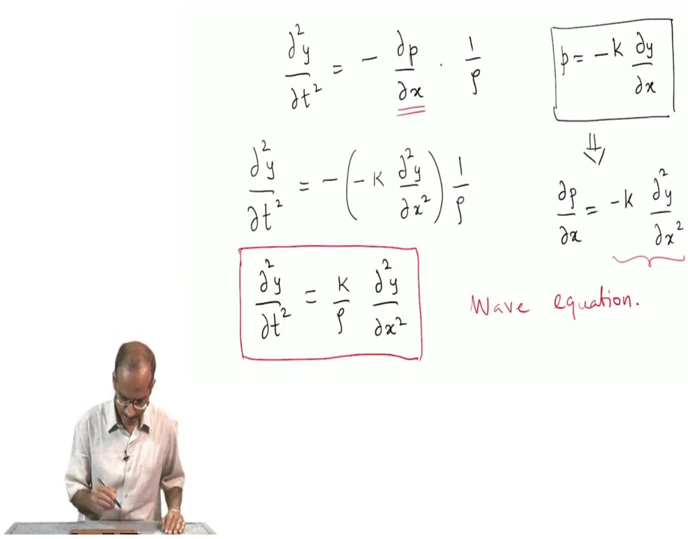

So, when you assume that the length of your parcel is infinitesimally small enough this entire equation can be rewritten as $\frac{\partial^2 y}{\partial t^2}$ would be equal to $-\frac{\partial^2 P}{\partial t^2}\frac{1}{\rho}$. So, I have this equation and from here I can take the derivative with respect to x on both sides; I would get $\frac{\partial P}{\partial x} = -K\frac{\partial^2 y}{\partial x^2}$.

And now we know what to do; all you need to do is to simply substitute for this $\frac{\partial P}{\partial x}$ by this. Hence my expression for $\frac{\partial^2 y}{\partial t^2}$ will be equal to minus of; of course, I have another minus sign which will make it positive $\frac{\partial^2 y}{\partial x^2}\frac{1}{\rho}$.

So, this can be simplified it to $\frac{K}{\rho}\frac{\partial^2 y}{\partial x^2}$. So, again what we have is indeed a wave equation and given that it is a wave equation, we also straightaway know what is the velocity of the wave that this equation represents let us do that immediately.

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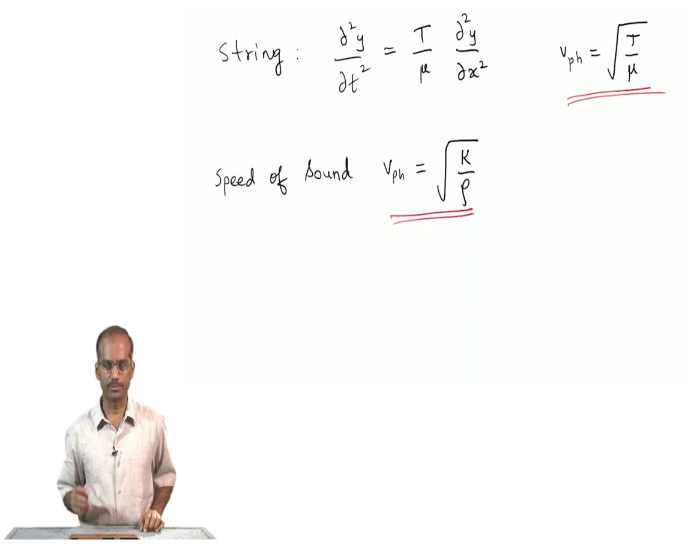

And that is equal to $\sqrt{\frac{K}{\rho}}$; K is the bulk modulus and ρ is the density of gas. Now let us go and replace this bulk modulus in terms of pressure.

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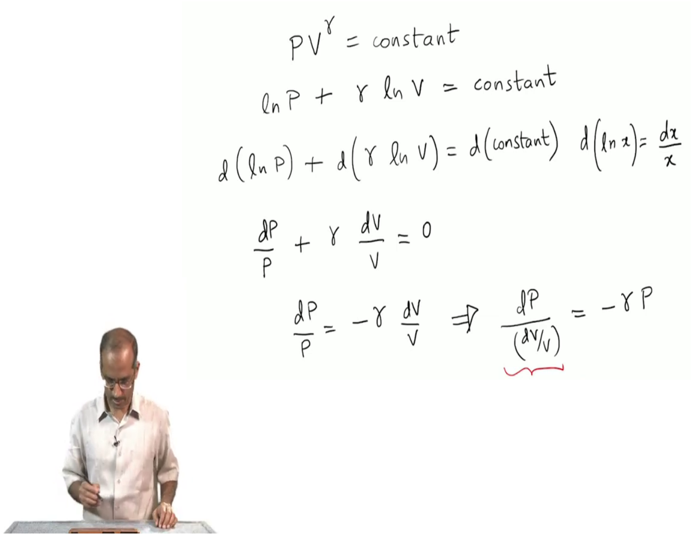

The starting point here is the ideal gas law for adiabatic process. So, I am going to start with PV^{γ} is a constant. So, take logarithm on both sides; we will have $\ln P + \gamma \ln P$ is equal to log of a constant; log of a constant is another constant, so let me just call it constant.

Now, let me differentiate throughout; if I do that the kind of relation that I need is the following that if I take $d(\ln x)$; I am going to get $\frac{dx}{x}$. Of course, if I integrate $\frac{dx}{x}$ that gives me $\ln x$; so it is just the reverse of that. So, I am going to differentiate throughout, so it will be $d(\ln P) + d(\gamma \ln V) = d(\text{constant})$.

So, this is going to give me $\frac{dP}{P}$ plus γ is a constant can be taken outside and $d(\ln V)$ will be $\frac{dV}{V}$ and taking a differential of a constant is going to give me 0 and this can be rearranged differently. So, $\frac{dP}{P}$ will be equal $-\gamma \frac{dV}{V}$ and this I want to write it differently in the following way. So dP, the change in pressure divided by $\frac{dV}{V}$. So, I am taking this $\frac{dV}{V}$ in the denominator on the other side and that will be equal to $-\gamma P$.

Now, if you remember clearly the quantity that is here on the left hand side; this one, of course I should have taken in the negative sign here as well if I had done then that is precisely the definition of the bulk modulus.

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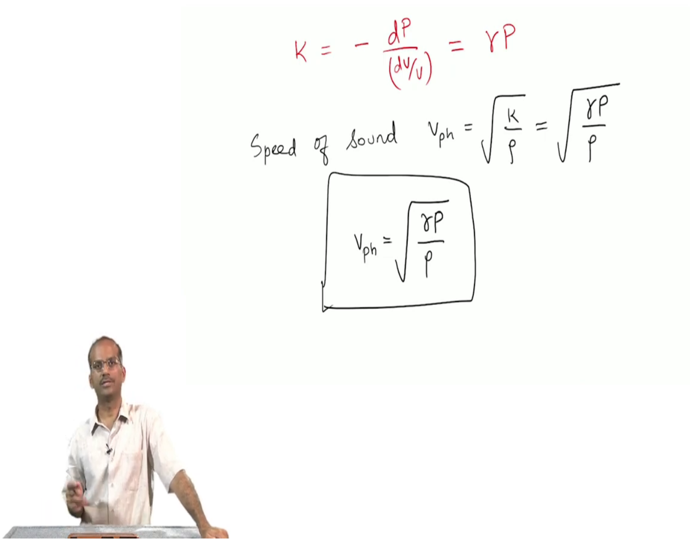

So, let me rewrite it. So, bulk modulus will be $-dP$ divided by a fractional change in volume and that is equal to γP. So, that is what we get from here.

So, now you can see what we have achieved; we are able to relate the bulk modulus to pressure and this γ is of course, the ratio of specific heat constants. Now, all I need to do is to simply substitute for K; back in my formula for speed of sound $\sqrt{\frac{K}{\rho}}$ which is equal to $\sqrt{\frac{\gamma P}{\rho}}$.

So, this is the standard formula for the speed of sound. As you can see or at least the way it appears the velocity of sound seems to be proportional to pressure and inversely proportional to density. Now that we have this expression for velocity of sound, we need to be able to put in the numbers and see whether it is reasonable or not and we can straight away calculate the speed of sound in air because all I need to do is to replace this pressure by the atmospheric pressure and density by density of air.

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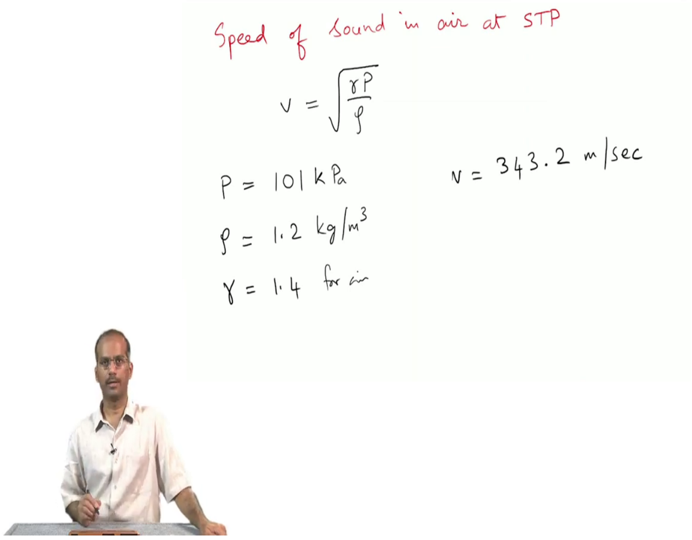

So, speed of sound in air at standard temperature and pressure. So, that will be; let me call it V it is just $\sqrt{\frac{\gamma P}{\rho}}$ and P is about 101 Kilo Pascals so that is the standard atmospheric pressure in units of Pascals and also at a temperature of about 20 to 21 degree Celsius and density is 1.2 kilogram per meter cube and γ of course, is 1.4 for air.

Now, when you put in all these numbers turns out that velocity is about 343.2 meters per second and this number 343.2 which we obtain completely theoretically has a very good correspondence with what is seen experimentally. So, it is something of an achievement that purely from models of displacements of air; one is able to write down a wave equation and obtain an expression a theoretical expression for velocity of sound and that has an excellent agreement with experiment.

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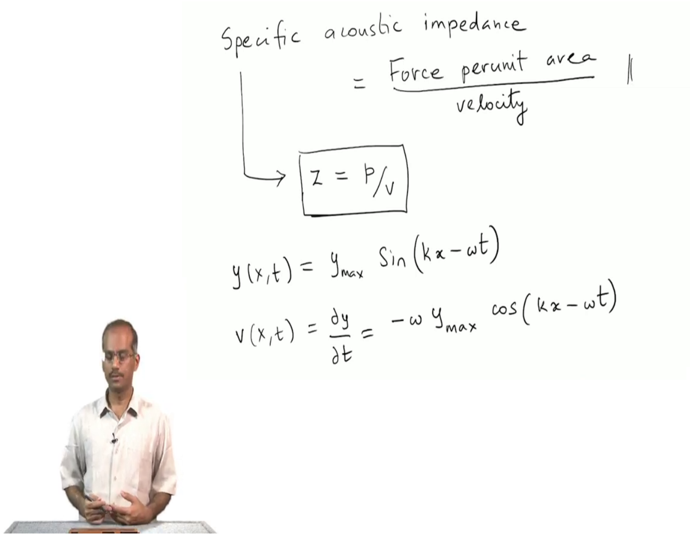

So, before we close this lecture; let us also look at specific acoustic impedance, it is a quantity we might need in a future lecture. So, impedance is a quantity that we have already met and we discussed quite a bit; we discussed quite a bit about it.

So, impedance is simply the resistance that the medium offers to the passage of wave. So, that is how we understood it and it is indeed true in this case as well. So, here we are talking of pressure disturbances flowing through the medium of air. So, air is going to oppose the changes and the pressure disturbances that is flowing through air. So, this acoustic impedance is simply a measure of how much it opposes it and specific acoustic impedance is simply the acoustic impedance per unit volume.

So, this is force per unit area divided by velocity and force per unit area of course, is the pressure. Hence I can write specific acoustic impedance as Z equal to P which is pressure excess pressure divided by velocity. So, let us get a simpler expression again for this specific acoustic impedance and the starting point is the solution of the wave equation. We derived an equation for pressure disturbances which turned out to be another form of wave equation and as I said we already know the solutions.

So, I can write out the solutions as y which is a function of position and time is $y_{max} \sin(kx - \omega t)$. So, y in some sense you could think of as; as the average of y_1 and y_2.

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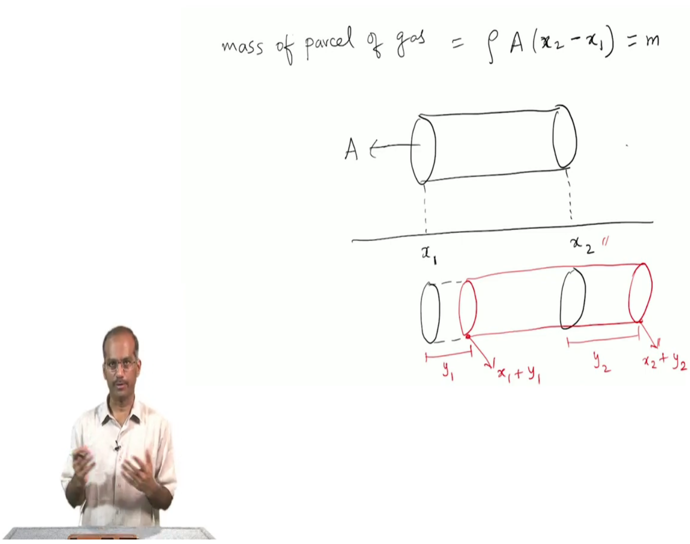

So, if you remember our first figure that we wrote down; so we have this displacement y_1 and y_2. So, y is in some sense the average displacement of this y_1 which is the displacement of the one phase and y_2 is the displacement of the second phase. So, here our variable is y which is the average of these two displacements.

So, what I have a solution is simply the average displacement of the parcel as a function of position and time. Now that I have the solution for y of course, I can also write a solution for v. So, in this case this is not the velocity of sound this is what would be called the particle velocity; so it is not the phase velocity now.

So, this v would be $\frac{\partial y}{\partial t}$ which is $-\omega y_{max} \cos(kx - \omega t)$ and the velocity that I have here in this specific acoustic impedance is the particle velocity; P is $-K \frac{\partial y}{\partial x}$.

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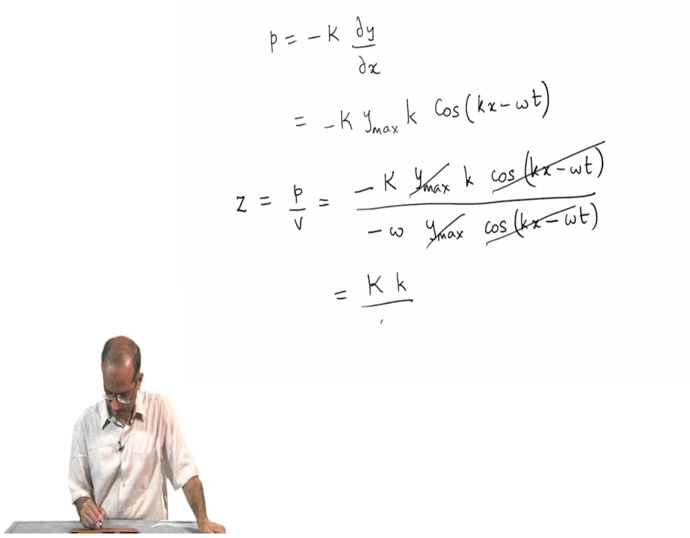

So, that will be $-K\dfrac{\partial y}{\partial x}$ we calculate starting from this y here and if I do that I am going to get minus $k\,y_{\text{max}}$ this k is of course, the wave number times $\cos(kx - \omega t)$.

So, I have the expression for P as well. So, now, I can straight away write specific acoustic impedance as $\dfrac{P}{V}$ which would be of course, this cos function will cancel here and here and y_{max} also will cancel. So, that is going to leave me with $\dfrac{Kk}{\omega}$.

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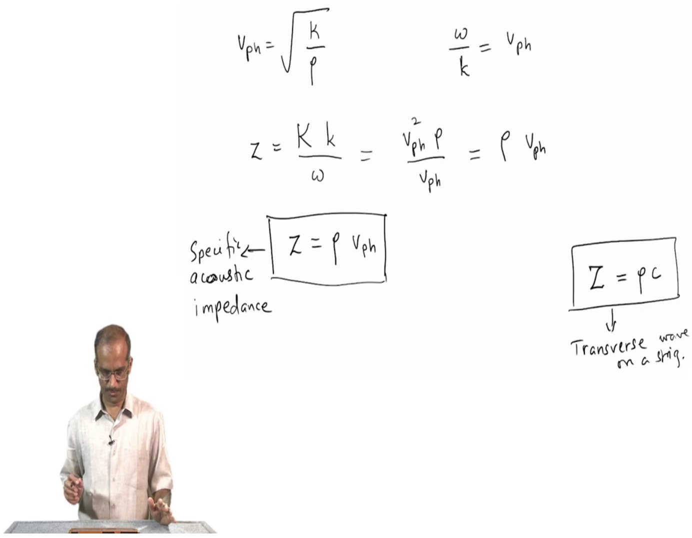

So, let us use both these relations and what we had obtained. So, is Z; we said is K divided by k divided by ω, hence I would get ρv_{ph}. So, I have the final expression which is really simple; density multiplied by the phase velocity. Irrespective of these differences you could see that the expression for impedance look similar; it is a density multiplied to phase velocity with the same thing in the case of transverse waveforms density multiplied to phase velocity.

So, clearly there are these analogies; even though now we are looking at longitudinal waves, there are many things which are similar to the kind of phenomena that we saw with transverse waves.

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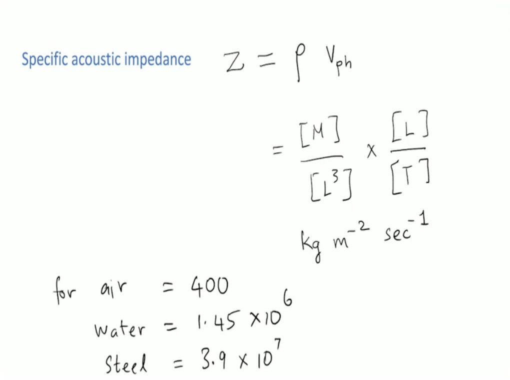

Having seen specific acoustic impedance, there is one final P is that we need to know what are the units of specific acoustic impedance and I have written down our standard formula which is ρv_{ph}.

So, we can easily work out the units in this case; ρ is of course, the density, so it will have units of mass per unit volume multiplied to velocity which will have units of length divided by time. So, in general specific acoustic impedance has units of kilogram per meter square per second. So, typically whenever you see numbers quoted for specific acoustic impedance in SI units it is in these units.

For instance the specific acoustic impedance for; for air is about 400; in these units kg per meter square per second and for the case of water specific acoustic impedance is about 1.45 into 10 power 6 and it is again kg per meter square per second and just for comparison sake, if you look at the case of some hard material like steel; the specific acoustic impedance is of the order of 3.9 into 10 power 7, the same units. So, that gives us an idea of to what extent different materials resist the flow of waves through them.