Waves and Oscillations
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Lecture - 14
Applications of Forced Oscillator

Welcome back. We are in the fourth module of the 3rd week. Throughout this week we have been looking at forced oscillators, the oscillators that have both a damping mechanism and our externally forced and one of the most important phenomena that we came across in this oscillator or in this class of oscillations as the phenomena of resonance. So, we looked at in detail how resonance comes about and so on. And in this module, we look at some applications of resonances.

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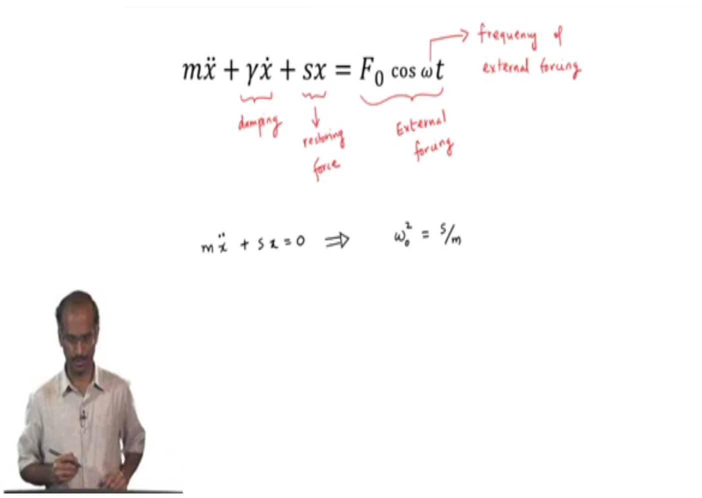

Let us begin by just quickly collecting what we mean by resonance. So, again we go back to our damped and forced oscillator. So, here I have the equation of motion written for you. Damping comes from this term, so γ is the coefficient of damping, and oscillations itself come from this restoring force term. So, that is the one that is responsible for producing oscillations.

Here, what I have is the external forcing, F_0 is the amplitude of external forcing, and we assumed that this ω is the frequency of external forcing. So, this is a simple sort of model for an oscillating system which is, both has a damping mechanism in place and also is externally pumped up and is forced to continue the oscillations. So, one of the central reserves that we saw was that if the frequency of external forcing, which is ω equals the natural frequency of the system; then, you can observe what is called the velocity resonance.

So, here whenever I mean natural frequency, as we saw before we are going to mean the frequency of the system without the damping and without the forcing. So, without damping and forcing the same system would simply be

$$m\ddot{x} + sx = 0,$$

and the corresponding frequency or the frequency corresponding to this I would denote by ω_0^2 which would simply be $\frac{s}{m}$, and s is the stiffness constant for the system.

So, whenever $\omega_0 = \omega$, you do see resonance, and as we also saw the displacement resonance one observes when ω is close to ω_0 but not quite equal it comes a little below the value of ω_0.

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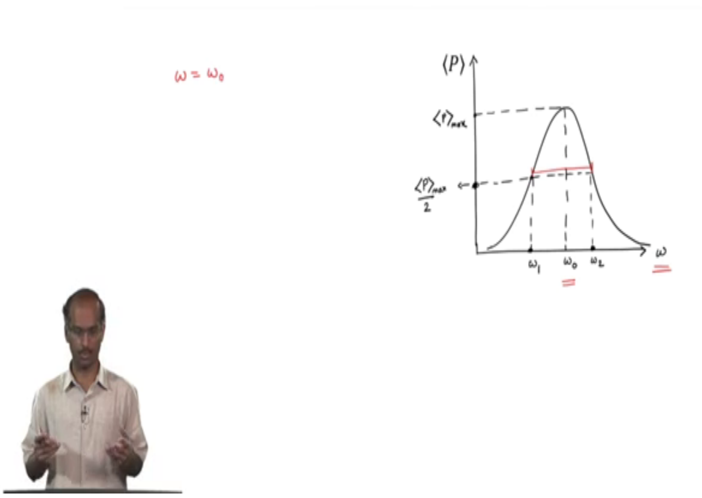

So, here is a pictorial representation of what we mean in terms of resonance. So, what is plotted on the x-axis is the frequency of the driving force, which is omega here. And on the y-axis plotted the mean power, the mean power let us say absorbed over one cycle or the mean power that the system uses up over one cycle and both these quantities as we saw are equal. So, what we see here is that when the value of ω is equal to ω_0, the mean power absorbed is maximum. So, that is an example of resonance. And, if you choose any other value of frequency away from ω_0, the more further you go away from ω_0, the mean power absorbed is going to be less, and also the mean output power is going to be less.

So, in this sense you basically say that if you are at the resonance resonant frequency which is $\omega = \omega_0$, the system, in turn, responds in a big way. And if you are not at the resonant frequency, you hardly see any response in the system. This is actually a very beautiful phenomenon. In fact, you can realize it by doing very simple experiments with the kind of things that you will have at home.

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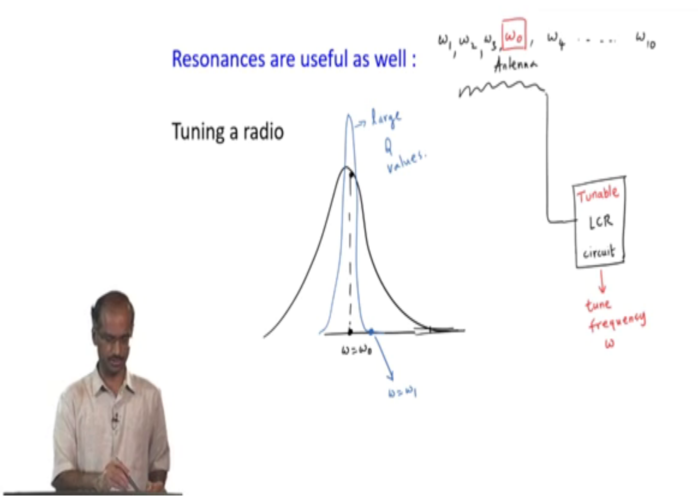

So, resonances are not just used for breaking glasses but one can also use them for doing very useful things; for instance, every time you tune a radio especially the AM radio you are using the phenomena of resonance. So, what happens is the following. So, you have radio stations which broadcast in some frequency. And of course, it is very easy to see what those frequencies are if you look at a typical radio or even your mobile phone, which should have a radio app you will see that different frequencies are listed.

And if you actually tune your radio to one of them you are more likely to tune in to one of the radio stations. So, those are the frequencies at which the stations are actually broadcasting. So, that is a wave that hits the antenna of your radio some kind of let us say an antenna here maybe it could just be a long piece of metal or even a long piece of wire. At its very simplest the output that comes from this antenna is fed into typically either an LC circuit or an LCR circuit and this circuit is tunable.

So, that it will be able to tune just to one station and this will happen if your Q factor or Q value is large enough, which is why when you say you have a high Q value; it also means that you have high fidelity electronics. In terms of our oscillator terminology, it simply means that this corresponds to large Q values. So, if you have that then you can even separate two frequencies that are close by. If your Q value is small enough your response curve is broad, it overlaps several frequencies, and if you use such kind of electronics you are going to hear two or even more stations at the same time.

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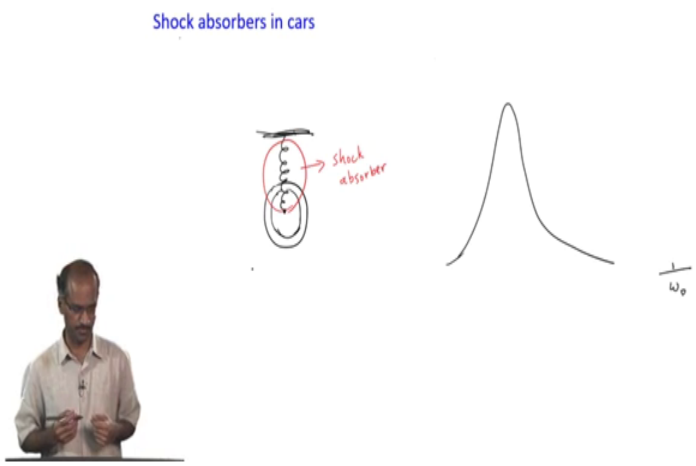

So, this is one example of a very useful application of resonance. Another useful application of resonance is in the design of shock absorbers in cars. So, you would have noticed that while traveling in cars or any other vehicle or even in trains. So, you notice that sometimes the car actually runs over some bad patch of road, the road is really bad but, still you do not feel the shock when you are sitting inside the car that is because you have this device called the shock absorber. So, the basic mechanism again uses the idea of resonance.

So, typically what is done is that someone has already studied the kind of bad patches on roads in general and they know that what are the rough frequencies, in which they transmit the shock. So, you then go back to your design elements in your car and say that I will create a shock absorber system using maybe a spring and damping mechanism which works like an oscillator which has both damping and forcing. The forcing in this case comes from the bad patch of road, if the road is really-really smooth, there is no real external forcing.

So, in this case, all you need to do is simply design a shock absorber, such that it will dampen it. So, the response curve should be in such a place that, the frequency which gets transmitted to the car is not amplified by the shock absorber. So, compared to the case of tuning a radio, where you really want to match and obtain the large response, so that the signal is strong enough. Here you want the signal to really have small response.

So, in other words, your response curve if it was like this, your shock absorber mechanism its natural frequency would be placed somewhere here. So, that they do not match, in which case whatever be the bad patch of the road and ups and downs of the wheel it does not get transmitted to the internals of the car it does not get transmitted to the person sitting inside the car it just gets damped out. So, that is another example of using resonance in a way that is very different from the way you use resonance in a radio tuner.

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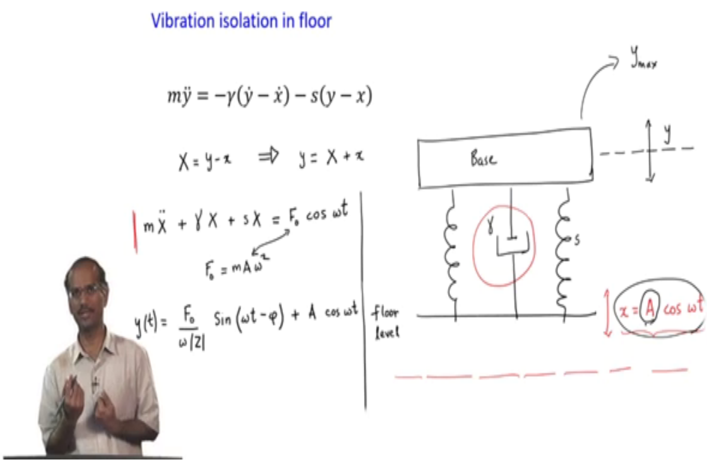

Let us look at it in a little more detail in the case of vibration isolation. So, typically this is used when you have to do, for instance you are doing an experiment. And the quantity that you want to measure is really very small. But then you do not realize that even when it appears that the earth is perfectly not moving, if you do a close measurement you see that all the time the earth is really shaking by small amounts that you do not feel it in real life. But, if you are going to make measurements of quantities length or velocities which are really-really small of the order at in which the earth is also shaking then you are going to get wrong results. In all such cases what you need is vibration isolation.

So, the earth may be vibrating by the small amount but I want to keep a table which should not amplify those vibrations. So, that is called a system for isolating vibrations and it is widely used in industries, in many labs, where you do experiments or measure quantities which are really-really very small. At its very basic it consists of something like this. So, what you have is a base here which is fairly heavy and sturdy, and let us say this is your floor level and this is the floor level that is possibly vibrating. And, this vibration just arises from small vibrations inside the earth we have no control over it. And let us assume that this vibration is essentially something like this. So, if you denote the displacement by x.

So, it is like $A\cos\omega t$, and ω is the frequency of this vibration. So, that is basically our floor. On the floor is mounted some springs like you see here and on the spring you have a heavy base, there is also a dampening mechanism here, and these springs have spring constant s or stiffness constant s and this γ here represents the coefficient of damping. And whenever there is vibration in the floor, of course, the base also vibrates but, the point of doing all this is that you minimize the vibration of the base, so that anything that you keep on that and do measurements would not be affected something like the shock absorber.

So, all the shocks are absorbed by the mechanism that you have underneath, it does not get amplified or conveyed about. So, this can be a very simple model of vibration isolation in floors and you can write down the equation of motion for this case. So, as you notice x is the vibration at the floor level, and y is the amplitude or the displacement at the level of this heavy base, and s is, of course, the stiffness constant, γ is the coefficient of damping. And this is the equation of motion and as you can see we can actually rewrite the equation of motion in a different coordinate system because what is important is $y - x$. And I can call it X.

So, if I do that this equation can be rewritten $m\ddot{X} + \gamma X + sX$, and this would be $F_0 \cos\omega t$. And what is the forcing here forcing comes from this term here. So, I could say that F_0 is simply $mA\omega^2$, which means that this F_0 is simply this quantity. So, I have all the necessary ingredients and if you look at it closely, this is like the equation that we solved before. It is simply the equation of that equation of motion for a damped forced oscillator, and I know the solutions steady state solution. So, I can directly write the steady state solution here.

So, if I write $y(t)$, for instance, from here I could write y to be $X + x$, and then $y(t)$ would be simply $\frac{F_0}{|Z|}$; remember Z is the impedance. So, I have the solution written down straight away. So, I urge you to compare this equation with the standard driven and damped oscillator equation and convince yourself that this is the solution that we should be getting. And, from here all I want to do is to say just is to ask for the ratio between the maximum amplitude of the base. So, base is something that is also going to vibrate, and the maximum vibration of the base let me denote by y_{max}. And, the maximum amplitude of the floor level is A.

So, I want this ratio to be less. So, $\frac{y_{max}}{A}$, want it to be small enough for my purpose.

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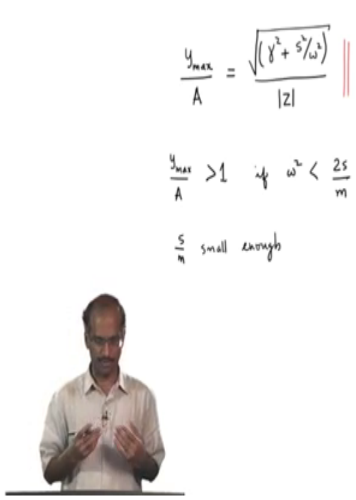

What we can show is that $\frac{y_{max}}{A}$, this ratio is given by this quantity here again I leave it as an exercise for you to get this expression. And, from here on it is one step to realize that $\frac{y_{max}}{A}$ will be greater than 1, if ω^2 is less than $\frac{2s}{m}$. So, for a given value of frequency, if you want to dampen that frequency you should essentially keep the value of $\frac{s}{m}$ small enough. In which case you can hope that this condition would be violated and vibrations of frequency ω will be dampened, and would not be conveyed to the base.

So, that is the basic idea behind this vibration isolation. There are many other considerations in this but, this is a very sort of simple way of realizing a vibration isolation system. And, at the heart of this is our simple model of damped and a forced oscillator. And, if you take care to look around you will see many many examples of resonances; it is used in things like nuclear magnetic resonance the NMR which is used quite a bit in many medical devices. So, clearly the idea of resonance is very useful in many walks of life. I would urge you to look around and see in large number of places were at the heart of some device you probably might see resonance in action.

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Let us do one quick experiment with some very commonly available things. So, I have a cell phone which will generate for me variable frequencies, and I have this empty bottle, you can pick up pretty much any empty bottle and see if we can see any sign of resonance here. So, what I am going to do is the following. So, this bottle is empty. So, just air molecules are inside the bottle and each of them the molecules are free. So, essentially if I try to send let us say sound waves they will oscillate and if somehow the frequency of waves that I am sending is equal to their natural frequency I should be able to see some sign of resonance. So, in this case I am going to start with the a frequency generator, it is one of the apps which you can put on your cell phone and let me show you how this works first.

So, you get an idea of the difference between the intensity level of sound with and without resonance. So, here I am starting at 72 Hertz. So, I keep changing, if you can see the numbers. So, it is now around 114, it is increasing. So, that is how I vary the frequency. So, now, you will slowly start hearing the hum, the hum of the frequency generator now the frequency generator is generating for me a sound wave of 267 Hertz. So, this is about 320 or so and so on.

So, what I am going to do is use this bottle here and I will see if I can. So, I am going to keep varying the frequency. So, the sound basically comes from the bottom of the cell phone here and it is directed right into this bottle. So, when there is a match between the frequency of the sound wave generated and the natural frequency of all these collective of molecules, then, you should see something happening. Let us see if we can pick out that signal.   .

So, you can start hearing that hum of the frequency generator. So, it is around 200 now, around 270. Now, it peaked somewhere around there, possibly somewhere around 340 or 350 there is a resonance. Let us see closely if that is the case. So, let me go back to 280, go a little more slowly. So, I am at 337 now. So, you pick up that high intensity sound now, around 346 Hertz**.** So, it is even higher at 345 but, if I keep it at 336 Hertz its very low and from the phenomena of resonance we know that if I go beyond the resonance frequency, let us say, to around 350 360 or so, which is where I have kept my frequency generator now, again the sound is very low.

So, let me once again show you that, if I separate out the phone the intensity is not that much, but if I keep it here it is quite a bit. And, if I change the frequency again, it is getting low. The molecules of air inside this bottle their natural frequency is somewhere around 340, 345 Hertz or so. And when I send in sound waves of similar frequency, you do see resonance happening. If the intensity were high enough, it can even throw off the bottle or even break the bottle. But these are kind of simple experiments you can do to check for resonance. In fact, if you want to be more quantitative all you need to do is to simply record the intensity of sound and you will again see resonance pattern in the response curve.