Waves and Oscillations

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Lecture – 01
Simple Harmonic Motion

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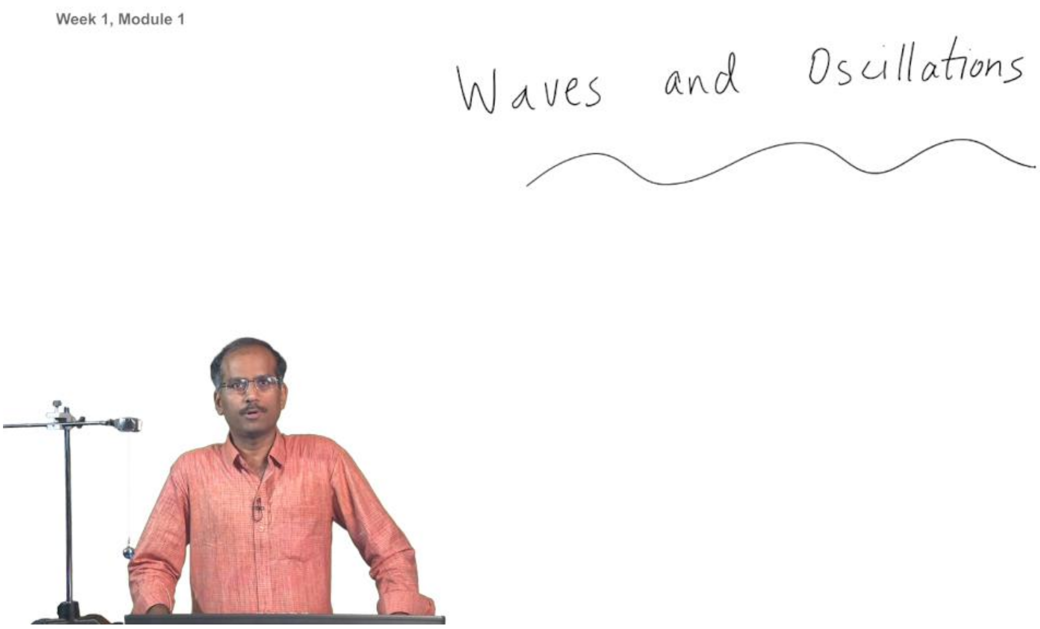

Welcome to the first module of this course on Waves and Oscillations, oscillatory phenomena is something that we keep meeting often in a real life we have seen things like for instance, pendulum in the clocks we have seen swings in the gardens that children play and I have a pendulum here and we keep meeting oscillatory phenomena again and again in various forms in real life.

So, this course is going to deal with oscillations of a single particles and if many of them act together and start oscillating together, they generate waves and, in this course, we will meet waves as well.

So, let us begin with what we mean by oscillation ok. So, I have a single particle very much like this bob of a pendulum that I have here and if I let it oscillate, it oscillates around some mean position to and fro.

So, that is an oscillation for us an oscillation that does not really go off to infinity or go off really far from where I am. For example, if I throw a stone from here it goes off very far from where I am and it is not going to return back to the original position that is not an oscillatory phenomenon.

So, we are looking at phenomena which are going to explore the same space again and again something like this so that will be the oscillations of a single particle. On the other hand, you can actually connect several of these particles together like they begin to interact with one another and you can make them oscillate. For instance, many of us would have tried doing things like tying a string between may be two walls and pluck it a little bit.

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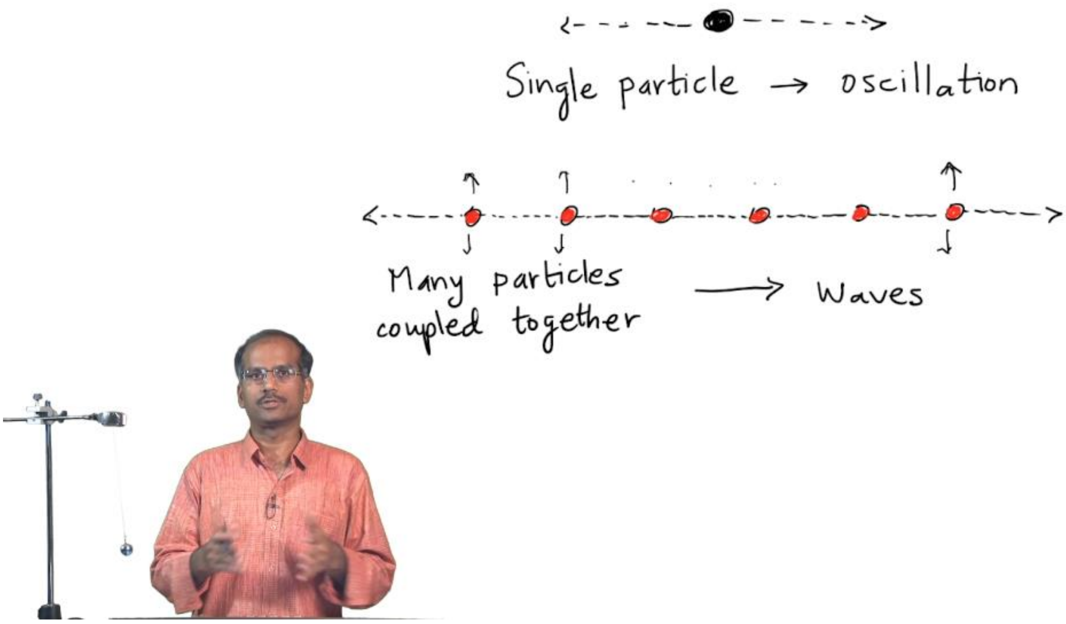

So, that is like closely packed particles together and when you pluck it you are basically oscillating one particle, but very soon it conveys the disturbance to the ones that are close to it and soon you see waves that are propagating in both the directions through the string so that is a phenomenon of waves. So, when you have many particles which are coupled together, they begin to interact with one another and wave forms are one of the emergent phenomena.

So, the remit of this course is to deal with both oscillations of single particles and waves which are essentially oscillations of many particles coupled together. Now, physics is an experimental subject in a sense that everything that we would like to learn is basically governed by what a real physical phenomenon does. So, we take our inspiration from real physical phenomena and decide how what are the important questions that we can ask of them and may be hopefully write a mathematical model for that and see if a model works correctly and make predictions about it.

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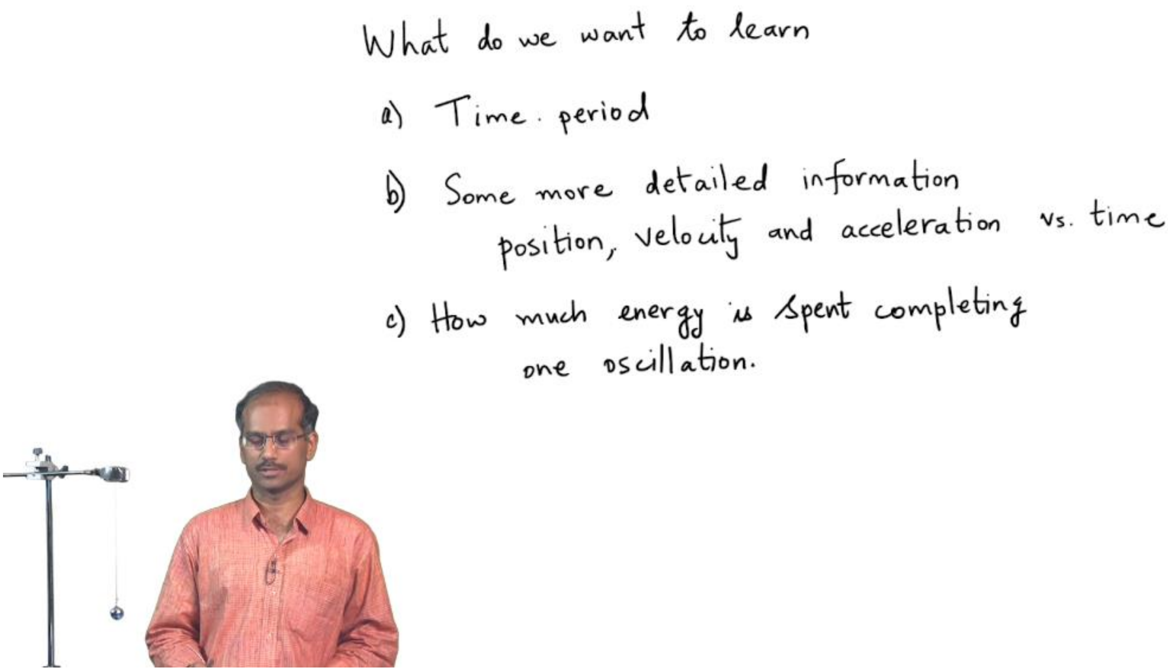

So, let us say that I have this system of string tied to a bob and this if I let it do this it begins to oscillate. So, when you see a phenomenon like this, the first thing to worry about is what are the interesting questions that we want to ask of this system ok. So, at first sight it appears that may be there is a time period to it in the sense that there is a fixed time when the particles begin from here goes there and comes back ok and hopefully it does not change.

So, maybe I would like to know what is the time period ok this quantity 'and, but that’s not enough that’s like a top-level question, as soon as I see I can realise that I need to know and understand about the time period, but there are more detailed information that I would like to get from here for instance, the position of the bob as a function of time. So, at a given instant of time where is this bob located and how fast is it moving the velocity and acceleration ok.

So, all these quantities as a function of time. So, this is fairly detailed amount of information amount of information maybe I would like to get about this ok and may be other question that I would like to know is about the energy ok. So, when I when I did this, I am putting it I am giving it some energy and I would like to know how much of the energy does this bob spend when it actually completes one full oscillation ok.

So, of course, there could be many more question at least these three important questions, we would like to learn and another thing which we will not probably see in the first week itself is the fact that when you start an oscillation and keep watching it after sometime, it is going to die down and it will basically come back to the starting point here like this and that is simply because of the dissipation. So, such damped oscillators are the subject of few lectures away and we will take it up in a later module.

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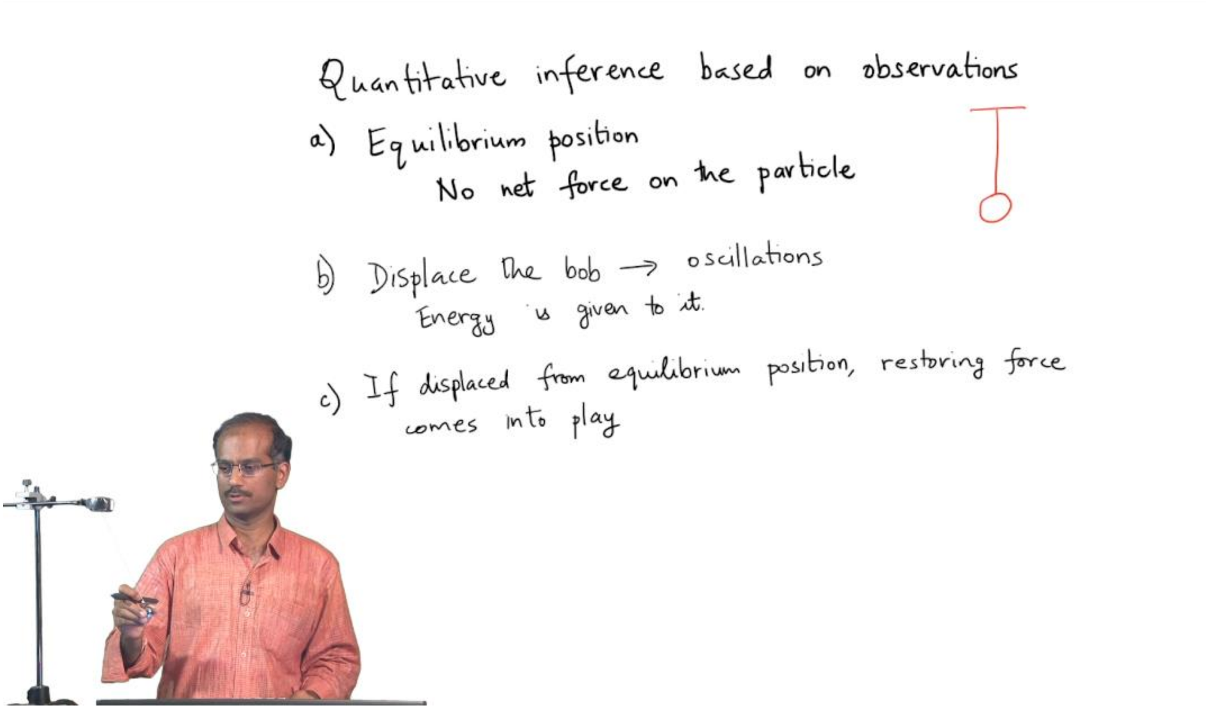

So, let us get back to our pendulum here and let's see, what are the inferences that we can make about this system ok. So, when nothing is done to this system, the position of the bob is what we can call the equilibrium position. So, at this position the bob here does not experience any net force which is why it does not move at all from there.

And it is very useful because if you want to measure any displacement like this, you could do it with respect to this equilibrium position you can actually put a scale here like this and measure coincide the zero of your scale with the equilibrium position. So, that any displacement would either be positive or negative on both sides ok something like this could have been negative and that could be a positive displacement and zero is where your equilibrium position is located.

So, in this position nothing will ever happen, the bob will stay as it is ok. Now, if I want it to oscillate, I just need to give it a little bit of displacement here and leave it and it begins to oscillate. So, that tells us that in the process of doing this, I have given energy to this bob ok. So, my way of giving energy to this system is simply to pull it aside a little bit and leave it. So, consistent with the amount of energy that I have given, the bob will start oscillating.

So, if I can give more energy by pulling it a little bit far out from its equilibrium position in which case it will also oscillate it will also show bigger displacement ok. So, that is second inference that we see ok and most crucial for what we are going to talk now is the fact that until I make the small displacement and give it some energy it doesn’t really try to come back to where it started from.

So, equilibrium position is where it is comfortable sitting when nothing else happens and if I move it either of these sides this side or this side, it tries to goes back it it tries to go back to the equilibrium position so; that means, that there is something that we can call as the restoring force a force that tends to restore the bob back to the equilibrium position ok. So, that comes into play only when I displace it ok.

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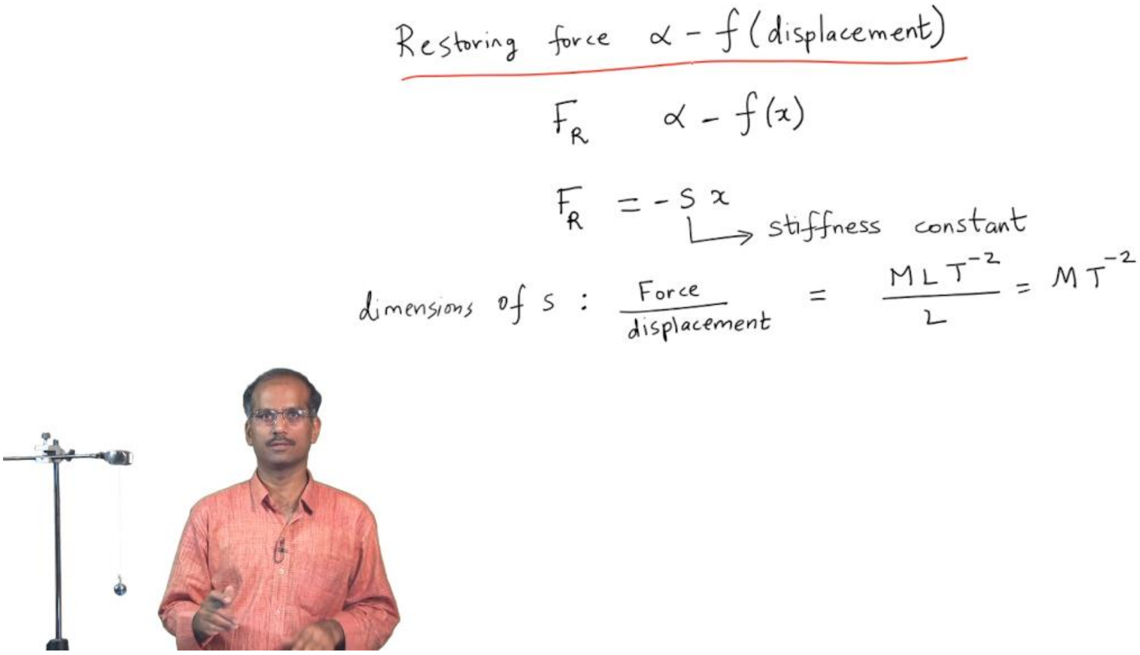

So, restoring force basically comes into play when there is slight displacement away from the equilibrium position and we should also note another crucial thing if my displacement is in this direction, the restoring force is in the other direction. So, the displacement and the restoring force are oppositely directed ok.

Now, we can make the first of our physical ansatz having been inspired by this physical phenomenon, that the restoring force is proportional to some function of displacement and the negative sign that you see here indicates that the restoring force and the displacement are oppositely directed. Ok given this we do not know what form of function this $f(x)$ is ok, but we know that when there is no displacement $f(x)$ will have to be zero because the restoring force is zero when there is no displacement.

So, the simplest assumption that's consistent with all these constraints is that $f(x)$ is linear in x. So, in other words you could make reasonable assumption that for small displacements at least the restoring force is proportional to the amount of displacement that you have given ok. So, which means that $f(x)$ is equal to x ok. So, you can plug that in here and then we can write an expression for this restoring force.

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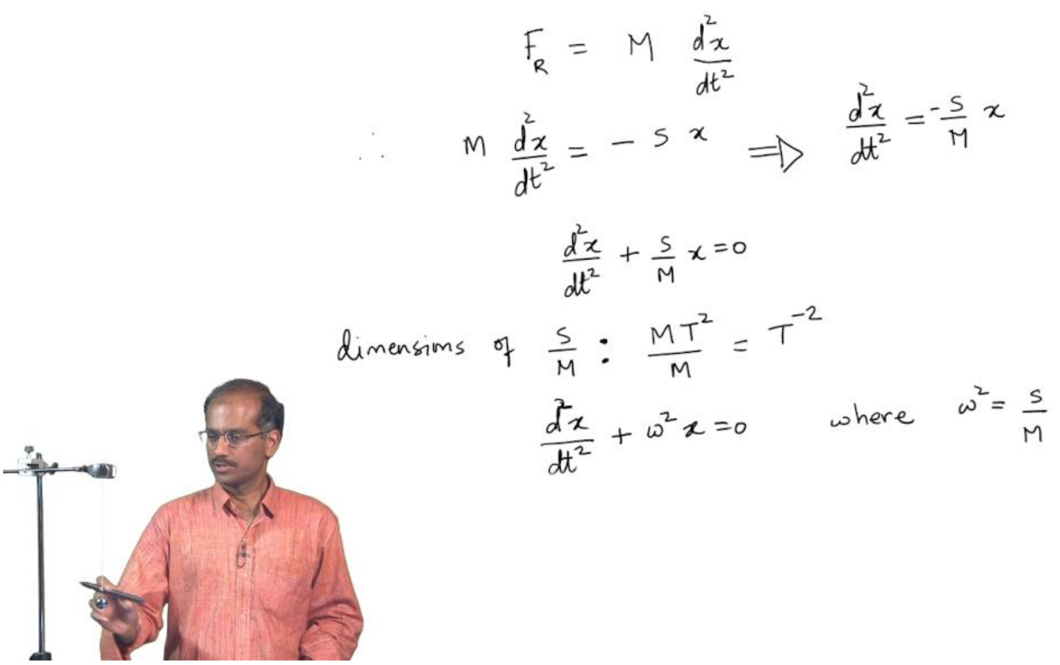

So, the restoring force is now equal to $-S$ times x. So, this S is a constant that I have now introduced to get rid of this proportionality sign here and S is what is called as the stiffness constant. So, before we say anything about stiffness constant let us look at the dimensions of this stiffness constant. So, S is simply f_r divided by x. So, it should have the dimensions of force divided by the displacement and force is mass into acceleration. So, I have MLT^{-2}divided by L. So, the dimensions of stiffness constant or S is simply MT^{-2} ok.

So, as the name suggest stiffness constant is somehow a measure of how stiff the system is. So, it is a measure of force that you need to put in to get a unit displacement. So, that is what this force by displacement tells us. So, the more the force that you need to put into get unit displacement tells you that somehow the system is very stiff you need to really put in lot more force to get even a let say a centimetre of displacement.

Now, we will use all these to write out our simple model ok. So, the restoring force is what gives you the mass times acceleration. So, I can rewrite the equation as mass times the acceleration and that's equal to minus $-S$ times displacement ok. Now, this can be easily transformed and written in this standard form.

$$\frac{d^2x}{dt^2} + \frac{S}{M}x = 0$$

So, d^2x/dt^2 which is the acceleration plus S/M multiplied by displacement is equal to zero. So, what I have is a second order ordinary differential equation that describes oscillations. So, you will notice an important point here, we were inspired by the oscillations of this bob to write this (Refer Time: 12:38) that restoring force is proportional to displacement and there was negative sign and from that we have obtained this, this is basically an equation of motion that describes the motion of this bob ok.

But if you look at this equation here, it doesn't really relate anything to the pendulum. So, in fact, nothing of the pendulum let say for instance length of this piece of string does not even enter this equation here. So, it is not clear that this describes this pendulum that I have here, but on the other hand this equation describes all the oscillatory phenomena until you stick to the important piece of initial assumption that we made that the restoring force is proportional to displacement with the negative sign.

So, before we go ahead and test it with the with this pendulum let us rewrite this equation in a slightly different form and something that is useful for us is the dimensions of this quantity $S/M$and S we already saw as the dimensions of, So, we saw that S has dimensions of MT^{-2} divided by M which means it is T^{-2}. So, the quantity S/M has dimensions of $1/t^2$ ok.

So, somewhere this quantity S/M is related to the time scale in the system. So, we will see what that time scale is and to make it easier lets rewrite this equation as

$$\frac{d^2x}{dt^2} + \omega^2 x = 0$$

where I identify this quantity ω^2 to be S/M

Motivated by what we saw with the pendulum, we made this ansatz that restoring force is proportional to displacement with the crucial negative sign there and then with few simple steps we arrived at this equation here ok this one and . So, it is important to remember that this is possibly valid for small oscillations and this quantity ω^2 that we have written down here is related to inverse of something that as information about the time scale in the system , this quantity T here is simply related is the time period of the system ok and in fact, in this problem like say this pendulum there was only one time scale in the system which is simply the time period or the time taken for the bob to go from one end to other end and back to the starting point ok.

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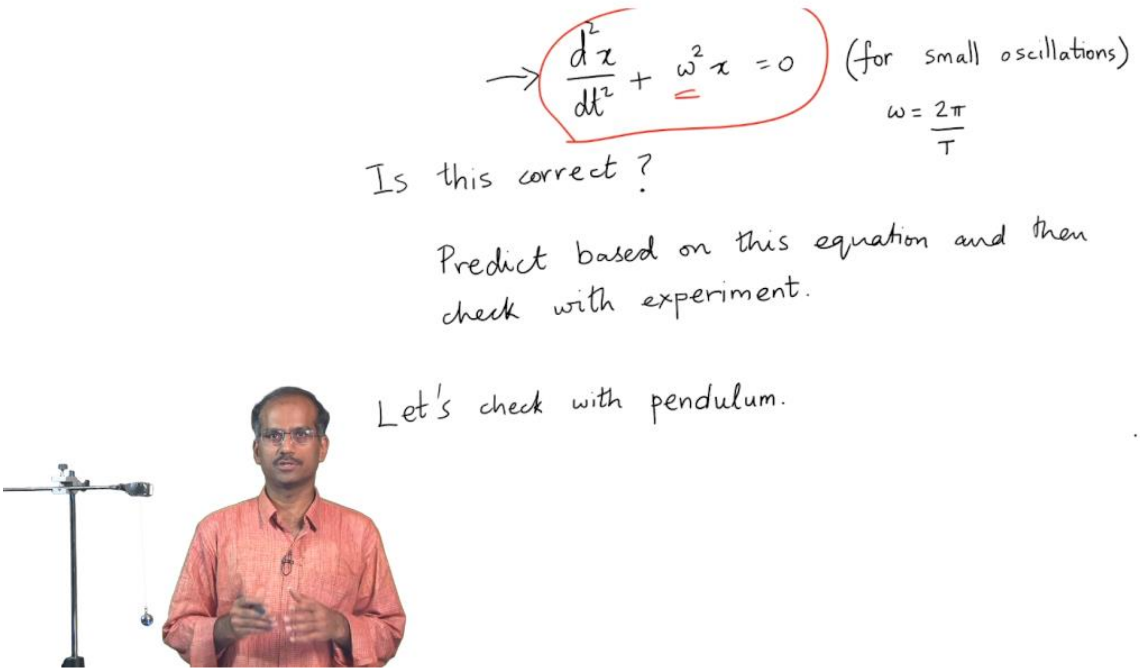

So, this capital T time period is that time taken for one complete circuit or one complete oscillation. So, now, we have the mathematical equation that describes small oscillations. Now, the crucial question to worry about is, is this correct? How do we know that what we have derived based on looking at the pendulum and our own intuition and we put together this simple equation, is this correct?

So, the way to answer this is by actually doing an experiment let us predict something based on this mathematical equation and then check with the experiment whether it matches or not. So, which is what we are going to do right now and let us check that with the system that we have here namely the pendulum, but again we should remember that the equation that I have written down here does not refer to pendulum at all, it is in general valid for all oscillatory systems provided you are in the regime of small oscillation.

So, at this point that is the claim that we have and now we are going to check this with pendulum.

So, when we go to pendulum now, we need to derive specifically an equation of motion for pendulum. So, I have drawn here a sketch of pendulum. So, I have a string of length L and there is a bob whose mass is M and if I pull it a little bit aside the displacement or the angular displacement is θ and the distance here is x.

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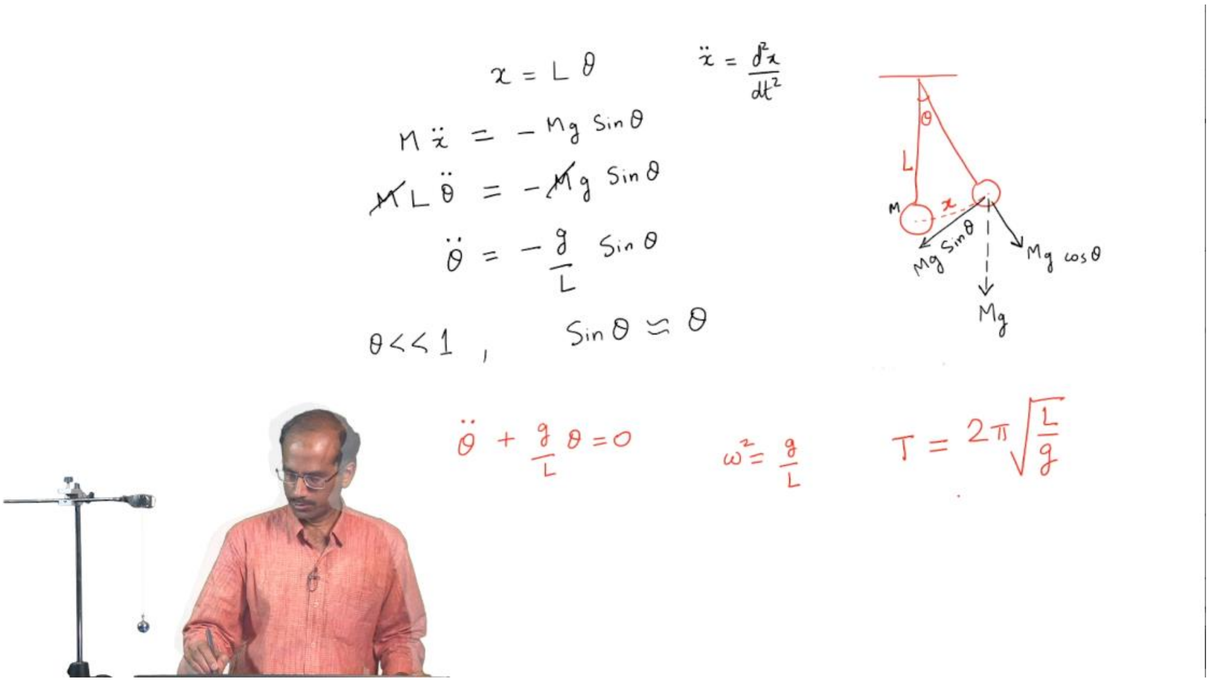

So, x and θ are related by x is equal to L times θ . Now, what are the various forces acting on this bob when you displaces it away from the equilibrium position when it is here. So, there is of course, the weight which is acting vertically downward that is mass times the acceleration due to gravity $Mg$and then this Mg can be resolved into two components, one that is along the direction of the restoring force and the other one which is perpendicular.

So, in the perpendicular direction it will be $Mgcos\theta$. So, I will leave it you to figure out how it comes it is simple resolution of the vector and along the direction of the restoring force that is along this distance x it is $Mgsin\theta$. So, now, we are ready to write equation of motion that will precisely describe the motion of this bob.

$$M\ddot{x} = -Mgsin\theta$$

So, $M\ddot{x}$ is equal to the restoring force which is - $Mgsin\theta$ and here $\ddot{x}$ can be replaced in terms of L and θ. So, that could be written as

$$ML\ddot{\theta} = -Mgsin\theta$$

and maybe I should have mentioned that this double dot indicates second derivative.

So, $\ddot{x}$ would mean d^2x/dt^2. So, this is a convention that we will use throughout this course ok. So, here now back to this equation this M and M will cancel. So, I can rewrite this equation as

$$\ddot{\theta} = -\frac{g}{l} sin\theta$$

So, if you go back to our equation for small oscillation that we wrote down it looks like

$$\frac{d^2x}{dt^2} + \omega^2 x = 0$$

but the equation that we have does not quite look like that, but we can bring it to this form provided we make the assumption that the angular displacement has to be small.

In the so, when I say angular displacement has to be small, I mean that θ should be much less than 1 in which case I can replace $sin\theta$ by just θ I could then write it as

$$\ddot{\theta} + \frac{g}{l}\theta = 0$$

Now, if you remember this equation that we have just written down is similar to the equation that I have here.

So, hopefully this should describe for us the dynamics of this pendulum, but we can go ahead and associate this g/L with ω^2 and since we said $\omega = 2\pi/T$ if you put in $\omega = 2\pi/T$ and simplify you can get an expression for T which is a time period of the pendulum as $2\pi\sqrt{l/g}$ So, this is the result that we will we will test it out in an experiment.

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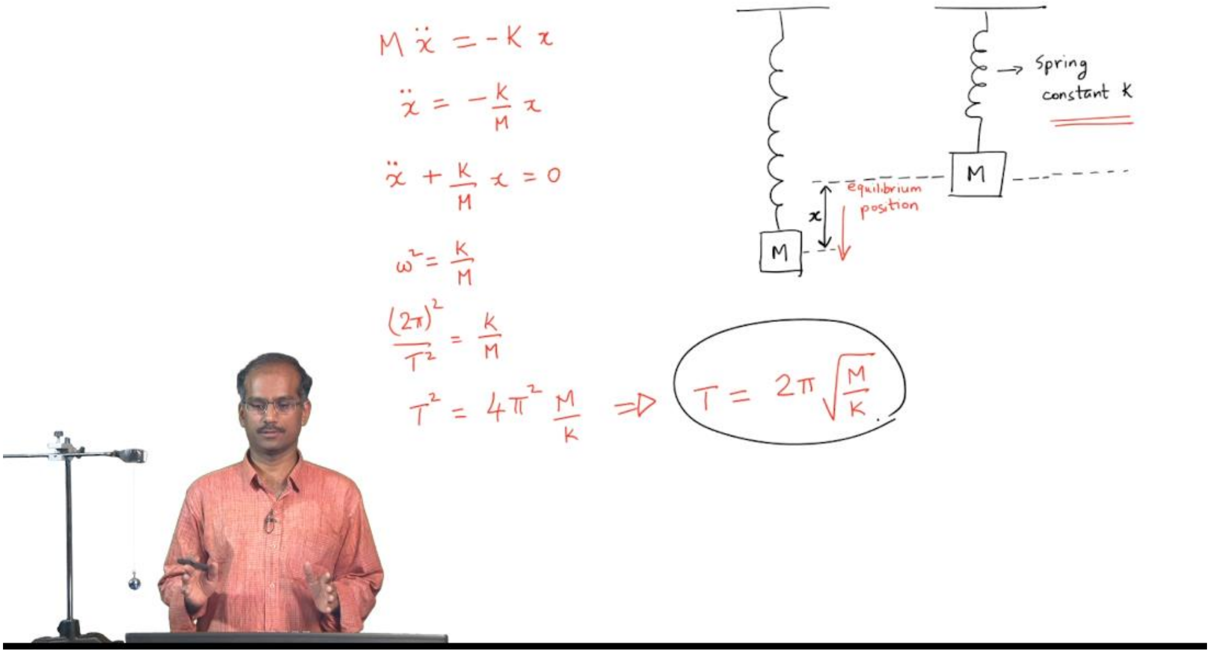

Similarly, we can also consider one more oscillatory system. So, in this case what I have is a spring and mass which is hanging from a support like this and if I do not do anything to this system roughly this is where the mass would stand as it as shown in this figure and as usual, we will identify that position of that mass M as the equilibrium position.

So, that is where the net force on the on the block is zero ok and if I displace it a little bit here in this direction by an amount x. So, the spring is, has expanded and when this happens there is going to be a restoring force which will take the block back to its equilibrium position so it will start oscillating.

So, here again we can go back and write an equation. So, once again it is mass times d^2x/dt^2 which I write as $\ddot{x}$ and that's equal to, it is actually proportional to displacement with the minus sign and the constant here is k and this k is the spring constant and here in this case it is very easy to bring it to the standard form.

So,

$$\ddot{x} + \frac{k}{M}\,x = 0$$

So, you can see that I can directly write of the value of ω^2which is k/M and I can write an expression for the time period. So, that is

$$\frac{(2\pi)^2}{T^2} = \frac{k}{M}$$

And

$$T^{\,2} = 4\pi^2 \frac{M}{k}$$

So, this will give me an expression for time period which is

$$T = 2\pi \sqrt{\frac{M}{k}}$$

So, once again we have an expression for the time period, this time for system of a spring in which block of mass M is hanging and it is executing simple harmonic motions.

To summarize this module based on the pendulum example that we started with, we made the ansatz that the restoring force is proportional to displacement with the negative sign. That simply is the central physics that we need to learn and from that we obtain an equation of motion which is shown here

$$\frac{d^2x}{dt^2} + \omega^2 x = 0$$

ok and of course, for detailed information we will have to solve this equation of motion which we shall do in the next module.

But even without solving it, we could still test the validity of this equation by simply extracting the information about the time period and matching it the time period that we get from this theory with what we get from experiment.

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$$T = 2\pi\sqrt{\frac{L}{g}}$$

$$L = 0.25 \text{ m}$$
$$g = 9.8 \text{ m/sec}^2$$

$$T \simeq 1 \text{ sec.}$$

So, before we compare with the experiment, let us see what we should expect theoretically. So, based on the equation that we just derived the time period depends only the length of the pendulum and the acceleration due to gravity.

So, in the setup that we have length of the pendulum is 0.25 meters and acceleration due to gravity is 9.8 meters per second square. So, if you put in all those numbers, it tells us that the time period expected based on a theory is about 1 second, now let us do the experiment.

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$$T = 2\pi\sqrt{\frac{L}{g}} \qquad T_{expt} \simeq \frac{10.22}{10} \text{ sec.}$$

$$= 1.02 \text{ sec.}$$

$$L = 0.25 \text{ m}$$

$$g = 9.8 \text{ m/sec}^2$$

$$T \simeq 1 \text{ sec.}$$

So, we will run the bob and oscillate it ten times and then compute the time period. So, when tenth oscillation completes, we saw that it has taken 10.22 seconds. The time taken for ten oscillations is 10.22 seconds as we just now measured. So, the time period is 10.22 seconds divided by 10 that is about 1.02 second.

So, we can see that there is a very good agreement between what we theoretically expected which was about 1second and what was experimentally measured which is 1.02 seconds. So, to within the experimental errors there is sufficiently good enough reason to believe that the assumptions that we made about small oscillations and the equation that we obtain for describing the motion of this pendulum are correct.