Non Equilibrium Statistical Mechanics
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Module 1
Lecture 32
Critical phenomena (Part 4)

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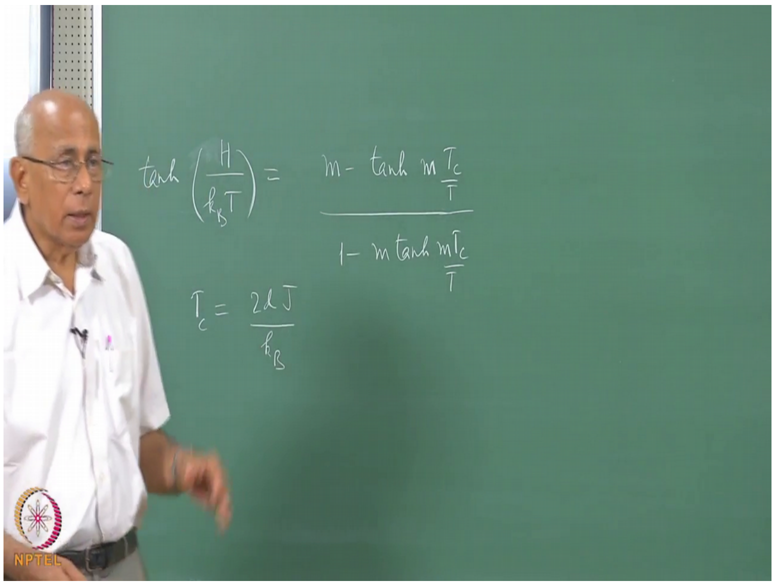

Right, so we go resume where we left off, we left off at the magnetic equation of state for blazing model in mean field theory and if you recall the magnetization particle in suitable units m this was given by the magnetic field if you recall H k Boltzmann T we solve for it and we discovered there was a solution of the form m minus tan hyperbolic m times Tc over T divided by 1 minus the product of this two, oh yeah tan hyperbolic is transcendental of this was m Tc over T and Tc was when we call Tc was equal to it certainly was 2d and then there was a J which is the exchange constant the coupling constant divided by k Boltzmann and I think that was it that was all, right?

So there is a dimensionality dependence here in this thing in trivial sense because it just includes increases the number of nearest neighbors, right? So now we are going to look at it in the critical region and from this we can extract all the critical exponent which we already saw in various ways.

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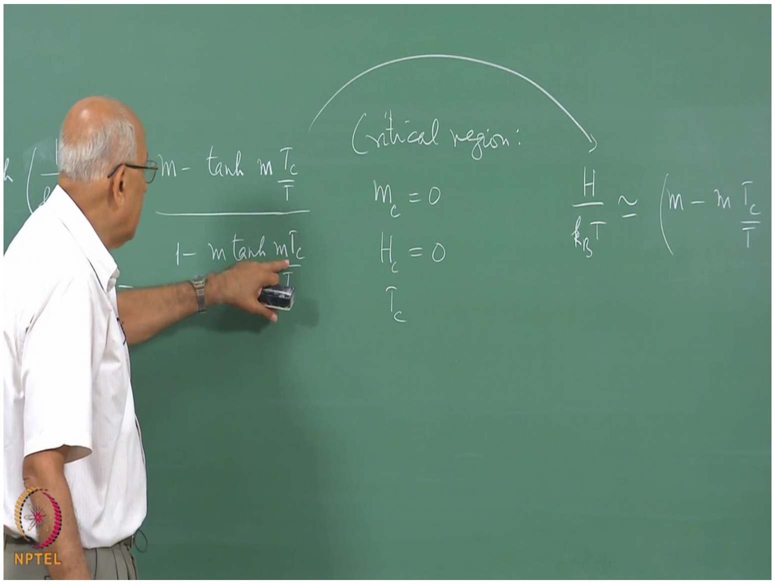

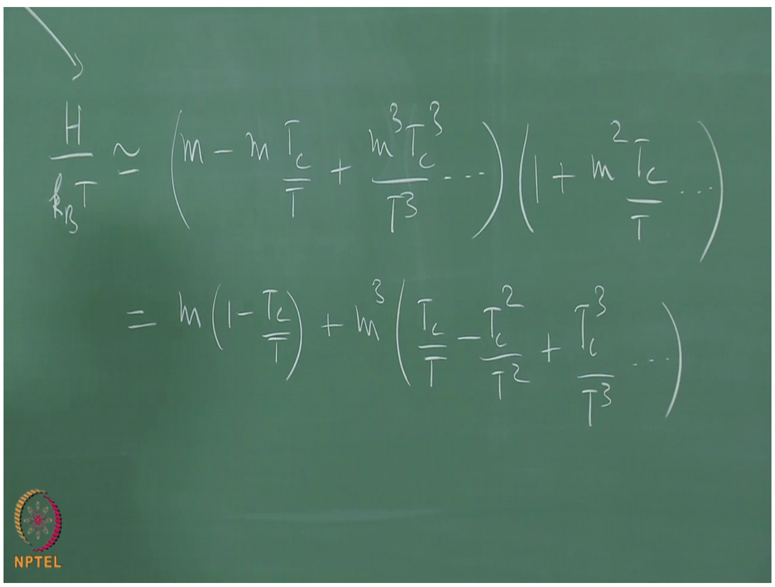

So critical region remember that in this problem the critical value analogous to Pc, Vc and Tc the critical values are m critical equal to 0 because it takes off from 0 from the paramagnetic phase to the ferromagnetic.

Hc is also 0 because remember that H equal to 0 corresponded to this flat line ending in a critical point in the H versus T plain. And of course Tc is non 0 it is some number given by this, so a critical region says when m, H are small very near 0 and T is very close to Tc that is the critical region. So let us see what this looks like to leading order this equation of state becomes H over k Boltzmann T is equal to m minus tan hyperbolic x here tan hyperbolic x goes like x minus x cubed over 3 the leading behaviors, so it is m minus m cubed Tc cube over T cubed in this case plus etcetera times now this guy here already has an m out here and it is going to be multiplied by another m, oh yeah m Tc over T yeah that is important otherwise I am not going to get the exponents.

You see they assume I make no mistake so they got rid of this, I have to prove them wrong so m minus m Tc over T plus m cubed Tc cube over T cubed dot dot dot times in the denominator you have 1 minus because tan hyperbolic is got be x minus x cube because it comes down and saturates so it got to be a negative and that becomes this divided by 1 minus m squared Tc over T and then the next term is T m cubed here and there is a m here so it becomes the m4 we will solve it out.

So it is 1 minus m squared Tc over T inverse which is 1 plus dot in this fashion, so if you write this out this is equal to m times this so this is 1 minus Tc over T that is this portion and then plus m cubed times the first term is Tc over T and then there is minus Tc square over T squared from this fellow and then this time here plus Tc cubed over T cubed dot dot dot, oh yeah there is 3 factorial, thanks.

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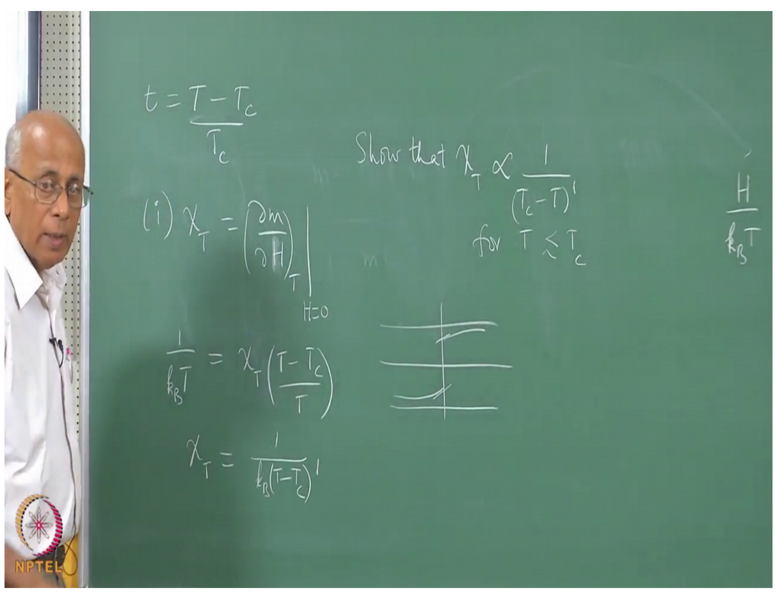

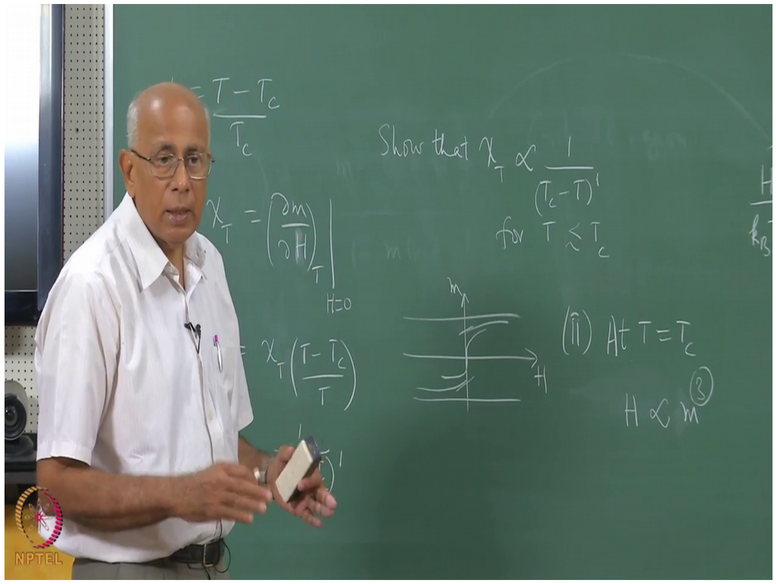

So this is m cubed over 3 in this fashion, okay so are ready to read off various results let us give this some name let us give this Tc over T some name we use little t I am going to use little t let us fix it is equal to T minus Tc divided by Tc so we could not principle write it in terms of little t this whole thing but what are the things we want to see immediately well the first thing we want to do first is to find this acceptability remember that kai T equal to delta n over delta H at constant T and H equal to 0 that was our definition.

So all I have to do is to differentiate both sides out here and this is m cubed already so it is clear that the linear behavior comes from here, okay and it immediately says 1 over k Boltzmann T is equal to if I differentiate both side with respect to H this is equal to kai T into T minus Tc over T or kai T in these units is 1 over k Boltzmann T minus Tc so I diverges at the critical temperature like 1 over T minus Tc.

Now this is for T greater than Tc so that this I positive I want you to show that the same result obtains for T less than Tc, so show that kai T is proportional to 1 over Tc minus T for T less than of the order of Tc just below Tc in that sense the constant of proportionality will not be this it will be some other number some number possibly. So it is the same critical exponent both are to power 1, I have used the fact that m is yeah so I have used the fact that this quantity is a ferromagnetic it is not a paramagnetic so it orders in the direction of the field in this acceptability has to be positive, right?

So implicitly I have assumed that T is greater than Tc you have to now solve for T less than Tc and find the right branch in m as a function of H this happens to be the right branch for t greater than Tc because this is the paramagnetic branch, yeah you have to solve that cubic, okay because this acceptability if for T less than Tc is the slow at the point when m is not 0 but has finite intercept and you have to find that root and then get back to this, okay either of the roots will do because it was the graph looked like this it was like this and like this and we have discussing this slope or this slope and that is going to go like this, okay so that is the good point I implicitly by using this alone I said when near I am equal to 0 that is only above the critical point below the critical point m is away the stable roots are m not equal to 0 in the absence of a field, okay they are the spontaneous magnetization roots, okay.

That is the first result, then the second one is we can also see what happens at T equal to Tc so we need this result to at T equal to Tc we want to get hold of this exponent so this is m versus H we want to get this exponent here how does it behave what is the power law we would like to find that. So you set T equal to Tc and of course you immediately see that this cancels this cancels this cancels this cancels and you are left with this guy.

So it immediately shows m is proportional to H cube, sorry H is proportional to m cube there is cubic point curve here on the critical isotherm that follows immediately from this because this term goes away and these two cancels and you are left with the leading behavior H proportional to m cube at the critical isotherm you know the words this is a cubic curve. Then, let us ask what happens to the spontaneous magnetization in the absence of the field.

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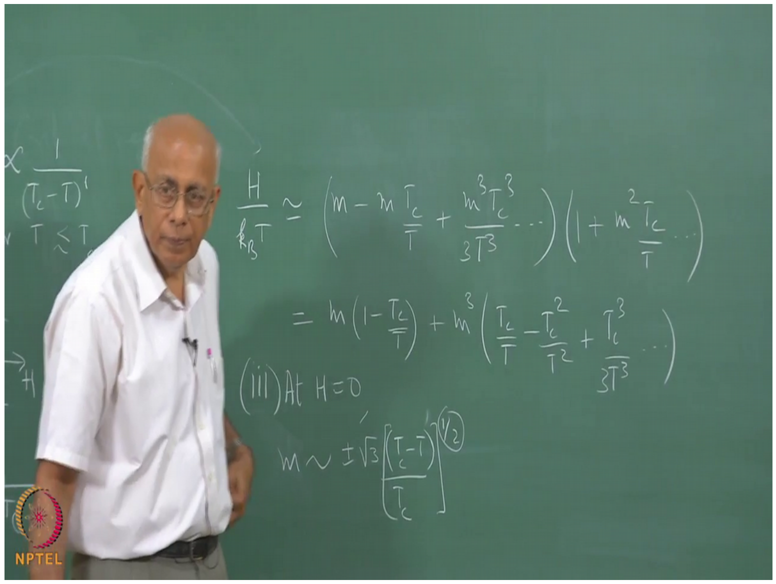

So we need to find out 3 at H equal to 0 now we are trying to find out here is T here is m m not and remember that it goes like this is what we had said this is Tc and we are trying to find the behavior here in this region having said H equal to 0. So this is equal to 0 and you have to solve this equation for m not with this set equal to 0 the root m equal to 0 is always a root as we saw but it turn out to be an unstable root we want the non-zero roots we want these roots not the 0 root here. So you can cancel out m for that and then I leave it to you to show this is fairly simple you cancel this out you get m squared then you move it to this side and remember now T is less than Tc so Tc over T minus 1 would be a positive quantity you move it to this side and that will be equal to m square times something or the other here so you get two real roots show that m goes like plus or minus square root of 3 times Tc minus T over Tc to the power this whole thing to the power half, of course it is immediately obvious, yeah that is right yeah you have to be little careful because this is similar term sitting here, exactly you have to check that is not 0 identically that is a straight forward to do.

There are several ways in which you can show from the exact equation that it is going to be square root singularity. So the critical exponent is a half now these are exactly the same exponents which you get for the (())(13:52) equation also in the critical region.

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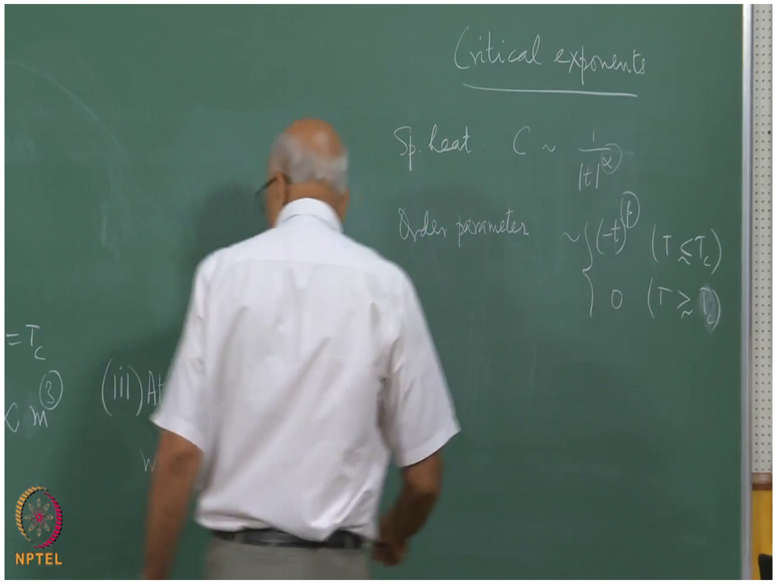

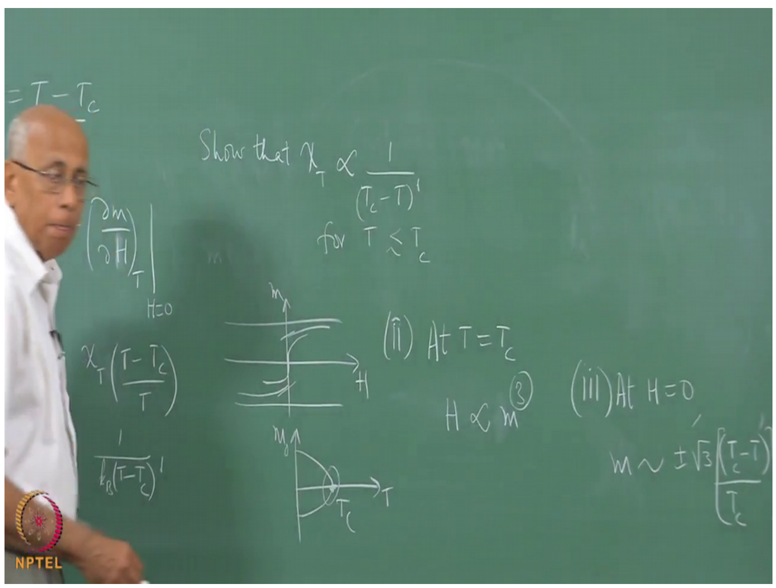

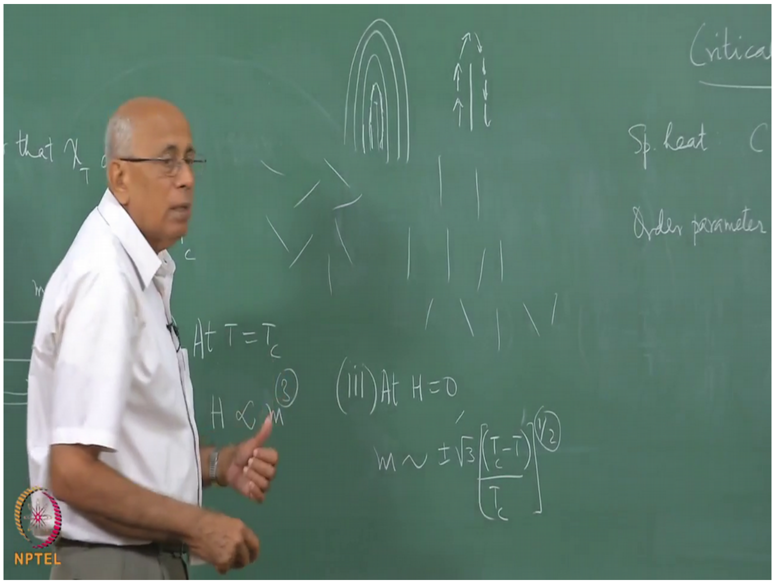

However they are not the experimental exponents experimentally what happens is these exponents are somewhat different, yeah I am going to do that that is exactly what we are going to do experimentally we have a whole set of critical exponents there is rigorous way of defining it but in realistic way there are various power laws what happens in the critical region near a critical point.

Now this critical exponents have values which are universal for different universality classes so called ising model universality class which is what we are dealing with here we started with ising model you have one set of exponents for the Heisenberg universality class you have another set etcetera etcetera. Now in the simplest instance and then you have what are called mean field exponents these are mean field exponents here, so this specific heat is supposed to behave C is supposed to behave like I am not specifying Cv or Cp I am not going to do that for a minute I am not going to do that this diverges like 1 over mod T minus Tc well let me write everything in terms of this little t out here this measures how far away you are from the critical temperature.

So this goes like 1 over mod T to the power alpha that defines the exponent alpha, then the order parameter either the magnetization the one that distinguishes the two phases and we are going to take typically one of them to be one order parameter in the high temperature phase to be 0 and non-zero below just in analogy with the magnetization. This order parameter goes in the critical region goes like mod T to the power beta in fact it goes like T minus Tc to it goes like Tc minus T, so minus T to the power beta for T less than Tc while it is 0 for T greater than Tc, yeah Tc.

So the order parameter is 0 and then it branches off in this fashion and we are talking about what happens in that region to the stable root, the order parameter there are many ways of defining there is no uniqueness about in order parameter you could take other things as well I mean in magnetization example you could ask why did not I take magnetization to the power 3 or 7 as the order parameter no reason why not but the most convenient and simplest one.

Now there is no reason why it should be a scalar, for example in not the ising class but the Heisenberg class the magnetization is a vector then there are situations where it is a planner vector no matter how many physical dimensions you are in for the lattice the magnetization itself magnetic moments can only move in a plain set that is the so called xy model. In the ising model they move only in one direction up or down so it is a scalar otherwise a two dimensional vector otherwise a three dimensional vector maybe in n component object in very complicated systems like in nothing like liquid crystal animatic liquid crystal for example it is an axis, liquid crystal animatic liquid crystal consists of rod like molecules which are arranged every which way in the disorder phase and if you lower the temperature these guys get order so on the average they all point in this fashion, of course there will be small fluctuations about it but on the average they point along this (())(18:23).

But there is no distinction between the head and tail so it is not an arrow unlike the magnetic moment and it is a very perform consequences follow because it is not an arrow but only the line and that implies that the order parameter is not a vector but an axis here. So in this case it turns out to be a tenser of rank 2, okay because that will specify an axis, okay headless vector it is also called a director, I did not named it, right?

It reproduces a line field and it’s got defects because it is a line field and so on if you look at your sum for instance I should not get (())(19:16) but if you look at your thumb you got words like this the thumb print and then at one point there is something like this this is a defect a topology defect on the surface here you should really look at it as something a point a line defect really but in two dimensions it is point defect here and the directors are supposed to be like this etcetera such a thing cannot happen if you had arrows because then it means that you are going like this then what happens on this line it is a point defect but happens on this line it is indeterminate completely.

So a 2 dimensional ferromagnet cannot have a point singularity of that kind but a liquid crystal can it is called a 180 degree declination and it is got physical affects and so on very real thing. So we are all carrying topological defects on our thumbs, okay. So this is a complicated order parameter if you go to more complicated substances like liquid helium super fluid liquid helium in helium 4 then the order parameter is a wave function of what is called the condensate a super fluid condensate that is now as you know a wave function is a complex number.

So it is a modulus and a phase that is the order parameter a complex number if you like, if you look at helium 3 we are the rare isotope of helium consist of fermions that too can become a super fluid and it is got all sorts of magnetic properties and so on that order parameter is pretty complicated it is SO3 cross SO3 cross O2 so it is some 9 by 9 or whatever fluids says some 18 dimensional object. So it is got a lot of physical information buried in it, but the order parameter can be very complex in the reaming cases you could ask what is the order parameter in a liquid gas as we said it could be the difference in densities between the gas and the liquid but what is it in the a crystal as opposed to a liquid because the liquid and crystal have practically the same densities most substances when they freeze they all become very much more dense in ice it actually expands the other way but they are equal to each within 10 percent.

So what would be a good order parameter in a crystal? Something that reflects the nature of the order namely that atoms only sit at regular intervals and so on. So it would be if you take the density of the crystal the local density wherever there is an atom there is a big spike and then there is nothing etcetera and you do its Fourier transform then it will have components at all the wave vectors corresponding to the reciprocal arties that set of amplitudes would be your order parameter, okay.

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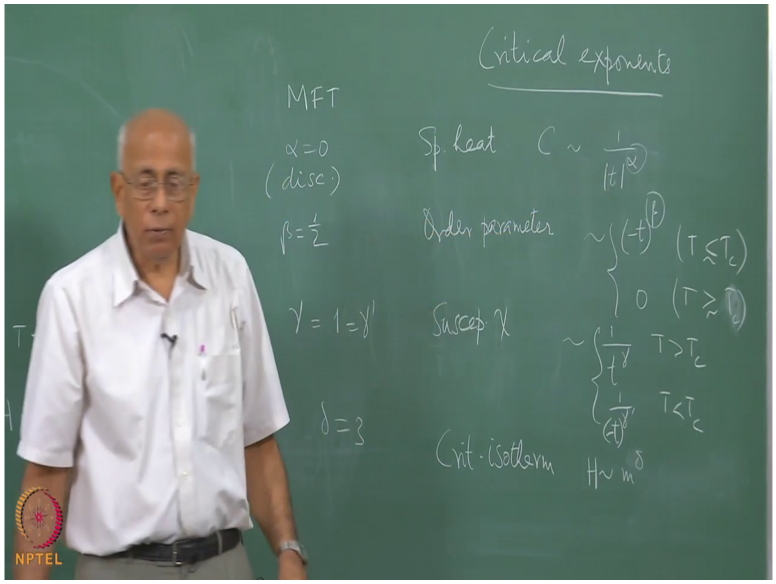

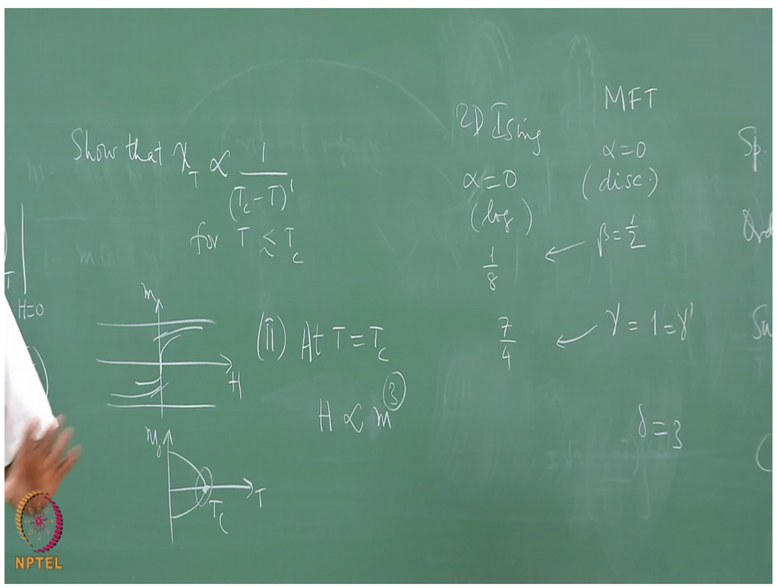

So it is not a trivial job finding the order parameter in many case in some cases but we know it when we see it and the order parameter exponent is called beta in mean field, so whatever we have done is called mean field theory this alpha is 0 it turns out that it mean field theory specific heat is predicted to mealy be discontinuous a finite jump and not divergent this says it is infinite as T tends to 0 when alpha is positive this becomes infinite on the other hand you have a discontinuity. The order parameter beta is a half in this problem then the so called susceptibility order parameter kai this goes like 1 over T to the power gamma for T greater than Tc and goes like 1 over T to the gamma prime for T less than Tc 1 over minus T to the power gamma prime, okay.

In mean field theory I just showed that gamma is one and gamma prime I asserted was also equal to 1, so the susceptibility exponent is 1, then you ask on the critical isotherm what does the critical isotherm curvature look like so critical isotherm this is delta because we found H is proportional to m cube P is proportional to minus v cubed and so on in mean field theory. In general this is some delta and in mean field theory this delta so alpha equal to this beta equal to half gamma equal to 1 equal to gamma prime delta equal to 3 there are two more there are actually a few more there are two more which I will introduce very shortly.

So we sort of extracted whatever we want from this thing here as much as we can but I must now we must now go back and ask where is all this coming from what about corrections to it etcetera but first some experimental facts in real life if you look at magnets like the 3 dimensionalizing model or the real liquids for instance then these exponents are very different, for example alpha is for liquids very close to 0 some small number 0.1 or something like that or less. In the case of super fluid helium, sorry in the case of 2 dimensional ising model alpha is 0 but the specific diverges logarithmically, so there is a log divergence there.

Now the specific heat itself depends on what is kept constant, it is either Cp or Cv or c with a constant field it depends on this system that we are looking at so that is why I did not specify which one it is for instance for the (())(26:50) model for fluids Cv continues to be that of an ideal gas in the critical region but Cp diverges and it is related to the divergence of a susceptibility in that case the compressibility.

So various possibilities this can happen, in 3 dimensional liquids data is not of the order of half but nearer 0.321, 0.325 something like that. This exponent gamma is of the order of 1.25 is larger this delta is of the order of 4 to 5 4.7. 5 something like that. In the 2 dimensionalizing model it is a very special model again exactly solvable all the exponents can be shown to be rational fractions exactly, for instance this turns out to be a log discontinuity 2D ising alpha equal to 0 but it is a log divergence log mod T minus Tc then beta is equal to one eighth (())(28:11) gamma is seven forth delta is what eclipse of the I mean I do not remember what is delta and I frantically trying to find the relations I mean there are relations between these exponents I am trying to see given this can I find the, for instance alpha plus 2 beta plus gamma is equal to 2 and the other are alphas here but it works here as you can see alpha plus 2 beta plus gamma is equal to 2 it will work and works here too but I am trying to think of what is the simplest, it will come back, okay.

But that is because it is dimension dependent the 2D ising model the 3D wants a much closer to real life on various cases. Now I want to get straight to the point that what is underlying this whole business and you will see in a minute where it comes from is the divergence as I said of something called a correlation length so we have to define a correlation length and that will give us a big handle on what to do next, okay.

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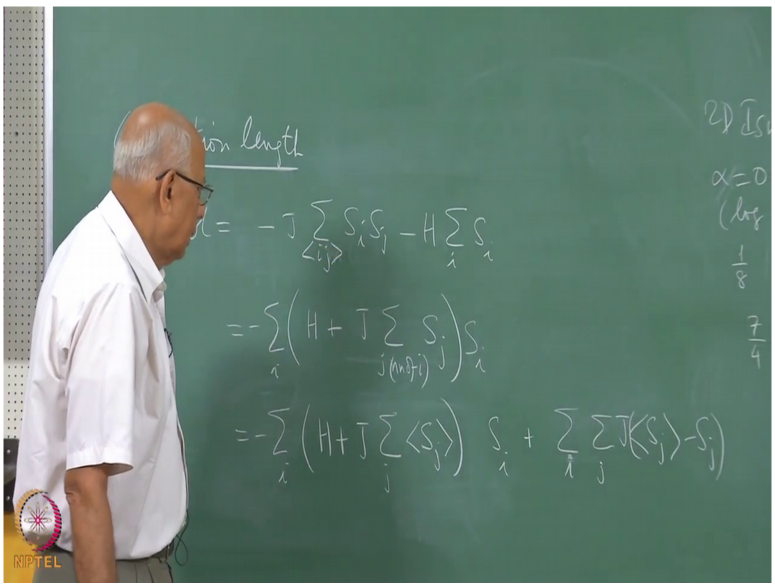

So let me define that what is that I mean by this correlation you see let us go back to the ising problem the magnetization or the moment at each side that is side is your is measuring the order parameter the average value of this moment, okay but you could ask in a thermodynamically homogeneous medium in equilibrium at every local side you do not have something which pints on the average, it is fluctuating all the time.

So there are fluctuations and you could ask given the average what is the next mean square what is the deviation look like from the average some kind of generalized co-variants you ask for and what would it be in the case of the spin problem you see if you recall we said that the affective field at every point and if you recall we wrote the Hamiltonian again I go back to this as minus J summation over ij Si Sj minus H time summation over i Si we started by writing this and then I said look this could be written as minus this field plus J times summation over j nearest neighbor of i Sj summation over i Si.

So the field that this guy is seeing this moment at the ith side is this this is exact in the ising model. In the mean field case we replace this fellow by its expectation value, okay in other words we wrote this field as equal to minus H plus J summation j nearest neighbor of Y there is J summation over i Si well it is not very elegant notation so let us summation over i Si so this is the effective field seen by the ith spin this guy times Si I added this instead of this so I got to put that back, right? So plus summation over i summation over J J Sj minus Sj, okay you could put a J ij here just in case it is in homogeneous etcetera.

So this term cancels out and I get back to this term, okay this is also in Si the whole thing acting on Si. Now this guy here represents the fluctuation about the average value of these variables here and mean field theory drops this fluctuation that is all its done is just dropped it, but we would like to know how important this is that is the fluctuation at every point.

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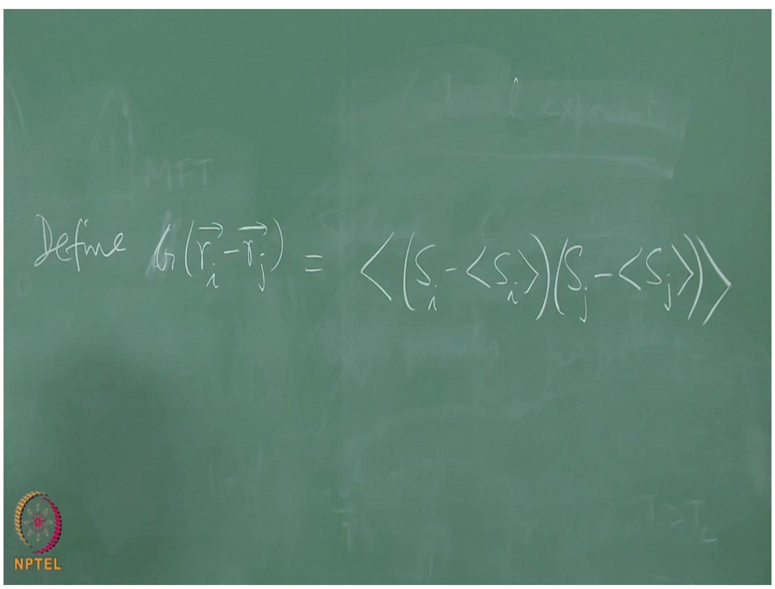

So now let us define auto correlation let us define let us call it G ri minus rj it is a function of the difference in position of ith and jth moments is equal to expectation value of Si minus average Si Sj minus average Sj, okay it is clearly by translational variance in the thermodynamic limit clearly a function of i minus j this is like delta Si delta Sj and it will be a function of i minus j of course you can also write this as equal to Si Sj minus Si Sj you can also write it like that by trivial piece of algebra, right? It is the generalization of the mean square deviation at some point but it is now specially dependent on two indices i and j, okay.

Now let us go back and ask what are these expectation values the next target is to relate this to the susceptibility that (())(35:18) give us the static susceptibility formula and you will immediately recognize linear response theory unit.

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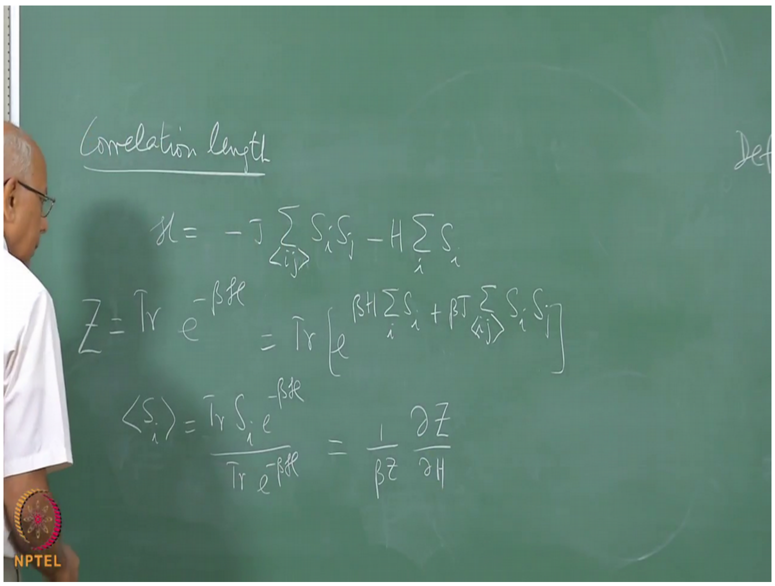

So the derivation I start with this again the density matrix, yeah expectations already there is no averaging of an average we solved the average, right? So trace so Z the partition function is traced either the minus beta (())(35:47) trace over the fact that each Si can be either plus 1 or minus 1 that gives you all 2 to the power n for n of them and then you take thermodynamic limit, okay.

So this is equal to trace e to the beta H summation i Si plus beta J summation ij nearest neighbors Si the trace of this whole thing, okay. Now what is Si itself equal to, this is equal to trace Si e to the minus beta H over trace e to the minus beta H which is equal to by the usual trick I want to pull an Si out here, right? So what should I do? I take a derivative with respect to H, right? That gives me the summation over i this guy, so let us do that equal to and there is a extra beta which comes out, right? So I have to divide by this way, so 1 over beta Z delta Z over delta H that summation over i Si because there is summation over i, right?

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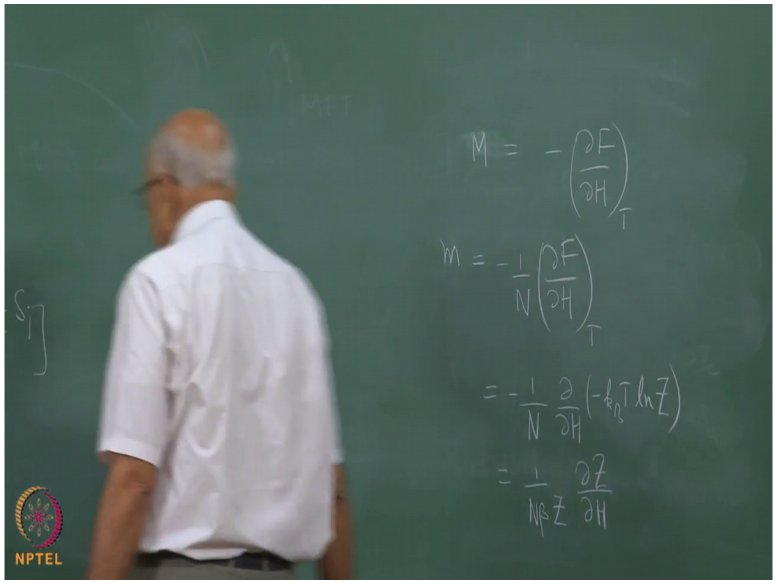

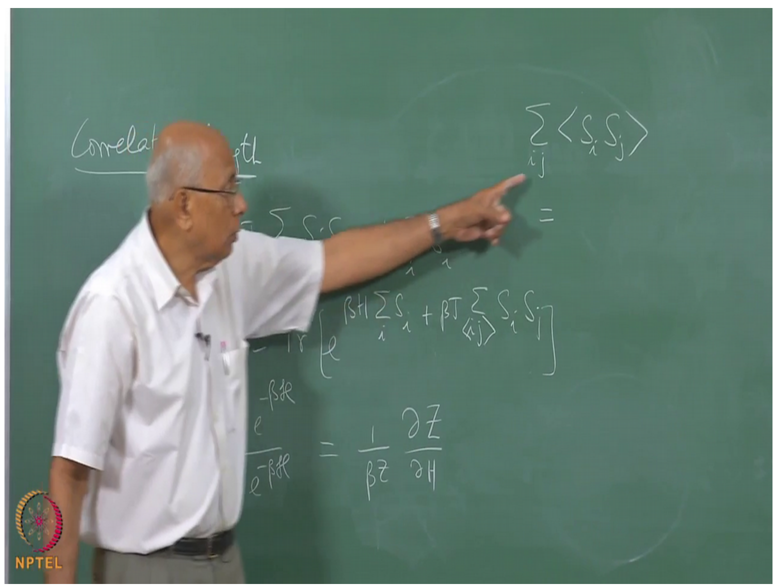

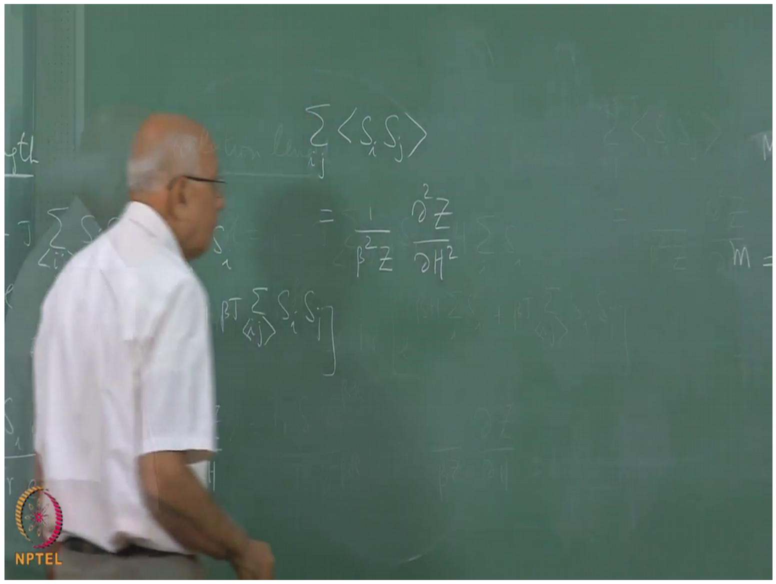

By the way you already know formula it is trivial you already know this from thermodynamics because you see let us connect it up it is useful exercise because remember that the magnetization M will appear in thermodynamics through MDH like VDP, right? And it will be equal to minus delta F over delta H keeping temperature constant. So m little this is capital M equal to minus 1 over N delta F over delta H at constant T that is equal to minus 1 over N delta over delta H of minus k Boltzmann T log the canonical partition function that is the formula for the free energy minus kth log Z.

So this is equal to 1 over N beta delta so this is equal to 1 over N beta Z delta Z over delta H, that is what I got here the same formula remember by translational invariance this expectation value is independent of i and the n of this follows, so each of them is 1 over n times this that is your M little m, so it matches this thermodynamic formula, okay. So all I have done is to write that show how that arises directly by doing this but now comes the interesting part, what is this equal to? What is summation over ij Si Sj equal to I do two derivatives with respect to H first I am going to pull down Si second time it will put down Sj it is not this term because this fellow the summation over j is restricted to nearest neighbor of i for each i but this what I am calculating here is over all i and j and that comes by taking this down and differentiating twice, right?

And that becomes equal to 1 over each time I pull down a beta so it should be 1 over beta square and then a Z D2Z over DH2, so now let us calculate our green function or correlation.

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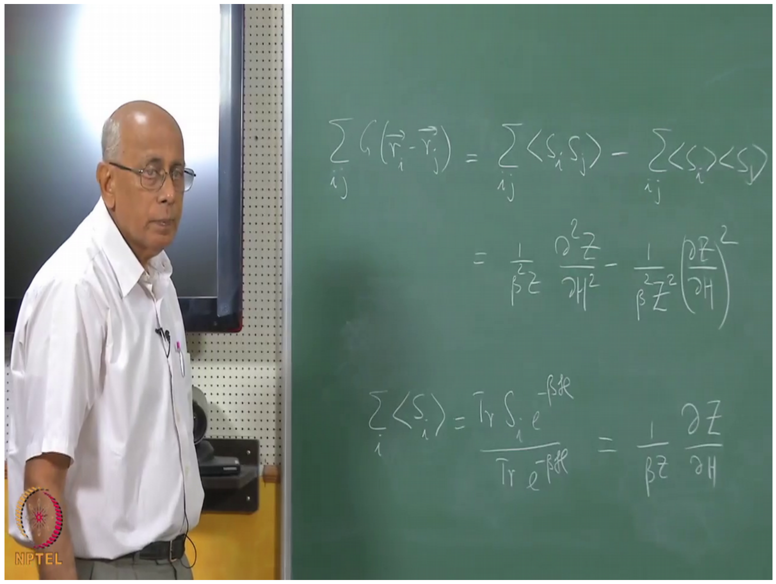

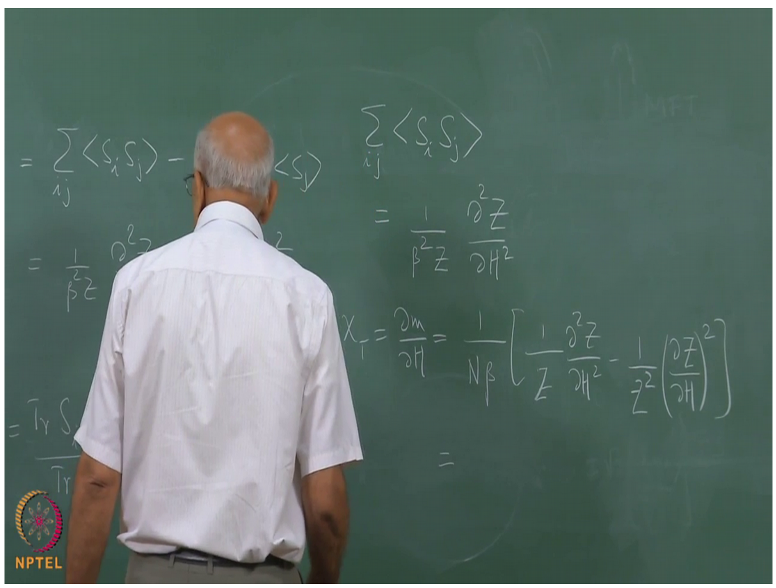

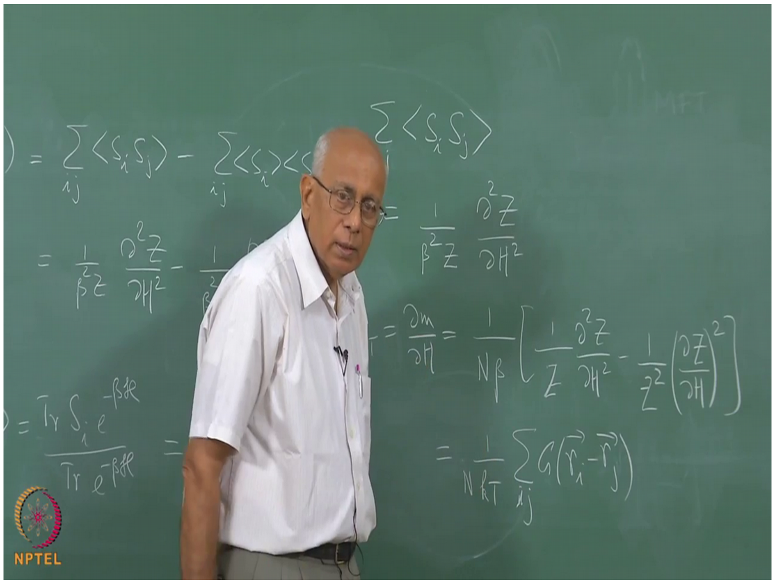

So the correlation function was G of ri minus rj this guy here summed over ij a sum over it this is equal to summation over ij Si Sj minus, minus what? Minus summation over ij Si Sj this square of this sum I call sum of ij Si Sj this (())(41:26), that is equal to what? Let us put this stuff N it is equal to 1 over beta square Z D2Z over delta H2 minus square of this 1 over beta square Z square delta Z over delta H whole square that is this guy this correlation or if you like Si minus expectation Si Sj minus expectation Sj same here.

What happens if I differentiate m with respect to H? I should get kai, so it is clear that kai T equal to delta m over delta H which is equal to 1 over N beta the derivative of this guy the field appears here in Z out here, so the first term is 1 over Z D2Z over delta H2 minus 1 over Z square delta Z over delta H whole square but this is just this guy here, isn't it? There is an extra beta. So it says you have to tell me where the beta goes, so this is equal to if I multiply beta so it is equal to 1 over kT summation over ij G of ri minus rj, there is also an N somewhere there is an N sitting where, 1 over N but this is a function of ri minus rj so I can fix the j in sum over ri fix the i sum over j's and then fix the next i sum over j's we are going to get the same sum so I can remove one of the summations and call this coordinates some relative coordinate the distance between i and j and I can therefore write this as equal to 1 over k Boltzmann T it is not for the k Boltzmann T summation over i G of Xi where Xi means you are centered at i and you are now calculating all the distances to all the other later sets.

In the thermodynamic limit you could actually convert this to an integral, right? I may put lattice spacing convert it to an integral and let us do that in diddum by the this is called the static susceptibility formula the fact that this guy here is equal to this correlation, does not that you see this measures what it does in an external field and this is now telling you what the auto correlation is, so it is exactly like linear response formula let us precisely make that, okay.

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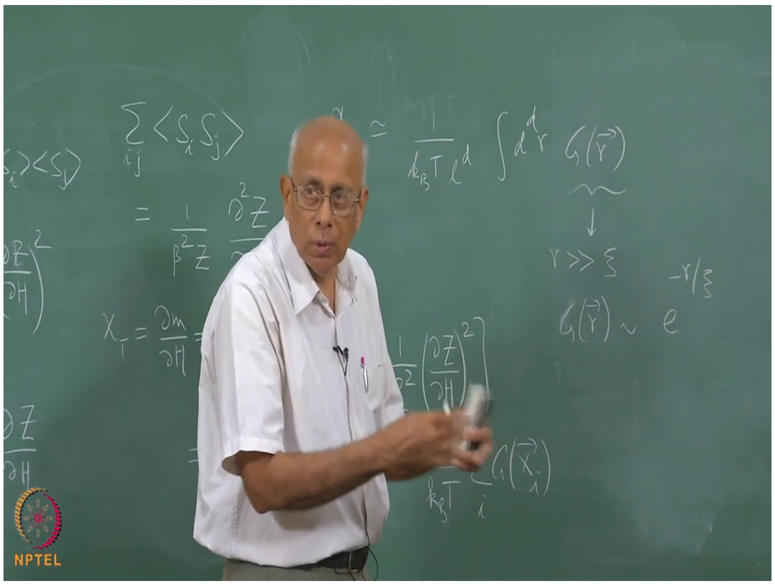

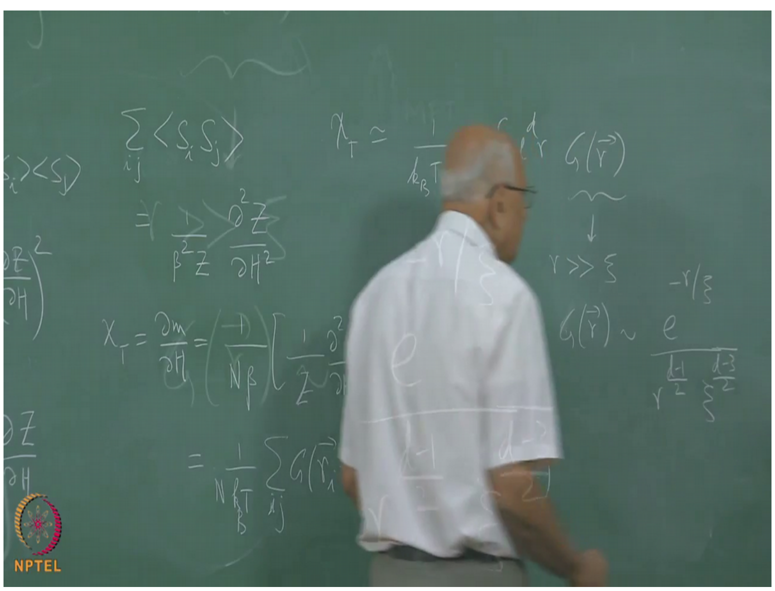

So now let us see what this does, so again we are on kai T is now approximately in the continue limit 1 over k Boltzmann times T now we have lattice with lattice constant l in d dimensions, denominator I think there is a beta square on top so that gives you, it is okay I mean I go back to linear response theory it is beta times a dot of 0 because you know what is beta 1 over kT, okay. So kai T is 1 over kT let us put a lattice constant l and it is in d dimensions and then you have an integral dd of r G of r.

Now we need a formula for this guy we need something for this correlation which requires hard work which requires little bit of work but let me state the result and if time permits we will try to derive it (())(46:33). We expect this is going to die down as r increases the correlation between the spin and spin is very far away is going to die down. Now it turns out that this fellow here for r much much greater than some correlation length xi G of r goes like e to the minus r over xi and this can be shown so I am going to assert the result and then we will see the consequence and then try to prove later on, divided by r to the power d minus 1 over 2 xi to the power d minus 3 over 2 away from the critical region T not equal to Tc and that is kai, okay.

So now look at what is happening if I put in what I already know for kai goes like T minus Tc inverse so it says T mod t upon minus 1 this guy goes like on that side this crazy integral this does not do any harm we just replace this by Tc but you got to do this integral, okay.

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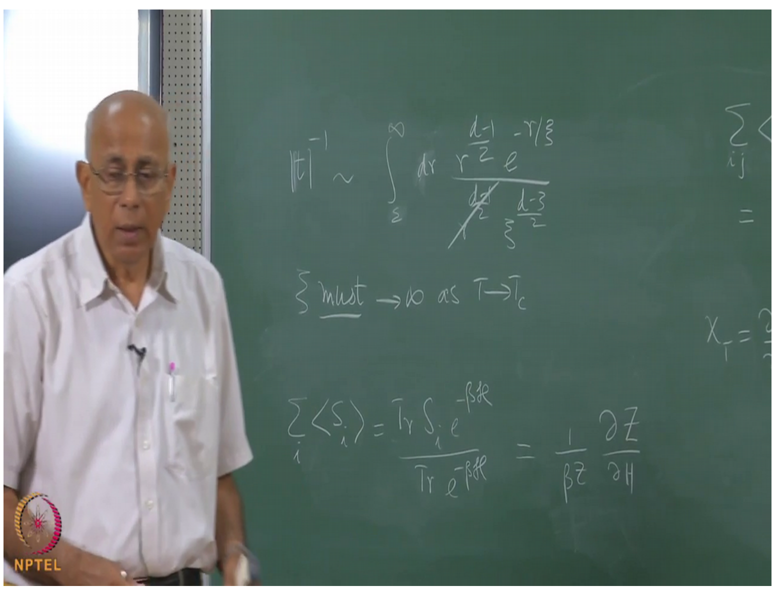

So you have to do an integral only the radial coordinate matter so it is 0 only what happens near infinity matters really r to the power dr r to the power d minus 1 e to the minus r over xi over r to the power d minus 1 over 2 xi to the power d minus 3 over 2 and this integral is 0 to infinity this guy some cutoff to infinity you do not need to, we do not care it is not 0 when this blows up.

So now there is a paradox you have this fellow by assertion I said that G looks like this exponentially dampt there is some powers of r floating around by the way this becomes d minus 1 over 2 this cancels against this, okay this is the phase space factor r to the d minus 1 and d dimensions. Now this diverges as T goes to Tc by this equal to an integral which has got this very strong converging factor here, what does it mean? Well xi certainly cannot go to 0 because if it goes to 0 this will kill it faster than any power here I do not care, xi blows up xi has to blow up so xi must tend to infinity as T tends to Tc this correlation between spins I already said the fluctuation effect is going to become extremely strong at the critical region and so much so any correlation length just diverges in an infinite system it goes to infinity itself, the question is how? That is easily seen from here because all you have to do so scale out by this so change variables let us change variables to.

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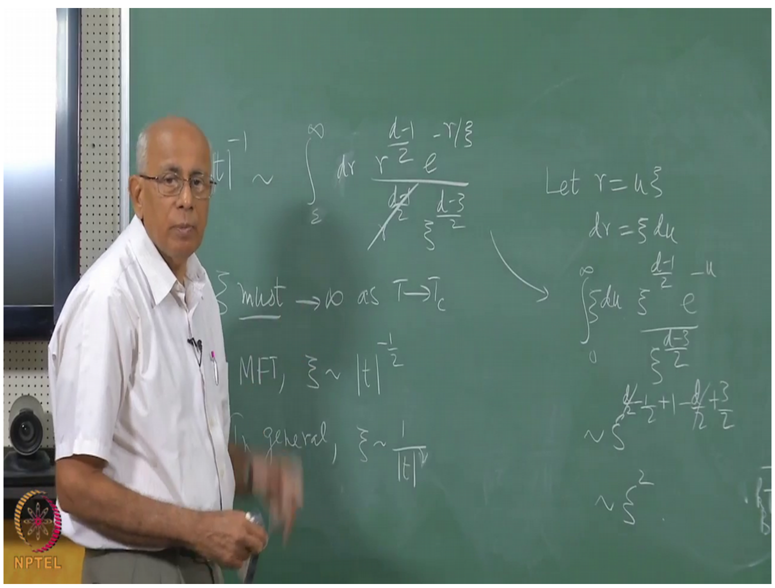

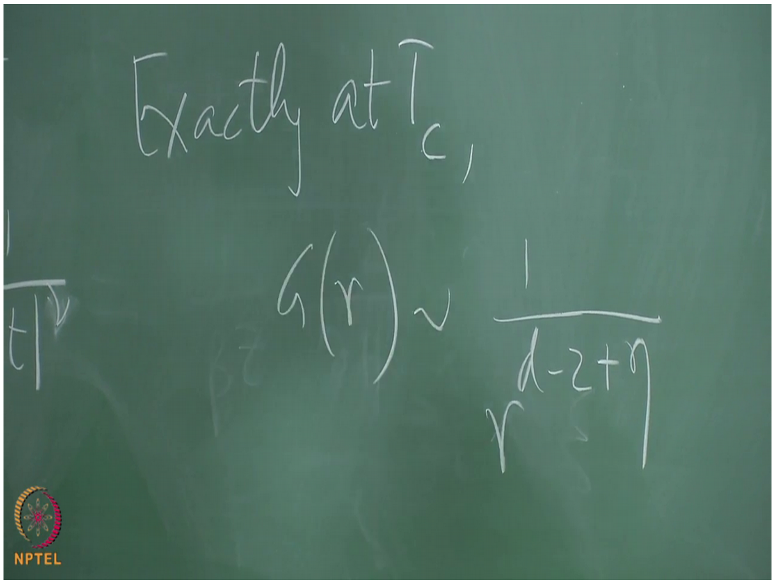

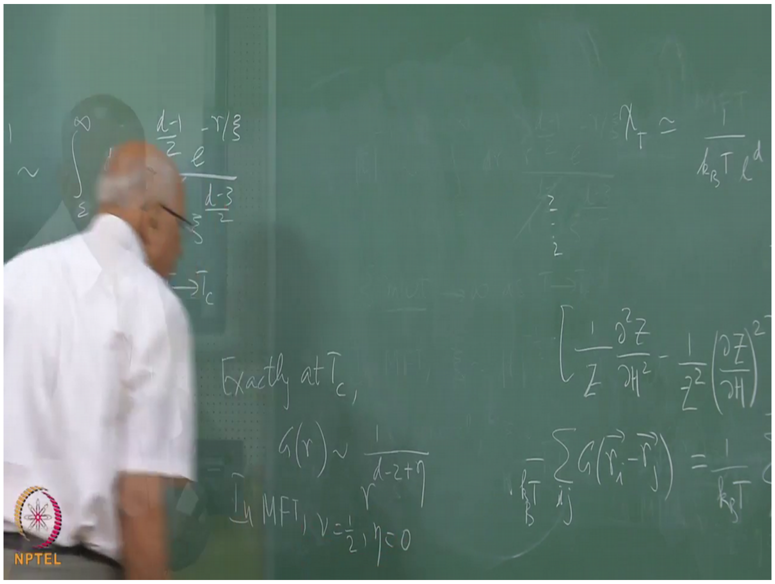

So this integral goes like integral dr as xi du so there is a du then there is a xi to the power d minus 1 over 2 E to the minus u and then divided by xi there is a xi d so there is that guy and then d minus 3 by 2 from 0 to infinity which is xi to the power du over 2 minus half that is this part plus 1 that is this part minus d over 2 that is this part plus 3 halves times a number, so this goes like xi this cancels 3 halves minus half is 1 plus 1 is 2.

So we find that in mean field theory in mean field theory xi goes like mod T to the minus half because I squared goes like 1 over T so xi goes like 1 over square root of T you know the words the correlation length diversion like 1 over Tc minus T to the power half that there is a in general xi goes like 1 over mod T to the power nu and the mean field exponent nu is a half in the framework of mean field theory, pardon me? There is no effect of d as far as this is concerned but we will see in a minute what really happens one could ask what happens at the critical point what happens to the correlation function at the critical point, what does it behave like?

I said this formula is true away from the critical point, so you could ask exactly at the critical point what happens, this fellow becomes infinite and then what happens, is this formula still true? This is the question we have to ask this is r over xi is r over infinity which is 1 this goes away so there is a power law so the question you are asking now is there a power law which is does it blow how does it blow up it turns out that you can show independently that at Tc exactly at Tc G of r goes like 1 over r to the power d minus 2 plus and exponent eta you have to introduce one more exponent eta is not as bad as it sounds because everything is in terms of the correlation function.

So the idea is that away from the critical point the correlation 2 function point correlation dies down exponentially with a correlation length at as you approach the critical point that correlation length diverges like with temperature in this power law fashion and at the critical point it becomes an algebraic function a power law decay which is d minus 2 plus another exponent eta, okay and all these exponents can be written in terms of eta and nu and I will write those relations down.

In mean field theory nu equal to half and eta equal to 0, so if you put those two pieces of information N all the other mean field exponents that we got will jump out automatically. So everything is now hinging upon the correlation a two point correlation and characterize by these two exponents eta and nu, okay. How this happens how this happens and how this happens requires more careful analysis we will try to do it, that is the starting point of the modern theory of equilibrium phased transitions this is the starting point the fact the recognition that what happens at that point the reason thermodynamics fails is because the correlation length becomes infinity you cannot neglect fluctuations, whereas thermodynamics neglects fluctuations it also say something more than that even mean field theory does not work it gives a wrong exponents.

And now you could ask what is the region how close to Tc should I be in order to see the new exponents this is given by a rule of thumb called the Ginsberg criterion which I will try to mention but it is not a rigorous statement means you have to take the case by case as you can aspect because what is exact is the university class in each case, so it will give you values of exponents but to tell you how good that value is or when the mean field starts becoming a bad approximation depends on the system so it is not the universal in that sense, okay but there are criteria which will tell you.

For instance if there are long range forces then mean field theory is very good, so if you ask what about this normal to super conducting transition in metals, metals becomes super conductors in the suitable conductions then it turns out that the temperature range in which the real normalization in which you need to correct the mean field exponents is of the order 10 to the minus 8 degrees which is negligible unless you hit exactly on the critical point it does not matter so you can get away with mean field theory because there is long range coulomb force involve in the property, okay but in other places spin models and so on it can corrections can start appearing much more significantly.

So that is not a precise question but there is such a criteria for this. So we will take it from the point in the next time I will introduce the relations between the exponents, scaling, generalize so much in this functions.