Nonequilibrium Statistical Mechanics
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Lecture no 26
Module no 01
The Boltzmann equation for a dilute gas (Part 4)

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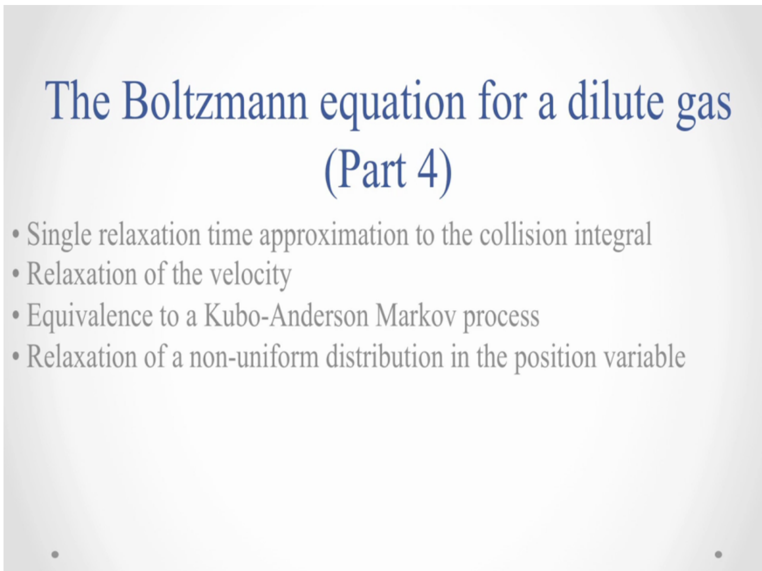

So today let us look at some of the consequences of the Boltzmann equation vis-a-vis is relaxation. One of the things that primary points to be emphasised with regards to the Boltzmann is that it gives you an evolution equation for the phase space density in new space for systems which are necessarily in equilibrium, in general out of equilibrium. And then the question arises, so how the system could relax to equilibrium under suitable conditions for instance if we switch of the external force, how it would eventually go to the Maxwellian distribution of velocity and the distribution in space should be uniform if we did not have any external potential acting on the system.

So the question is, how does this relaxation proceeds? And this equation should presumably give you an exact answer, so let us look at such phenomena. Let us call it Relaxation to equilibrium from the Boltzmann equation, but of course uhh the full Boltzmann equation is too difficult to solve. Even if you linearize it, it is still too difficult to solve, you need a systematic approximation procedure for in powers of some correction to the distribution from the equilibrium distribution that is a long story. So the question is uhh can we make a quick crude approximation and get some 0^{th} order idea of how relaxation occurs in various situations, so this is what we are going to do today.

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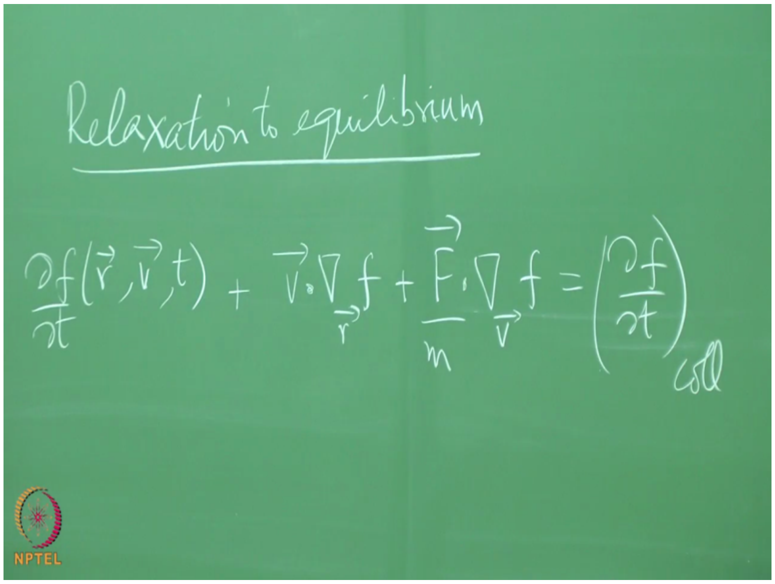

Talk about different situations where you have relaxation to equilibrium in a very simple particularly simple mechanism. First of this, before I do that lets write the Boltzmann equation down just for your to refresh your memory. So it is delta f over delta t that is in general function of r, v and t + v dot Del with respect to r of f + if there is an external force, it is F over m dot gradient with respect to v of f and that is equal to the collusion integral delta f over delta t collusion, so all the physics is contained here, this is basically Kinematic but this is where all the physics is.

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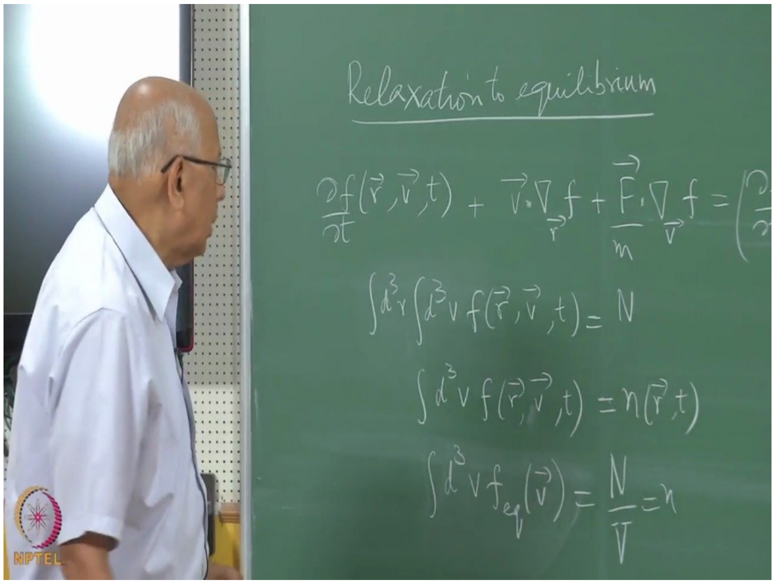

Now of course we know that it is not… It is horribly complicated, but even after we make a linear approximation, it is still as I said very messy, so let us make the simplest possible approximation. You have seen already that if you switch off the external force, there is no force at all then this thing here tends to an equilibrium distribution which is f not remember the rotation, I call it f not but let me call it f equilibrium of v was equal to well… Recall on first we should write down normalisation. The normalisation of this f was integral d 3 r integral d 3 v f of v, t = N the total number of particles in this system okay.

If you integrate over v alone so if you integrate v 3 v f of r, how should I write this? r, v, t, v alone then you get a function of r and t which is equal to, well it is the number but it changes with the number density but it changes as a function of position and time, so this is by definition equal to n of r and t right. If you integrate over r alone, d 3 v f of r sorry what I have write down… over r… r, v, t. How did I write the normalisation down? Did I mess up the normalisation? So this is okay as a stance, but how did I get the number density?

(Student is saying something is not audible from 5:06 to 5:12)

Oh yes, so we wrote integral d 3 v f equilibrium of v was equal to N over V equal to n that is the point, thank you. So let me define this non-uniform density by just integrating all the velocity alone this distribution function. And in equilibrium there is no t dependence, there is no r dependence if I integrate over d 3 v I should get N over V okay, so that is the normalisation.

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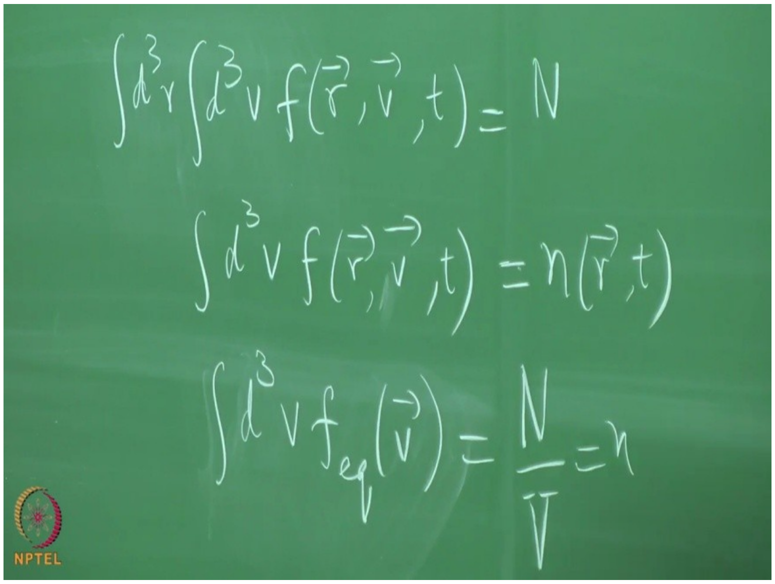

Now what we like to say is that remember that f I need a little more notation, f equilibrium of v was equal to n this thing here in space for the special part and then the velocity part was the Maxwellian the distribution. So normalise distribution was m over 2 Pie K Boltzmann T to the 3 half e to the – m v square over 2 K Boltzmann T, so that is the equilibrium distribution, this is a constant okay.

Now this function is going to keep appearing all the time, the Maxwellian distribution in the velocity alone okay, so let us give it a name, let us call this equal to is identically equal to n times let us call this W of v to show that this is the Maxwellian distribution because in equilibrium everything is in the absence of an external force with distribution in space is uniform so you have a uniform density, number density n per unit volume and then multiplied by velocity distribution which is the Maxwellian distribution and this is normalise to unity so we know that integral d 3 v W of v = 1 it is the reason for this factor 2 Pie K T whatever it is right.

Now, what the approximation does simplest approximation to the right-hand side is to say that collusion cause, if you have a small departure from equilibrium, collusion cause you to go towards equilibrium, they help you to equilibrate. So if your distribution f of r v, t is a little bit away from f equilibrium then if you make the single relaxation time approximation, the so-called single relaxation time approximation, so let us write that down. Single… When if you start with the uniform density the system is completely in equilibrium and you ask if I (())(8:38) a little bit away from equilibrium, the velocities are not terminalise, they are little bit away from equilibrium, how would these velocities equilibrate is a question, right?

Yes, 0 force case, I will do a single system with a force prevent but 0 force case. So that is the reason, whenever this happens whenever this is a function of r, you know that this is probably a force present but even in the absence of force instantaneously there could be density fluctuation and there are, so in general I will call this n of r and t, r, t. But when you list equilibrium in space and time I mean in velocity and position then the whole thing is just a constant here okay.

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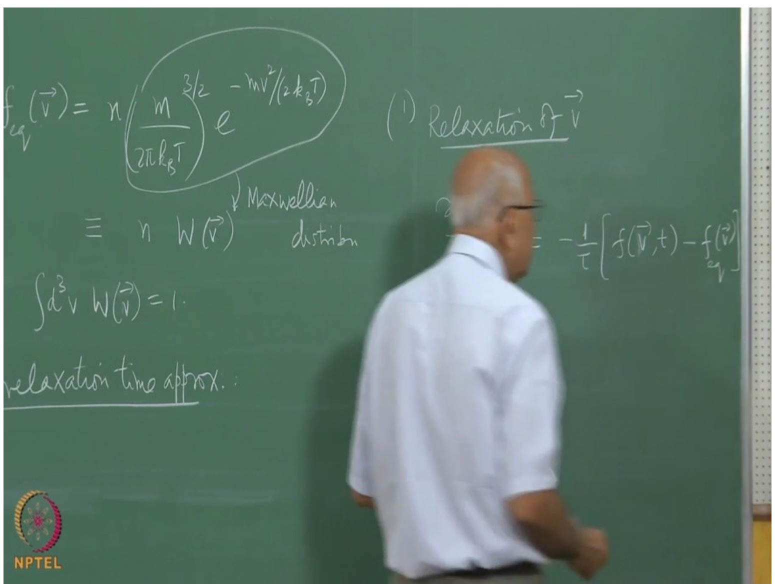

Now how does the velocity relax that is the 1st question we want to ask, so let us call 1, within the single relaxation time approximation relaxation of the velocity that is given by the following. Now there no space dependence because it is uniform throughout, the velocity supposed to relax to equilibrium. Now we already know there is a model for this, this was the Langevin model when I wrote down the Langevin equation and then I argued that if there is white noise which causes collusion (())(10:13) the effect of collusion is mimicked by white noise Gaussian white noise, et cetera, then you have a Fokker Planck equation and the solution to that was the (()) (10:23) distribution.

And then the constant Gamma the fiction constant determines how things relax to equilibrium and the velocity correlation time was an exponential single decaying exponential with the relaxation time Gamma inverse that was that model okay. But now we do not have that model at all, there is no stochastic force or anything like that, this is a completely self consistent system and we are not saying that the mass of this particle is different from the masses of rest of it, we are not saying that at all, no such a function, but we made a huge approximation to this in the single relaxation time approximation.

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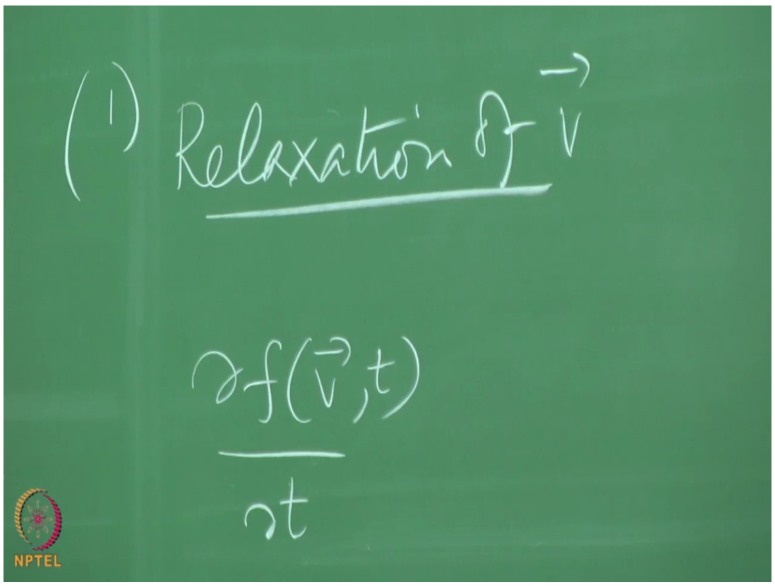

So now this f is going to satisfy Delta f of v, t over Delta t no external force, the distribution is uniform in space so this term is missing and we only have this and this is equal to the collusion integral and what single relaxation time approximation says is that this is equal to there is a relaxation time bought in the parameter Tau just one of them, so it is – 1 over Tau and inside is just f of v t – f equilibrium of v so that is the approximation.

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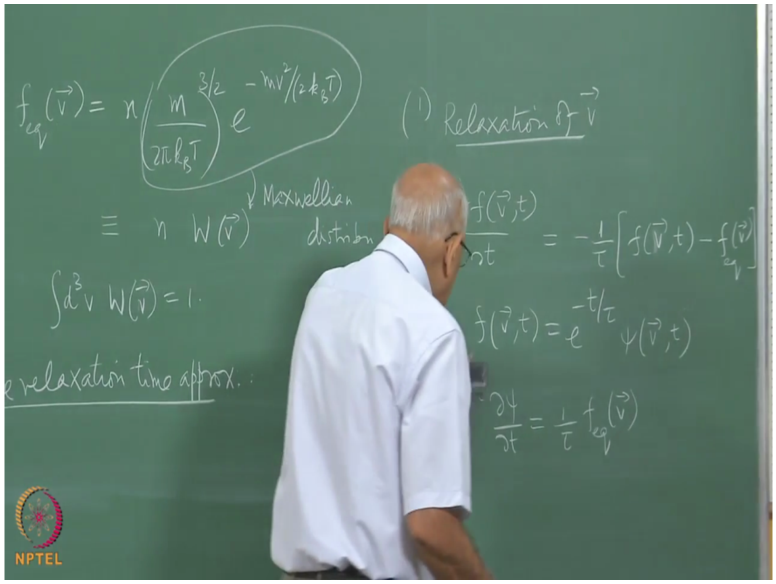

If we replace… By saying the deviation at any instant of time of the velocity distribution from the Maxwellian distribution is negligible is very very small and then what you have if exactly like in radioactive d k very similar to that, this difference divided by Tau and it has got the right dimension. It is the dimensionality of Delta f over Delta t except this f is discretised and said there is a difference between the distribution instantaneous distribution and the equilibrium one divided by the timescale Tau, so this is an extremely simple model okay. What does it predict now?

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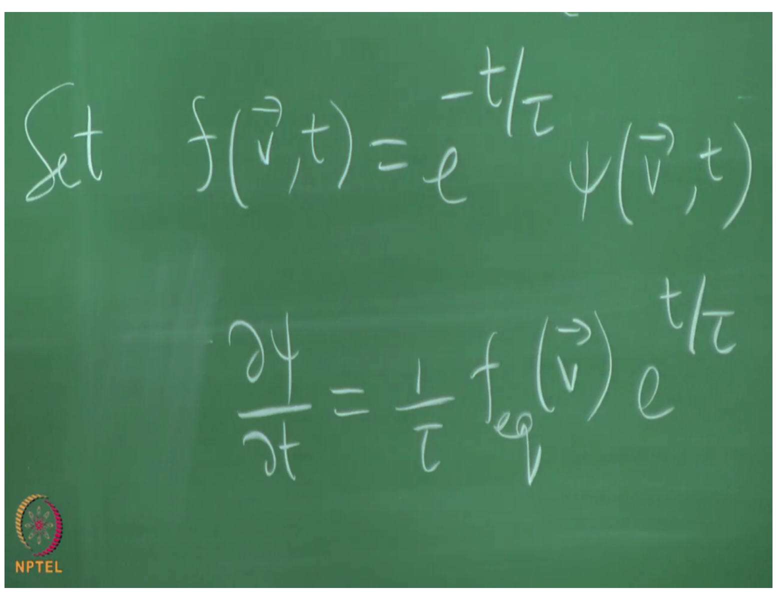

We can solve this, this is a trivial equation to solve first order equation so it is trivial to solve and of course what one should do is to write set. Well we can even write down the integrating factor or else f of v t = e to the – t over Tau obviously it is going to relax with that characteristic timescale Tau multiplied by something else, so let me call this Psi of v, then the equation on the left-hand side is the first term is – 1 over Tau times this whole business which you cancel against that in the right-hand side and it says e to the – t over Tau Delta Psi over Delta t = the first term cancels, so it is equal to 1 over Tau f equilibrium of v, moving this to the right-hand side.

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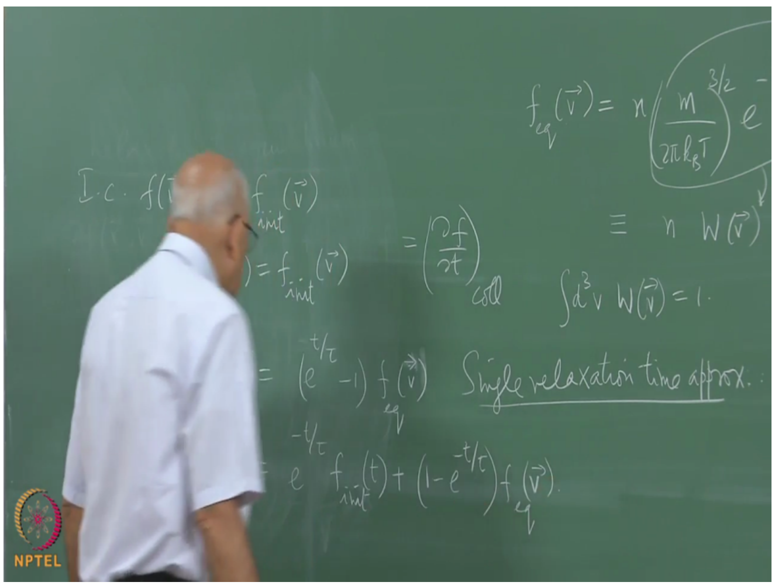

But you have to tell me what the initial condition is, you have to tell me what is the initial velocity distribution, so let us put that in so initial condition f of v, t equal to some initial distribution from prescribed initial distribution. So that immediately implies that Psi implies Psi of v 0 is f initial because this term becomes 1 at t = 0 trivially. So what is the solution to this thing? It says Psi of v t – f initial of v solving this differential equation uhh you have to integrate this from 0 to t, put t prime and integrate 0 to t so that integral becomes Tau cancels as you can see so it becomes e to the t over Tau – 1 times f equilibrium of v, which implies that f of v, t equal to…

I move this to the right-hand side and multiply this by e to the – t over Tau so it gives me e to the – t over Tau f initial of t +, I multiply this by e to the – t over Tau so this is 1 – e to the – t over Tau f equilibrium that is the solution, which is exactly what you would expect. At t = 0, this thing is equal to the initial value because that vanishes and as t tends to infinity, this term goes away, that term goes away and you are getting the f equilibrium yes.

“Professor–student conversation starts”

Student: It is very close to equilibrium?

Professor: Exactly, so this will work as long as this is very close to equilibrium, so that is the whole point that you replace this entire collusion integral by saying that the system is very close to Maxwellian distribution.

Student : (())(16:38)

Professor : Pardon me.

Student : t = 0

Professor : Whatever you prescribe, there is no reason why that should be Maxwellian. For consistency of the approximation, this too should be fairly close to the Maxwellian distribution of course.

Student : Does it also means that the initial time will be close to equilibrium time will be close to the time when the (())(17:00)

Professor : No, that is not the point, there is a characteristic timescale Tau, we do not know anything about it. We simply said this entire collusion integral has been replaced by this discretised version okay, we do not have any angle here about what this Tau is at all okay, but we need to do so find some way of finding out how good is approximation is to find some way of measuring this Tau if it exists. Now in practice that is not going to happen, but you can see what is what we can do what we should, let me indicate how one should go about it in the general problem.

“Professor–student conversation ends”

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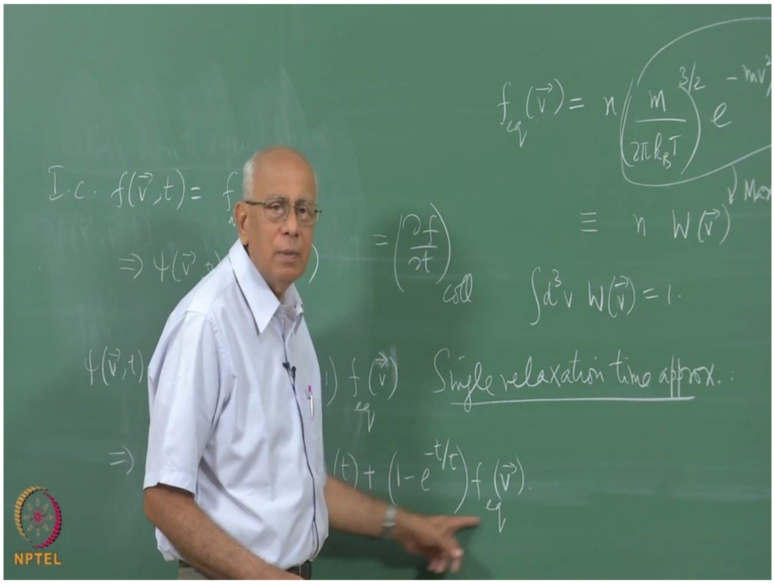

In general case we are looking at function of v, this quantity is function of v and t okay. Now we know that the equilibrium distribution is a Maxwellian is this fellow equation okay. Now, any arbitrary function of v where v run each component of v runs – v infinity to infinity can be expanded as Gaussian Gaussian in v times mid polynomials. So in principle you can say this quantity here is the super position of all the mid polynomials times the Maxwellian distribution with coefficients which are possibly time-dependent, which are certainly time-dependent that is how the equilibrium will happen.

Now this is the crudest approximation to that, it says that in some sense it just says the polynomial talking about is 1 okay. You have taken the constant is no mid polynomial anyway, there is no function of v anywhere, you just have this function of v and that is it, this is already the Maxwellian okay. So there is a systematic way of justifying I mean finding out what this approximation means but in physical terms it says that if you replace this entire collusion integral by saying that we do not care what is happening in the inner mechanism, there is some effective time Tau on which the system equilibrates then this is what happens as far as the algebra is concerned.

But now how far this is good, how good in approximation this is, can we get an (())(19:15) on Tau can we measure it and so on are not answered questions within this framework as yet.

“Professor–student conversation starts”

Student : Since we are talking about molecular dynamic, can’t we say that Tau is also order of molecular collusion time?

Professor : We are going to find out, we are going to find out if this Tau appears anywhere else and then this cover how what Tau could possibly be okay certainly yeah.

“Professor–student conversation ends”

It will not be a single molecular collusion time because there are many many timescales involved here so this is some effective timescale on which the velocity is thermalised okay. We already know from the Langevin model that the timescale on which the velocity thermalises in that model seems to have something to do with the viscosity of the medium. But that was dependent on the assumption that the Langevin equation was valid which is itself true only if the mass of that particle is much much higher, that is not obvious here it is not obvious here. So it is clear as a spectrum of relaxation time and you have taken the lead in contribution that is really what has happened.

Now the question is, can this also could this result have not produced in any other argument any other way? Is there a simple model which will produce exactly the same result some stochastic model which will produce the same result? It is clear this is not the (())(20:39) distribution at all, so it is not certainly the Langevin model at all, it is very different from that. The Langevin model has this (())(20:49) distribution which is again a Gaussian where the system tends to the Maxwellian distribution exponentially with timescale gamma inverse but it is very different looking from this. Here it is just a either the… this is v sorry…

And incidentally this if I start with a particle like within the Langevin with a given velocity v 0, this of course would be Delta of v – v 0, it should be replaced by delta function. But I am allowing a little more general solution by saying that it could be a distribution in itself, not all fixed at 1 velocity. Is there some way of producing this thing here by a simple model assumption on random nature of the random velocity v, can we do this at all, not the Langevin model? Now the answer is yes, but it must be a much simpler model than the Langevin equation because that involves the Fokker Planck equation which we solved.

This thing here looks like it is much more trivial thing to solve right? There is an equation for the distribution function which is 1st order in time and involves the same thing out here, so it just looks like a Markov process, you just assume it to be a Markov process but it looks like it is a discrete Markov process but it is continuous because v is continuous but it looks like v is changing through a jump process of some kind, let us see that is validated or not.

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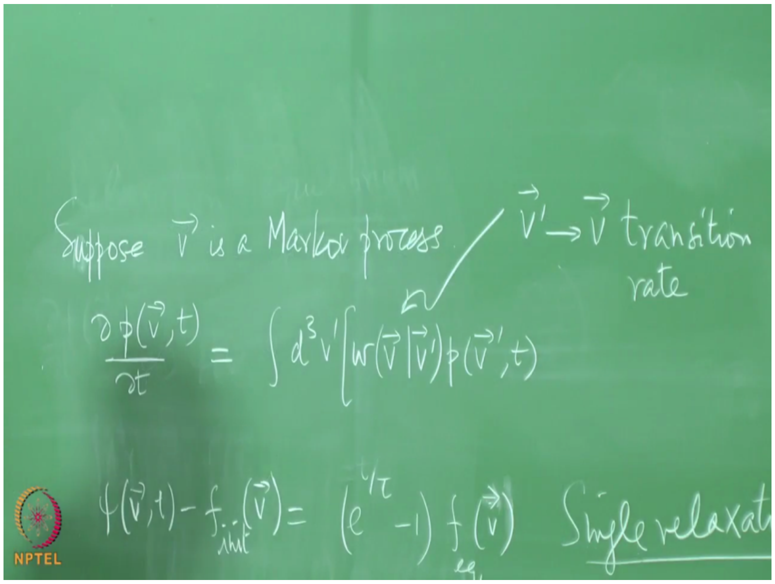

Suppose v underwent suppose v is given by Markov process suppose. So what I am trying to do is to argue that there is an effective model but v is just taking to the Markov process by which you will reproduce this exact result. So that is another way of understanding what is the meaning of this single relaxation time approximation. Suppose v is a Markov process, collusions will take me to some other value, each component of v varies continuously – infinity to infinity, the moment you say it is a Markov process when its probability density Delta I need to use another symbol for it since I am reserving f for the symbol we used in the Boltzmann equation, the distribution in new space, let us just call it p okay.

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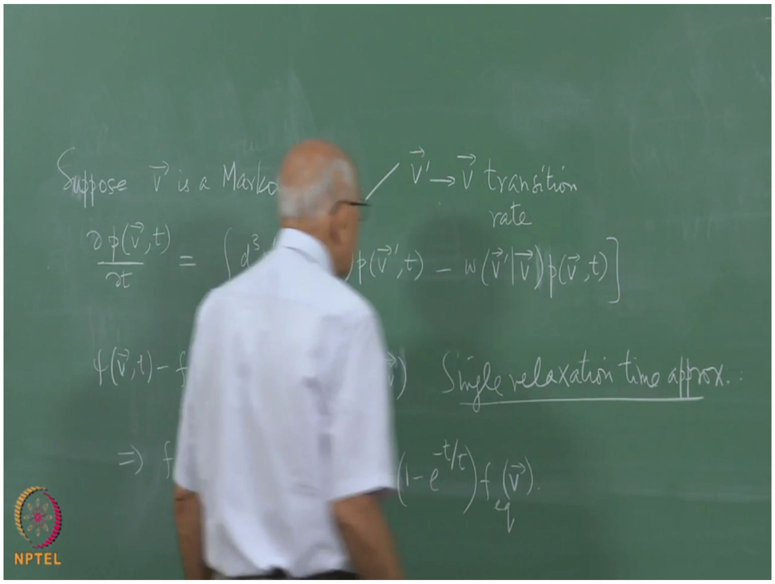

So Delta p of v and t divided by Delta t must be equal to on the right-hand side we need to write it rate equation because it is a Markov process. So in general you can write this and it is a continuous process d 3 v prime on this side times the gained term and the lost term that is what you do for any Markov process for which you can define a transition probability per unit time right? So times the probability that you reach the velocity v prime at time t multiplied by the transition rate per unit time that you hit v given v prime starting from v prime. So this guy here is the v prime to v that is the gained term, and the lost term in the rate equation is – W you jump out to be that is the Markovian master equation.

Now the question is, can I produce this solution by writing a model for W? Okay that is the question being asked, detailed balance must obtain in equilibrium, so what sort of function should I put in here such that in equilibrium this will become p equilibrium that becomes p equilibrium, what sort of function should I put in here such that detailed balance will obtain. Well, let us look at the physical process, you have an initial velocity v in this transition rate, you have velocity v and is getting knocked out into any other velocity v prime. Now what is the effect of this collusion?

One possible approximation is to say the collusions are extremely weak, so whatever you started with that change is very slight in a given collusion. In the limit of no collusion it will not change at all but in the limit of v collusion it will change for very small by a very small amount Delta v.

To start with v, the collusion will connect you only to those velocities which are within range Delta v of the initial that is the weak collusion approximation. It turns out that collusion leads to the Fokker Planck equation okay assuming that there is no memory in this collusion and it is a Markov process. I am not going to do there, there is a way of systematically deriving from this master equation the Fokker Planck by making the so-called weak collusion approximation okay.

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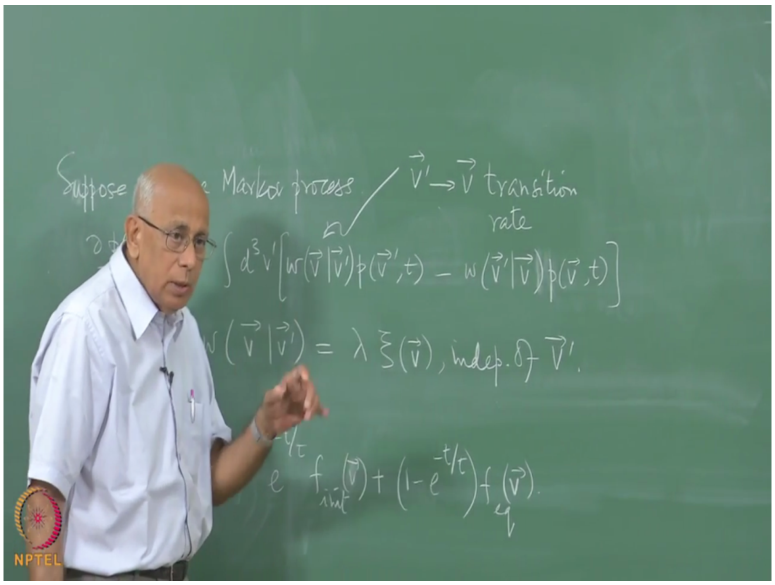

That is another possibility and that is a very strong collusion approximation, in other words it says that look each collusion is such that it thermalises it at once okay. In other words, it says that this quantity Lambda v, v prime the transition rate from any velocity v prime to any other velocity v does not depend on the initial value at all, the collusions are so strong that given collusion is immediately thermalised okay. That is one possibility that the transition rate is independent of the initial state, it depends only on the final state. So suppose that were true suppose but equal to we need this is the transition rate so you need a constant of dimension time in a let us call it Lambda times some function.

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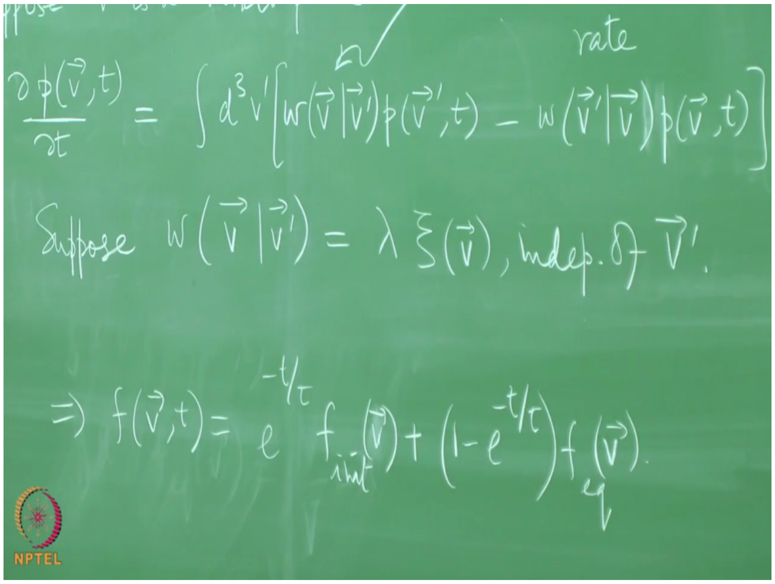

I should not use the function Psi, I should use some other function, what is the good what is the good, Zi of v independent of v prime suppose. So the transition rate every time there is a collusion and average rate of collusion is Lambda, the velocity changes. Whatever be the initial velocity it goes to final velocity v depends only on the final velocity okay. Suppose this is true and now you impose detailed balance in equilibrium then what will it imply?

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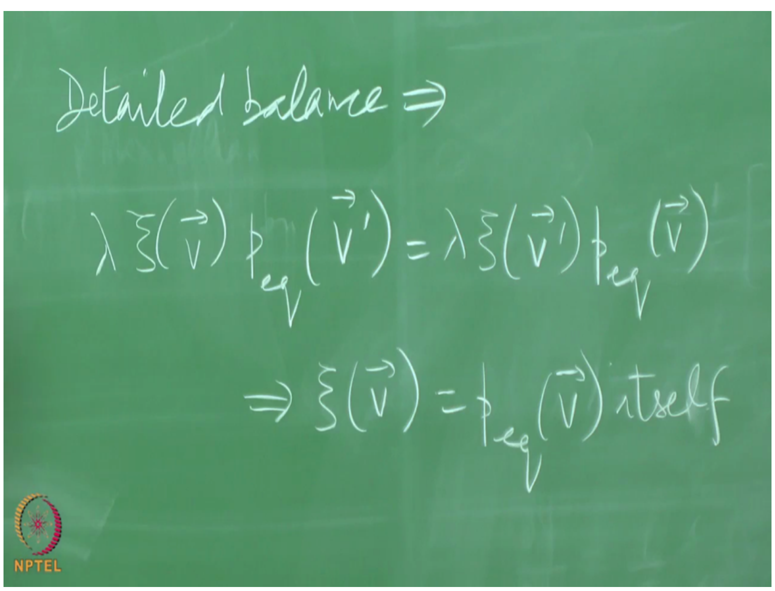

This will imply detailed balance implies then with this assumption this assumption, if you like t tend to infinity, this term is 0 the time derivative, it goes to equilibrium this becomes p equilibrium of v prime and this becomes Lambda times Zi of only the final state of v, p equilibrium of v prime must be equal to Lambda times Zi of v prime p equilibrium of v and there is only one solution to that, which is that this is p equilibrium of v itself, so it is obviously the strong collusion approximation.

"Professor–student conversation starts"

Student : (())(29:50)

Professor : In one shot yeah, so what it is saying is the following.

"Professor–student conversation ends"

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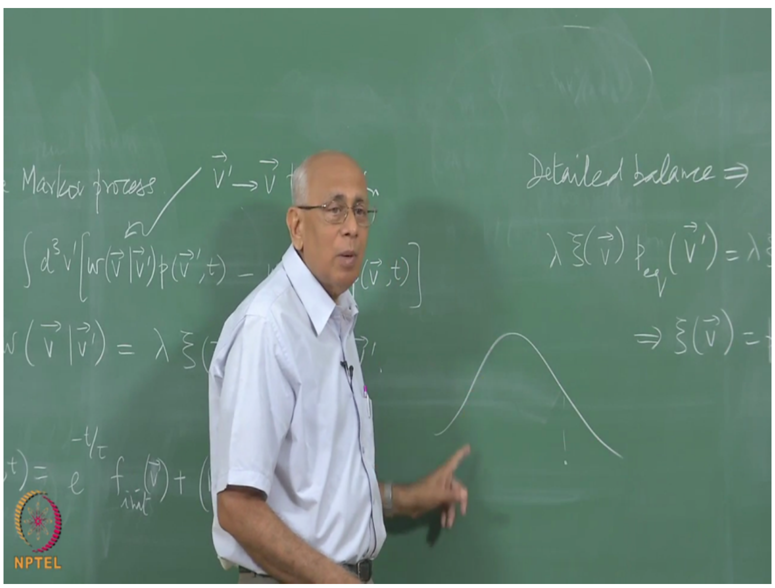

You have an equilibrium distribution, in every collusion the velocities reset from whatever its value to some value drawn from the equilibrium distribution with those probabilities. So if your distribution is Gaussian and you are at this point you have this initial velocity, after collusion the velocity new velocity is reset to be one of these velocities with this distribution with this probability is chosen okay, so that is the implication of the statement here. Random process in which a Markovian process continues Markov process in which the transition rate is independent of the initial state and is drawn from the final state alone and is the function of final state value alone and more over there is detailed balance, this is called a KuBo Anderson process okay.

So what we have shown so far is that if you assume a Markov process for this guy and Kubo Anderson process with detailed balance then this equation simplifies anonymously and look at what happens to it.

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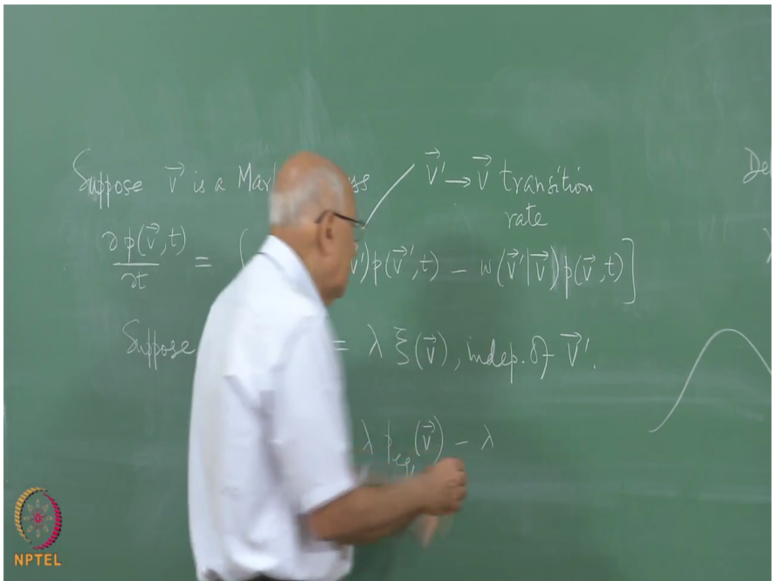

Exactly, apart from that factor N, there is a factor N for our phase space distribution, this is only in the velocity, so I have distinguished it by writing p equilibrium separately. So look at what happens to this equation, you have d 3 v prime, this is a function Zi of v p equilibrium of v and then you have d 3v prime p of v prime t which is equal to 1 because it is going to be normalized. So this immediately says, Delta p v, t over Delta t = Lambda times this fellow here is p equilibrium of v – Lambda times now this is an integral d 3 v prime of p equilibrium of v prime, which is 1 on this side times p of v and t.

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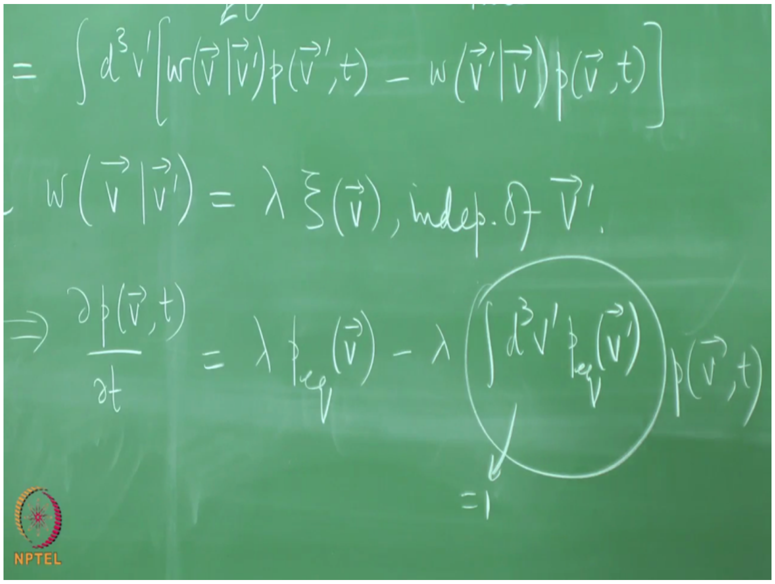

So it is clear that when the identification Lambda equal to 1 over Tau you get a single relaxation and approximation, so exactly the same so the solution is exactly the same okay. So one way of interpreting the single relaxation time approximation in the weak in the Boltzmann equation with a linearised Boltzmann equation to say it is equivalent to saying that the velocity V is just a Markov process a Kubo Anderson process, nothing more than that okay, this gives exactly the same solution as you can see.

Now you can ask where can I tweak this a little bit? Can I try to improve this model by saying look each collusion need not take you to the velocity drawn from the equilibrium distribution so there could be one limit in which you have very weak effect of collusions another this is a strong collusion limit in which it immediately thermalises in some sense okay.

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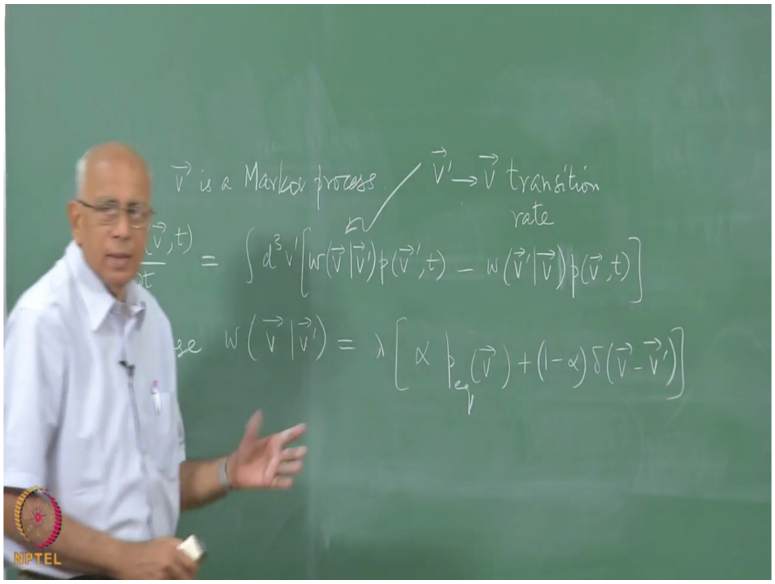

So is there anything in between, I leave it as an exercise to you to show the following, a simple exercise that is to say independent of this, if you say that this W is not this, but is equal to Lambda times let us say some Gama times, I think Gamma is a bad number, Alpha times a function of Zi, this is p equilibrium so we may write it like that p equilibrium of v + 1 – Alpha times at Delta of v – v prime, we could do that right. This interpolates, when Alpha is 1, you have the Kuba Anderson process, when Alpha is 0 this goes away and you have no collusion at all, does not do anything, this will not equilibrate at all and remain where it is, but this here with Alpha equal to 1 equilibrates with the time constant Lambda inverse okay.

So now figure out what happens, you can easily satisfy yourself that this again satisfies the detailed balance condition, so it is a kind of interpolation model between the strong collusion limit and the 0 no collusion limit in between, where Alpha is any number between 0 and 1 okay.

Such a system with strong collusion, the system should (())(35:51)

Ah, it does not equa… That depends on Lambda, which depends on Lambda.

(())(35:59) the problem with the collusion timescale which we know is too fast.

Right, so it does not equilibrates in 1 collusion, it simply says that the transition rate is drawn from the equilibrium distribution independent of where you started with. Yes you can but this is an arbitrary parameter, this is an arbitrary, yes there are models for collusion broadening in gases and so on where you have used strong collusion limit seems to be the correct limit to work with. Yes there are physical systems which display this behavior. So the point that I am trying to make is if you did not have this strong collusion limit, it is a rather trivial solution, it is equal to the single relaxation time approximation and the Boltzmann equation the linearised Boltzmann equation.

On the other hand, if we did not have this but you have this alone, there is no collusion there is no physics if no equilibration at all, but if you tweak this and make this a weak collusion limit then you get a Fokker Planck equation okay. So it is possible to derive the Fokker Planck equation from the Boltzmann equation by making a single relaxation time approximation and weak collusion approximation okay. I am not going to do that, that is little bit of machinery but I am not going to do that, I just wanted to point this out okay. But now you could choose this Alpha to be between 0 and 1, what you think will happen?

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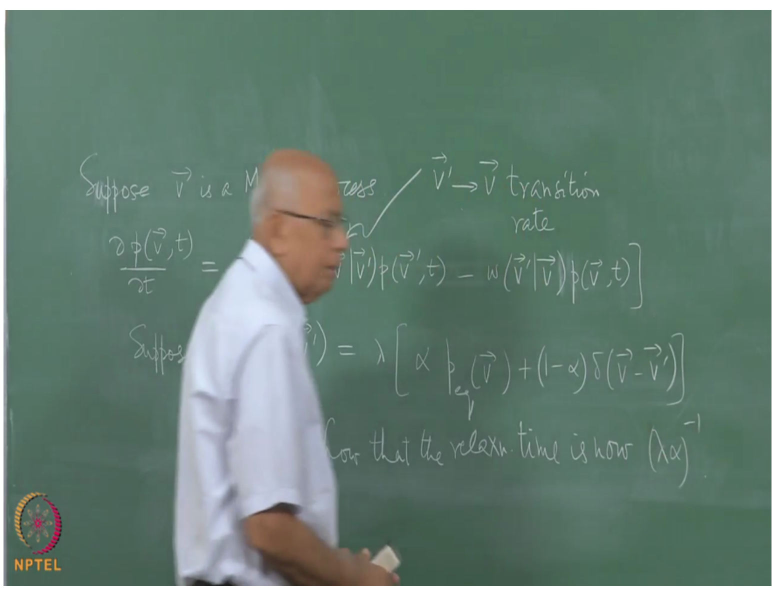

This model also satisfies the detailed balance condition for any Alpha between 0 and 1, so what do you think will happen to the solutions? Again it should be solvable completely because there is a delta function here. All it does is to rescale time; it just buys you some time it does not do anything.

2nd term (())(38:15)

2nd term does not do very much.

The integral (())(38:15)

Not entirely, but this one here means okay. So it just changes Lambda Alpha and then because it is Lambda Alpha inverse as you can see, if Alpha was 0 that means collusion will not change the velocity at all so there will be no equilibration, which is equivalent to saying the relaxation time must become infinite, so it is not surprising that it appears here in this place okay. So the reason purpose of introducing this was to show you that relaxation phenomena can be modelled in a systematic way from the Boltzmann equation okay. Now let us look at the other thing we did with the Langevin model…

Student: I think it also tells you why Lambda should not be a collusion model, assumption is quite often…

Absolutely absolutely yeah it is not the collusion time, very emphatically no it is not the collusion time between collusions no, emphatically no impact we will see what this Tau is in the single collusion approximation okay. This was just a model, the Lambda is completely arbitrary here, but let us go back to the single collusion thing and look at the other phenomena which was, we have a case in which you have a non uniform initial distribution the velocity has thermalise and now the system defuses. That was a famous diffusion regime where the mean square displacement went linearly with time and the velocity correlation time had long died down and you are looking at longer time scale. Let us see how that comes out in this Langevin in the Boltzmann equation.

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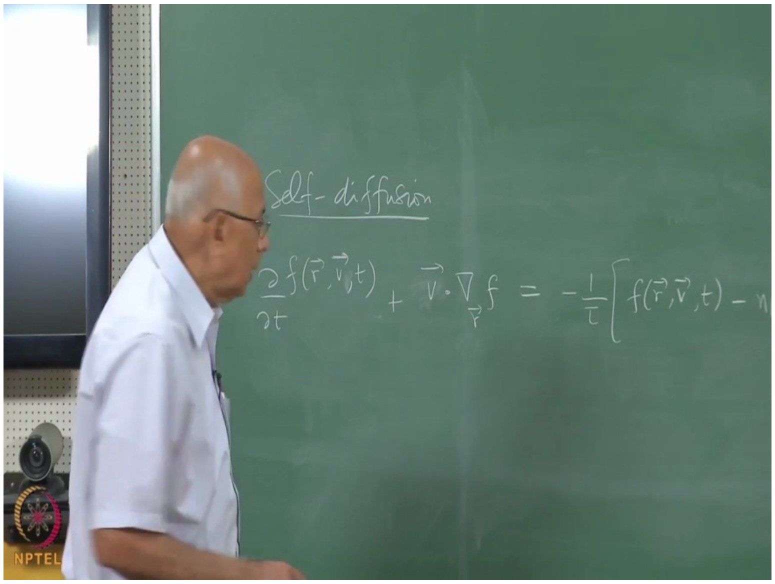

So this is the smoothing out of a nonuniform initial distribution in other words, diffusion self diffusion this time because the particles of the medium are themselves defusing. No external force as before and the Boltzmann equation is Delta f over Delta t + v dot gradient with respect to r of f, this term is very much present because this is a function of r, v and t. That is equal to the single relaxation time approximation and what would you write it as this time is 1 over Tau f of r, v and t – what would you write this time as? There is an r distribution, there is definitely an r distribution right? We are trying to, okay but the velocity has thermalise

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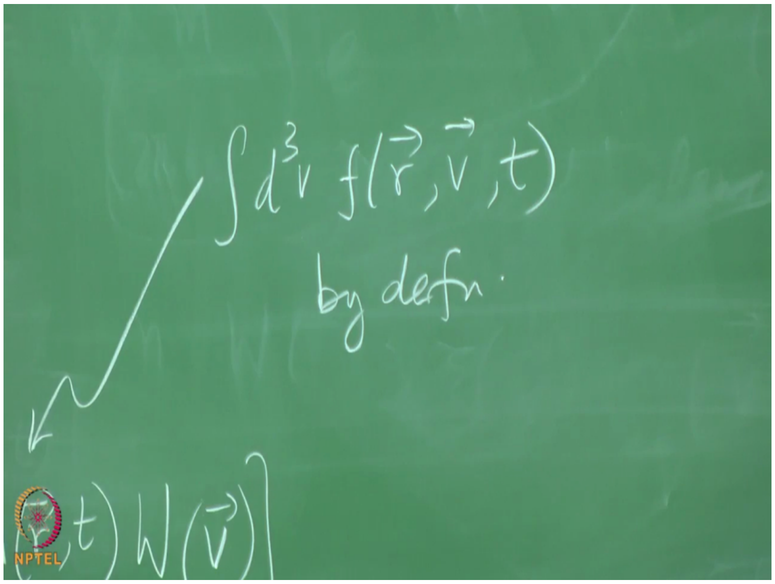

So you write it as small n of r, t times W of small v and recall that this fellow here is integral d 3 v f of r, v, t by definition, nonuniform distribution the velocity has thermalised, it is the Maxwellian. We want to know how this guy relaxes, how would you solve an equation like this? Well, it has got both space and time derivatives so clearly you are going to do the do Laplace transform with respect to time and Fourier transform with respect to space okay.

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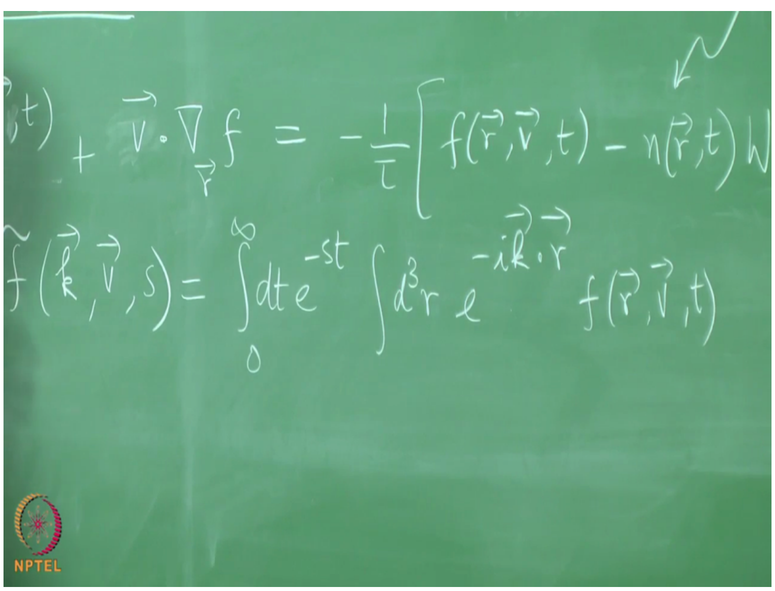

So it is going to define and f tilde of k, v and s to be integral 0 to infinity d t e to the – s t integral d 3 r, e to the – i k dot r f of r, v, t. I do not want to put two tildes I mean it stands for okay this stands for Fourier transform with respect to r, Laplace transform with respect to time of this, we plug that in here in this point. Then let us write the equation down directly, oh by the way I need to tell you what is the initial value of this fellow okay, what is the initial distribution?

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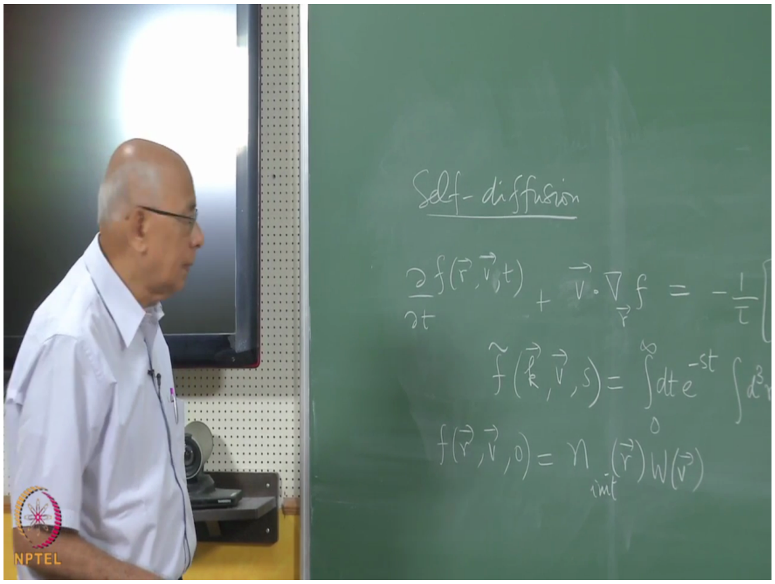

So I have to tell you what is the initial distribution is, so let us put n f of r, v, 0 = n initial of r, whatever be that initial nonuniform distribution that you have plugged in times W of v of course right. So what is the equation you get this, let us call this f tilde when I do the Fourier Laplace transform, so f tilde s times f tilde of k, v, s – this guy here that is the formula of Laplace transform of time derivative, but I am going to do a Fourier transform with respect to space, let us call that n tilde of initial of k. W of v just is there as a spectator this + v dot gradient, what is this guy going to do?

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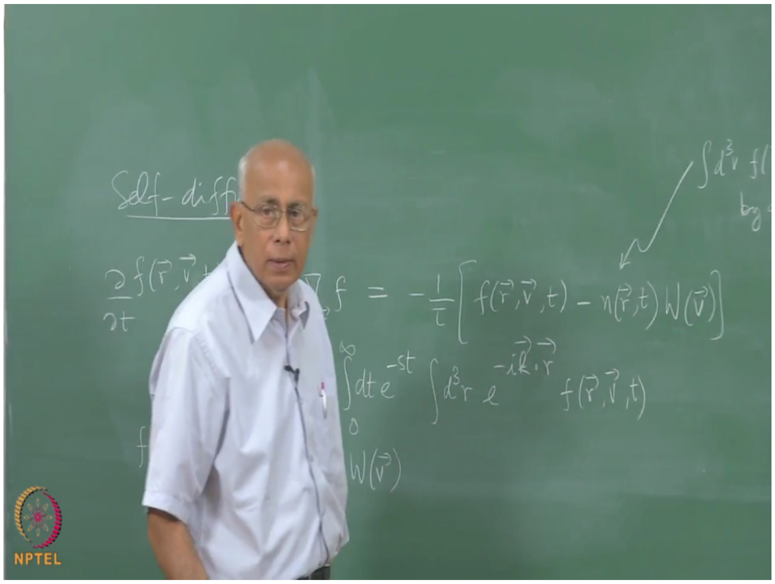

Gradient with respect to (())(45:12) and – i k so we got – i k dot v times f tilde of k, v and s that is the transform of this fellow here okay. This is equal to on the right-hand side equal to 1 over Tau – 1 over Tau F tilde of k, v and s. It should be + so I expand this in terms of I expand this fellow yeah I expand this fellow and then it if f tilde. I I leave it you to put in all the 2 pie factors and stuffs like that okay, yeah.

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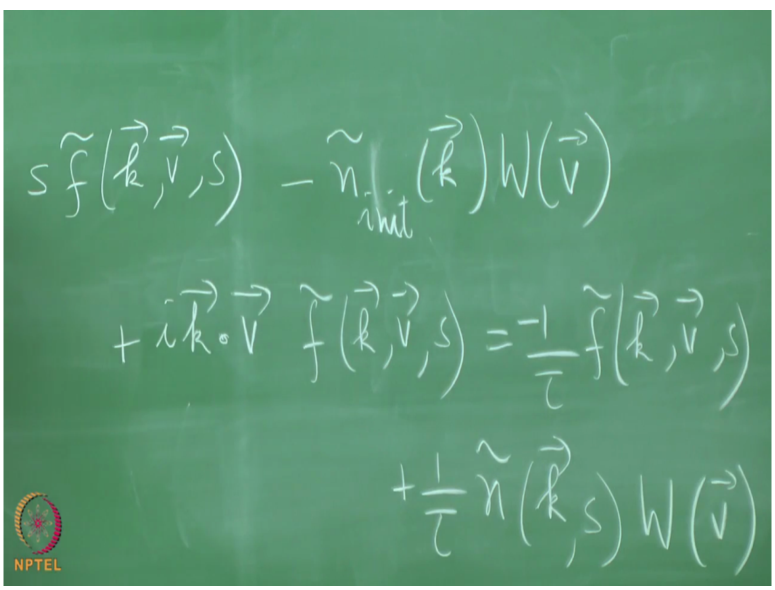

When I go to the Fourier transform, in everyone of these terms is 1 over 2 Pie, the inverse transform that cancels out but this is a + because what I am doing is to write this fellow in terms of as 1 over 2 Pie d 3 k this guy to the + i k dot r so + i k dot v is equal to this this term + 1 over Tau the Fourier transform of this + 1 over Tau n tilde of k and s W v. Now this looks like a hopeless task because we do not know what this is and we do not know what this is, what should I do? This is presumably given to me, this is the known function.

Integrate it.

I integrate both sides with respect to v, what happens then? So what happens to integral d 3? Notice that integral d 3 v f of r, v, t integrated over v is n of r, t. If I take Laplace transform this becomes n of r, s, if I do Fourier transform it becomes n tilde of k, s. So I integrate both sides with respect to v in which case you are going to get this thing here and this is going to be integrated over with respect to v on this side but you got be little careful in doing this. This n initial will move to the right-hand side here and there is this guy so we should not yet integrate over v, we need to pull various things and then do the integral over v, let us write all the f guys together.

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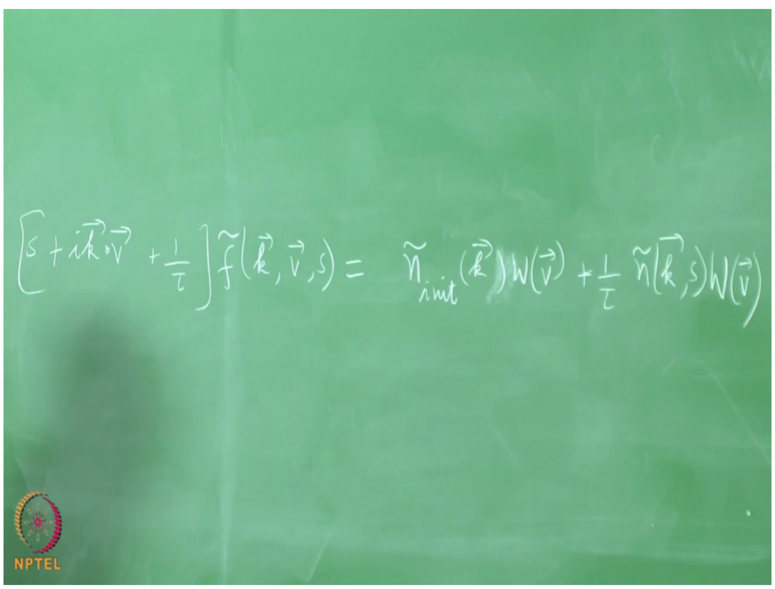

So there is s as this term + i k dot v so this term is gone, this term is gone, + 1 over Tau Times F tilde of k, v and s = so this term is gone, I think I have written 1 extra term, we seem to have an extra term somewhere, this is equal to let see where this takes us, n initial tilde of k times W of v that is certainly there + last term + 1 over Tau, now where is the problem?

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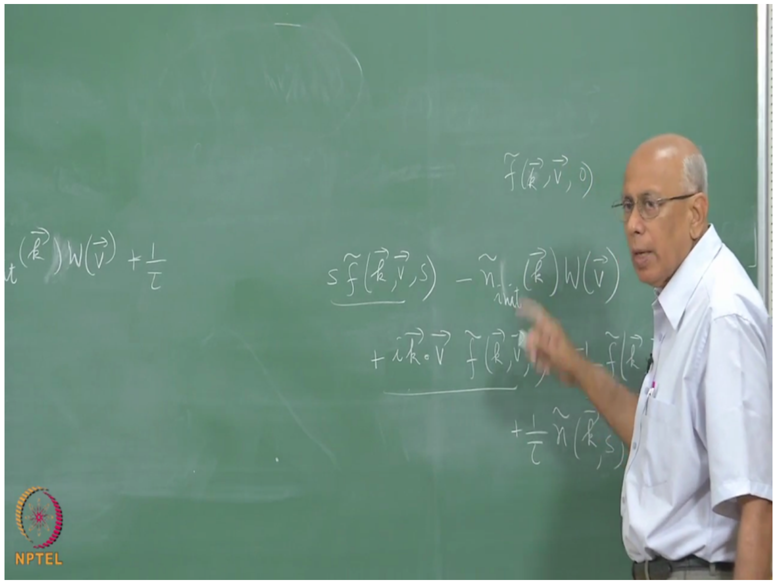

"Professor–student conversation starts"

Student: (())(49:33)

Professor: No no no, what did I do? Was this quite right, how did this come about?

Student: It is d by d t.

Professor: This is equal to this is f of r, v it is f tilde of k, v and 0 is not it?

Student: Yes.

Professor: This term is F tilde of k, v and 0 and which we wrote as the initial distribution in r times the equilibrium distribution in v and then the Fourier transform with respect to that which was this guy so this is okay this is okay + n tilde of, yeah so that part is all right.

"Professor–student conversation ends"

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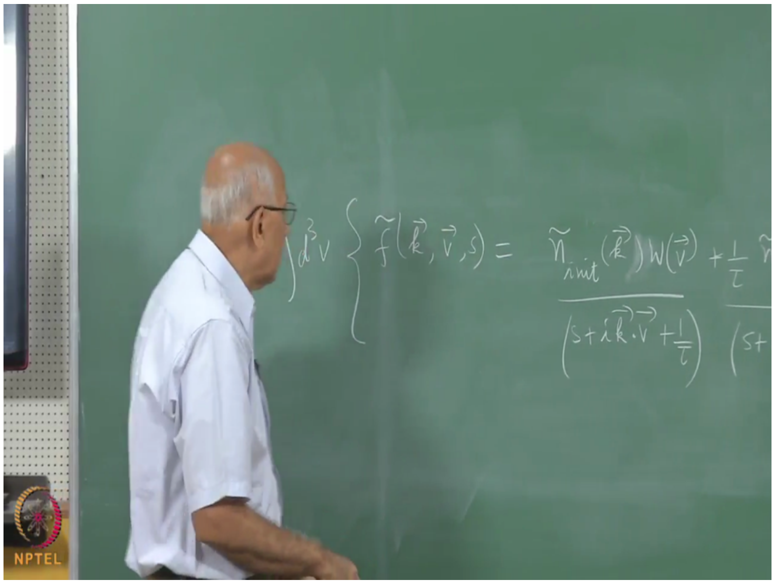

1 over Tau n tilde yes k and s W of v, so this is gone that is the equation right. Now let us move this to the right-hand side so this is divided by s + i k dot v + 1 over Tau s + i k dot v + 1 over Tau, we are going to leave the rest of the completion of this algebra d u so I move this there and I integrate over v okay. Then if I do this on both sides d 3 v of this whole thing of the entire thing both left and right hand side then what is this quantity equal to? I have integrated over this so this is n tilde of k, s so I get an equation, which says n tilde of k, s = the integral of W of v alone is 1 but you have got this in denominator so you cannot do anything with it.

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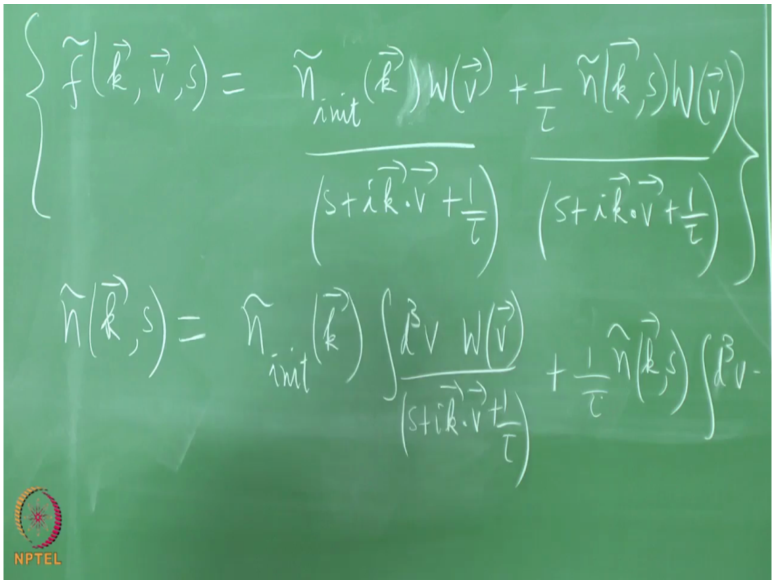

It is equal to n initial tilde of k integral d 3 v W of v over s + i k dot v + 1 over Tau is this fellow sitting there that is some number it depends on s and it depends on k some function of s and k right + the same number once again because there is W of v over this guy 1 over Tau n tilde of k, s times the same integral d 3 v W of v blah blah blah. Call that integral something or the other and bring it to the left hand side and you have an equation now of n tilde of k, s. If you invert the Laplace transform, you get n tilde of K, T, if you invert the Fourier transform you get n of r, t in principle.

Now do it analytically as formidable but you have a closed equation for it completely and what we will do next time, we should take this and see how therefore we are going to get the diffusion coefficient in the so-called hydrodynamic limit. First of all you have a relaxation time Tau here, we have seen that the velocity is relaxing with the relaxation time Tau. We know the diffusion regime is when your timescale is much bigger than the relaxation time right, so in terms of Laplace variable we need not even invert the Laplace variable, the diffusion regime would mean s times tau this guy here would be very very small s is small compared to 1 over Tau that corresponds to long time, small s is large t okay so this guy here should be equal to much much less than 1.

And k should be very-very small also because you are looking at hydrodynamic modes, you are looking at Long wavelength fluctuations not on very short length scale. Long timescale Long timescale should give us then the diffusion coefficient okay, you will see how that emerges from here systematically. We get actually more information from this equation but the basic trick is the following, the basic trick is you write the single relaxation approximation by saying collusion integral is some – 1 over Tau times the difference between the distribution function as the asymptotic form or whatever, and then you solve that equation in a self consistent way.

In this case the trick was to integrate over v and then get self consistent equation for n tilde of K, s which is what we are trying to find n of r, t and what is done is to find this Laplace Fourier transform first okay, so we will complete this, we will do this we will try this out and then we will see how a systematic approximation procedure will give us the diffusion coefficient okay. Then there also remains a case of what happens if you apply a uniform but time-dependent force on the system? How will it take it out of equilibrium?

So we start with an initial condition that specially uniform and the velocity is thermalise and then say I am going to switch ON a force, which does not depend upon the position in the simplest instance but on time how the system is going to go out of equilibrium again in the single relaxation time approximation, so we look at that as well. And those are the things which these are the things which will help you to find things like the viscosity, the diffusion coefficient, the thermal conductivity and so on as I said before okay, so let me stop here.