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today we will try to motivate a very important equation in kinetic theory, namely the Boltzmann equation which is along lines somewhat different to the ones that we have been following so far. We looked at the Langevin model, generalised Langevin model, a lot of linear response theory, etc. But then the question arises as to whether there is a more basic way of deriving densities and phase space, in particular the one particle density, the simplest of such densities. Well this was a problem of kinetic theory, great deal of work was done in the 19th-century as well as 20th-century on this. A lot of problem still remains to be solved in this area and I will try to give you a flavour of how even in the simplest of instances the problem reduces to something which is fairly complicated and is fairly non-linear as you will see.

I am not going to give a rigorous derivation of the Boltzmann equation but I will point out what the physical assumptions are, very reasonable physical assumptions, justifying which is going to be much more complicated but even without that just the mechanism involved is something which could be explained in reasonably simple terms. So let us look at that. Let us look at what the problem is and how it is tackled.
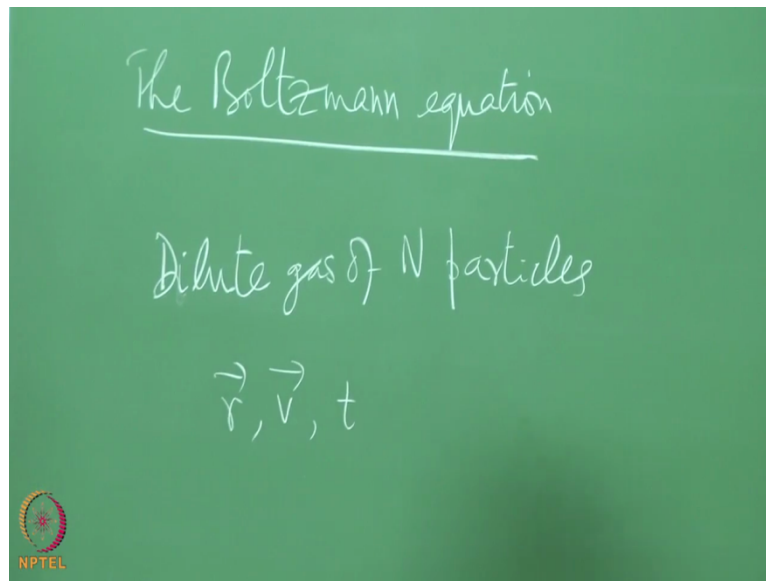
this problem arose initially the context of Extremely Dilute gases when people were trying to derive from 1st principle, things like the Maxwell distribution of velocities, etc. At the very foundation of statistical physics, Boltzmann worked out what is now known as the Boltzmann equation in 1 form or the other and he motivated much of what happened later on. And this continues to be of great importance in many many applications including plasma physics, fluid dynamics, etc. etc. Now what is the problem is we are going to look at? We have in mind a gas, very dilute gas of particles which of course interacts by Collision among the particles.

to understand the collision themselves, one would probably have to look at the quantum mechanics nature of these particles which is outside the purview of what we are going to do now. But given the fact that particles scatter of each other, there is some collision cross-section which you can compute in principle, the question is what does it say about uncle particle densities. So the rest of the argument has to do with the physical arguments on how phase space density evolves. Suppose you have N particles, a very dilute gas of N particles, we are not going to get into things like quantum statistics or anything like that, it is a very classical kind of gas, very dilute gas and this gas is in thermal equilibrium.

the question then arises, if you start with some initial conditions, which of course means that when talking about more special initial conditions, the question is how does the gas equilibrate. Or if you start by disturbing the gas a little bit, by applying the feel of something like that, how does it go back after you set it off, how does it go back to equilibrium distribution. So this is the question asked. Now of course to actually deduce what is

happening to all the particles, every one of the particles in this gas is a formidable task. We need to know conditional probabilities or densities, given a particle layer, what is the probability of a particle there and the probability of yet another particle there and so on.

that is formidable but we could say all right, all these particles are on equal footing and let say, let us look at the phase space of any one of these particles, given particles here. Now this phase space is described by 3 special coordinates and of course 3 momentum or 3 velocities. Since we are going to talk in terms of collisions, let us look at velocities instead, that is the conventional way it is done. So we will say that the particle is described by specifying its position and velocity at any given time t. Now this space of all possible R and v and R is inside the volume of the container if you like, that space, the phase space of a single particle is known as mu space.

So the space occupied by these quantities is called mu space and the idea is that this mu space is the same for all the particles. Any one of the particles will have the same set of possible values of r, of possible values of P. So the single particle phase space is what I call mu space. then the particles of course are of discrete nature but would like to have some kind of continued description of these particles. So let us look at some numbers, at normal r temperature and pressure in a dilute gas, you have about 10 to the 25 molecules per cubic metre.
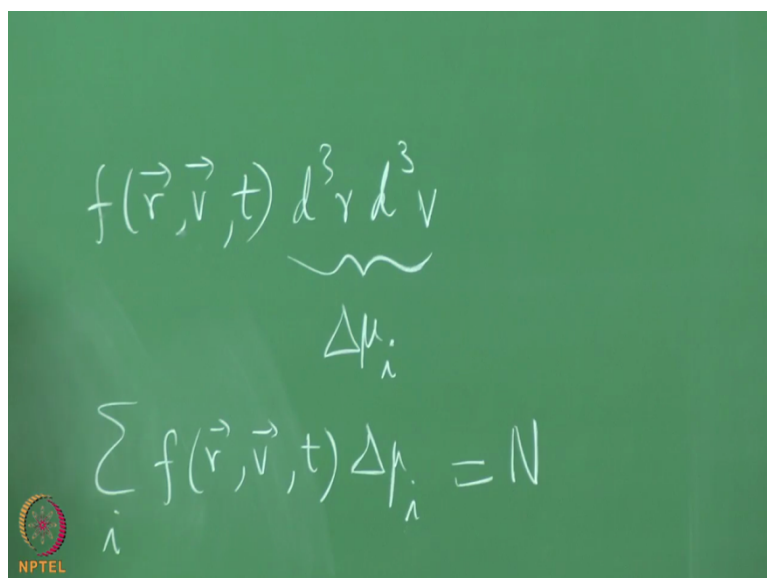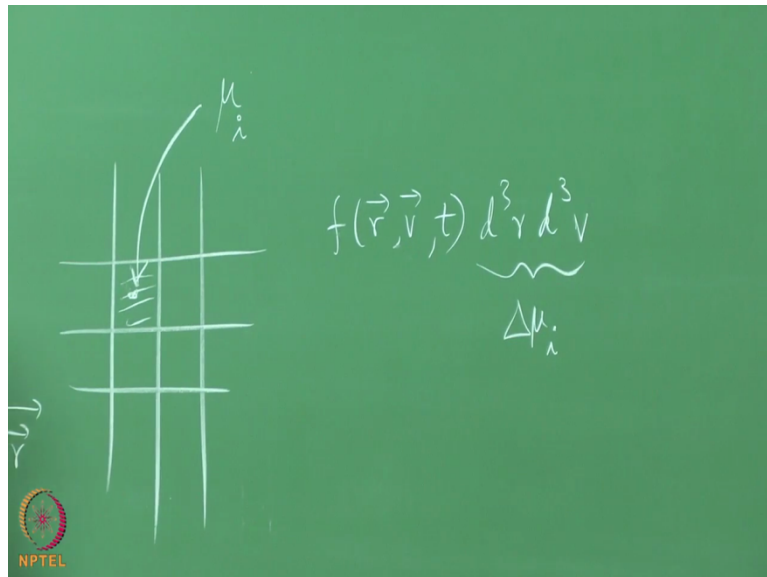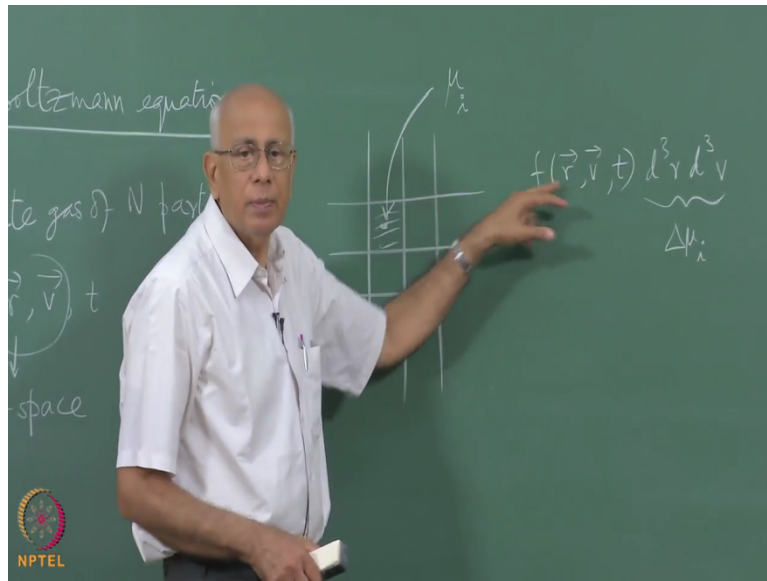
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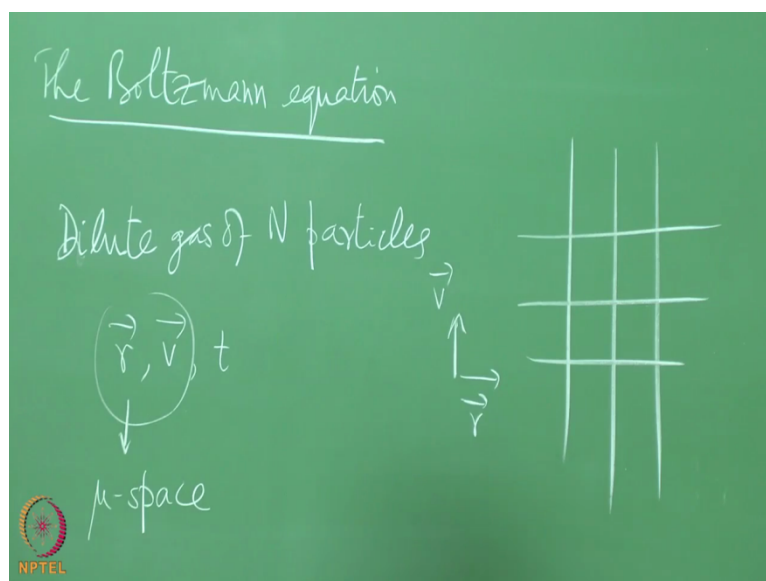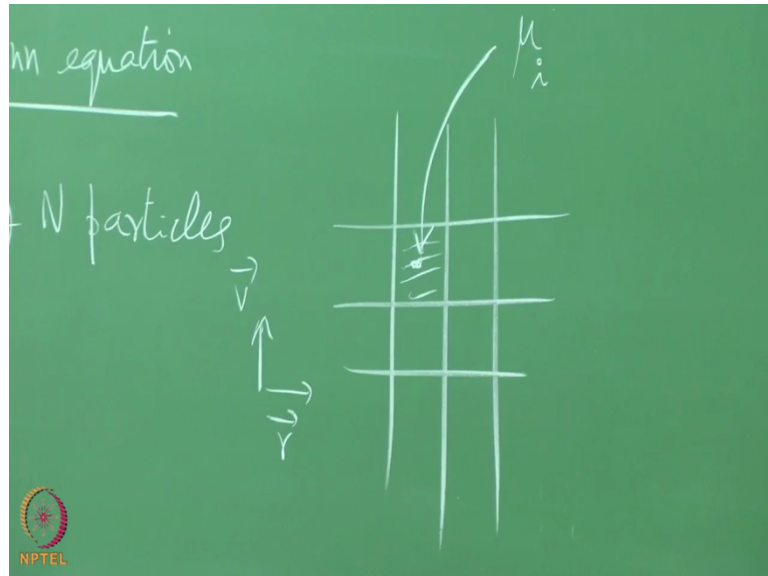
this space if you know coarse-grained it, if you try to make it into small cells, you break up this space into cells, so let us suppose schematically you have the velocity in that direction and the coordinate in this direction schematically, pardon me, the coarse-grained mu space. the space, the phase space of a single part, one particle phase space. If I coarse-grained it in some fashion, then, well since in physical volume I have already said that each cubic metre has 10 to the 25 particles. If you look at the physical volume of say 10 to the -15 metre cube, that is 10 to the -5 metres on each side of a small cube, 10th of a millimetre, hundredth of a millimetre on each side, you still have 10 to the 10 particles in it.

So in that sense these particles form practically continuum even though you are looking at a point. A point as small as volume of 10 to the -15 metre cube which is pretty much like a point. But even then you have a very large number of particles in it. So let us suppose that you coarse-grained it in this fashion and this thing here is what I called mu sub i, that is the ith sell in the mu space. And I label it by the points of its centre. then I can ask at any given instant of time t, what is the number of particles in this cell. Let us call that number of particles f of r, v and t, where r and v are the coordinates here in this point of the Centre of that cell.

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$$f(\vec{r}, \vec{v}, t) \, d^3r \, d^3v$$

$$\underbrace{\qquad}_{\Delta\mu_i}$$

$$\sum_i f(\vec{r}, \vec{v}, t) \, \Delta\mu_i = N$$

$$f(\vec{r}, \vec{v}, t) \, d^3r \, d^3v$$

$$\Delta \mu_i$$

$$\sum_i f(\vec{r}, \vec{v}, t) \Delta \mu_i = N$$

$$N = \int f(\vec{r}, \vec{v}, t) \, d^3r \, d^3v$$

$$d\mu$$

So this is the number of particles in this volume at time t when multiplied by D3 R D3, so let me call this, give it a short name. Let us call this Delta mu i, that is a measure, phase space measure of the ith sell. So this is like a density, number density if you like. And it is a local quantity and the idea is that you do not have very sharp variations of this f as you move from one cell to the neighbouring cell. So you can treat this as a continuous variable. And now the question is what is the total number particles in the gas? the total number, remember that each particle, each of the N particles is represented by some, somewhere in this space, in this mu space because each of them has an R and v and therefore there is some kind of point in this space.

And when you add up all these numbers, you are going to get the total number. So it is clear that if I integrate D3 r, D3 v, F of r, well if I take this and sum, I sum over i F of r, v, t, Delta mu i , some lower i come all the cells i. this must be equal to N and the limit in which these volume go to 0, this measure goes to 0, this goes to an integral F of r, v, t, D3 r, D3 velocity and let me call this is D mu. Shorthand for that, I call it D mu. And this must be equal to N.
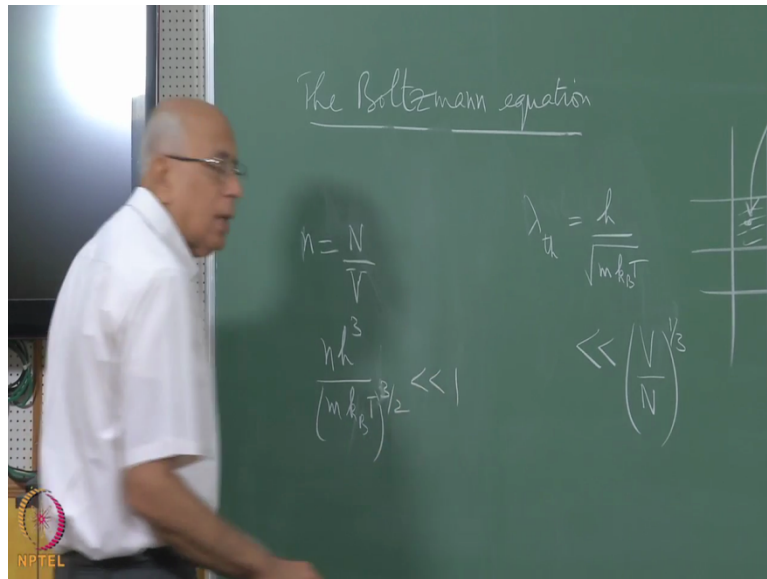
If I integrate this quantity over r alone, right, what happens then? then I get a volume, if I integrate over r, I get a volume and that will give me N over v. But whatever this is the constraints, this is a normalisation of the single particle density. And the whole point of kinetic theory is to find out what this F is. Okay. So that is the task to calculate this F of r, v, t. Because from that it is like a density for phase space density for a single particle, from that I should be able to find out physical properties, transport properties. I am not given anything at the moment, except this condition and I have tried to find this I have been issued.

It will satisfy some kind of differential equation and I have to tell you what it is and of course the standard problem would be, given the value of F of r, v, t at t equal to 0, initial condition, find out what it does at later times, that would be the typical problem. So the whole target is to find an equation for this. Just as when we did density matrices functions in classical or quantum mechanics, we found there was an equation for the density operation, the Louisville equation, it says Delta rho over Delta t was equal to the Poisson bracket or commutator of rho with h or h with rho.

We want a comparable equation for this F here. Now we are going to assume that the gas is dilute, what that means is that, it is so dilute that, to spread apart, that the blowing wavelength of each of these particles is much smaller than the thermal wavelength. that of course as you know is the standard criterion for non-degenerate in the quantum mechanical sense for non-degeneracy in the quantum sense, namely the thermal wavelength, so schematically what it means is you have little fuzzy objects which are individual particles and this fellow should be at a distance considerably greater than the fuzziness in the position of each of these particles.

So you want this, say lambda thermal which is equal to Planck's constant divided by the average momentum but in a free gas, the kinetic energy is proportional to Kt, it is spread over M is proportional to kt, so P is proportional to square root of M Kt. So you want this square root of MK Boltzmann t, this fellow here must be much much smaller than the inter particle separation. Now the volume of the particle is v over M and that is like a three-dimensional, it is a cube of the inter particle separation. So you want this to be much much less than v over N to the power one 3rd. Okay.

That is the condition under which classical statistical mechanics will apply, okay.

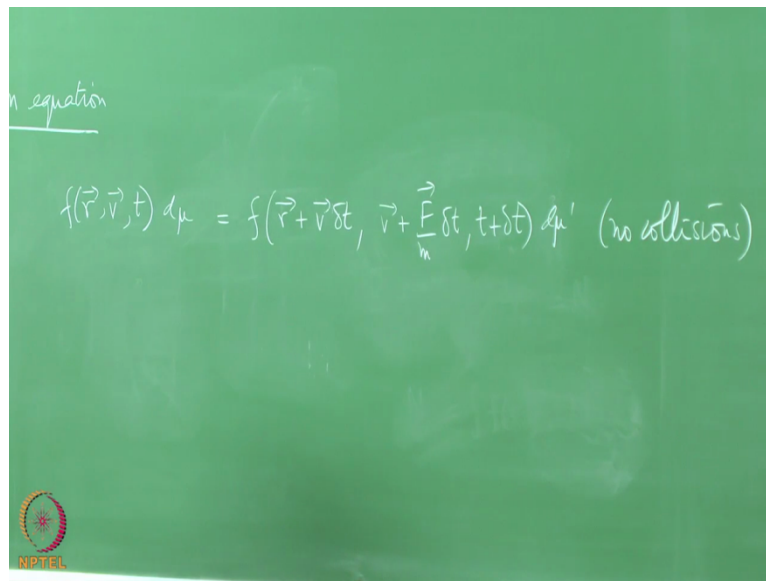(())(14:00).

Should not I have a…?

(())(14:06).

Yah, because this is saying that once you prescribe for me how the particles distributed in this phase space, then that is the end of it, right.

I think it will more comfortable if you add (())(14:21).

Yes, if you like, I already said that R and v have the coordinates of this point here, the middle point. Yes, sure. Okay. So if I bring it to this side, it says the number density if I define n equal to N divided by v, number of particles per unit volume, it says that N h cube over MK Boltzmann t to the 3 halves much much greater than 1. that is a condition for classical statistics as you know. Low densities, high temperatures, then this becomes much much less than 1. For the gas in this room for instance, nitrogen gas at normal temperature and pressure, this quantity will be of the order of 10 to the -6 or -7 or something like that, when you are in very good shape. Okay.

On the other hand if you go to extremely low temperatures and higher densities, then you are in a very degenerate situation. Right. If you go to something like a Newton star, this quantity becomes enormous. Even at relatively high temperatures you would still have, it will get reversed in the other direction, okay. And then you have to go for quantum statistics. So we are going to work in this regime in this classical, completely classical regime. Okay. Now the matter is very simple, it would not have collision at all, 1st of all throughout the going to neglect to start with collisions with the walls, because now we have to them deal with the atomic structure of the walls and ask what the interaction is.

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We are going to ignore that and look at what happens in the bulk 1ˢᵗ. And then this is what happens to this F. If you consider what happens to F of r, v, t, D mu, D3 r, D3 v, etc. and ask what happens to these as t goes to t + Dt, okay. then a particle in this cell at this position, maybe get out of this Cellcom it would acquire a new position and a new velocity depending on what force there is on this particle. Let us suppose you impose an external force on the particle, not collision, an external force on this particle, some F, capital F, and that force could vary with time but very slowly compared to the timescale of molecular motion.

Could change with position, you do not care, whatever, then this quantity all the particles here would move into a new position, coordinates would be equal to F of r + v Delta t, okay and v becomes v + F over M Delta t and the time is of course t + Delta t. that is the density multiplied by D mu prime, where mu prime refers to the measure of the cell corresponding to these coordinates. Okay. So this is clearly conservation of number, nothing has happened, these people, these molecules are moved into some other cell in the phase space, okay.

We will assume Delta t to be quite small and the cells to be adequately small, the limit we are going to take the limit in which all these things become point objects but that is what this conservation law is to start with, in the absence of collisions, no collisions. Okay. But now… Yah.

To the left-hand side of this equation (())(19:05). But for that in a given cell the number of particles can always change, right?

It will.

So the total number…

That is why it is changing with time, sure.

But then it imposes the fact that in a given mu cell itself, the number of particles are same before and after that…

No, it is a different cell.
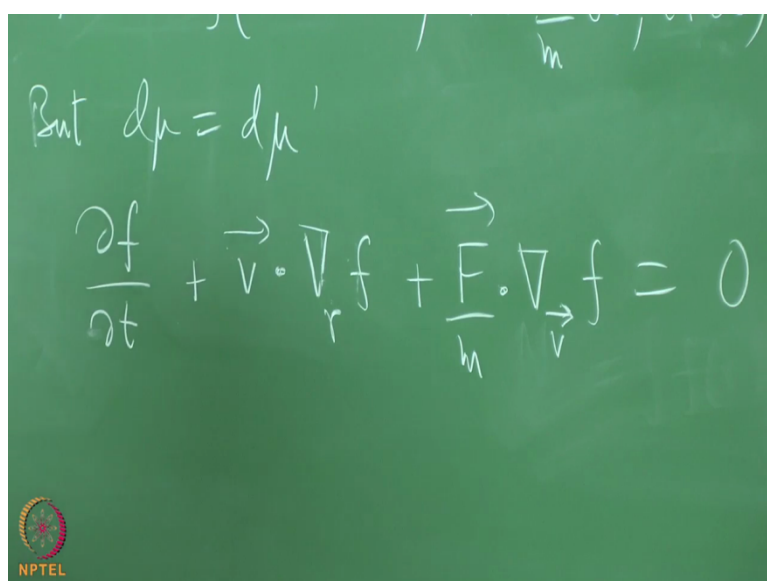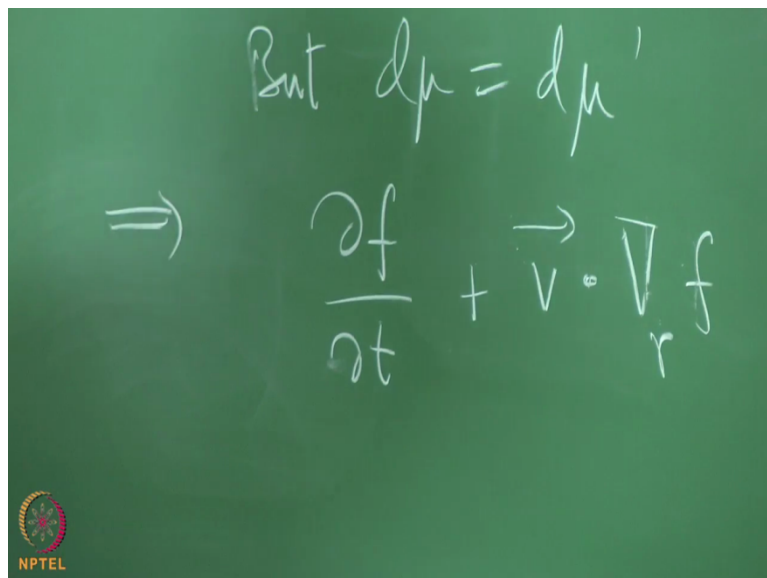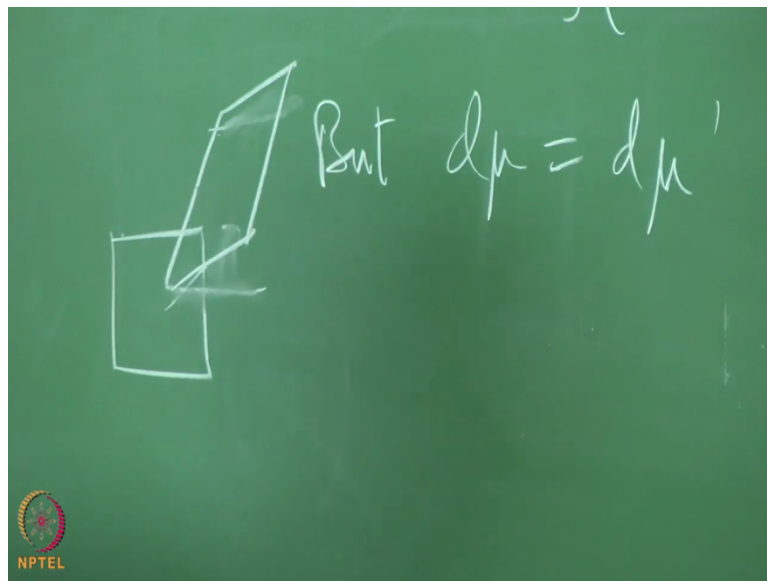
All I am saying is this set of particle moves out, right.

(())(19:38).

This is the way it evolves in time, in a sufficiently small interval Delta t, the velocity changes by v + F over M Delta t, this will be acceleration. this is the change of momentum divided by mass, that is a change of velocity. So that is the whole point, maybe a complicated equation of motion but we are looking at the very infinitesimal interval of time. Okay. So that is all it is in the absence of collision. But we also know that if this happened in phase space, imagine for a moment you working in phase space and Hamiltonian dynamics in R and t, then we know the motion is such that phase space volume elements are preserved under time, it is like an incompressible fluid, okay.

But we are working with velocities here, but this is still true and if you look at Huang's book for instance, statistical mechanics, than a nice derivation is given of this Boltzmann equation. It points out that this is real true even in terms of the velocities. Essentially in this problem it is kind of trivial that it is true because what happens is is in the volume elements you have this, initial volume element in R and p, you have this, then since r changes by r + v Dt and v changes by a similar kind of thing. this goes over into something that looks like the originally shifted, it looks like this. Okay. And you can show this area as equal to this area by simple geometry, which I am not going to do now.

You do not get Dt is much much less than (())(21:42).

But $d\mu = d\mu'$

But $d\mu = d\mu'$

$$\Rightarrow \qquad \frac{\partial f}{\partial t} + \vec{v} \cdot \nabla_r f$$

But $d\mu = d\mu'$

$$\frac{\partial f}{\partial t} + \vec{v} \cdot \nabla_r f + \frac{\vec{F}}{m} \cdot \nabla_{\vec{v}} f = 0$$

I have stopped all collisions, I said we do not have collisions. the problem will arise when we are going to include collisions, which you have to but I am trying to see what happens purely geometrically without collisions, if you ignore collisions. So let us put it on underway, this is in the absence of collision which is equivalent to saying this is in the case in which the particles can go right through each other without affecting each other. Okay. Imagine they flow through each other, they do not know about each other's presence. So the scattering cross-section is 0, then this would be true.

But we have to include the scattering cross-section which we will do in a moment. Okay. So since D mu equal to D mu prime, right, it implies that this is equal to that, explicitly, right. But what does that mean? We can now go to the limit in which Delta t goes to 0 implies this F is equal to this F in the limit in which Delta t goes to 0. And let us expand, taylor expand this, the 1st term cancels and retain to first-order in the infinitesimal, that immediately tells you Delta F over Delta t, if there is an explicit t dependence. And then when you differentiate with respect to this, you get the convective derivative part, gradient r with respect to r, f.

And when I differentiate this, I will get + F over M dot gradient with respect to v F equal to 0 in the absence of collisions. Okay. But now there are collisions, there certainly are collisions, the whole physics is because there are collisions, equilibration occurs because there are collisions. So this is wrong and what has been left out is precisely due to collisions. this differs from this precisely by the extra term due to collisions. So this is, so that is our 1st equation and everything is sitting here, everything is in this term here. Pardon, pardon? Normal derivative with respect to what?

(())(24:52).

this is the way in which F changes in time, the change in F in a time Delta t due to collisions is Delta F over Delta t times Delta t, I have just cancelled out the Delta t on both sides.

What had the subscript (())(25:09).

It is a contribution to do this F due to collisions. We do not know what is happening to this F due to collisions, so now we are going to ask what is going to happen, right. So you had a certain number of particles in this D mu in here, here is the molecule, in the time between t and t + Delta t, the molecule can come and hit this and throw this out of this cell. So it will deplete it. On the other hand there maybe a collision here between 2 molecules and in the

time between t and t + Delta t this fellow me get in here, that will add to this F. And you therefore have to take into account both these fellows out here.

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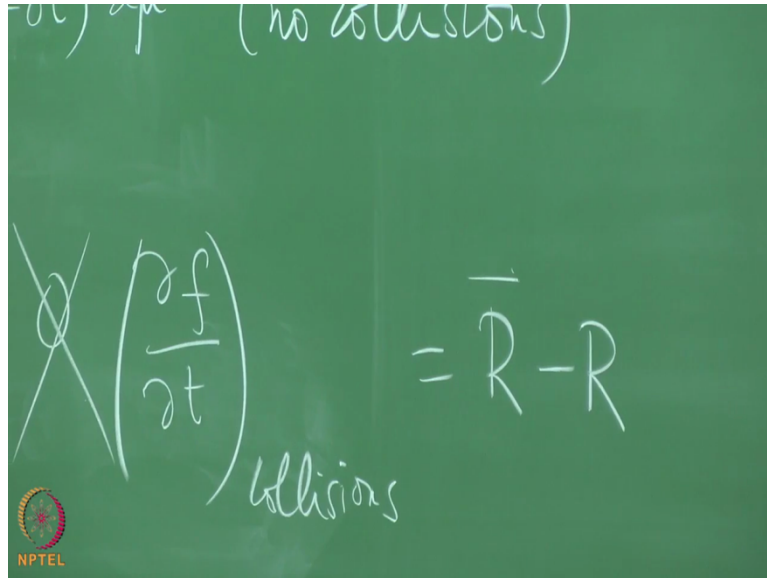
things which get in will increase this F and things which get out will decrease this F, right. So this is equal to R bar - r, this is standard notation and I have explained what these R and R bar are.

they have 2 bodies I think (())(26:19).

Exactly, 2 or N bodies, we do not know, we do not know as yet, we are now going to start systematically making approximations. So R Delta t equal to number of particles that, oh yes, we now make an assumption, we now make a crucial assumption that the cell is so small that any college and takes it out of the cell, we have are going to count the number of collisions, so we need to know what these collisions do. You may have a collision but it still leaves is in the cell, are very mild collisions. But we are going to say the cell size is so small in the limit of 0 volume that any collision takes it out of the cell. Because finally what is happening, we are trying to find a local differential equation, a completely local equation at each point.

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So some kind of continue approximation is being made and the idea is that any collision kicks you out of the cell. Okay. So number of particles that leave D mu, our cell at the point R at the volume, at the velocity v, okay. So that is R Delta t, okay. And similarly R bar, number that come in into D mu due to collision in this time interval. It is a very standard kind of assumption, that you say that these collisions are such that any collision gets you out of it but there could also be some collisions that brings you into it. this is like, these assumptions which in the case of Markov processes, there is again under is a loss term in any rate equation, it is pretty much in the same spirit here.

We have still not said anything about the dilute, we have to now explain where this dilute business comes in. We have only assumed that any collision, the cells are sufficiently small that collisions knock your profit, right. Now the question is what sort of collisions are talking about? Now in the simplest instance, let us assume the elastic collisions, so the energy is conserved in these collisions, there is no other absorption. When 2 molecules collide, they come out with, total kinetic energy is the same before and after the collisions. Okay. And we have an elaborate theory of scattering in quantum mechanics to describe scattering processes.

But the simplest instance we will assume that the collisions are binary collisions. Namely, only 2 are rectangular, the idea is that the probability of 3 particles scattering off each other at the same instant of time is 0, effectively 0. This is a very good approximation. Yah?

Says we take a time over (())(30:15).

there will be fluctuations but the assumption is that, look at the cell size that we are talking about. It is sufficiently large, that you are in the continue approximation as far as the number of particles is concerned. It is not as if you are looking at a cell size in which all of sudden 10 to the 15 particles may become 10 particles for instance. that is not going to happen ever. Okay. the probability of such a large number of fluctuation in negligibly small, okay. It is always true, I mean the gas in this room for instance, it is binomial distribution in thermal equilibrium, so the probability of something which is very faraway from the mean is going to be extremely small. Okay.

So then you would need , then you said there are 2 conditions that we imposed on the cell size, one of which is the fact that if there is a collision that is taking place, it is move out of the cell, so that is like the mean path length of the molecule should be larger than the cell size itself, after the collision it does not actually move out of the cell.
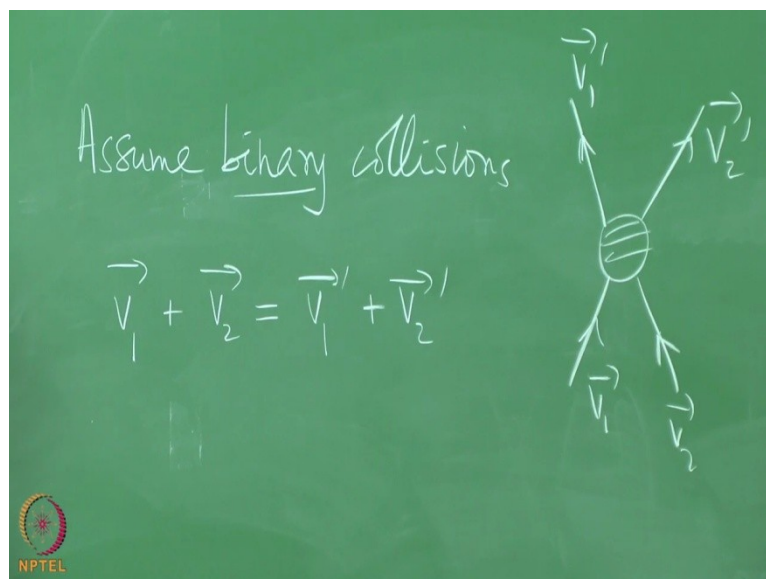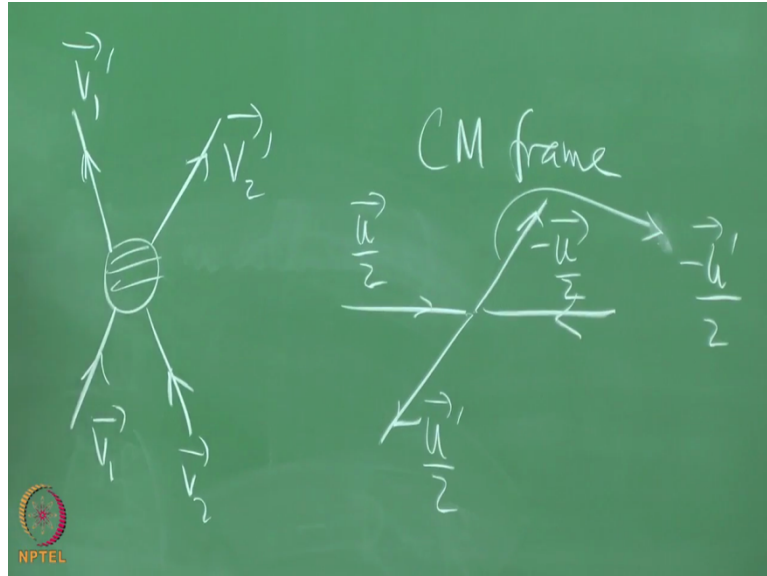
Yes.

So then you need the intersection of the fact that the cell size needs to be large enough so that…

Yes.

So is that…?

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Yes, that is built-in, yah, yah, absolutely, we will see what where this gets us. So the question is, is it going to be exact, then the answer is no. It is not an exact equation but we know the precise conditions under which the approximation is valid and for dilute gases it will turn out to be an extremely good one. Okay. Now the crucial assumption is this business of binary collisions, that the collision events are all 2 partition collisions, okay. Now what happens in into particle elastic collision, very simple, if you have a particle with velocity v1 colliding with a particle with velocity v 2, then these particles go to new particles, the same particles go out with velocities v1 prime and v2 prime for instance.

So there is some collision event happening in which you have v1 coming in, v2 coming in there and then this goes off v1 prime, and v2 prime in this fashion. then the concentration of momentum in this case, there assumed to be equal mass particles, is of course that v1 prime + v2 prime. that is the 1st equation and the 2nd one says that the energy is concerned, right. So it says v1 square + v2 square equal to v1 prime square + v2 prime square. Now in this situation it is such obviously sensible to go to a frame of reference in which you are in the centre of mass, frame of reference, so that it looks like to fellow is coming in and 2 fellow is going out in this fashion.

So in the centre of mass frame of reference, the collision will not look like this at all, in the CM frame, it looks like 2 particles are coming in, in this fashion with equal and opposite velocities, so this comes in with say some U over 2 and this is - U over 2 and then after collision they go off in this fashion. So this is U prime over 2 and this fellow is - U prime over 2. Okay. And how is that achieved? Well, you go to a frame of reference in which the U,v, the centre of mass is the velocity, capital v which is half v1 + v2 to start with.

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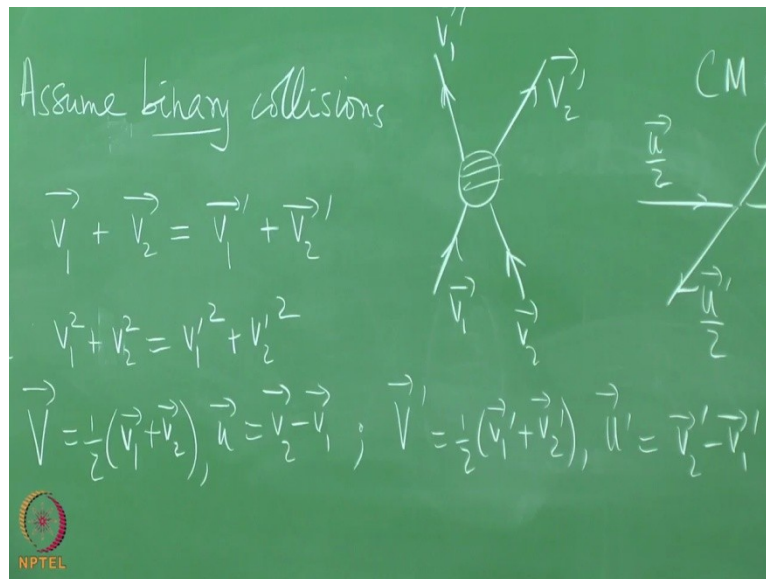
So let us convert v equal to half v1 + v2 and U is v2 - v1, that is the valid velocity. Okay. Similarly further prime coordinates, so v prime equal to half litre v1 prime + v2 prime and U prime equal to v2 prime - v1 prime. then what is the content of these 2 equations? Well the 1st one of course is that capital v equal to v prime, that is trivial, total initial velocity here is equal to the same thing out there. And this thing here, if you substitute for little v1, little v2 etc. in terms of capital v and U, it becomes the cross term, v dot whatever cancels out and you are left with U equal to U prime, but the magnitude of the relative velocity is the same, it is unchanged in an elastic collision.
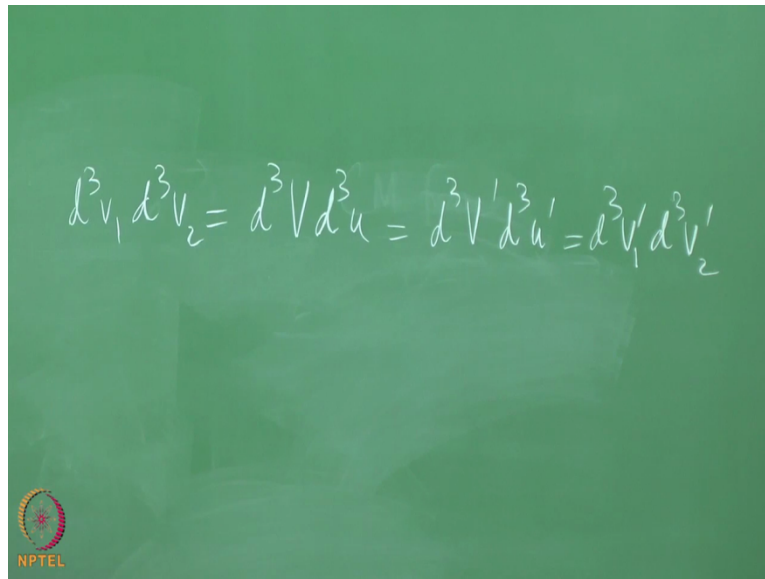
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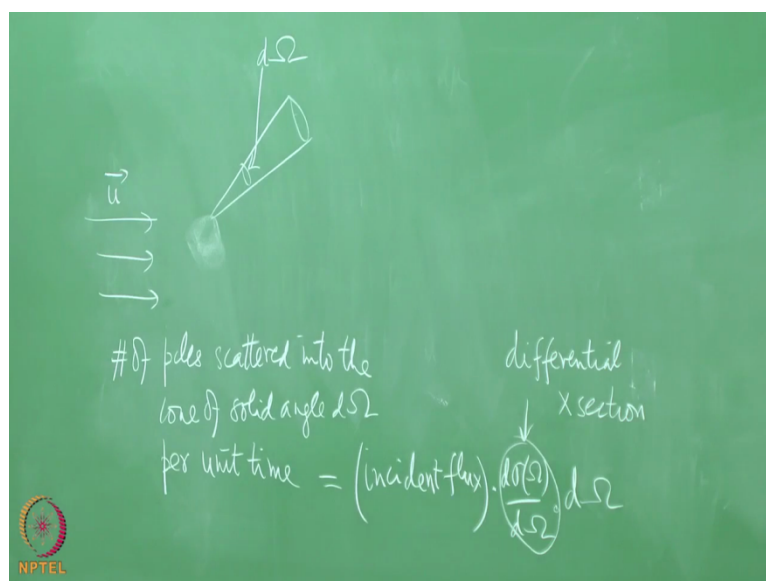
So for all practical purposes, you can view this as scattering of particles of velocity U in a fictitious potential, whatever that potential be, okay. And we know how to do the scattering theory in that case, okay. So you have a centre of attraction, Centre of force for example, some potential and then you an incident beam of particles coming in with a velocity U. If you take this quantum mechanically, then particles scatter off it in all possible directions and one of the fundamental problems of, the fundamental problem of quantum scattering theory is to find out the flux of particles in any solid angle. Okay, that depends on the nature of the potential, it depends on the spherical symmetry or not, etc., it depends on the potential.

But it is a 1 body problem effectively. Classically too we can solve the problem, you give me the potential and I will tell you what the scattering in different directions is. Okay. Rutherford scattering for example, from a coulomb potential it will tell you what exactly the different cross-sectional is in various directions. So that is a problem in scattering theory and it essentially says that at the end of the day, if you ask what is given in the incident flux in some direction, what is the number of particles, what are the flux of particles that is coming out in the solid angle d Omega at an azimuthal angle Phi and polar angle theta with respect to this incident direction here.

So this problem is a problem in scattering theory, okay. And what this, how is that solved? the dissolved by saying that this number of particles, the flux, let us say number of particles scattered into the cone of solid angle d Omega per-unit time is equal to what? Well it will depend on the incident flux, the number of particles coming in per-unit area per-unit time to start with. So let me call that incident flux equal to incident flux multiplied by the scattering cross-section, the differential scattering cross-section, right. So this is equal to d Sigma which depends on omega over d omega, times d omega because I am asking for a number in d omega.

$$d^3v_1 \, d^3v_2 = d^3V \, d^3u = d^3V' \, d^3u' = d^3v_1' \, d^3v_2'$$

And this is the differential cross-section, this quantity here. In this soup particle scattering geometry, a lot of interesting things happen. 1st of all it is easy to show that the measure does not change, that, it is, this is simple exercise, I wanted to show this from these algebraic equations again show that D3 v1, D3 v2 equal to D3 v D3 U, if I change variables, that is equal to D3 v prime, D3 U prime, okay, which in turn is equal to D 3 v1 prime, D3 v2 prime. So this volume element in phase space in velocity space does not change under the scattering. the rest is just, is just simple algebra, it simply says counting, just counting. Because we need to find out the following, yes.

Let us look at the term R 1st, we have R bar - R Delta t. Let us look at what gets out of this phase space cell, in a time Delta t. What is that going to depend on? It is going to depend on the incident flux, so let us pretend that we, we know this quantity by solving this scattering problem and let us pretend that v1, we fix v1 at some value and ask what are all those v2s which will cause this scattering out into some cone or the other. What are all those v2s, so for a fixed v1, we want all the v2s that will hit against this v1 in that phase space cell is time t to t + Delta t and take it out, okay.

So it is equal to the incident flux multiplied by scattering cross-section. But the incident flux is provided by what? It is provided by F of R v2 because that is what is hitting against the v1 at time t, right. Multiplying, multiplying the differential cross-section and any other factor. Well you have to integrate over all these fellows here, D3 v2 and integral. We do not care what angle it gets scattered into, so you have to keep, before we get to this, we will not finish

with the incident flux yet, we need to have all those particles which are within a mod U of it because of per-unit time. So mod v2 - v1 and then d Sigma omega.

So this is going to be the R for a fixed v1, yes. No, the condition and v2 comes from, that this is going to depend, this is a function of U, it will be function of the energy and of course the angles. Because what is happening is that if you have a scattering centre here, in this geometry, in this geometry, I ask how many particles are going to go here. that is going to depend on what the initial velocity is, or initial energy is because this is elastic scattering. U prime is equal to U, so it will definitely depend on mod U and then relative to this direction of U it will depend on what this scattering angle is, it and it will also depend on what the phi is in this direction. I have included both of them in getting Omega, okay.

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$$d^2 v_1 \, d^2 v_2 = d$$

$$\int d^3 v_2 \; f(\vec{r}, \vec{v}_2, t) \, |\vec{v}_2 - \vec{v}_1| \, d\sigma(-\Omega)$$

afn. of u

$$d^3 v_1 \, d^3 v_2 = d^3 V \, d^3 u = d^3$$

$$R = \int d^3 v_2 \int d\sigma(-\Omega) \, f(\vec{r}, \vec{v}_1, t) \, f(\vec{r}, \vec{v}_2, t) \, |\vec{v}_1 - \vec{v}_2|$$

If it is a spherically symmetric potential, then it will not depend on the phi or it will still depend on theta. And of course on the incident energy which is measured by mod U, right. So that is the, that is sitting here, this is providing the factors in the flux, the number per-unit time, it depends on how many of them can reach this point, the particle v1 to be scattered out, okay. So this is going to be the R term, okay. But that is for one particle, but all those which are sitting in this phase space within the volume and D3 R going to get scattered out. Right. So you must multiply this one again by f of r, v1, t.
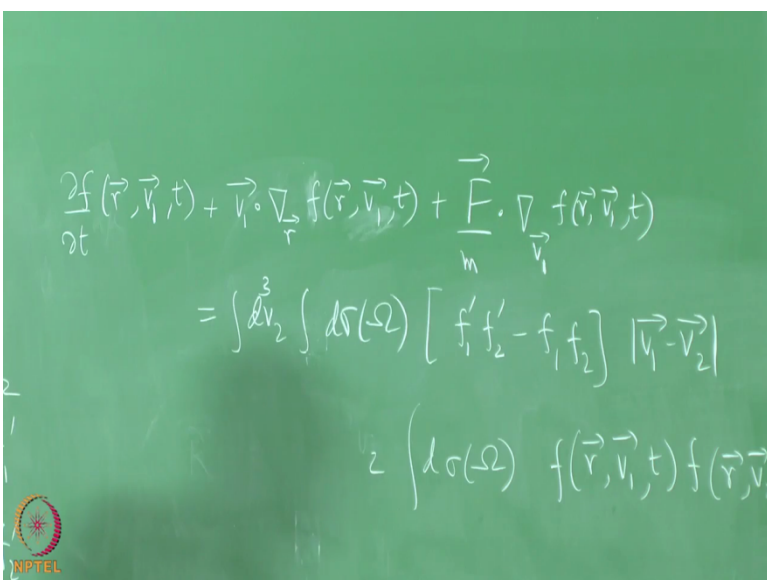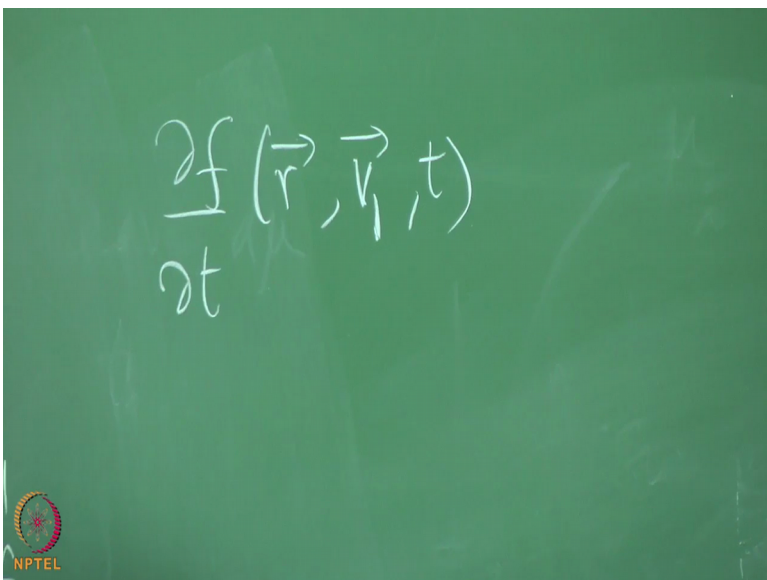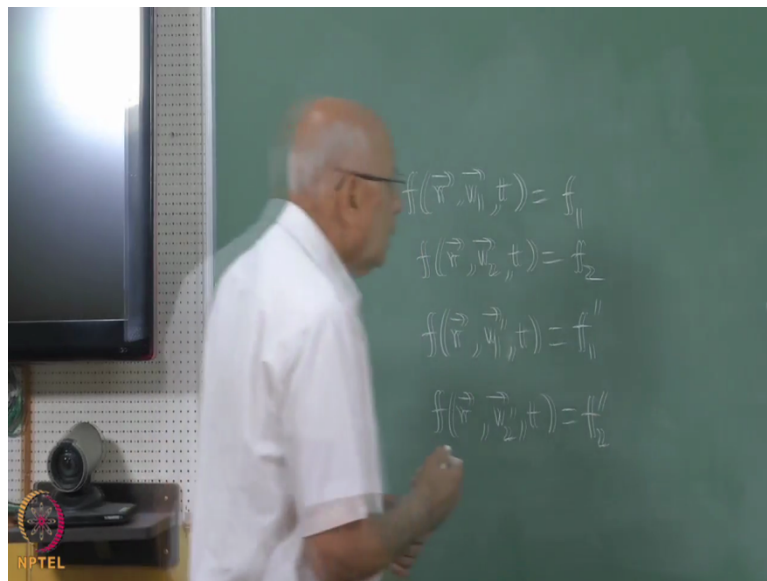
that is number of scattering centres?

that is the number of scattering centres. So R is going to be proportional to this. We do not care in what direction these particles get scattered out, so there is also an integral over this, so I bring that fellow downhill and then write this. Let us put this in, but there is an integral over D3 v2. All the molecular details of the actual scattering process, actual collision, etc. is sitting here. So the computation of this is a separate story altogether. But in a very rough sense because of this binary approximation, you have 2 of these fellows sitting here. Okay.

Mod v1 - v2.

And mod v1 - v2, absolutely, that is what couples the 2. Okay. So this is what R is proportional to, it is essentially R. But what is R bar, which is, those which are getting scattered outside to get into this phase space, okay, how are we going to find that? Well, we argue in the following way, if you have a scattering process, it is equal to, it is actually equal to. Okay. If you have a scattering process in which v1 + v2 goes to v1 prime + v2 prime, then assuming that the dynamics is time reversal invariant, the scattering process in which v1 prime + v2 prime combined to give you v1 + v2 as the final product is also possible and has the same probability, okay.

So by the, by symmetry, time reversal symmetry it turns out the scattering cross-section for that process is same as the scattering cross-section for the original for this process. Okay. It is exactly the same, so there are some intermediate steps in between where you have to show that this is really true and so on and so forth. But we will take this, let me just make a statement that owing to time reversal symmetry + the overall rotation symmetry of this and inversion, reflection symmetry of this scattering process, the other scattering cross-section for v1 prime v2 prime scattering into v1 and v2, it is exactly the same.

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$$\frac{\partial f(\vec{r}, \vec{v_1}, t)}{\partial t}$$

$$\frac{\partial f(\vec{r}, \vec{v_1}, t)}{\partial t} + \vec{v_1} \cdot \nabla_{\vec{r}} f(\vec{r}, \vec{v_1}, t) + \frac{\vec{F}}{m} \cdot \nabla_{\vec{v_1}} f(\vec{r}, \vec{v_1}, t)$$

$$= \int d^3v_2 \int d\sigma(\Omega) \left[f_1' f_2' - f_1 f_2 \right] |\vec{v_1} - \vec{v_2}|$$

$$\approx \int d\sigma(\Omega) \, f(\vec{r}, \vec{v_1}, t) \, f(\vec{r}, \vec{v_2})$$

So therefore we introduce notation, so it is conventional to call f1, f1 of r, f of, f of r, v1, t, it is conventional to call it f1, v1 prime, t equal to f1 prime and similarly f of v2 prime, t equal to f2 prime. Then R bar is exactly the same thing, except that you have prime sitting here. It is out that this is exactly the same factor with suitable changes. Remember that mod v1 prime - v2 prime was the same as mod v1 - v2, okay. So that is also put in as rules. And you end up with an equation which essentially says that Delta f over Delta t of r, v1, t, that is our fixed velocity to start with, + v1 dot Dell r, let us put f outside, v1, t + the external force F over M dot gradient v1 f of r, v1, t.

On the right-hand side is equal to D v2, D3 v2, integral D Sigma, differential cross-section and then inside f1 prime, f2 prime - f1, f2, mod v1. Notice what these quantities are, f1 is this quantity, f2 stands for this quantity here and that is integrated over the v2 is integrated over, so all those v2s are such that the relative velocity is v1 - v2. So integrated with this weight factor and this quantity depends on that modulus on this modulus here, it is a function of that, as well as the angles. Similarly for the prime quantities where these prime stands for v1 prime, v2 prime, such that v1 prime - v2 prime modulus is equal to v1 - v2 modulus.

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So those quantities v1 prime, v2 prime are determined in terms of v1 and v2, okay. But ultimately these are the fellows scattering into the volume, these are the fellows scattering out of the volume, that is a measure of these quantities here and the binary business comes here. Now let us stand back and look at this, the moment you see an equation like this, 1st of all it is an integral differential equation to start with. this is a famous Boltzmann equation in this language. Okay. But what is the most striking thing about it? All these quantities are very

similar to the kind of things we had and we wrote down various Fokker-Planck equations and so on.

But now this has nothing to do with that, there is no randomness or anything, we are just saying that we are going to look at all deterministic processes such that the phase space density changes due to collisions. But the most striking thing of all is that this is non-linear in this f, so it is a non-linear integral differential equation, extremely formidable to solve mathematically and even to show the existence of solutions under suitable conditions. So already even with the simplest of assumptions with binary collisions and so on and so forth, even if you knew the dynamics, even if you knew the scattering cross-section here, you still had the problem that you got to solve this integral differential equation which is highly non-linear. Okay.

So that is the root of the source of the problem, source of the difficulty. Now where did this question of dilute coming? Well, we assume that these 2 things are independent of each other, so we assume that 2 particles which collide have no correlation at all, no memory, no correlation at all. those who collide once are gone forever, after that they do not see each other again, okay. And moreover it is uniform across r, independent of r, the same geometry applies. So in that sense we are using this so-called molecular chaos assumption of Boltzmann, that there is no correlation in this kind and there is independent of velocity, position space, the velocity distributions act in this multiplicator manner.

Now of course if you have correlation between particles, this is not true anymore. So if you go to dense gases, this is not true, even with binary collisions this kind of assumption is not true anymore. then there are many cases where it is particularly you not true at all. for instance if you imagine gas, and elastic gas of equal mass particles on a line, one dimension, then you know that one dimension, when you have equal mass particles, when 2 particles collide, they just exchange velocities, right. So if you have one particle A between 2 particle B and C, then all the collisions of A are always with B or with C.

So the number of recollisions, all collisions are recollisions in some sense. At best there is one more particle to pitch, you can collide in between 2 collisions with one neighbour on one side. It is the opposite exchange, it is highly correlated, it is extremely correlated, controlled entirely by recollisions but the Boltzmann equation assumes there are no recollisions at all, it has completely neglected it. In 3 dimensions it is pretty much there, it is clear that it is pretty

much valid, this assumption is valid. When 2 fellows collide with each other, they are gone after that.

So this is the most elementary way of, sort of motivating the derivation of Boltzmann equation, I have not gone through all the steps but very roughly this is what it entails in this case. the rest has to do with approximation, how to linearise this, what are the assumptions one makes to linearise, etc. for example in plasma physics there is a very famous approximation called the Vlasov equation which follows from the Boltzmann equation. But that gets into kinetic theory and that is not my purpose is really. One problem that one should tackle with this is all the transport coefficients, so there is an elaborate theory which tells you how to compute the viscosity of this gas, dilute gas using, starting with the Boltzmann equation in this fashion, making suitable approximations.

But I thought I mention this thing here because our earlier assumptions for all stochastic in nature. But now we are dealing with a situation where we do not have a tagged particle that is much more massive than the rest of the particles, all of them on equal footing completely. So that is one of the reasons why the equation is not expected to be as simple as in the case where you have a very heavy particle and is distinct from all the rest from the heat path. Okay. So we will stop here today and any questions and comments we will take it next time.