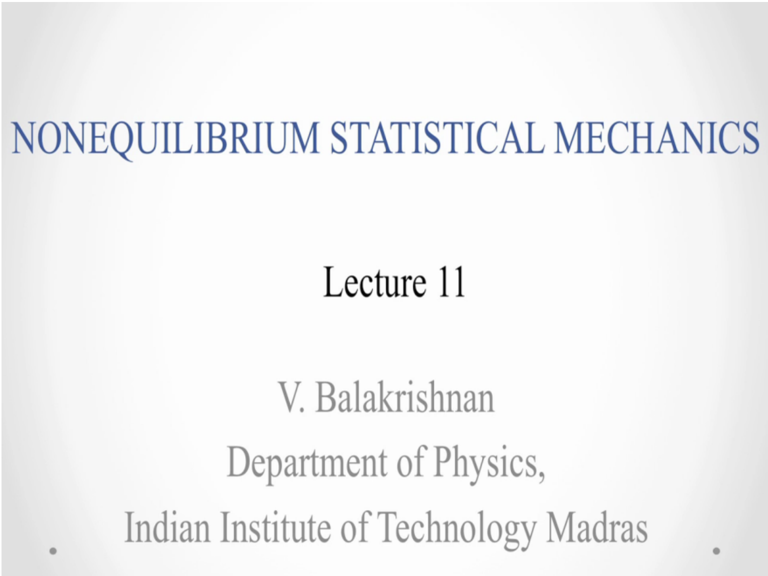
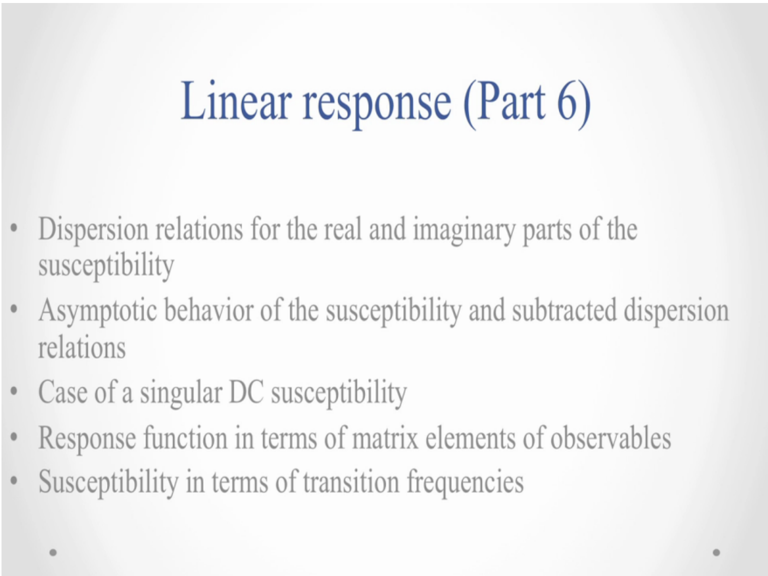
Non Equilibrium Statistical Mechanics
Prof. V Balakrishnan
Department of Physics
Indian Institute of Technology Madras
Lecture 11
Linear response (part 6)

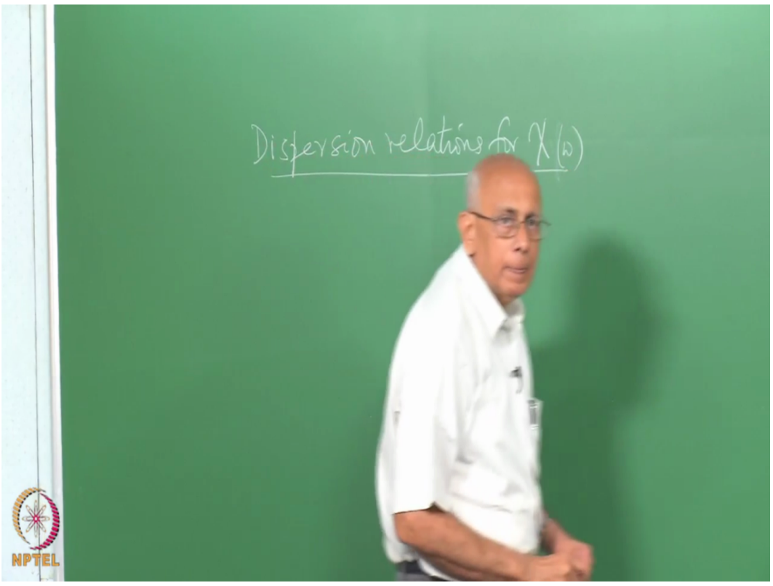
(Refer Slide Time: 0:12)



(Refer Slide Time: 0:14)

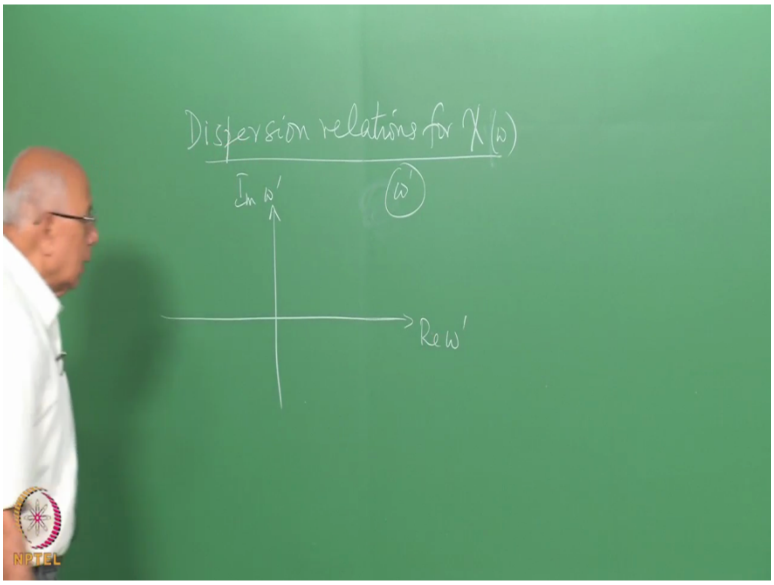


(Refer Slide Time: 0:24)



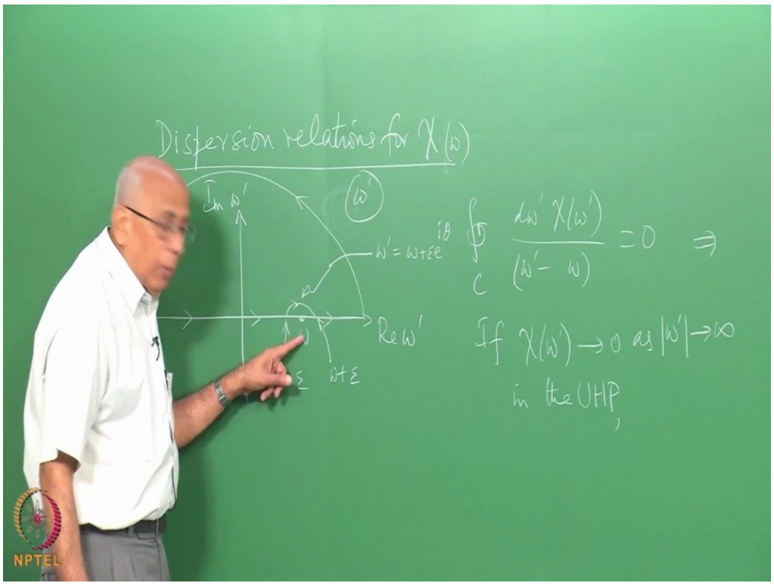
Right, so we were exploring the consequences of the fact that the generalized susceptibility Kai is analytic in the upper half plane in the frequency.

(Refer Slide Time: 0:55)



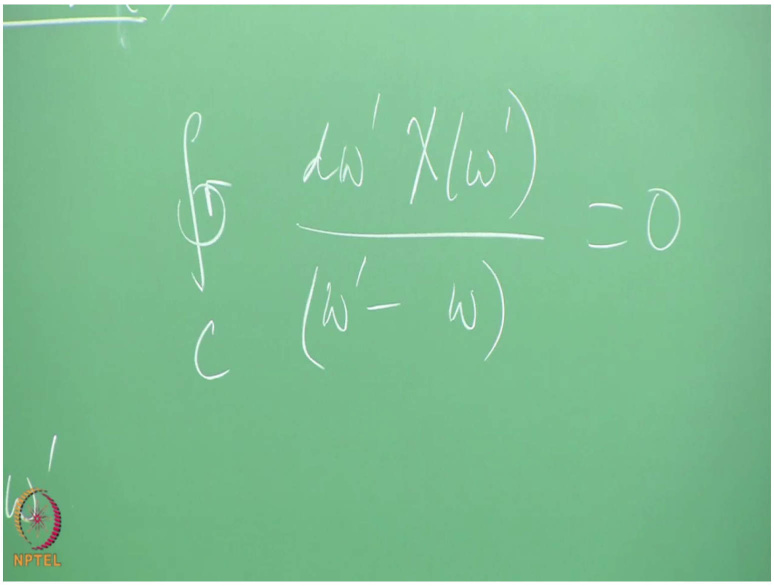
So if I draw the frequency plane this being the frequency with the real omega here and imaginary omega let us call it prime, so this is the omega prime plane out here. Then the point was that it will start with any fixed real frequency omega and we discovered that the integral over the close contour C of d omega prime Kai of omega prime over omega prime minus omega over this contour.

(Refer Slide Time: 4:17)



So all the way from minus infinity coming in and then a little indentation, a semicircular indentation in the upper half plane through an radius Epsilon say and then back on the real axis and all the way down and closed in this fashion, this is equal to 0 and with the condition we needed for this was that this integral, the integral have to vanish as omega prime went to infinity anywhere in the upper half plane.

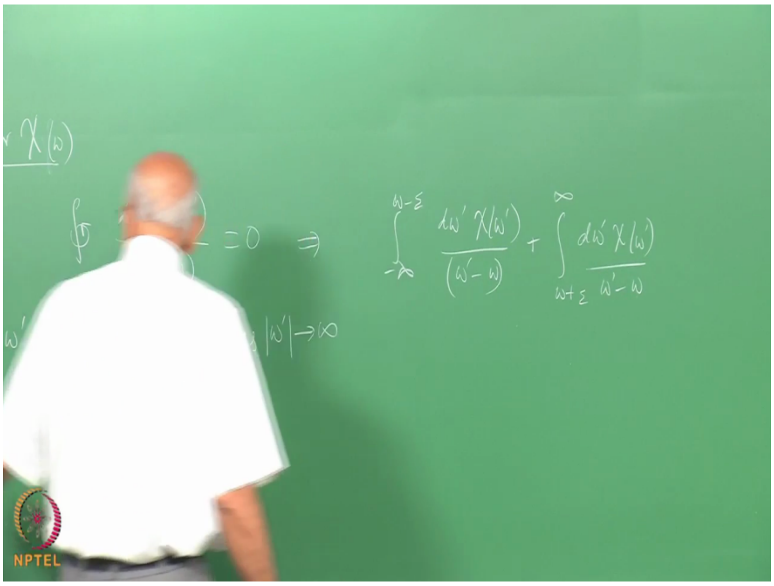
(Refer Slide Time: 1:58)



So if Kai of omega prime goes to 0, if Kai of omega prime goes to 0 as mod omega prime tends to infinity in the upper half plane then this contour C could be blown out all the way to infinity this contribution would then vanish because this is going to give you a capital R into the i theta, this is going to give you a capital R they 2 cancel each other and then if this goes to 0 the answer goes to 0, okay.

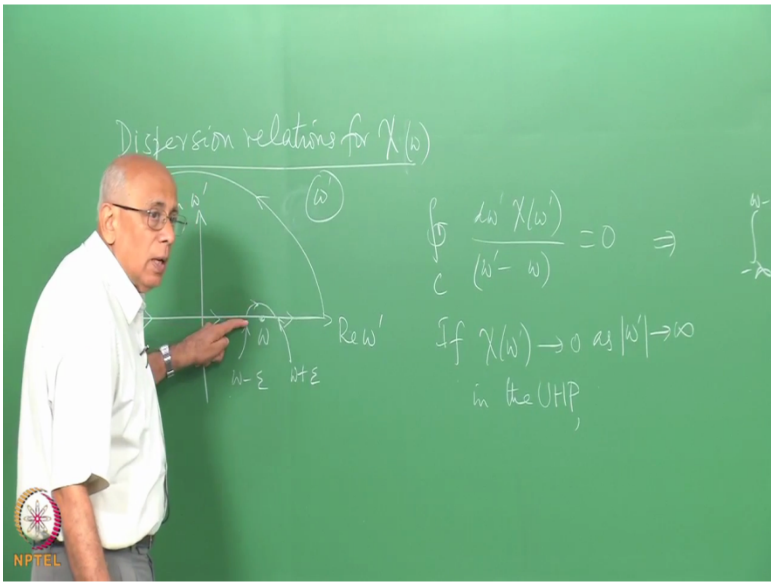
So a sufficient condition for this integral to converge and for this contribution goes to 0 is that this be true that Kai vanish in the upper half plane. As you can see it suffices if Kai vanishes along the real axis because on the imaginary axis you actually have extra convergence factors. So if this is true then this 0 implies that the integral from minus infinity up to this point which is omega minus Epsilon and this is omega plus Epsilon up to infinity plus this semicircle is 0.

(Refer Slide Time: 3:54)



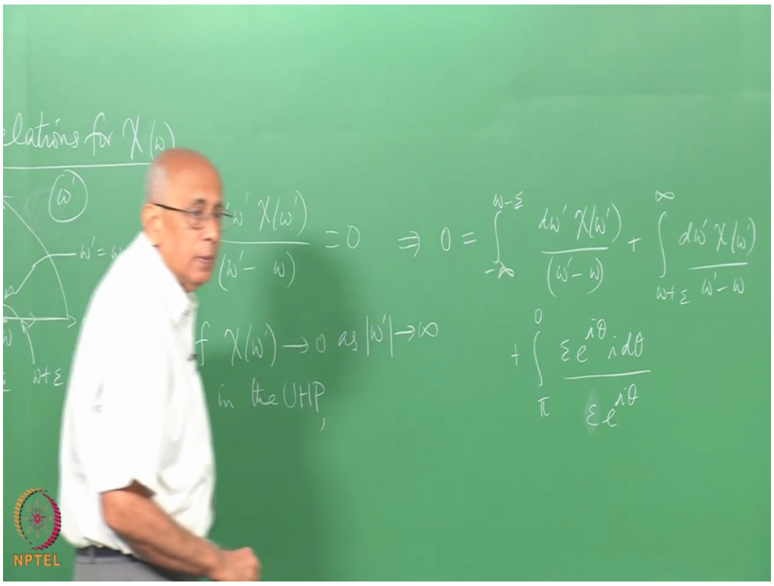
And since the arc integral from this in that contribution from the semicircle is 0 anyway you end up with this statement that integral from minus infinity to omega minus Epsilon d omega prime Kai of omega prime over omega prime minus omega plus an integral from omega plus Epsilon to infinity the same thing.

(Refer Slide Time: 3:57)



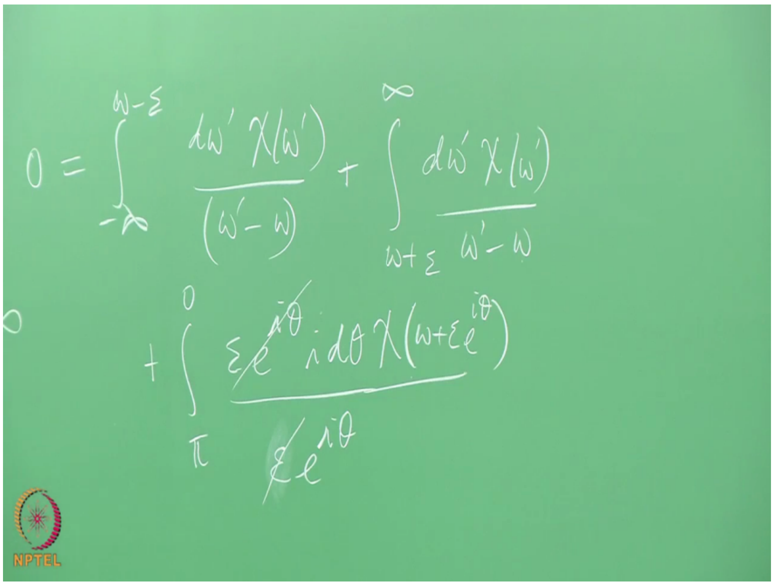
These 2 plus this contribution this little contribution is as follows plus an integral from on this contour on this little contour here omega prime equal to omega plus Epsilon e to the i theta. So it is circle about the point omega, so omega prime is omega itself plus this little complex number Epsilon e to the i theta and the integration variable here is theta running from pi to 0 it goes the other way.

(Refer Slide Time: 4:40)



So this is equal to Epsilon e to the i theta i d theta that is what d omega prime is, I just differentiate this quantity and then the integral runs from pi up to 0 divided by omega prime minus omega that is equal to Epsilon into the i theta and that is it. So 0 is equal to this whole thing, okay.

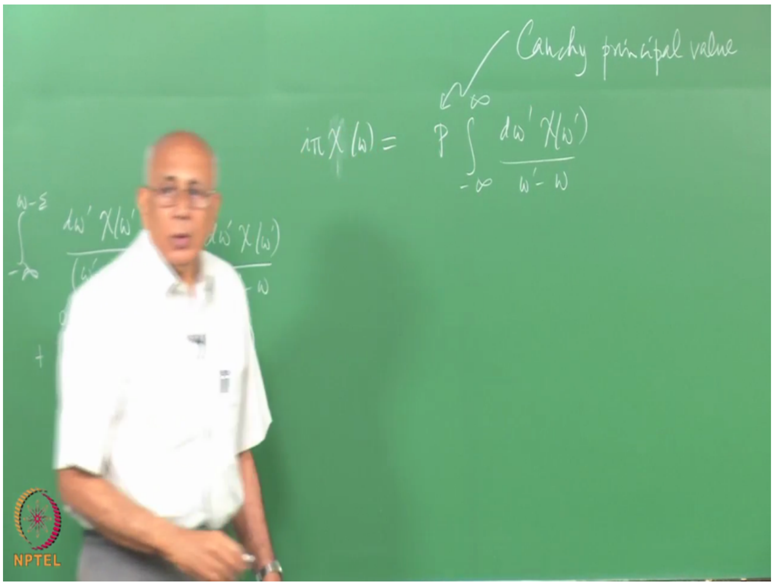
(Refer Slide Time: 5:24)



And this cancels, this term cancels here and the integral is minus i pi times oh! Sorry there is Kai of omega plus Epsilon e to the i theta there is of course sitting inside there.

So in the limit in which Epsilon goes to 0 this becomes the Epsilon cancels the theta cancels and you are left with i pi times Kai of omega, right? Minus i pi because this is from pi towards.

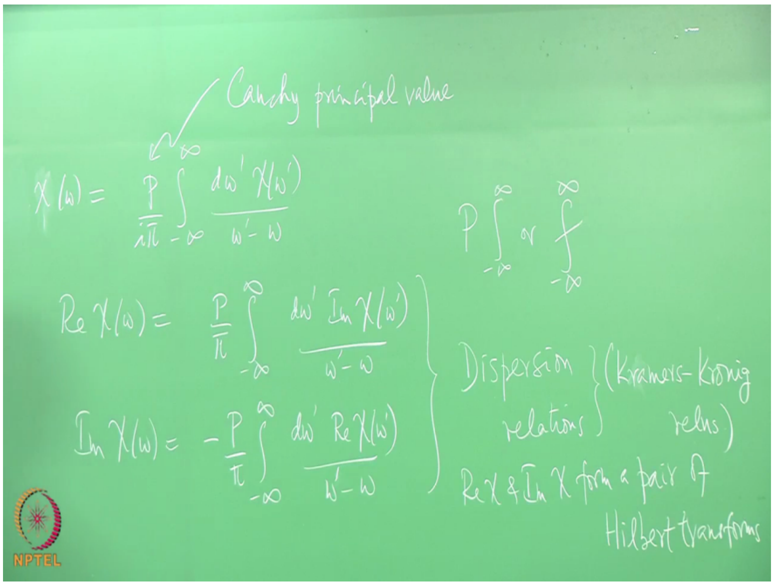
(Refer Slide Time: 6:42)



So this finally says that i pi times Kai of omega is equal to an integral from minus infinity to infinity d omega prime Kai of omega prime over omega prime minus omega living out a small portion which is symmetric about the point omega and this is called the principal value integral this is called the Cauchy principal value. So this is equal to, let me just write it as P and this stands for Cauchy principle value.

So this was an invention of Cauchy's, he discovered that in many cases when you have an integral with a singularity on the real axis on the axis of integration on the contour of integration in general the real axis then if you leave out a small symmetrical portion about this a little segment which is symmetrically situated about the singularity in this case at omega and take the limit as Epsilon goes to 0 then that can be finite and it is called the Cauchy principal value.

(Refer Slide Time: 7:27)



In this case it's finite and it's equal to this susceptibility at this point apart from this factor here. So if I bring the i pi to the right-hand side over i pi. So look at what has happened? We did an excursion we met an excursion from real values of omega to complex values but we are back to the real axis because this contribution went away we are back to the real axis and what we have done is.

We have succeeded in expressing this susceptibility at any real frequency as an integral over all other frequencies except from (()) (7:50) interval about that point that frequency, okay. This sort of thing is called a dispersion relation we will write a simpler form of this in a minute but notice that you are back on the real axis, so there are no complex frequencies here it is completely physical, right?

All we have to do, now is to take real and imaginary parts of this, so it follows that the real part of Kai of omega is equal to it comes from the imaginary part this is real plus i times imaginary and the i cancels and you get P over pi integral minus infinity to infinity by the way I have written this as P integral minus infinity to infinity. Sometimes it is written like this minus infinity to infinity with a slash over the integral that notation is also used just to show you that leave out a symmetrical portion about infinite decimal symmetrical segment about the singularity and take the limit, okay.

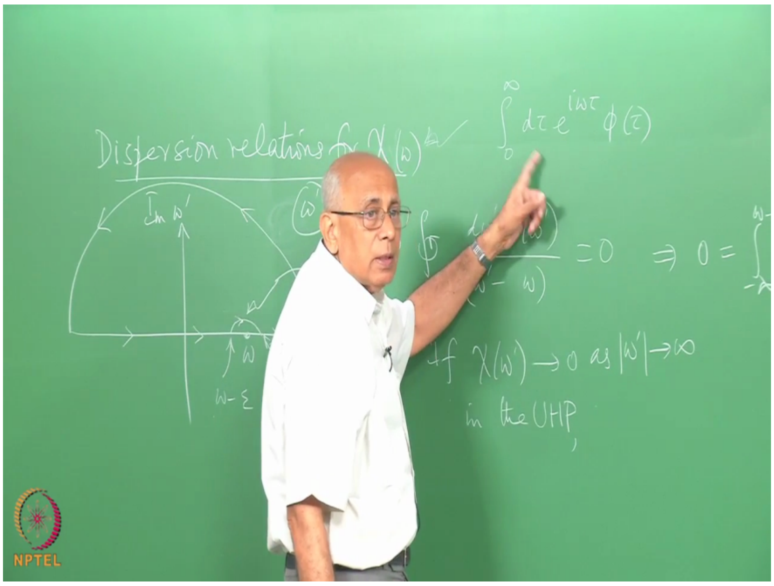
That limit is not always guaranteed to exist but in this case it does, okay. So this is equal to d omega prime imaginary part Kai of omega prime over omega prime minus omega, the singularity of the integral integrant at the point omega prime equal to omega is avoided by the principle value supposed to be left out omitted and similarly imaginary part of Kai of omega equal to, that comes from the real part but there is a minus i here and you take this up, so it is equal to minus P over pi integral minus infinity to infinity d omega prime, real part of, so this is the reason why asserted in the beginning that this susceptibility cannot be purely real or purely imaginary.

Because if it were so then being an analytic function it would just vanish identically, you cannot have this to be identically 0 or this to be identically 0 because then the whole thing is 0, okay. These relations are called dispersion relations, they are also called occasionally for historical reasons these are the people who introduced these relations first into physics Kramers Kronig relations.

Dispersion because it was introduced by Kramers and Kronig first in the context of refractive index, it is an optical susceptibility if you like refractive index, the complex refractive index is just the optical susceptibility and they introduced within that connection. Now any 2 real function, any 2 functions real valued functions of a real variable such that you have supposed say 2 functions f and g then the fact if you have f of x prime d x prime over x prime minus x principal value equal to g and g is the inverse relation they hold a pair of Hilbert transforms.

So the real and imaginary parts of a causal, linear retard susceptibility from a Hilbert transform pair, okay. So the real part is a Hilbert transform of the imaginary part and imaginary part is a Hilbert transform of a real part, notice there is a minus sign. So in principle if I substitute for imaginary Kai of omega from here in this, you will have one more integration to do, in principal you should get an identity, in other words that intermediate integral should turn out to be a Delta function, okay which it will, I am not going to show it over here but it will. So these 2 guys real Kai and imaginary Kai form of a pair of Hilbert transforms.

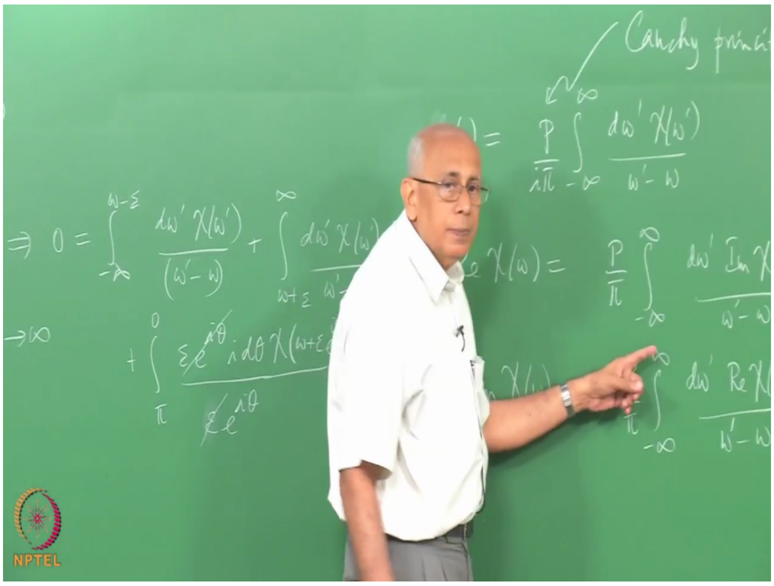
(Refer Slide Time: 13:08)



Now what is the reason for this whole thing happening? Well it happened because this is an analytic function in the upper half plane and where did the analyticity come from? What is the origin of this business? Remember this got represented as an integral 0 to infinity d tau e to the i omega tau pi of tau. So it got represented in terms of this one-sided Fourier transform and then the argument was if this is true for real omega then for omega with positive imaginary part? It is certainly true an analytic.

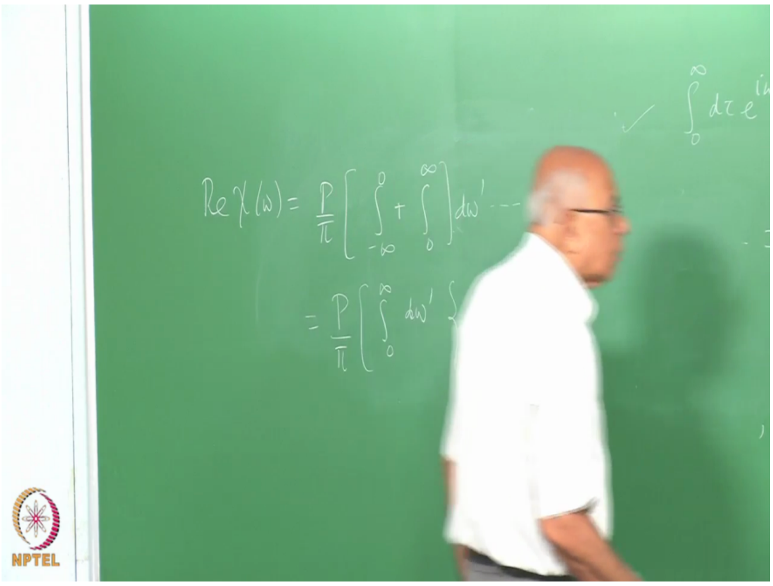
So it arose from this and where did this come from, why is this 0 and not minus infinity or anything else? Causality, so in all cases dispersion relations are a consequence of causality, finally that is what is doing it. So the physical reason why the generalize susceptibility satisfies dispersion relation is because it is a causal response, okay. And this has profound implications in other parts of physics especially in particle physics, Quantum field theory and so on anywhere where dispersion relations appear and they appear in the large number of places but finally it is traced down to causality, okay.

(Refer Slide Time: 14:14)



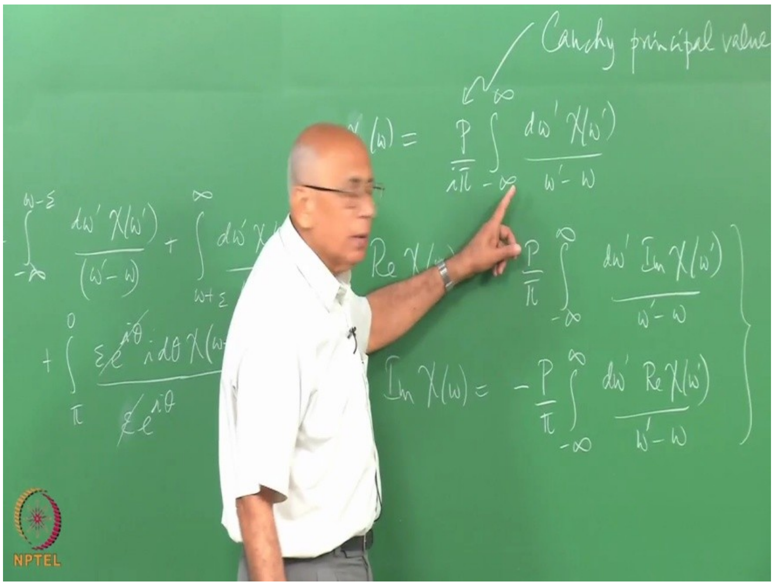
Now you might say look this is an integral over negative frequencies also whereas physical frequencies are positive but there the fact that this is an anti-Symmetric function comes to our aid. So we can convert this integral into something that runs over from 0 to infinity. So let us do that.

(Refer Slide Time: 14:40)



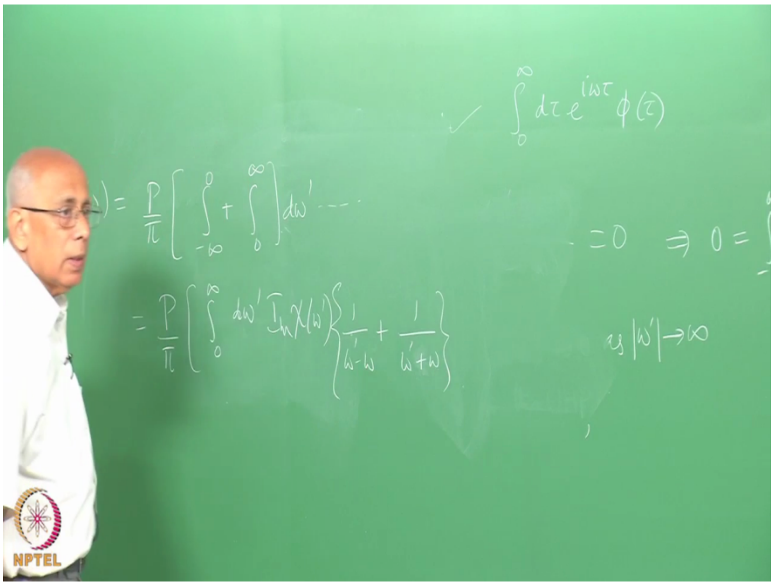
The real part therefore equal to P over pi there is an integral from 0 to infinity and then the portion from minus infinity to 0 I am going to write in this form, is minus infinity to 0 plus integral 0 to infinity times d omega time blah blah blah, in this term so I am going to write this as P over pi I am going to change variables to minus omega prime when I pull out a minus sign due to the Jacobian, this becomes infinity to 0 and those 2 minus signs cancel.

(Refer Slide Time: 15:42)



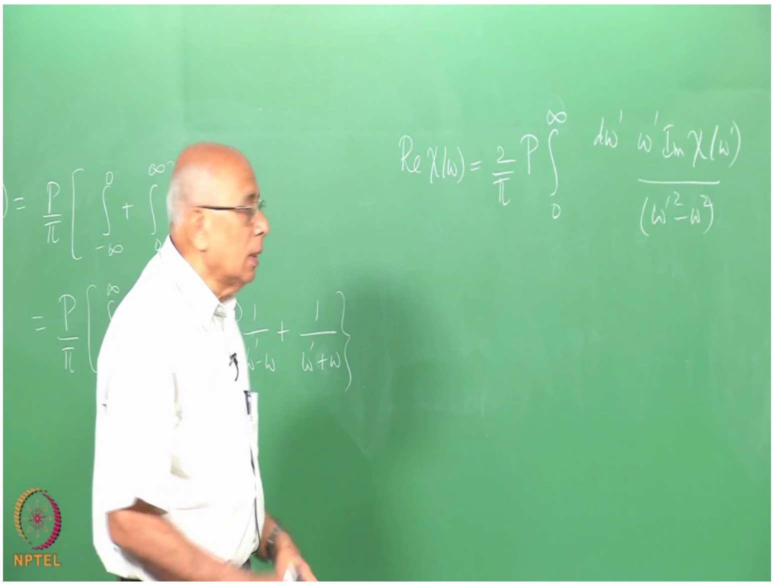
So both integrals look like 0 to infinity d omega prime and then in the first integral I have Kai of omega prime over omega prime minus omega that is integral from 0 to infinity in this formula and then minus infinity to 0 remember I change variables to minus omega prime. So let us write that as, so this is imaginary plus imaginary Kai of minus omega prime divided by minus omega prime minus omega because I change variables.

(Refer Slide Time: 16:40)



But this fellow is an odd function, so it is equal to minus imaginary Kai of omega prime that minus sign cancels with this and you are left with the following, you are left with imaginary I of omega prime times one over omega prime minus omega plus 1 over omega prime plus omega but this omega cancels and just gives you 2 omega prime.

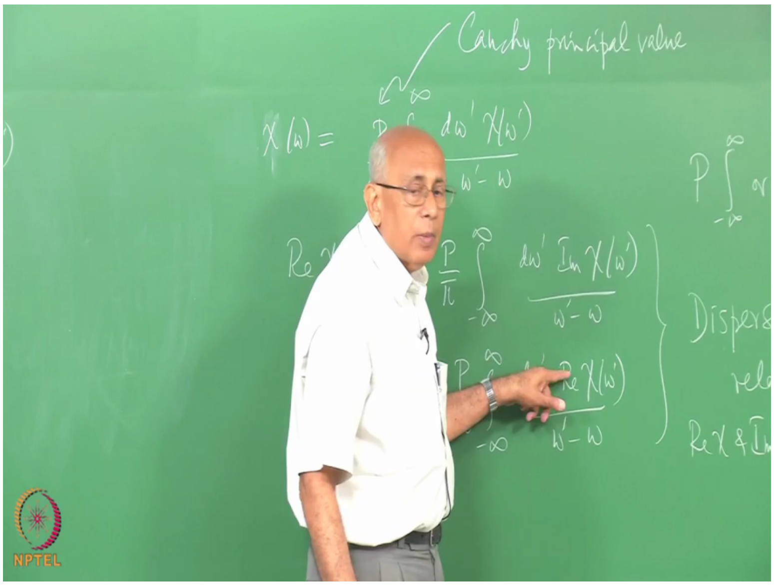
(Refer Slide Time: 17:02)



So therefore I have a nice representation which says that in general we have useful representation which says real Kai of omega equal to 2 over pi principle value 0 to infinity d omega prime omega prime imaginary part of Kai of omega prime because when I add these 2 I get 2 omega prime divided by omega prime square minus omega square. Now everything is physical it says this denominator has a simple pole at omega prime equal to omega, the one at minus omega is outside the region of integration and you have to take the principal value in the pole at omega prime equal to omega and integrate this quantity, okay.

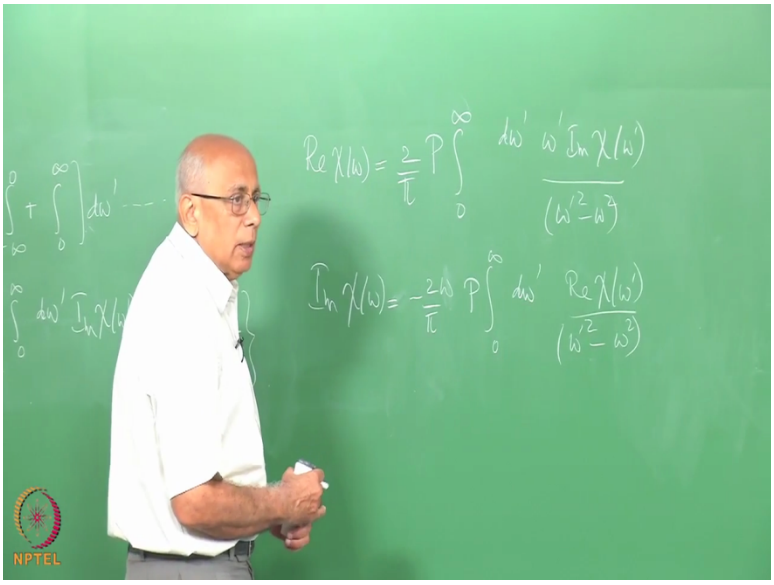
Its guarantee to converge, at infinity this is going to produce capital R, that is going to produce capital R, this is going to produce R square they cancel each other but this fellow goes to 0 and the integral exists. So this is a physical form of the dispersion relation. Similarly you do the same thing for imaginary for Kai of omega equal to still be 2 over pi and there was a minus sign which is going to persist, so minus 2 over pi times principal value from 0 to infinity d omega prime.

(Refer Slide Time: 18:31)



Now you are going to subtract one over omega prime minus omega minus one over the other because this fellow is symmetric, so it does not produce an extra minus sign.

(Refer Slide Time: 18:56)



When you subtract that you get 2 omegas, so this becomes an omega which is outside the integral and then the real part Kai of omega prime, okay. That is a very useful form of the dispersion relation, so this is the one that you would use in practice, okay. In practice what happens is the physical use this is put to is, suppose you know the susceptibility or you have measured the susceptibility both the real and imaginary parts because they have physical meanings.

You measured it in some frequency range and you could approximate it by 0 or something like that outside this range then to find this susceptibility at any point outside the range you could use this formula to first order it will be correct even if you cut it off that sump up a limit here, okay.

“Professor -Student conversation starts”

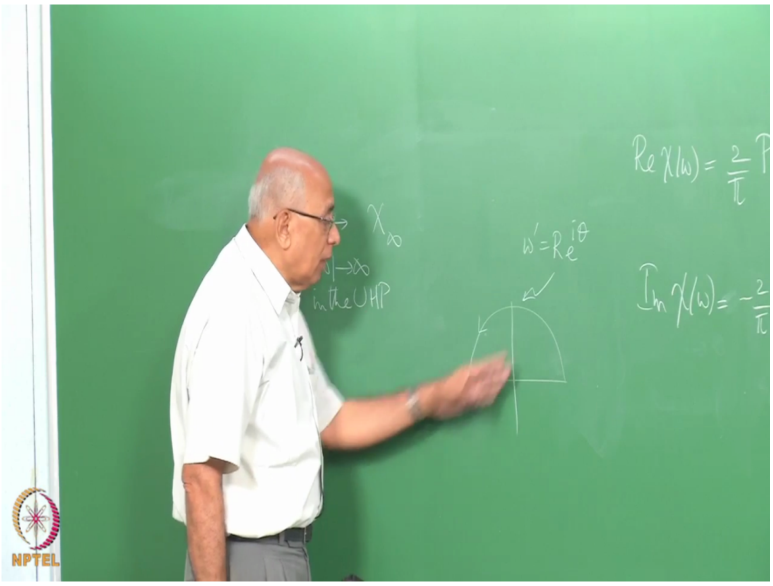
Student: How the second integer looks better than.

Professor: Yes that is the consequence of the fact that one the real and imaginary parts that is the reason, yes sure that is exactly true but it is not really better because these functions behave differently asymptotically, okay.

“Professor-Student conversation ends”

Now you could ask the other questions we slurred over some points we said Kai of omega must go to 0 at infinity, suppose it does not, you still have to deal with this situation because that can happen in many cases.

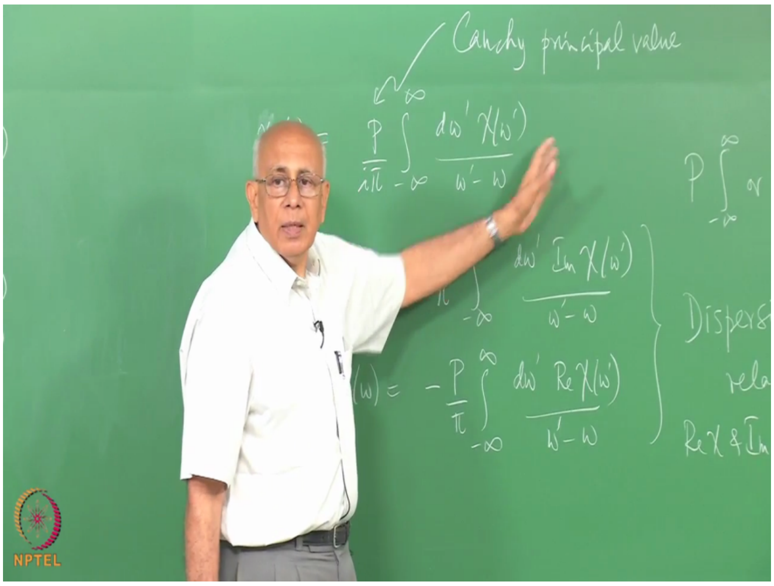
(Refer Slide Time: 20:25)



Suppose it does not go to 0 but it goes let us say on the upper half plane suppose not Kai of omega goes to some number, some Kai infinity as mod omega goes to infinity in the upper half plane, suppose this were true then the contribution from the semicircle is not 0 that is all but you can still work out what the contribution is because now you notice that this infinite semicircle is here.

On the semicircle Kai goes to Kai omega Kai sub infinity and then you have on this circle omega prime equal to capital R e to the i theta, so d omega prime is going to produce R times i d theta and so on and in the denominator you have the same thing omega prime minus omega is going to produce an R. So you will end up with a Kai infinity as a constant in integral so fine.

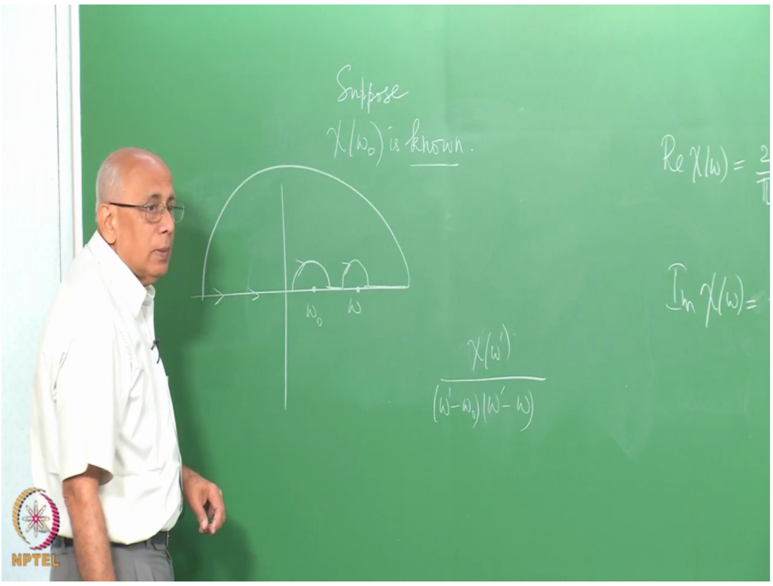
(Refer Slide Time: 21:32)



You will still get dispersion relations but you get an extra contribution here plus something time pi times Kai infinity or something like that, so you can still write it down still compute but now suppose you say well I do not know Kai infinity I just know that it does not go to 0 but I do not know or suppose it is a function of the angle. Suppose the limit that it goes to does not go to 0 but suppose it is a function of which way you go to infinity you know handle on that.

I have assumed here that all through uniformly it goes to the same constant but if it depends on the angles and you cannot do the angular interpretation, right?

(Refer Slide Time: 22:18)

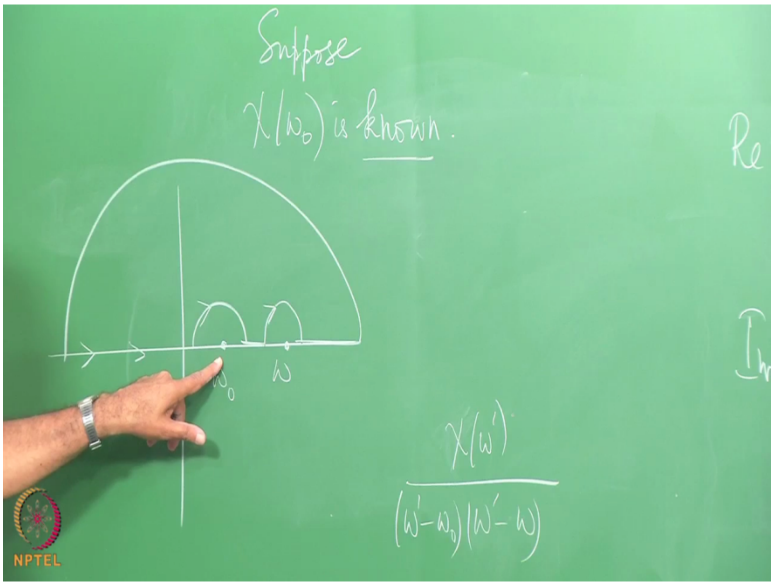


Then this is the trick you do in that case, what you do is to modify the function whose analytic behaviour you are looking at. So your omega prime plane here is omega choose some value of the frequency at which you know the value of the susceptibility. So let us suppose there is a point omega not and let us suppose that Kai of omega not is known or measured.

So suppose, so you know the value at this point and now I am trying to derive dispersion relations. So the function I am going to look at is not Kai of omega prime over omega minus omega prime minus omega this is not good enough because it does not converge fast enough but if I multiplied by this omega prime minus omega not then I am in good shape because there is an extra omega prime capital R sitting from here and the d omega prime will cancel against this.

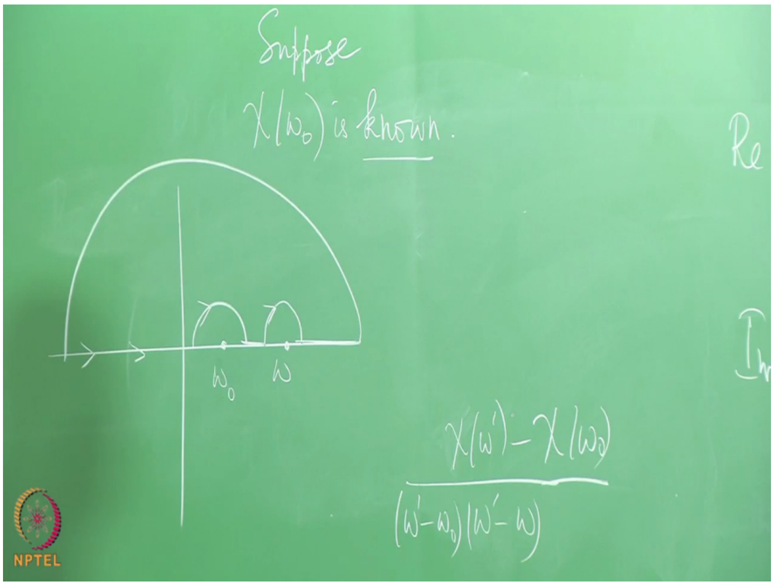
There is 1 over capital R here and there is this fellow who goes to a constant some constant at infinity, so there is 1 over R and this contribution will still vanish but now you will say that is not good because there is a singularity at this point now. So what you will have to do is to consider the contour this way and then the indentation here and then another indentation there and then all the way to infinity, okay.

(Refer Slide Time: 24:08)



So that is one way to do this, another way to do this, so what would that amount doing? What would this contribution be finally? It would be i pi or minus i pi whatever it is, Kai at that value sitting and then Omega not minus 1 would sit there.

(Refer Slide Time: 24:39)



You can save yourself some trouble by considering not this function but this function this is a constant, this function if this is got a simple pole at omega not than this function is well-behaved at omega not, okay.

So I write a dispersion relation for this function, keeping track of the fact that this is a complex number in general.

“Professor -Student conversation starts”

Student: Basically you are making it a reversible direction.

Professor: Yes I convert this singularity into a removable singularity by subtracting this one here.

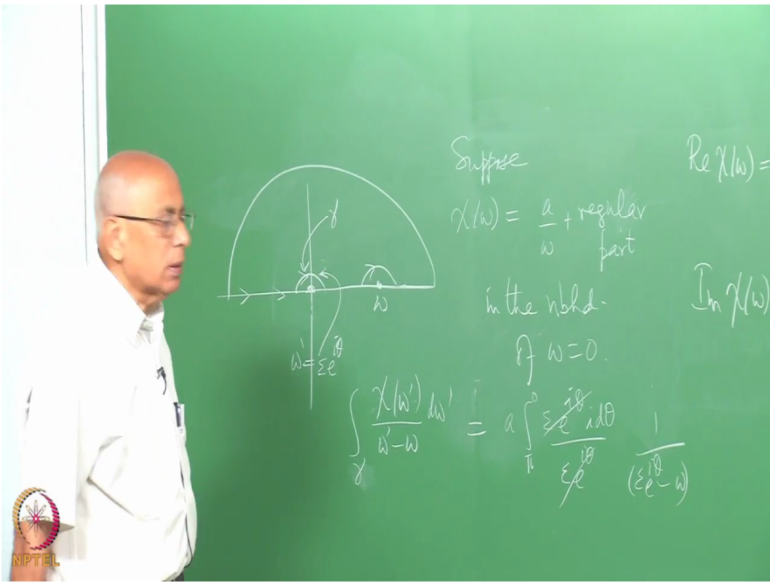
“Professor-Student conversation ends”

So since I have subtracted this, this is called subtracted dispersion relation and this is called the point of subtraction. So there are several ways of fixing this problem of Kai not going to 0 at infinity subtracted once, this assumes that Kai is such that it goes to , does not go to 0 at infinity but goes to a constant. Suppose it goes like omega prime itself what would you do? Then I have to subtract at 2 points and so on.

So each time you add a denominator it improves the convergence, okay. And then it is a doubly subtracted dispersion relation and so on, as long as it does not have an essential singularity at that point at infinity you are in good shape, as of now there is no, so if it blows up like some polynomial like some power of omega prime as omega prime goes to infinity we are okay.

Blows up exponentially you cannot write dispersion relations, okay. So these are techniques just techniques for getting rid of singularities but this tells you what the basic idea is or there is one more very important case where you have to deal with this situation which is as I mentioned it happens so happens in many cases that the dc susceptibility is actually divergent that it blows up namely you apply steady force to the system forever and the response becomes exploit it divergences, okay.

(Refer Slide Time: 27:00)



What you do then? That implies that Kai of omega has a pole or singularity at the origin and simplest case it has a simple pole, right? So suppose that happens, suppose so this part is taken care of.

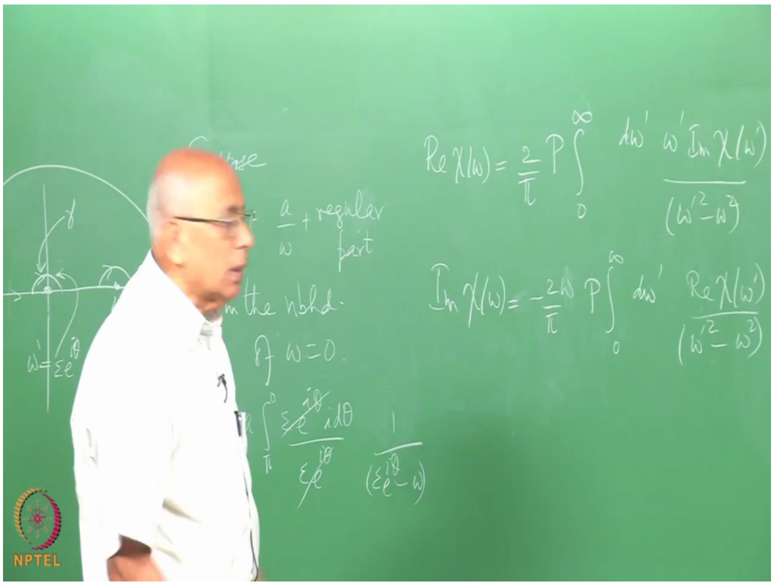
Now suppose Kai of omega has a simple pole at the origin, so it is of the form some residue a over omega plus regular part in the neighbourhood of omega equal to 0. So suppose it has a simple pole at this point at the origin, here is my omega then the thing do is to go back and consider this contour out here and consider the original function itself. So you have Kai of omega prime over omega prime minus omega and you look at it over this contour.

So indent this in the upper half plane, so as to avoid the pole and stay in the region of analytic behaviour of this function and include the contribution from here in the contour integral, this indentation is going to give you minus i pi Kai of omega, what is this going to give you? What is the contribution going to be? We are going to have to integrate remember that on this contour omega prime is just equal to Epsilon e to the i theta plus 0 because it is at the origin.

So this thing here integral b omega prime over this little semicircle, so let me call this little semicircle little gamma little gamma this fellow here is going to become equal to this is going to go like a over omega prime plus the rest of it, right? So the leading term is going to be a and then an integral from pi to 0 Epsilon e to the i theta, i d theta that is d omega prime divided by omega but omega is Epsilon e to the i theta. So Epsilon e to the i theta, that is the Kai the behaviour of Kai with an a here and then there is this factor which is harmless.

So that factor is 1 over Epsilon e to the i theta minus omega, so the pole contributed 1 over omega prime which cancels gives you this Epsilon to cancel this, so this fellow goes away and this is i pi or minus i pi times a and the rest of it will follow with a minus omega. So the whole integral will have a contribution which is essentially a over omega which it should because they are writing a representation for Kai and if it is got a singularity residue a at omega equal to 0 there better be an explicit term a over omega that is what is happening here.

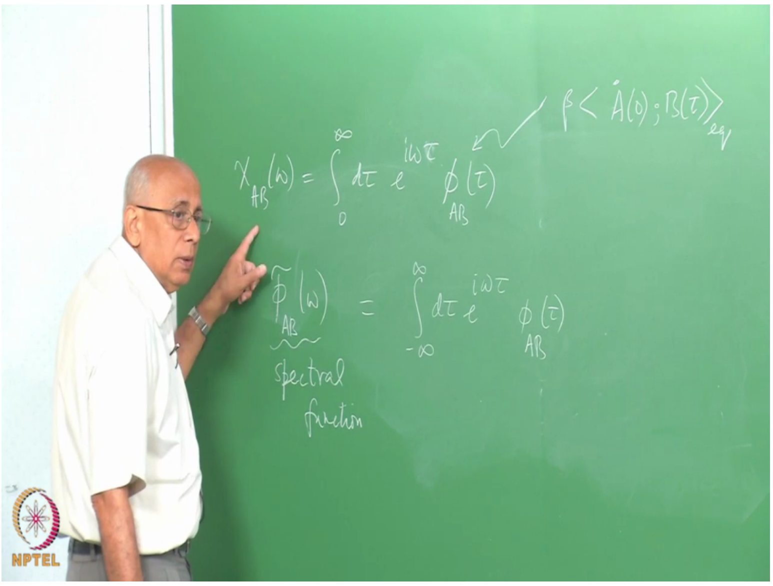
(Refer Slide Time: 30:55)



So tacitly the point of subtraction has been the origin in that sense. So in all these cases we know how to deal with this situation but the physics of it is that causality leads to dispersion relations Kramers Kronig relation for that generalized susceptibility, okay. And that is a general statement and you can write it in terms of physical frequencies using this which is then usable for numerical evaluations.

Okay, now let us come to terms with what is this susceptibility actually is? We have to look at the response function a little more carefully, so let us do that, we want to attach some physics to the whole thing, so given for instance quantum mechanical system can I say what the structure of this response function is? What does it really look like etc? In particular I want to be able to write things in terms of this spectral function.

(Refer Slide Time: 31:41)



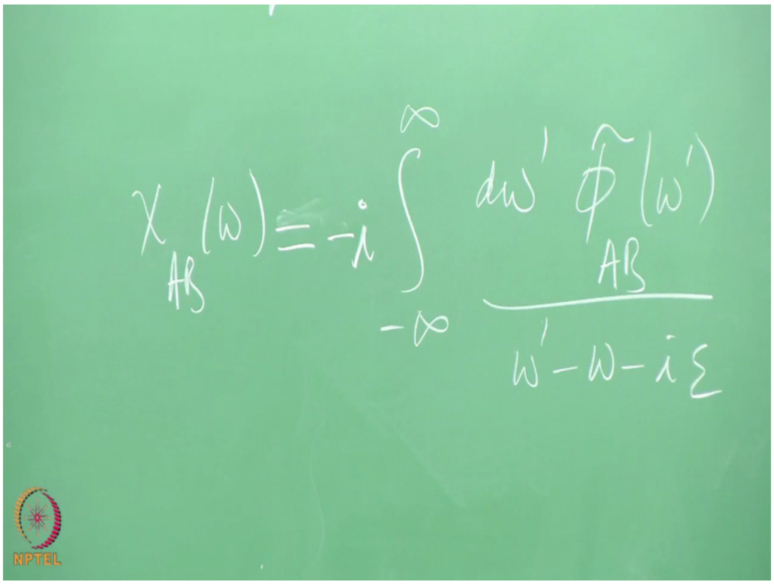
If you recall I pointed out that just to write these formulas out again Kai AB of omega is integral 0 to infinity d tau e to the i omega tau phi AB at tau this guy here be showed was the canonical ensemble f is equal to beta times the equilibrium the canonical correlation between A dot of 0 and B of tau we already defined this quantity the classical case is subsumed in this as a special case and this was analytic in the upper half plane etc etc.

We also had defined a Fourier transform of this quantity of omega and I call this the spectral function for reasons which will become clear now, this was the Fourier transform of phi the response function, so this was equal to integral d tau minus infinity to infinity e to the i omega Tau phi AB of tau, this will talk not very much about what is this quantity for tau less than 0?

Because the susceptibility just involves this right here, I will come back to this we will deal with the question of how to define it for negative values of tau? But notice that this spectral function this Kai here was also related to the Green function, we found out what the Fourier transform of the Green function was and the Fourier transform of the Green function which is this multiplied by theta function of tau.

So I called it GAB of tau that quantity it is Fourier transform was the susceptibility, exactly the susceptibility. We also found the relation between this fellow and this fellow by writing a theta Fourier representation for the theta function and if you recall that was a pi AB of omega was equal to an integral for minus infinity to infinity d omega prime phi AB tilde of omega prime over omega prime minus omega minus i Epsilon in the limit in which Epsilon goes to 0 from above and there was some i factor somewhere here.

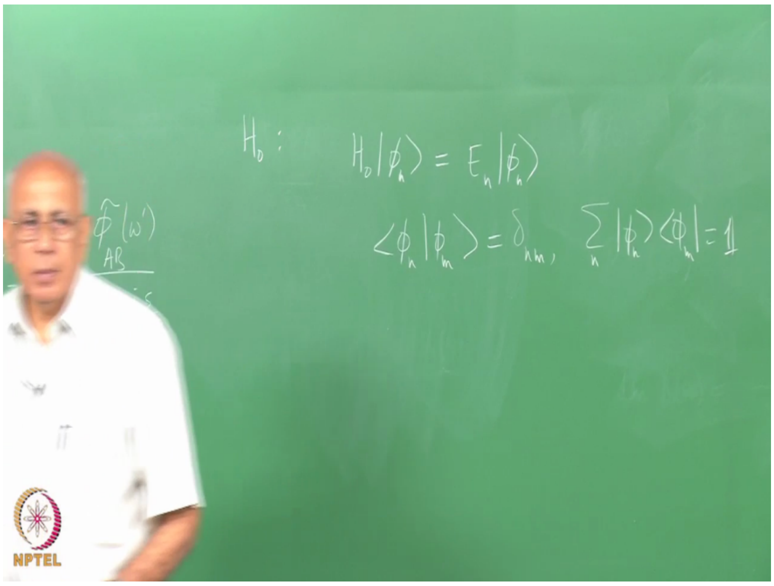
(Refer Slide Time: 34:43)



Would you check and let me know there was an i dependent on the Fourier transform Convention which I have fixed once and for all but I am pretty sure there was an i here somewhere, i or minus i or something like that. I just want to keep this straight, okay. So now let us see what the content of this response function is in general?

What would it imply? We will take a specific case and I will do this then the simplest notation possible and then we can add fields to it later. So we will look at the quantum mechanical system to start when and take A and B to be summation operators their physical observables or operators corresponding to physical observables and then I am going to assume that there is a discrete spectrum of the Hamilton in h not in this system, just so that the notation becomes simple.

(Refer Slide Time: 35:23)



So let us suppose that this is Hamilton in h not has a complete set of states labelled by a quantum number or set of quantum numbers let me call n for collectively, so I have H not on normalised Eigen function phi n is En on phi n and n runs say 0, 1, 2, 3 whatever that discrete spectrum just for notational convenience and then this is an orthonormal basis, so let us say phi n phi m equals to delta nm and similarly sum over n phi n phi m equal to the identity operator.

So it is a complete set of states and it satisfies orthonormality, okay. Are you familiar with the terms completeness in orthonormality, okay. So given any state vector of this system you can always write it as an expansion in terms of these phi n's here, this summation here is supposed to some over states labelled by this set of quantum numbers n and not the energy levels because there could be degeneracy in general.

So every time I write a sum like this it is not over the energy levels per se but over the states of the system, okay. Then this response function phi AB of tau, if you recall this quantity was equal to the equilibrium expectation value of A of 0, B of Tau equilibrium that was one of the formulas we had for the response function and in the quantum case this stands for 1 over ih cross A of 0, B of tau stands for this.

So let us calculate this in this basis, we need to compute trace that is what this thing here means, so let us calculate that in the basis phi n, you can compute a trace in any basis you like but let us do it in the basis of Eigen states of the unperturbed Hamiltonian h not.

"Professor -Student conversation starts"

Professor: Pardon me, no it is still a commutator.

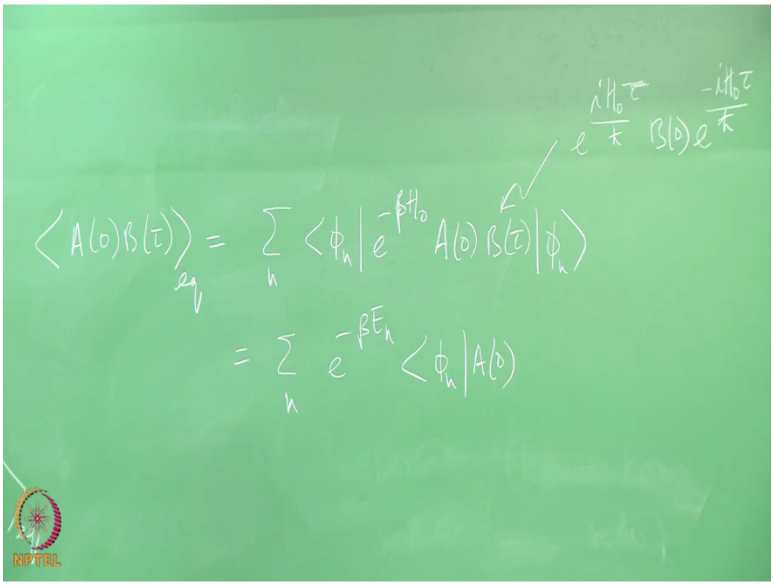
Student: Yes but I mean it is A dot intervention.

Professor: No, no, no the response function is not a dot, after I compute, right? Then it becomes dot and so on, what happens then you get rid of the commutator and you get this kubo transform or whatever it is and then it becomes a dot but before that it is just a commutator.

"Professor-Student conversation ends"

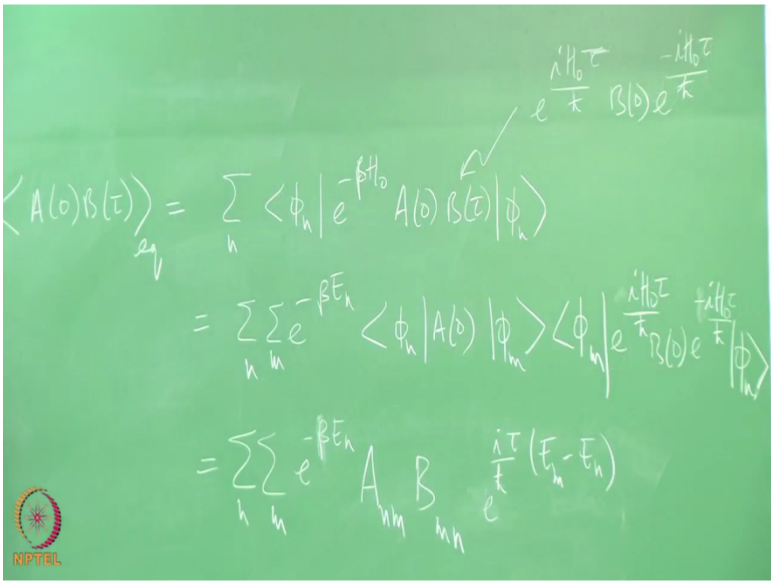
So let us calculate this, let us calculate A of 0, B of tau equilibrium this is equal to that the first in the commutator we will just compute this number this is equal to trace which means a summation over n phi n e to the minus beta H not that is the equilibrium density operator A of 0 B of tau phi n which is equal to a summation over n e to the minus beta is not on phi n is e to the minus beta e n because it is an Eigen state.

(Refer Slide Time: 39:43)



So e to the minus beta E n phi n A of 0 and now for this B of tau, let us write this as e to the power i h not tau over h cross, B of 0 e to the minus i H not tau over h cross that is the meaning of B of tau, so that is the Heisenberg picture operator but that B of 0 is the Schrodinger a picture operator because that is at t equal to 0, that is the way we define the Schrodinger of picture, okay.

(Refer Slide Time: 40:37)

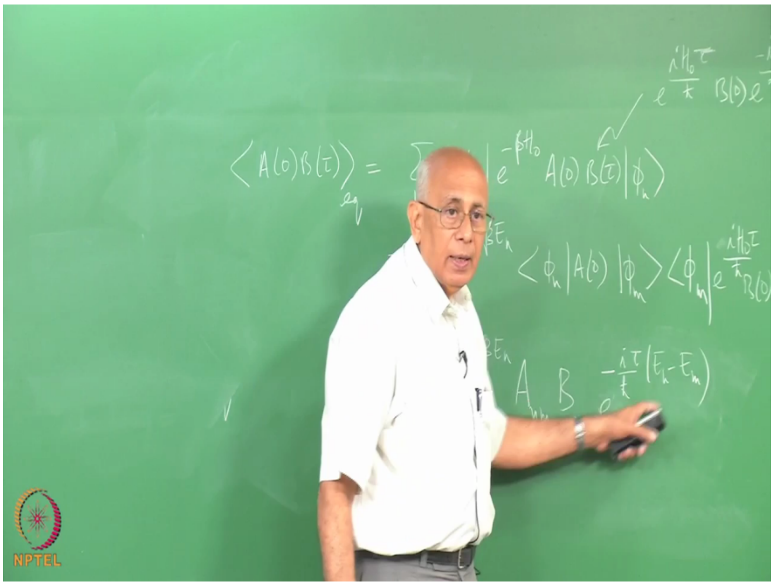


So this fellow here I have to put this in but what I could do is in between I put phi n phi m and sum over m that is the identity operator and then I put this in e to the minus i h not tau over h cross B of 0, e to the of plus i and minus i h not tau over h cross times phi n. This is equal to a summation over n, a summation over m, e to the minus beta En on this side, what would you call this?

This is Schrodinger a picture operator the time independent Schrodinger a picture operator and if I represented in the basis on by the Eigen states of the Hamiltonian unperturbed Hamiltonian that is the n mth matrix element, okay. So this is just Anm there is no time dependence that is just a number, okay. Some complex number in general, does it have to be a real number?

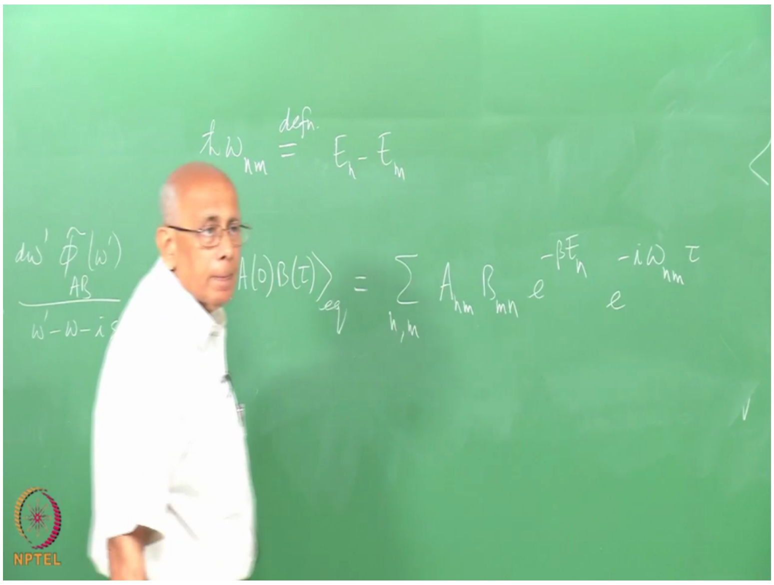
A is the hermitian operator does this have to be a real number? No, not true in general, does the diagonal element of A have to be a real number? Okay, alright. So this thing here Anm and then here I can pull out an e to the i this becomes e sub m and this fellow becomes e sub n when you take it outside and what is left is B? In n and then in e to the power i over h cross tau and then you have an E m minus e n and that is it. So let us give this a name, let us write this as e to the power minus E n minus E m, what does that give you?

(Refer Slide Time: 42:40)



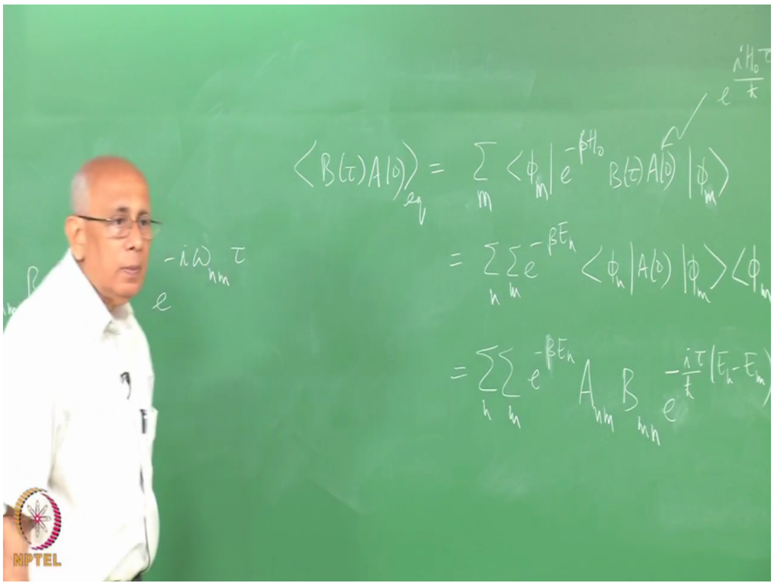
That says, let us now it is natural to do the following this is the energy difference between the n and the states its call that omega nm times h cross.

(Refer Slide Time: 42:51)



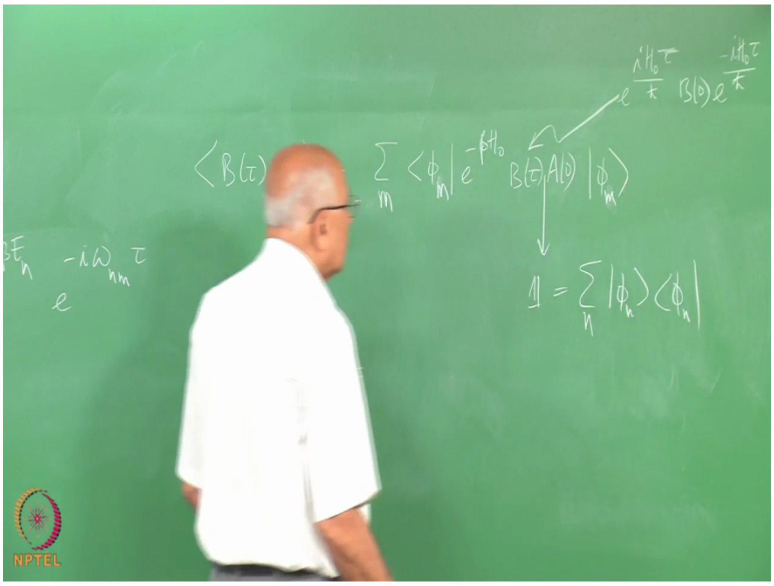
So I define h cross omega nm by definition equal to En minus Em it says A of 0 B of tau in equilibrium can be written in compact form summation n, m Anm Bmn times e to the minus beta En and then e to the minus i omega nm tau. So it can be written in quite a compact form.

(Refer Slide Time: 44:21)

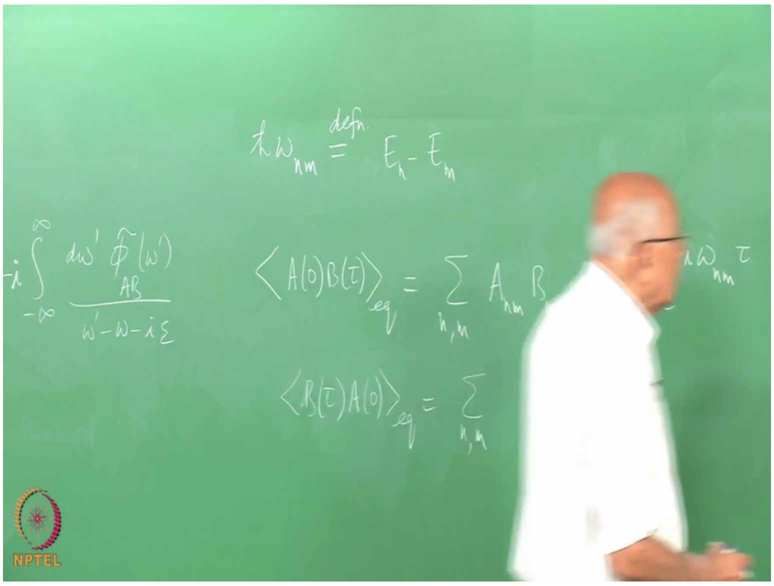


Next up is to wait for the other part of the commutator, right? So let us write the other way point B of tau A of 0 and this is now going to have B of tau A of 0 and since I want to match with that let us take the sum here to be over m and sum here in between to be over n and this fellow becomes B out here we have to work this out properly.

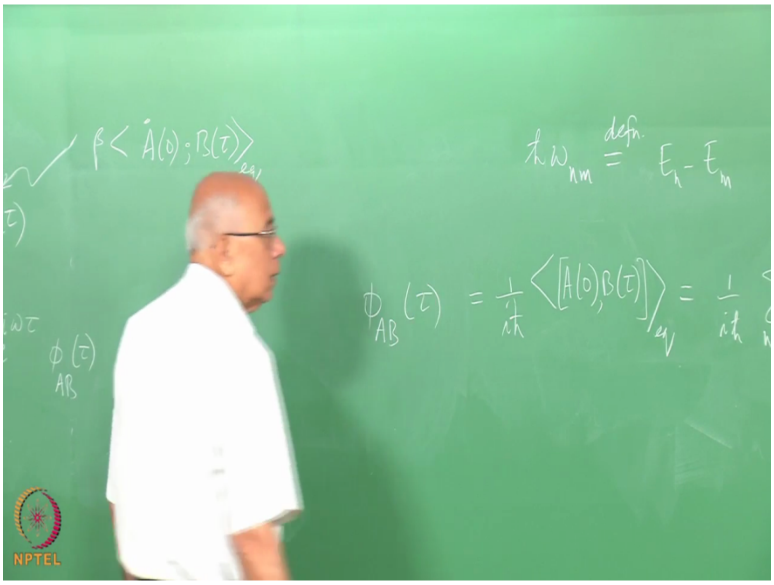
(Refer Slide Time: 44:44)



(Refer Slide Time: 45:16)



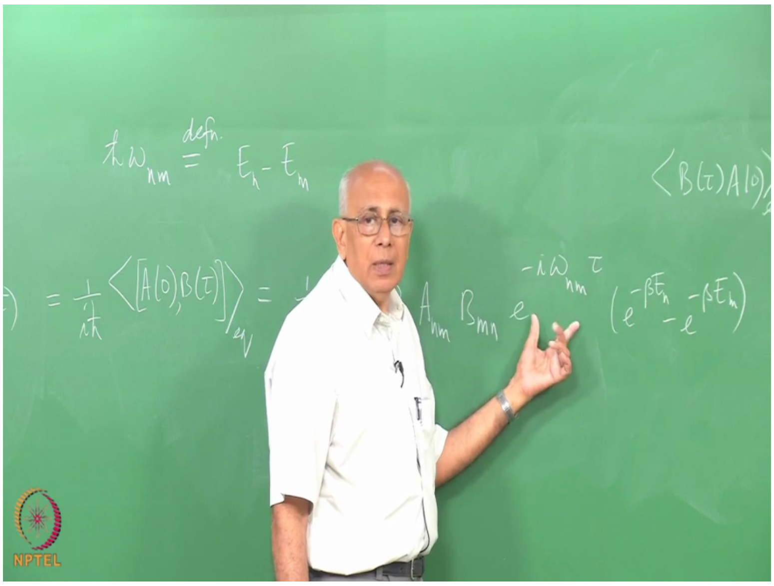
(Refer Slide Time: 46:48)



So this is just A of 0, this fellow here is that now introduce a complete set of states your by putting i into the summation over n phi n phi n therefore you can write-down directly, what this quantity is? B of tau A of 0 equilibrium equal to summation over n, m you are still going to get Bmn and then you going to get Anm which is the same as this, so this is still Anm Bmn but now this is going to hit this and give you e to the minus beta Em.

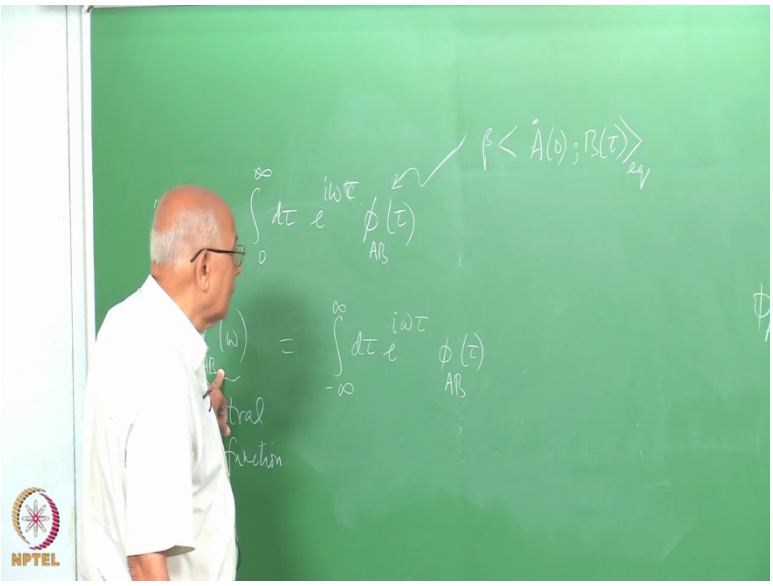
So this is going to be e to the minus beta En and the rest is going to be exactly the same as before. So therefore what does the commutator do? Let us divide by 1 over ih cross because that is what the response function is, all these portions are common but this becomes e to the minus beta En minus e to the minus beta Em and this goes away. So this is equal to 1 over ih cross summation over n summation over m Anm Bmn times this product that is it, Right? And this is equal to phi AB of tau.

(Refer Slide Time: 47:08)

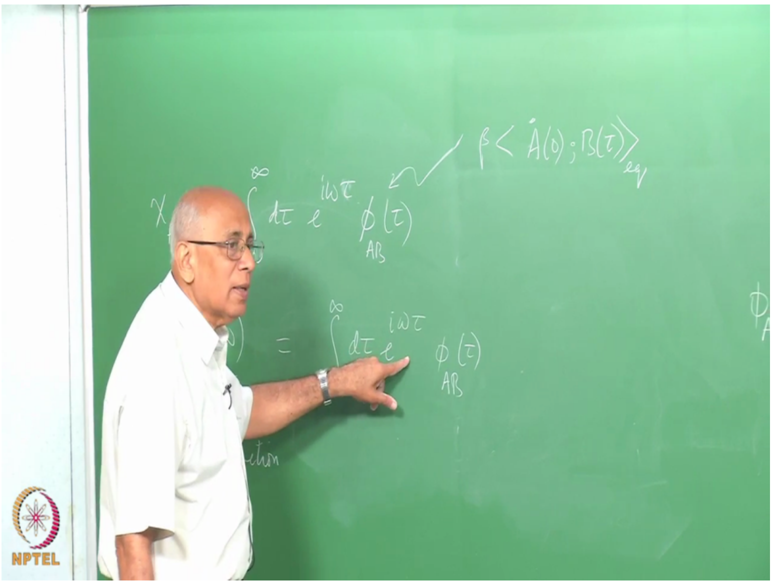


So we have now explicit equations in terms of physical quantities the matrix elements of these operators, you have representation for these operators then all that rigmarole about going to the interaction picture and so on and so on, finally ended up with just these quantities the Tau dependence is sitting here and the temperature dependence is sitting here and this is a set of exponentials that is it.

(Refer Slide Time: 47:28)

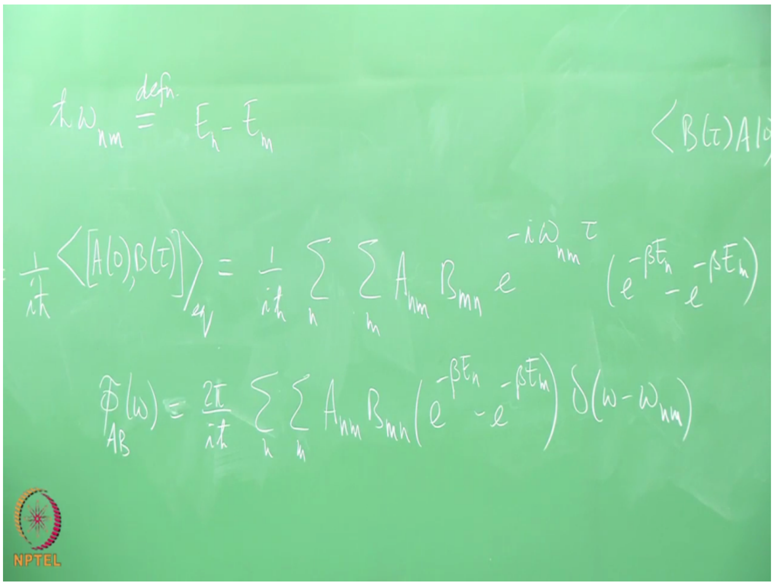


(Refer Slide Time: 47:30)



Now it is trivial to calculate what the correlation function is? What the spectral function is? What is going to happen? You want to do this integral and there is a minus out there, exactly. So what will this be? I am going to stop with that today, phi AB tilde of omega therefore equal to 1 over ih cross summation over n summation over m Anm Bmn E to the minus beta En minus e minus beta En times e to the i omega minus omega nm integrated over all values of tau that is a Delta function 2pi times delta function.

(Refer Slide Time: 48:32)



So this is 2pi, so the spectral function is a function of frequency has speaks at all these points, okay. If the number of the levels are very close to each other it has a large system at many levels then it is going to look like a continuous spectrum, okay but these are the characteristics frequencies the next thing we will do is write down the susceptibility based on this representation.

So it will give you all explicit omega dependent thing involving the transition frequences of the system these are the transitional frequencies of the system between which transitions another due to a perturbation, okay. So and then we will interpret this further and so let me stop here today.