Mechanics, Heat, Oscillations and Waves
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Lecture – 43
Summary

We have come to the end of the short course on Mechanics, Heat, Oscillations and Waves. And what I would like to do today is to kind of given overall picture of all that we have been through all that we have studied in some detailed during this course and also put matters in perspective and suggest what we should do next and in preparation for a future course.

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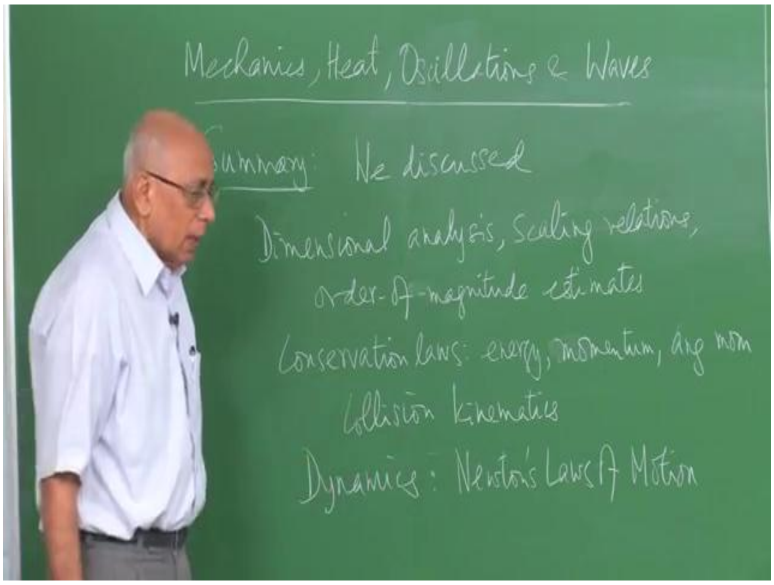

We started by discussing dimensional analysis, scaling relations, order of magnitude estimates for physical quantities. So, I made the points here that we have a certain tool at our disposal called dimensional analysis, which helps us guess the combinations of physical quantities which could appear as answers to questions we ask about physical problems. So, this is a very powerful relationship, this is a very powerful tool it has it is limitations, but as a first order guess or 0'th order guess or what the answer is it often gives us very, very wonderful answers, surprisingly powerful answers for questions which appear to be very difficult a priory.

Scaling relations is the kind of generalization of dimensional analysis, I did get too much into it. But, the reason I mentioning it is, because you should be aware that these are the tools this is the starting point and these are the tools with which one analysis real life problems. Conservation laws was what we looked at next and I use the vehicle of collision kinematics two body collisions, elastic collisions in order to bring home the effective power of the laws of principles, such as the conservation of energy, momentum and angular momentum in mechanics.

I also took care to point out that the origin of these laws lies deeper the conservation of energy follows, because of time cancellation in variance. In other words when a systems behavior in time does not depend on the absolute origin of time, when the laws are independent of time in that sense then you have the possibility of conservation of energy. Similarly, the fact the space is homogenous to start with in the absence of fields or forces and so on leads to the fact that you have conservation of momentum under suitable circumstances.

And the fact that space which is three dimensional is isotropic, has empty spaces as the same properties in all directions, again unless broken by the appearance of a field or a forces or something like that you have then the possibility of angular momentum conservation. And use properly these put a great deal of constraint on the kind of solutions that one can have for physical problems, they also help us in arriving at the solutions.

Because, constants of the motion in general in dynamics mean that every time you find the constant of the motion, you have gone closure to a solution that much closer to a solution. As I said I use collision kinematics we did not put in any dynamics except to say that two bowls or two particles collide with each other elastically, we did not say anything about the nature of the contact forces between them and yet you are able to derive the great deal of kinematic information about the momentum after the collision or the energies after the collision using just the conservation principles.

Dynamics have just one point here, recall that in this case where you have two particles colliding with each other and then going off in opposite directions, these are in arbitrary directions, this particular problem involved the knowledge of the two initial momenta and the calculation of the two final momenta is what was needed to find out what is the final momenta after the collision.

But, there we discovered that the conservation of energy momenta and momentum was not enough to determine the problem completely, you needed to put in some dynamical information and I did that by saying, well let us assume that one of the particles moves after some given angle beta to the initial direction of the projectile. And then it terms of this beta we were able to derive the other quantities, to find the beta itself you need to know a little more about the dynamical process here and that is the process that is the problem of scattering theory in general just to complete what I had said earlier.

We then looked at the Newton's laws of motion, the three laws of motion and I pointed out that it is not as if the first law is a special case of the second or that the third law can be derived from the second, no such thing. The second law contains dynamical information, it is the basic equation of motion in Newton in mechanics, the first law essentially tells you what an inertial frame is. And the third law is a relationship which Newton discovered for the first time, namely the force of particle one and two is minus the force of particle two and one, this leads to the conservation of angular momentum of the system as a whole.

On the other hand, today our understanding is that spatial homogeneity leads to the conservation of at linear momentum of the total linear momentum and that implies Newton's third law as the special case, when you restrict yourself to two body forces, two body problems.

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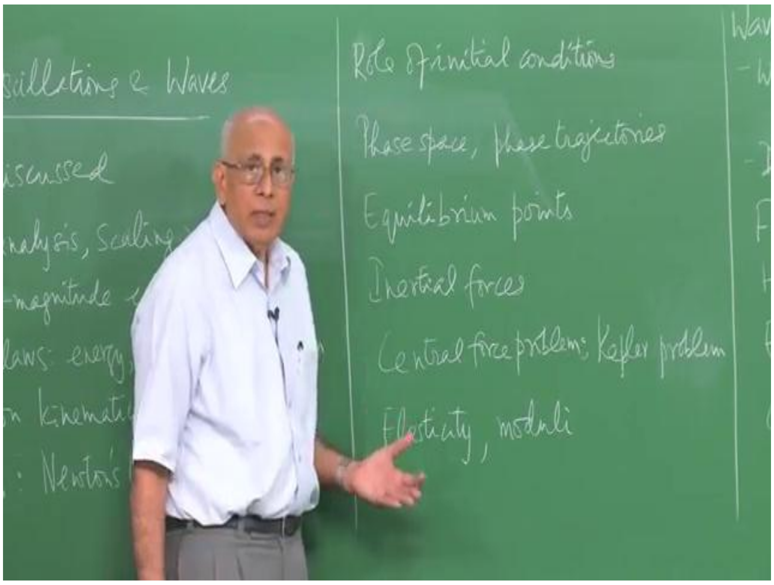

So, so much for Newton's laws of motion, in solving these equations of motion I pointed out carefully the role of initial conditions, in other words dynamics happens in the space of not only the coordinates of all the objects concerned, but also their initial velocities or momenta that is an important thing to understand. The equations of motion of Newton are second order equations in the displacements or in the position vectors and they require two initial conditions, the initial velocity as well as the initial position.

So, role of initial conditions is crucial in fact, the number of initial conditions you need for a dynamical problem tells you the dimensionality of the phase space in which this motion occurs, the dynamics occurs. So, the next thing we did was to introduce the concept of a phase space and then, the concept of phase trajectories pointing out how huge number of initial conditions, in principle an infinite number of initial conditions on any phase trajectory could be described by one common trajectory.

For instance, if you had simpler harmonic oscillator the x p plane in the x p plane with motion in phase space, in this case the phase plane was essentially on an ellipse and you could start anywhere on the ellipse and the periodic motion would cover this periodic orbit over and over again. So, that is one way of combining a lot of initial conditions into one description, but the phase space analysis is much more profound than that and in higher dimensionality is it leads to a great deal of insides into the dynamics itself.

So, the idea of phase space and phase trajectories is intrinsic is basic to the understanding of dynamics of all kinds of systems. We also took a brief look at the importance of equilibrium points both stable and unstable equilibrium points and these points in some sense or the markers if you like around which dynamics takes place and these are the points which characterized systems, dynamical system. This is the idea of equilibrium points and the generalizations which we are not talked about in this course at all.

Characterize if you like dynamical systems in general, we took a brief excursion to looking at what a called inertial forces. In other words, the forces or pseudo forces that appear when you write Newton's laws in terms of frames of reference which are accelerated. In particular we looked at the rotating frame of reference with respect to the lab frames say and discovered that appearing forces or accelerations in this case the centrifugal accelerations and the Coriolis acceleration arrows very naturally from the fact that the coordinates themselves, the coordinate access themselves you are time dependent with respect to an inertial frame of reference and had an acceleration.

And the case of rotational motion was particularly revealing, because it is automatically brought out the existence of very real effect the Coriolis effect as well as the centrifugal force, which acts some bodies in rotating frames of reference here. I also mentioned that depending on your frame of reference the way you look at the weightlessness of an astronautic satellite or our weightlessness even though we are in falling freely falling in the force and the gravitational pull of the sun as they earth orbits around the sun can be understood once one analysis the problem correctly in any given frame of reference.

In the sun's frame of reference we are in free fall, the earth and everything on it is in free fall. So, we do not fell this force directly, because we are essentially freely falling under it from our point of view, we do not feel the sun's gravitational pull, because we have the force of gravity due to the sun, the centrifugal force is balanced by the centrifugal force which we feels and where in a rotational frame of reference. So, the same thing can be translated to case of an astronaut in an artificial satellite around the earth.

Next, we looked at the central force problem in particular, we looked at the Kepler problem which corresponds to a one over r potential or the inverse square law of gravitational force for instance or coulomb's law and we discovered that this particular problem has special features which are summarizing Kepler's laws of planetary motion, namely that the closed orbits or ellipses with the sun or center of attraction at one focus that equal areas or covered by the radius vectors in equal times in the orbit, which is near nothing but, the statement of the conservation of angular momentum about the center of force.

And Kepler's second law I pointed out is applicable to motion in any central force, in particular is applicable even to hyperbolic orbits and not necessarily restricted to elliptic orders. But, in general when you have a central force there is no talk on the particle which is subject to central force. And therefore, the angular momentum about the center of attraction or repulsion is conserved as a function of time.

We saw that Kepler's third law which said that the square of time period is proportional to the cubed of the mean distance from the sun or the mean radius of the orbit is actually a consequence of the one over the r squared law of force and if this one over r squared where replace with some other force law like r to the power n. Then, you get a different code and code Kepler law third law even if you did not have closed orbits.

In particular when you had a harmonic oscillated force, namely the force was proportional to the first power of displacement then we discovered that the time period is independent of the amplitude of oscillation which is a special case of Kepler's third law if you like or some law similar to it and that is a well known fact feature of simple harmonics motion. There was a brief excursion into elasticity of solve it and the elastic moduli.

The key point here was I pointed out that there are for isotropic solid, there are two independent elastic moduli and there is a relation which will relates the young's modulus, the bulge modulus and the sphere modulus or the young's modulus, the bulge modulus and Poisson's ratio or young's modulus and the bulge modulus and Poisson's ratio and so on. So, you have two independent elastic moduli and stability thermo dynamic stability essentially gives you certain constraints.

Such as the fact that the Poisson's ratio must lie between minus 1 and plus half although for most standard materials like metal it lies around one third or so. But, theoretically it cannot be less than minus 1 and greater than plus half due to thermo dynamic stability here.

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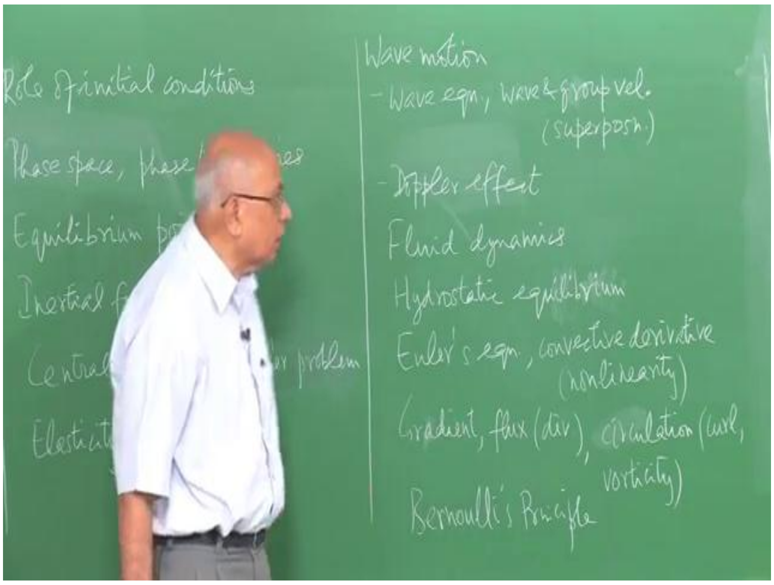

We went on to wave motion and there I wrote down the basic wave equation even though it involves some partial derivatives and then introduce the idea for wave or phase velocity, group velocity, the super position of different kinds of waves. Since, you have a linear equation, super position applies and then we also took a brief look at the Doppler's effect I wrote down without any derivation the formula is concern, I also wrote down the corresponding formula for the relativistic case or the Doppler effect in the simplest instances.

The next topic we looked at was fluid dynamics which now concerned if you go back here ((Refer Time: 13:05)) we started with the dynamics of individual particles or bodies and then we went to a continue on the case of wave motion, like waves on a string or wave sound waves and the medium like air and then fluid dynamics tells you about the dynamics of these continuation media in particular liquid and with gave a condition for hydro static equilibrium and after that I derived Euler's equation which is writing down Newton's equation for a fluid element.

And we discovered that it was natural to introduce in the time derivative of the velocity the idea of a conductive derivative.

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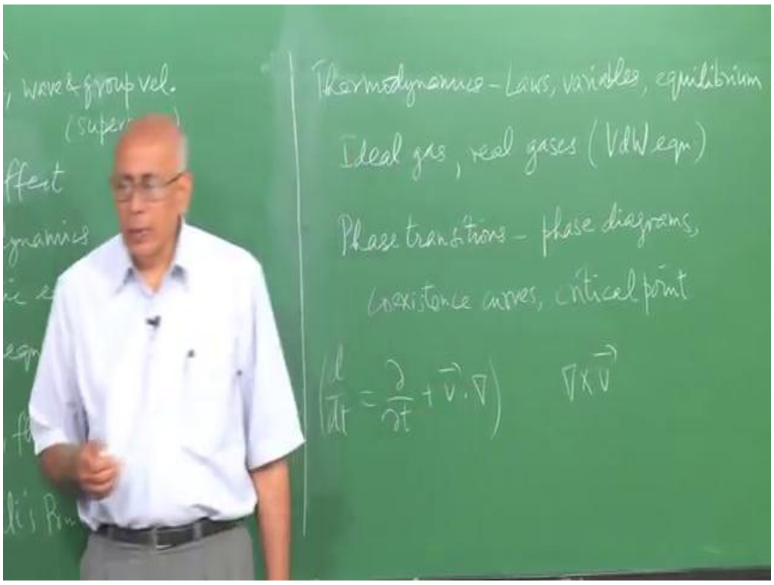

So, basically we ended up saying that the total derivative d over d t symbolically something like this, when applied to the velocity and that led to a non-linearity in the equation of motion. We appeared in the quadratic form and that I pointed out then that leads to great deal of complications of fluid dynamics. Fluid dynamics also offer to us and natural way in which we could introduce certain concepts of vector calculus which appear everywhere in physics. Such as, the gradient when I introduce the gradient of the pressure to discuss hydrostatic equilibrium.

For instance and then the flux of the fluid as a fluid flows, which is actually the most physically transparent example of a flux that, we can think of for a vector field. The velocity of the current the fluid flow, let to the concept of the divergent of a vector field. So, while the gradient told as how to construct the vector field from a scalar function, the divergent tells us how to construct the scalar field from a vector function, from a vector field and a vector field from another vector field came out when we consider the circulation of a vector field, which also let to the idea of the curl of a vector feel which got related to the vorticity.

And if you recall I pointed out here that the vorticity of a fluid in fluid flow which is the curl of the fluid velocity, this is the vorticity this quantity had the physical significance at any point of being twice the local angular velocity of a fluid element sitting on that fluid element on watching how it rotated. So, this gave us the idea of a local rotational aspect to the fluid flow and we gave some simple examples of what happens when you have such kind of rotation motions, such as a bout floating down a river due to the fact the velocity has a transverse gradient while the velocity moves in the x direction as a function of the y direction that transverse direction you have different velocities.

So, as you go down a little raft rotates is rotated and this differential in the velocity leads to the idea of the curl of the velocity field. Vortices again this is mechanism by which energies dissipated in fluids and finally, leads to turbulence in fluids is intimately connected to the idea of turbulence is intimately connected to the idea of vorticity in fluids the very complex object. There was something analog us to and a suitable cases for steady fluid to conservation of energy, which was a came out in the form of a Bernoulli's principle which is use very often in practice provided you have first stream line flow which is steady.

We went on after that to look at systems with a very large number of degrees of freedom typically of the order of Avogadro’s number degrees of freedom. And then that it became natural for us to introduce some statistical methods and I pointed out that what we call some more dynamics is really the sins of averages. So, you do these statistical methods and statistical mechanics and focus on the average values are physical quantities which are macroscopic quantities, averaged over very large number of microstates or configurations of the microscopic entities involve and this let to the laws of thermo dynamics.

So, we identified intensive and extensive thermo dynamic variables and then we wrote down the first two laws combined them to into one particular law in the simplest instance and focused on the case of the simple fluid one component fluid. I pointed out that thermo dynamic equilibrium or thermal equilibrium is what we assumed in the case of thermo dynamic systems that system is in thermo dynamic equilibrium, which means that the average values of macroscopic quantities are time independent.

So, that is what is meant by thermodynamic equilibrium, in contrast to equilibrium in dynamics this is for microscopic degrees of freedom individual degrees of freedom. Whereas, that say something nothing to do with mechanically equilibrium as such this has to do with the steady value or the time independent value of averages of macroscopic quantities themselves. So, in thermal equilibrium we have the two laws of thermodynamics and these laws enable us to get a great deal of information about the system based on considerations which really in some sense or a generalization of what I wrote here of scaling relations I did not explicitly mention it.

But, the fact that the internal energy of the system is a function of it is entropy the volume and the number of particles. But, it is a homogeneous function and it is function of each of these extensive variables to degree one I did not mention this explicitly, but that is used in thermodynamics in order to derive relations going beyond the set on the laws of thermodynamics. So, it is use in conjunction with the laws of thermodynamic to derive very powerful relations between these thermodynamic variables.

I use the ideal gas, the classical ideal gas as a kind of theoretical laboratory to get some feel for the way thermodynamic variables, behave and what one can say about these variables. We talked a little bit about the possible kinds of specific key that you could have in an ideal gas, once you subjected it to the class of processes called polytrophic processes of which the isothermal, idiopathic, constant volume and constant pressure cases are all special cases.

So, we got as I kind of general law for this specific key general formula for this specific key for a process in which p v to the power n is a constant, where n ran although way from the 0 to infinity here. And then we moved on to real gases in other words we took some cognizance of the fact that real molecules would actually attract themselves at long distances and also have a certain range of sort range repulsion between themselves.

And when you took those into account we had an empirical equation the Van der Waals equation to describe this gas. But, as a bonus we discover that this equation of state actually describes not only the gas state, but also possibly the liquid state of the system. So, it allows for it is had two branches if you like and it allows for the fact that below a certain temperature called the critical temperature, you have the possibility of separation in to liquid on gas.

And the Van der Waals equation help us understand to some extent, how this a change of phase occurred. We went on and looked at a little bit at phase diagrams the t v plane and v t plane and most importantly the p t plane to understand what the phase diagram of the generic simple single components substance looks like namely the crystalline solid phase, the liquid phase and the gas phase and we took some look at the properties of the coexistence curves.

The significance of the slope of the p t curve which is the coexistence curve given by the Quasi Lagrangian equation, which related the slope of these curves the way the boiling are malting point changed with pressure it related it to the change in entropy under this transition divided by the changing specific volume. I also pointed out that the liquid gas fair of coexistence curve ends in a critical point, where the liquid loses it is distinct to characteristics such as the existence of latent heat of vaporization that goes to 0 at the critical point that this surface tension of existence goes to 0, the meniscus disappears and the distinction between gas and liquid is lost.

In the case of water this happens fairly high it happens at something like 600 or 647 Kelvin and it happens up to more than 200 atmosphere pressure. But, there exist such a critical point there, such a point does not exist between the liquid and solid crystalline solid the coexistence curve it does not end in a critical point it cannot, because if you did you would be able to continuously go from the very symmetric phase at the liquid is too the much reduced symmetry phase of a crystalline solid periodic array and this is not physically possible.

So, in this case we can make a general statement based on symmetric considerations that there can be no critical point in such a coexistence curve. Whereas, in the gas liquid case there is such a critical point thinks always end in a critical point. I finally, mention that the critical point understanding that requires you to go for beyond it is thermodynamics for say it is in fact, a point where the normal thermodynamic fails for certain technical reasons and you need to use sophisticated statistical mechanical techniques to go beyond it.

Going beyond that we have a lot of applications and condense made a physics, which a properly understood only with the help of statistical mechanics, which is a crucial subjects which you at some stage and take who studied you certainly under take some stage. What is a left now in the next course hopefully would be the rest of the standard curriculum and basic physics, which would comprise things like electricity and magnetism and optics in little bit of modern developments in physics.

Modern is really a should be taken with little advisement, because modern world it is more than 100 years old, quantum mechanism is almost 100 years old, certainly the idea of the quantum is now more than 100 years old, 110 years old. So, it is really not all that modern is really part of mainstream physics now and that is what we now look forward to studying.