Mechanics, Heat, Oscillations and Waves
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Lecture – 31
Wave Motion Travelling and Standing Wave

We have seen some properties of the most basic kind of periodic motion, namely simple harmonic motion and I pointed out that simple harmonic motion is the very model or the prototype of all kinds of periodic motion, more complex periodic motion. Natural extension of oscillations or periodic motion is the idea of waves, both standing as well as travelling waves. So, we will now consider, what we mean by wave motion and some of it is fundamental properties.

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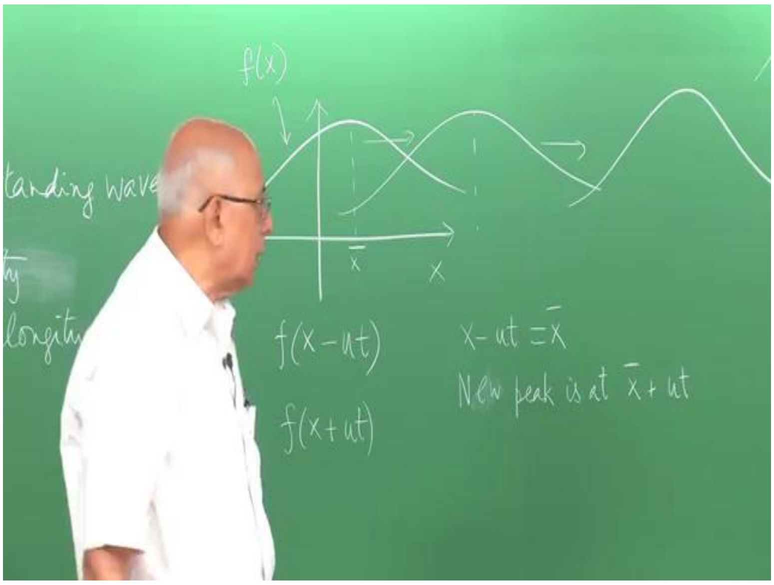

Imagine that you have a function f of x sketched in this particular fashion. So, it is some kind of disturbance could be anything at all and we will come cross physical examples of it. As a function of some variable x, independent variable x and it has a shape of this kind. So, this function here is f of x, it is got a peak and then, it dieses down on both side for instance.

Now, what would happen, if this disturbance changed it is position with time, it move for instance in the positive x direction with some assign, pre assigns speed u for instance, what we did look like. Well, at a later instant of time this thing would look like that and then, I as time progresses, it would look like this and so on. Such that, the peak goes on shifting and the way to described this travelling pulse.

If you like is to consider, instead of this function f of x to consider the function x minus u t, where u is got the physical dimensions of the speed and t is the time, because for instance, suppose this peak occurred, when f of x is some x bar. So, f of x has a peak at the point x bar. At a later instant of time, the peak as shifted here and it shifted to a point given by x minus u t equal to x bar, because, that is where the peak is.

When the argument of this function is at x bar, which would imply that, the new peak is at x bar plus u t. In other words, peek on the shifting and we would say that this pulses moving forward with constant speed given by this quantity u here. So, this functional form once you specify the function f of x, this functional form replacing the argument by x minus u t would correspond to a travelling pulse moving in the positive x direction with the speed u. That is the fundamental observation of a trivial one.

What would happen if the pulse moves backwards? Well, you then have to consider f of x plus u t, because now the new peak would not be at x bar plus u t, but x bar minus u t on this side. So, that would correspond to something moving in the backward direction, in the direction of negative x. So, with this basic observation that a function of a combination function of a space coordinate and time of this form describes for us a moving pulse in time that basic observation helps us to understand, what wave motion is all about in the general case. Now, of course, wave motions such as that you have thing that is periodic in space and if it moves, then it is periodic and time as well. And how do we describe that, well the most basic periodic functions or the cosine and sine.

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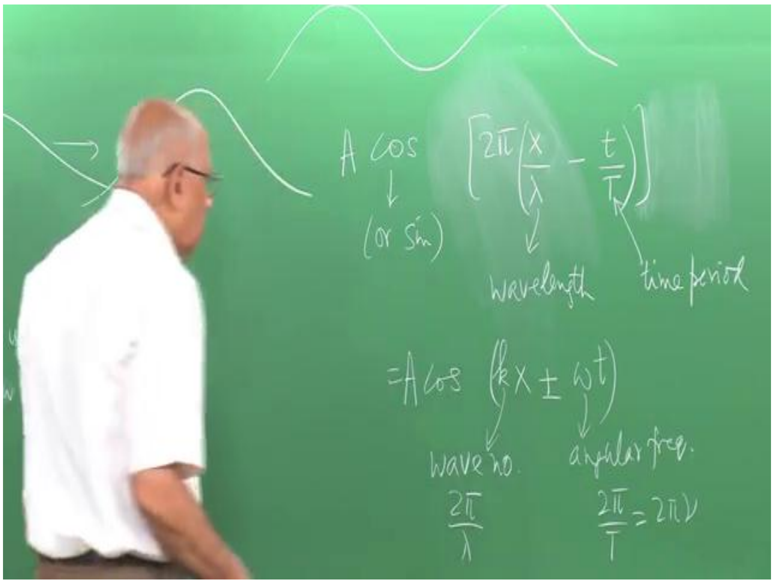

So, one could consider instead of a general f of x, one could consider the cosine, which could also be a sin or sin does not matter or some combination of the form x minus u t. This would be a periodic function in x for a fix t and a periodic function in t for a given value of a x. On the other hand, the dimensions are all wrong here, because it is clear that x must have dimensions of the length for instance and u t and t has dimensions of time.

So, we need to put in the argument of the cosine quantity which have no physical dimensions at all. And that is done by saying, suppose I want to pattern to repeat in this fashion I want this pattern to repeat after distance lambda for instance. When, the way to do is to write this exploit the fact that the cosine is periodic with period 2 pi and write it in the form cos 2 pi x over lambda that makes this quantity dimension less. And similarly in time, if you want to periodicity capital T make this minus on this side t over capital T.

Now, that is a periodic function in space with period lambda or wave length lambda and this is called the wave length and this is the time period, it is periodic in time with the time period capital T here. Either, sin or a cosine of this gives me a basic periodic function of this kind, I could of course, add to this whole thing, I could add some quantity here plus some phase delta.

And that is not alter the periodicity properties of this function at all, it would be determine this delta would be determine by it is saying, what is this function not disturbance at some specific value of x and t. Once, you fix that this quantity delta gets fixed. So, the general form that we write down is some function of this kind and can be rewritten in many ways. For instance, you could write this also as equal to cosine of if you called 2 pi over lambda in new quantity k and this becomes x minus 2 pi over capital T, you call it omega, then it becomes t here.

So, this quantity k is called the wave number and it is 2 pi over lambda and this quantity is the angular frequency and it is equal to 2 pi over t, which is also equal to 2 pi times nu, where nu is the plain frequency. Very often in physics, we call omega the frequency itself, it is actually strictly speaking the angular frequency and nu is the frequency the reciprocal of the time period capital T.

But, this distinction it should not cos any confusion, because by context, we know we mean, I will always use the symbol omega for the angular frequency and little new for the frequency itself. So, these are dimension less quantities and this is the basic model of a period or a sinusoidal wave, with wave number lambda have k and angular frequency omega here.

So, once we have this in place, we can have waves are different frequency super forced and so on and so forth. On the other hand, a wave which is travelling the backward direction would correspond to plus sign there. So, cos k x plus or minus omega t refers to travelling wave in the positive x direction for the minus sign negative x direction for the plus sign. Whether we write this as k x minus omega t or omega t minus k x does not matter as long as cosine is there, for the sin in just changes an overall sin. One could also put an amplitude for this wave no reason, why it should oscillate between plus 1 and minus 1 in magnitude in size, when put a here, which would be the amplitude of this wave. So, this is the basic model or basic function, which describes a moving wave, a moving disturbance, which is periodic in both space as well as time.

Now, comparison of that with this thing here, immediately tells us that the speed of this wave is easily determine the speed of the wave corresponds to saying, I take a particular part of this wave, say a crest and ask, what is this speed at which the crest moves or a turf and what is the speed at which it moves. This waves moves without changing it shape and therefore, whichever point I choose on this inside in a given wave length, I ask what is this speed with which that point moves across on the x axis.

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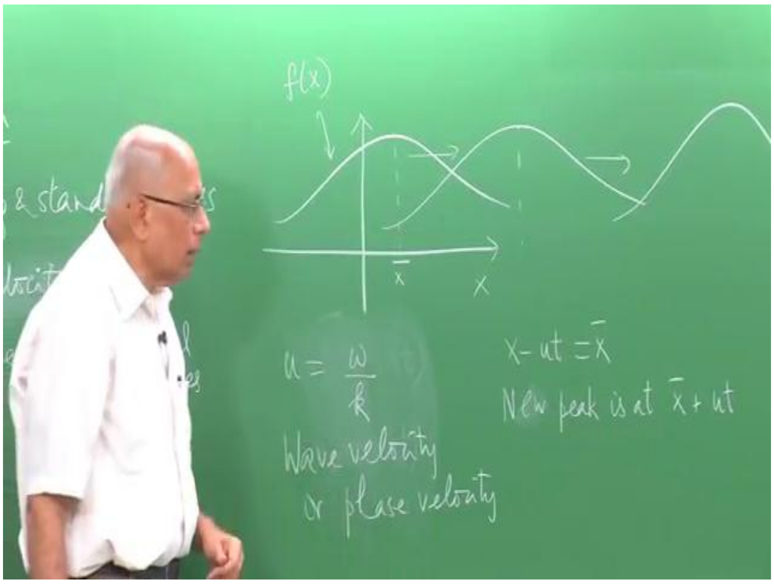

And a simple comparison of that form with the general f of x minus u t shows that the speed of the wave corresponds to u equal to omega divided by k all I have to do is pull out to k. And then, it is x plus omega over k t and omega over k is coefficient of t give me the speed of the wave here immediately, this thing here is called the wave velocity or another name for it is the phase velocity. So, it tells me the speed at which any given phase of the wave such as a maximum or a minimum or any point in between moves.

So, whatever a given phase, the speed at which it moves is called the wave or phase velocity here. The reason for this distinction is that very soon will come across another kind of velocity or speed associated with this sort of wave call the group velocity and we will come to that in it is done. But, here is the very simple idea that the phase or wave velocity is just the ratio of a coefficient of the 2 of x and t respectively with the proper sin here. Having set this, let us move on the some examples of such waves, the most standard example of all has to do with waves transfer waves on a string.

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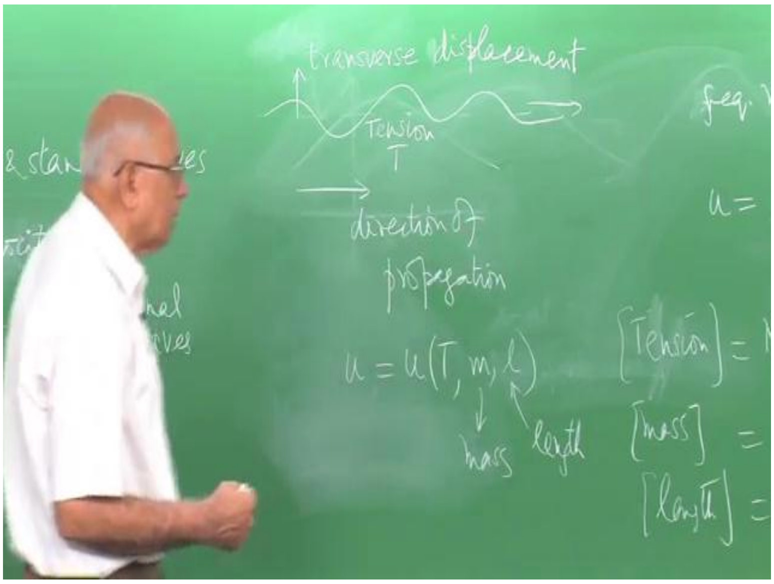

So, if I have a string and I the string is stretched and then there is tension on it and I let it move in this fashion some disturbance moving on this string, the direction of propagation is this the x axis say. On the other hand, the disturbance from the undisturbed string which would be just as a straight line is in the transferred direction. So, this is the transfers disturbance or transfers wave of some kind, transfer displacement.

And the question is, where does this come from, what is the way in which we can identify, what is the speed of the wave, what kind of transfers displacement you can have and how do I find the speed of this wave. Well, we have three properties associated with this wave we just obvious from the wave we have return in down.

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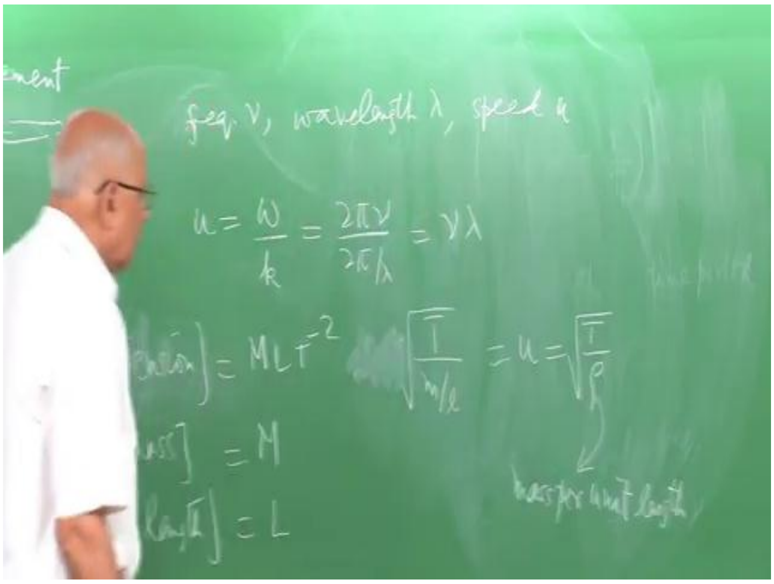

We have a time period t r of frequency nu a wave length lambda and a speed u and we need to know what is the connection between these three quantities and that is very straight forwardly done, the wave we wrote it down remember that we said u was equal to omega divided by k which is equal to 2 pi nu divided by 2 pi over lambda equal to nu lambda.

So, the relation between the speed of a wave, the wave length of the wave and the frequency nu is simply that the product of the frequency times the wave length is equal to the speed here that is the basic fundamental relationship which we learn in elementary physics. Now, how do we determine this speed that is the property which depends on the kind of wave you have and the medium in which it is propagates medium if any in which it propagates.

For instance, in the case of the string we could derive what the speed is by dimensional arguments, if this is string has a tension T here then it is clear that what the speed can depend on is a material property of this string which is under this tension. So, it must depend on the tension T I am for using the same symbol for the tension and the time period T.

But, this is standard notation and I am not going to bring in the time period any more I am going to always talking in terms of the frequency or the angular frequency. So, capital T will stand for tension from now on. So, this frequency or the speed u must be a function of this tension it should be a function of the mass of the string. In fact, it is a property which is the string as a whole, so it is not the entire mass that matters and as will see in a minute, but the mass per unit length.

So, let suppose the string has the mass m and the length of the string is l and this is the length of the string then simple dimensional arguments to find a quantity of dimensions speed is very trivial. Because, this quantity T physical dimensions of the tension, so it should be write it down this form this tension is a force which is M L T to the minus 2 and then the mass m has dimensions m. So, it is clear that to find velocity you need to divide this out tension over mass will kill this, but you got in L here on the other hand if you divide by the length as well this is equal to L.

So, if you form the combination T divided by M over L then that quantity has dimensions L squared T to the minus 2. So, the square root of this has dimensions u, so this must be u times and constant. So, it immediately tells us that the speed of waves on the string is equal to the square root of the tension of the string divided by the mass per unit length or the linear density mass density of the string multiplied possibly by a constant which will turn out to be unity.

We will derive this x at expression a little later and it turns out that in this case this formula is exact in this turns out to be T over rho and this rho is the mass per unit length must being. So, that tells us that this is a material property it is a property now of the property of the string itself, namely how much tension it subject to and what the mass per unit length of the string is. In the case of an actual metal string in which waves transfer wave propagate depending on the kind of waves you have some elastic modules of this string is of material of this string is what is going to play the role of this tension there something related to it.

What about thing like sound? What about sound wave? Well, there we have a formula which is actually quite easy to derive, as you know sound waves are created in air for instance or in a gas or created by changes in the local density. So, compressions in rare of action of this gas in a periodic fashion lead to sound or rare is propagate in the form of sound.

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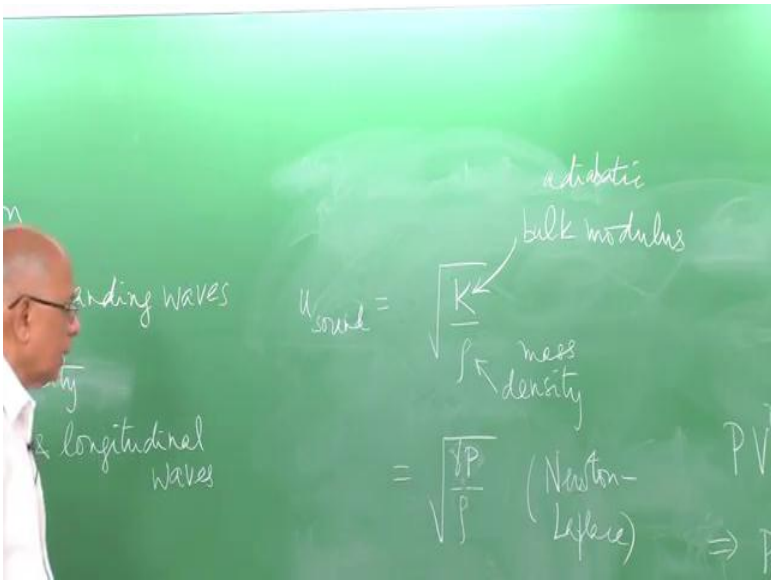

Now, what can this speed of sound in medium be dependent on, this u must dependent on a elastic property of the medium namely air in this case and in this case, because it is a pressure wave it must depend on the bulk modules. So, this again must be equal to the square root apart from the constant of the bulk modules k which has dimensions on stress or force for unit area, if you recall divided by the density of the string of the gas, the mass density.

And let see this is true or not, the way to do this is to write the formula down times the constant and once again it is possible to show that the constant is one, this is what dimension analysis gives you. And once I have formula for the bulk models of a gas you have done this is the speed of sound in this gas now what is this bulk modules equal to.

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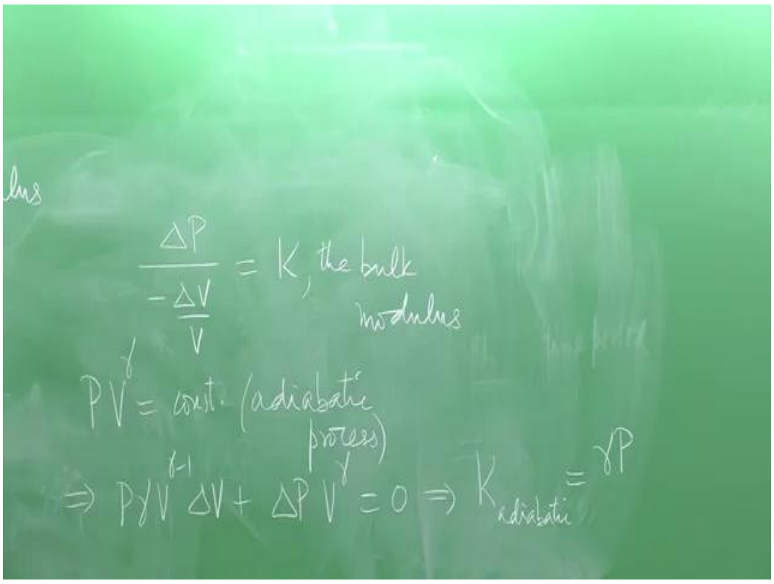

If you have a change in pressure delta p and that corresponds to in change in volume delta v and increase in pressure will decrease the volume. So, delta v is negative minus delta v divided by v is the string, so this is the bulk modulus this is equal to k the bulk modulus and we need to compute this. Now, you have to tell me whether the process through which this gas gets this changes in pressure the rare fashion and compressions is adiabatic or isothermal.

Well, in the simplest approximation if it is isothermal then we know that P V is a constant for in ideal gas. So, it says P V equal to constant at a given temperature fix T this would imply of course, the P delta V that is V P equal to 0 or P equal to minus delta P over delta V over V which means that the isothermal bulk modules, which I should really call k subscript T to show that it is isothermal is equal to the pressure itself.

So, it says this is equal to this square root of the pressure divided by rho, since k isothermal equal to the pressure for an ideal gas, this was Newton drive this formula for the first time and the set that the speed of sound u sound in a gas is given simply given by this square root of the pressure of the gas divided by rho. However, it was soon seen that this is not quite correct experiment did not match this once this speed of sound start getting measure to some accuracy, there is a small change in this formula which is quite measurable and the matter was fixed by Laplace.

So, pointed out that the changes the rare of action and compressions are happening so rapidly that the process is essentially adiabatic or not isothermal. Once that happens then what we must be put here is not the isothermal bulk modules which is the pressure itself, but the adiabatic bulk modulus and the adiabatic process has if you recall P V to the gamma is constant, this is what an adiabatic does for an adiabatic process this will immediately imply that P gamma V to the gamma minus 1 delta V pus delta P times V to the gamma equal to 0.

And now it is clear that this will imply that k adiabatic is equal to gamma P and not P. And therefore, if you put that in the correct formula for the speed of sound in a gas should be the adiabatic bulk modules and that is given by gamma P over rho. So, this is the correct formula for this speed of sound in a gas and this is due to the Newton and Laplace as modified by Laplace. Notice that for an ideal gas which is the case we are consider here the speed of sound is actually dependent only on the temperature of the gas and that is easy to see.

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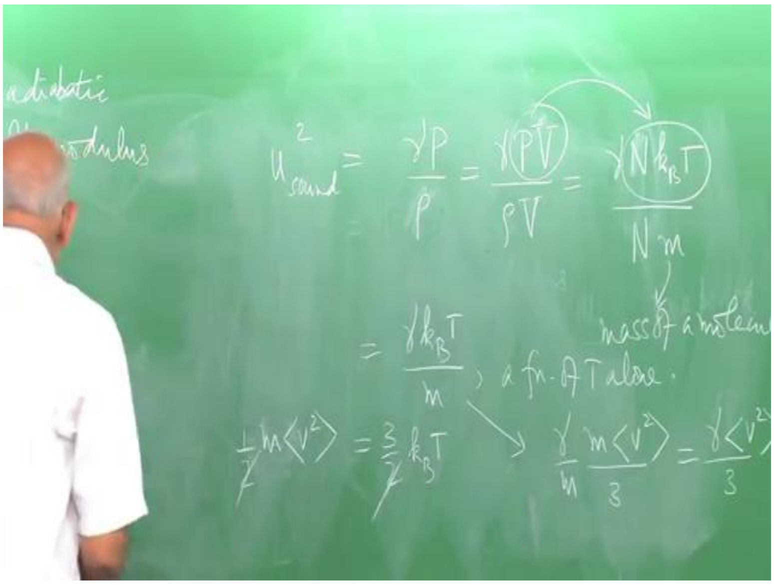

Because, if I write a u sound squared to get read of squared root sign it is gamma P over rho the density this is equal to gamma P V you can multiplied by V by rho times V multiply both sides by V. But, then P V for an ideal gas of n particles is gamma times N times k Boltzmann times T that is the ideal gas equation of state P V is N gamma k Boltzmann T and we are going to study this when we study heat in sort while divided by the density of the gas, the mass density multiplied by the volume which is the total mass of the gas which is equal to the number of particles in the gas multiplied by the mass of each molecule of the gas, this is the mass of the molecule.

And therefore, this thing here becomes equal to gamma k Boltzmann T divided by m which is a function of T alone. So, we have this interesting feature, important feature that the speed of sound in a gas is essentially a function only of it is temperature as long as you can make the approximation that the gas is as an ideal gas does not depend on any of the other thermodynamic variables. And one can further interpretive this in the following way after all we know today that heat is the form of random molecular motion.

In fact, what we call heat is essentially in these systems the kinetic average kinetic energy of the molecules due to the random motion that the under grow. So, this k Boltzmann T can be rewritten in the following way, know that m v squared average kinetic energy one half m v squared of a gas, we know that this quantity is equal to 3 half k Boltzmann T this is called equip patrician theorem and I will come back and see how we can established this in a fairly simple way.

So, if you put is in out here then k Boltzmann T can be removed if favor of m v squared the m cancels out and this becomes also equal to gamma over m the two to cancels out here and k Boltzmann T is m v squared divided by 3 which is equal to gamma times v squared divided by 3. So, we have a way of relating this speed of sound in a ideal gas to the ratio of the specific heats c p over c v times the mean square velocity or mean squared speed of the gas means squared velocity in this gas.

So, it is related two fundamental quantity namely a microscopic quantity in this case and it is of the order of this quantity here because these quantities are of order unity. So, it actually is making a very direct contact with what is happening in the system at a molecular level in the speed of sound from the very simple argument here. We will come back to waves in three dimensions what these waves are like and so on as we go along.

Now, let us go back a little bit to waves on a string, we have already seen that the speed of the waves on a string is controlled essentially by the material properties, the square root of the tension divided by the mass per unit length. What controls of frequency? Well, I take this string and swing it out and whatever frequency I like.

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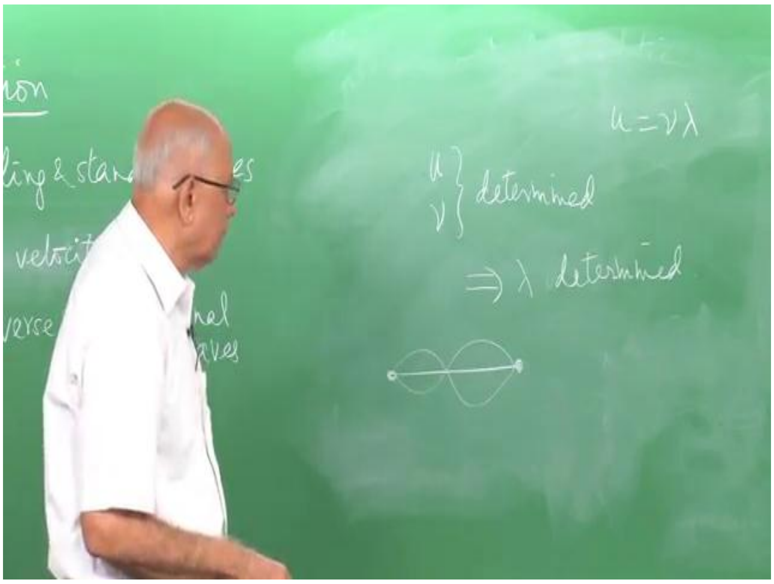

So, in that case the speed is fixed the frequency nu at determined by me determine by us for a free string if I swing it up and down at some given frequency the speed of the waves on it the pulses on it with determine by the tension and the mass per unit length the frequency is determine by me. Therefore, this implies that the wave length is determined on the other hand, suppose I take the string and clamp it at both ends here and then I pluck this and set up waves in it.

So, you could perhaps have a wave which is look like this and other function of time this goes up and down this time, one of the possible modes on this string is of this form. Now, it is the wave length that gets determine, because you must have nodes at these two points. So, the boundary conditions fixed the wave length or restricted to a set of wave length possible to wave lengths all those wave lengths are allowed which have nodes add the two end points and then the frequency gets determine, because u is given by the tension and the mass per unit length.

So, when we talk about free wave propagation in a medium in general it is the frequency of the source that determines of frequency and light passes through to represent for instance, it is the frequency of the sources that determine of frequency of vibrations and the wave length gets determine depending on what the speed in the medium is. On the other hand, when you have a camping medium of this kind and you set up standing waves in it.

Then, the wave length gets determine by the boundary conditions and it is a frequency that is determine by the connection with essentially says that u equal to nu times lambda we will look at examples of both these So, the next step is, to see what are the possible wave lengths that are allowed in this case here and let us do that next.