Physical Applications of Stochastic Processes
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Lecture-19
Power spectrum of noise

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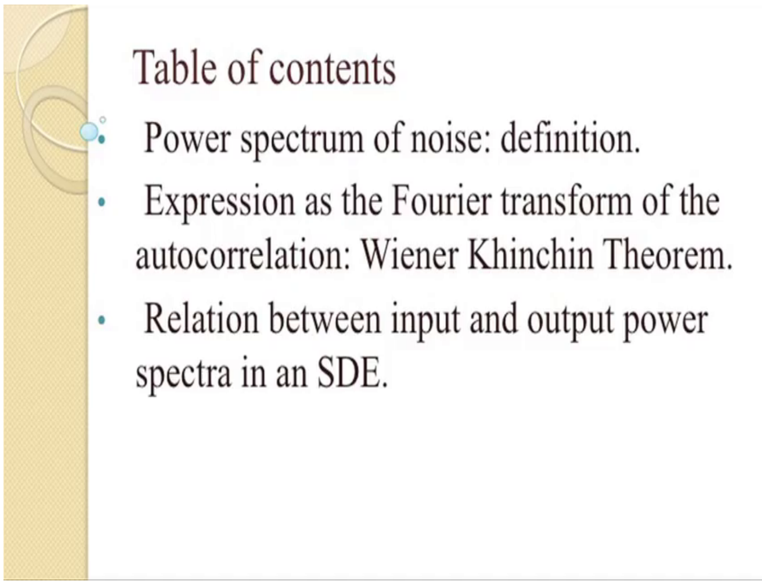

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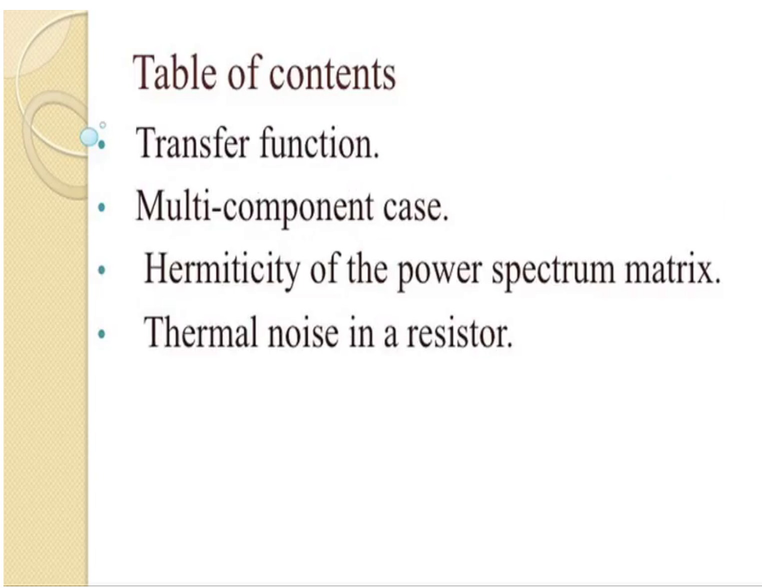

Right, so the last time we talked about the properties of particle defusing while in the presence of a magnetic field and we saw that the different components of the velocity of this particle were correlated to each other and we also saw that the diffusion coefficient in the direction of in the direction transverse to the magnetic field was reduced from the normal free diffusion coefficient okay. Now let us turn to the problem of the analysis of noise in general and I would like to introduce various kinds of noise.

But we need some tools to understand this so for a while let us look at some general formalism I would like to introduce concepts like the power spectral density and then the Wiener Khinchin theorem and so on which help us analyze noise in general without any specific reference to whether the process is Markovian or anything like that okay. Now one of the key tools in this kind of analysis is the exploitation of the fact that noise in general under suitable conditions is stationary that you have a stationary random process.

And then a great deal of simplification occurs if the process is not stationary it is generally non stationary then what one does is to look at it in windows of time where it is essentially stationary and then approximately you can assume the process to be stationary and go through with the formalism I am going to develop now so let us first look at what this entails what is meant by the power spectrum of a noise.

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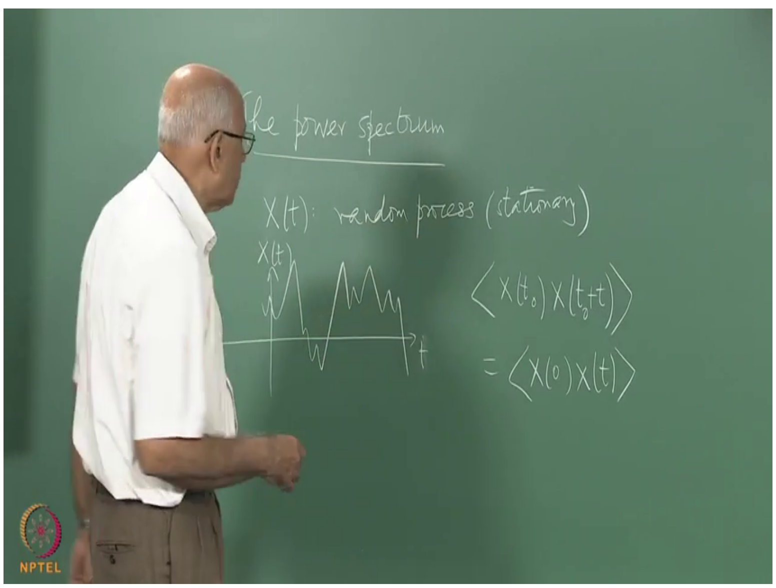

Now we have in mind in the simplest instance some random process as a function of time which is noisy random so it is got, a very, very irregular time dependence and for to be specific let us call this noise X of t or I of t for instance it is easier to write X of t a random process and we will assume it to be stationary then one could ask if you plot this X of t in a typical realization as a function of time you plot X of t you are going to get some highly irregular curve we saw in the case of Brownian motion it was so irregular it was not differentiable anywhere.

But in other cases the process could be differentiable for instance but in any case it is unpredictable in some specific sense because it is noisy okay. Now, one would like to first of all to understand any regular curve like this based on our experience with very complicated curves in which I could not sound for example is to further analyze the whole thing and ask what do the Fourier components look like what is the frequency content of this noise.

But this is not always trivial because this function needs to be absolutely integrable before you can have a Fourier transform. So, it is possible that the Fourier transform of X of t does not really exist in that sense on the other hand we have a much more powerful tool which is the autocorrelation of this function of this random process and that is a much smoother function as we have seen function of t.

So, what you do is to take X of t naught X of t naught + t the product and take its average over all realizations and then this if it is stationary is a function of t in general expected to die down as t becomes very large. Now we will assume that all this process has 0 average so it simplifies the writing of the formulas otherwise I would have to subtract out the mean each time and write correlation functions.

So, we would like to look at a correlation function like X of t naught X of t naught + t and if it is stationary this is of course = X of 0 X of t and we could ask what is the Fourier transform of this tell us now turns out that there is a very deep connection between what the process itself does and what the Fourier transform of the correlation function does base to this is the content of the so called Wiener Khinchin theorem which I will write down we are not going to prove it rigorously.

But I will motivate it and we will go through some of the steps to see what is entailed so what one does is to take this thing and look at it over a long period of time okay.

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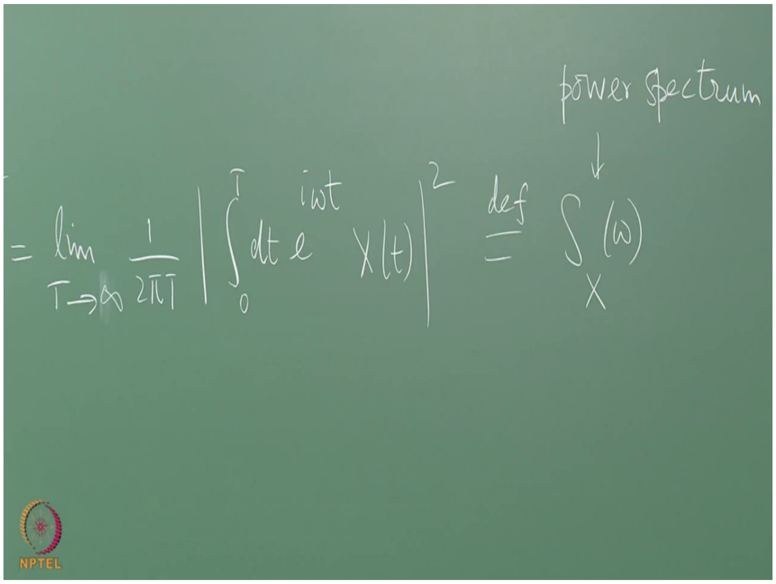

So, if you took at various instants so 0 to t some long instant of time you look at it at various instants of time so this is t1 this is t2 and so on. And compute e to the power I Omega ti at those instants of time multiplied by X of ti you have sampled it at those instants of time, times some infinitesimal interval of time around it in this fashion I do sum over this i = 0 to n and let n become very large and take the average the mod square of this quantity.

So, that it becomes real and take its average 1 over t1 over let us put a 2 Pi also limit as t tends to infinity so consider this. So, I am trying to do a kind of Fourier transform what I am doing is waiting this with e to the I Omega ti multiplying by the time interval and summing all these pieces together okay. Of course this thing here is also = limit in the limit in which these intervals become infinitesimal becomes 0 tends to 0.

You know this is 1 over 2Pi t integral 0 to t dt e to the i Omega t X of t mod square. So, there is a function of Omega. Now let us try and see what this function gives us and what it is going to become = it will turn out and this is the Wiener Khinchin theorem that this quantity is = the Fourier transform with respect to time of this correlation function, so that is our target we would like to establish this that that limit will turn out to be this one the Fourier transform of this quantity here.

But this thing here is defined as the power spectrum so this limit whatever it is, is = by definition the power spectrum of this random variable SX and it is a function of Omega and we will see what information it contains, pardon me I am sorry yeah e tends to infinity sorry. Of course so let us see how this arises, now the proof is subtle it is not a trivial theorem it is subtle but we are going to slur over the important part of it the part that is really requires a little bit of justification.

We go through just the algebraic manipulation but it will motivate how this result arises to start with so we look at that integral.

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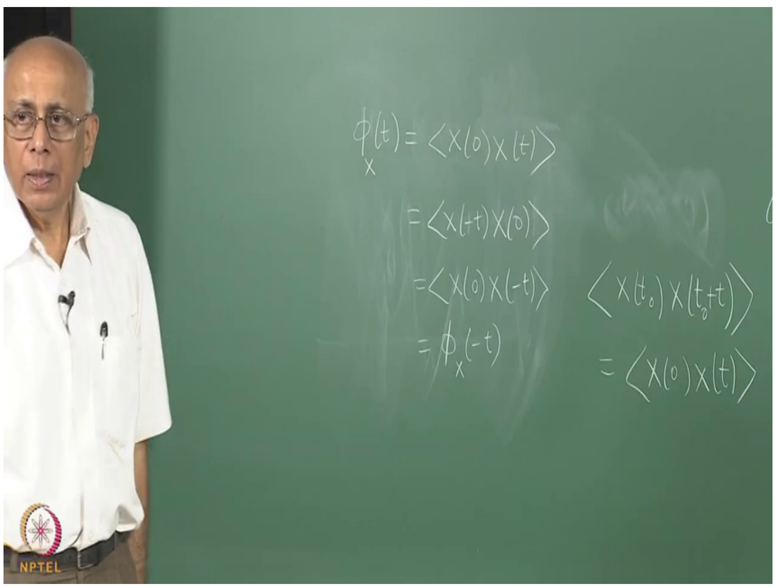

But before that a couple of properties of this thing here of this correlation function. So, let me call this Phi of t, Phi X of t to show that it is for the variable X in fact they are soon going to have different random processes here and here okay different components say of a vector random process for instance. So, I need a little better notation but now when I come to it we will be careful.

So, this is = X of 0 X of t but notice also that because of stationarity I can add a t0 to the argument say without changing anything but I can subtract any amount without changing anything, so this is also = X of -t X of 0 if I just subtract t from the argument of each of the time each of the time arguments right but this is X of 0, X of -t if these are classical variables if their quantum variables there are operators then we have to be very careful there is a formalism which will tell you what this the correct answer is?

You cannot commute these things randomly but the fact is there is a classical variables in this level and therefore this is = Phi X -t so the first piece of information we have is that in the simplest instance of a scalar process single component stationary process the autocorrelation function is a symmetric function of the time. This is why when we computed it for the velocity in a length of one component when the one component of the velocity for a line Langevin particle we found e to the -gamma modulus t for the correlation there is a symmetric function we are going to exploit this as we go along.

If you have more than one component of course then the symmetry property becomes a little more complicated and we will come to that in its time okay.

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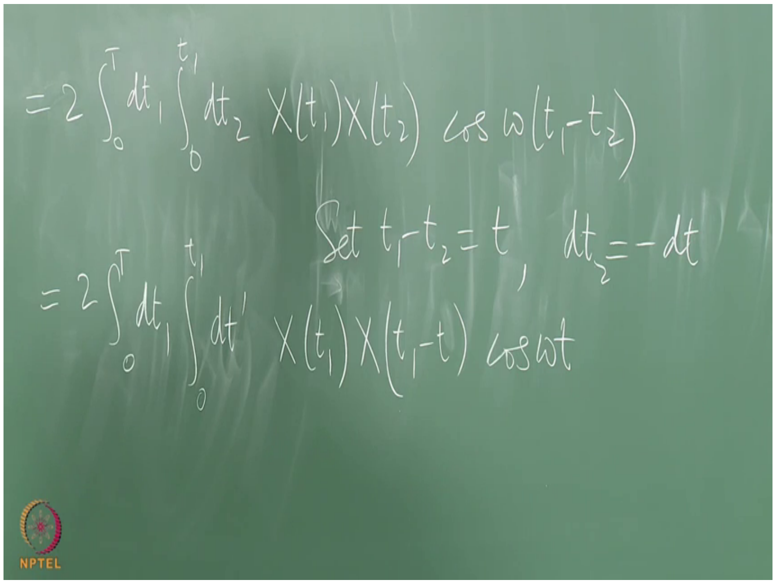

So, let us look at what this SX of Omega becomes the limit part I will omit and let us just look at the integral let us look at that thing alone so you have 0 to t be t1 e to the i Omega t1 integral 0 to t dt2 e to the i Omega t2 with a - sign because I want the complex conjugate of that X of t1 X of t2 that is what this quantity is this modular squared is = that now a whole sequence of manipulations.

First of all I can also write this as integral 0 to t d t1 0 to t d t2 and then X of t1 X of t2 no averaging or anything like that is being done I am just sampling the time series as we go along yeah times e to the I Omega t1 - t2 but this is a symmetric function of t1 and t2 right and this quantity is real. So, the imaginary part must vanish identically and indeed it does because I imaginary part is sine t1 -Omega times t1 - t2 that will be odd under the interchange of t1 and t2 and it will vanish okay.

So, that is a trivial statement this is cos Omega t1 - t2 right, it is a real quantity so the imaginary part must vanish identically okay. But you can also write this because it is symmetric under t1 and t2 and the range of integration is symmetric 0 to t in each of them you can write this as twice the integral from 0 to t1 and then the next step is obvious change variables from t2 to t1 -t2. So, let us put set t1 - t2 before the time so, dt2.

So, this guy is twice integral 0 to t dt1 integral and when t2 is t1 at 0 and when it is 0 its t1 so it is again 0 to t1 dt prime X of t1 X of t2 but t2 is t1 - t prime correct me if I am making a mistake we have to be careful miss t1- t prime cos omega let us set that = t that is easier because it is going to come out on the left so the t1 - t cos Omega t simplifies the notation okay. Now let us interchange the order of integration.

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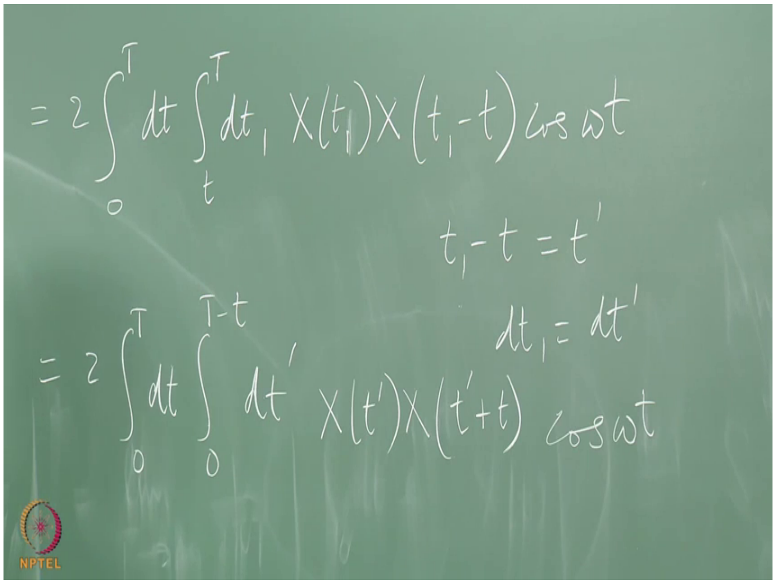

And what is this going to be by interchange the order of integration. This is = twice integral well t1 runs from 0 to t and t prime runs from 0 to t1 so if I interchange t prime will run from 0 to t and t1 will run from sorry t to capital T, so this is going to be 0 to t dt integral t to capital T dt1 X of t1, X of t1 - t cos omega t okay. You know the obvious thing to do is to change because of this thing here change variables to t1 - t right.

So, let us put t1 - t = t prime, so dt1 = dt prime that is = twice integral 0 to t dt integral where does this go t1 is t so this is 0 and capital T - t dt prime X of t1 - t is t prime and then X of t1 is t prime + t oh we forgot the cos, cos omega t. So, which is twice integral 0 to t dt, let us pull out this cos Omega t because it does not involve t Prime and then an integral 0 to t - t dt prime X of t prime X of t prime +, now look at what is emerging you got precisely the structure that you need for the correlation function.

If it is stationary because it is saying take any instant of time t prime and take X at that time and X at time t prime + t staggered multiply that to and keep doing this summing over all t prime's okay. And this is the step which requires rigorous justification but if this random process has this property of Ergodicity namely it takes on all the values in its available sample space given enough time an infinite number of times over and over again.

Then the time average of that integral is = the ensemble average over some prescribed distribution over a distribution for the stationary variable which we have not specified. So, this property is known as Ergodicity.

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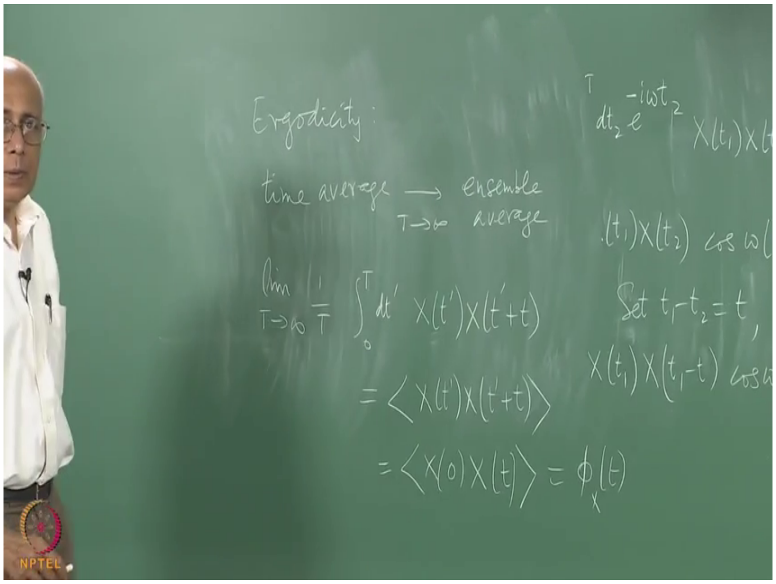

Let us write it down I am average over a very long time in the limit t tending to infinity tends t tends to infinity to ensemble average. This is at the root of equilibrium statistical mechanics if you think about it because it says that long time averages of the system given enough time all the accessible microstates are accessed by the system and the average over all of them is = an ensemble average over some prescribed distribution which you have to find.

So, what is actually being done what you actually measure in experiments are averages time averages what you compute using the rules of statistics or statistical mechanics are ensemble averages and the article of faith is that one is = the other this requires proof rigorous proof and it is the property of Ergodicity in the context of random processes you have to specifically check that this is true in a given instance let the property that this is that the random process is indeed Ergodic.

This is possible to do once you know a little bit about the statistics of the process you can do it we are not going to prove it we are going to assume that this is true and then this quantity limit t tends to infinity 1 over t integral 0 to t dt prime X of t prime X of t prime + t is indeed = this quantity is = the ensemble average of this quantity X of t prime X of t prime + t that is the property of Ergodicity way that we are using okay.

Then of course the power spectrum reduces it becomes we have done a little bit of sleight of hand here I mean to change limits I have shuffled limits here I have taken the limit the inside here and then said argued that this guy is in fact the ensemble average but this can be made rigorous this is the part that I am slurring over but it can be made rigorous the fact is that it is physically clear that it is if you got a city is valid it is this quantity this integral integration which is a time average because of this is = this correlation function here okay.

And of course this thing here by stationarity = X of 0 X of t = Phi X of t is how we defined this correlation.

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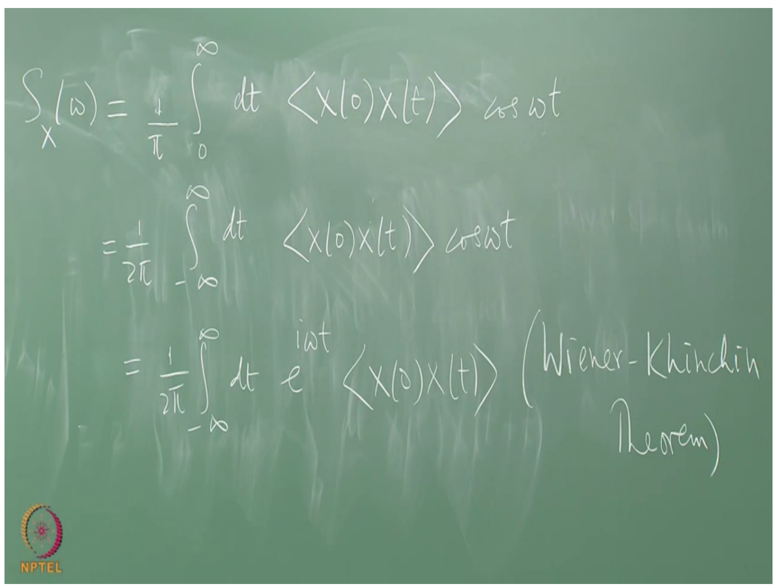

So, finally it tells us that SX of Omega that we have is twice the integral from 0 to infinity because remember there is a t gone to infinity limit of dt said property it out explicitly X of 0 X of t cos Omega t, but we already saw that this is a symmetric function this Phi X of t is a symmetric function. So, you could also write this as = integral -infinity to infinity dt there is a 1 over 2 Pi right so this is 2 over 2 Pi 1 over Pi and it is = 1 over 2 Pi dt X of 0 which of course is = 1over 2 Pi this is the Wiener Khinchin Theorem okay.

Sometimes the there is a wrong impression that the Wiener Khinchin theorem simply says that the power spectrum is defined as the Fourier transform of the correlation function no not true there is a non-trivial theorem here it requires proof which again I emphasize we have not given you have only done what some of the manipulations. But it is possible to show that if the process is Ergodic and stationary.

Then the power spectrum defined as that sampling integral squared is = the Fourier transform of the correlation function autocorrelation function. Now of course we have assumed stationarity here we have assumed all the properties of Ergodicity a stationarity etcetera but it is an exceedingly useful theorem in this form it is very useful either in this form or in this form okay you can write it either way you like. Let us see what it tells us what it specifically does let us look at some specific instances.

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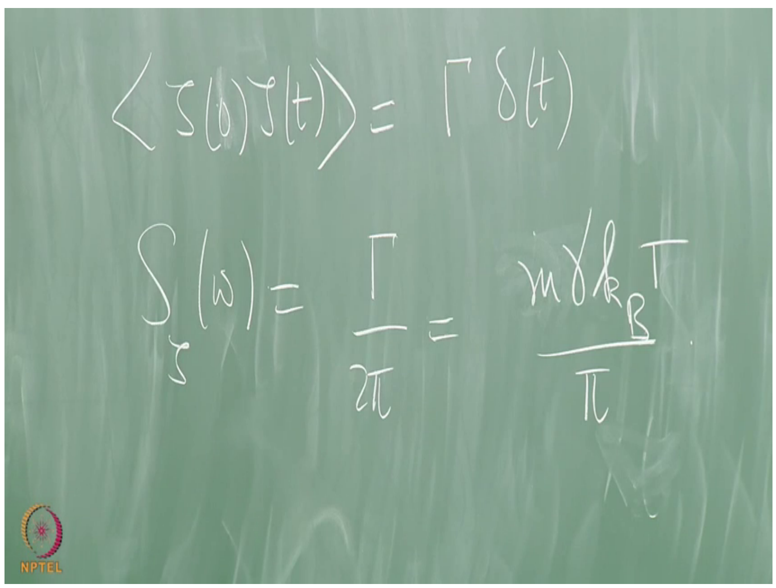

So, let us look at the line Langevin particles the particle that we talked about including the Gaussian white noise. So, if you recall our equation was mV dot + m gamma V in the one component case was = square root of gamma over square root of gamma times Eta of t where this was a delta correlated Gaussian white noise okay. Let us let us call this whole thing let us call this 0 then we know what the correlation of Zeta of t is Zeta of t is a 0 mean process satisfying rate of 0 Zeta of t = capital gamma delta t.

It is Delta correlated and it took this strength in here inside here. So, what is the power spectrum of this S Zi Omega = what is this = all we have to do is to use the way in a Khinchin theorem namely substitute it in there and that is it that is the end of the story right. So, if I put in here a gamma times Delta function at t = 0 it just brings out the gamma nothing else right. So, this is = gamma over 2 Pi and remember this gamma was 2m little gamma kt with two cancels.

So, it is gamma m in gamma k Boltzmann T over Pi and that was it and we could ask what is the power spectrum of the output variable of the velocity itself what does that look like etc.

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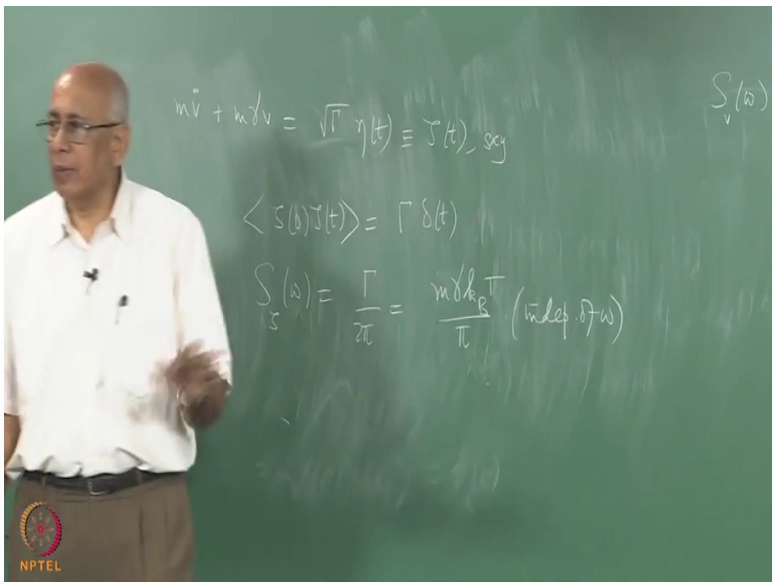

What is that going to be so the output variable has got Sv of Omega = once again this is = integral from 0 to infinity or 1 over 2 Pi -infinity to infinity dt e to the i Omega t times V of 0 V of t but we know what we have 0 V of t is in equilibrium we computed it its kt over m times e to the -gamma mod t we computed it. So, let us write that down this is = by the way you could write this out as twice 1 over pi this guy here so it is = k Boltzmann T over m Pi times dt e to the i Omega t e to the -gamma whatever sorry cosine.

Once you have written the symmetric part then it is just cosine Omega t once you have written it as 0 to infinity it is a cosine, so this is = e to the -gamma t so it is = gamma kt over 1 over gamma squared + that is not white noise white noise is something whose power spectrum is constant because it is Delta correlated and you immediately get a constant this is independent of Omega but this has got a Lorentzian shape you know maybe it drops down.

Essentially what the power spectrum does is to measure the intensity of this noise in a given window about any frequency Omega in a small window Delta Omega about Omega tells you how much of the noise if you like? How much of the amplitude this intensity is sitting there? Right and this says that it drops as a function of Omega variable this is unrealistic because it says it is got, the same power everywhere for all frequencies from 0 to infinity which is obviously unphysical. The moment you put a finite correlation time it will drop to something like that.

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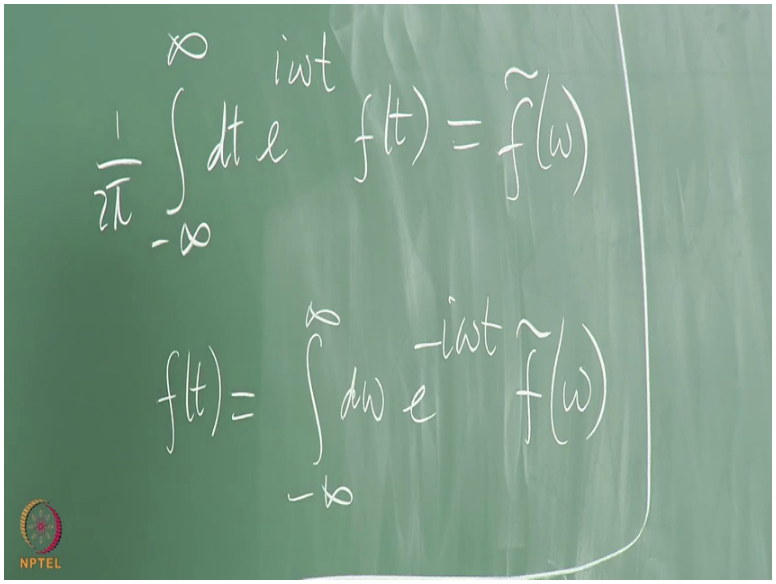

Is there a connection between this and that there should be because there is a connection heuristically and take Fourier transforms on both sides look at what is going to happen by the way our Fourier transform convention was to say that if you give me a function of t then 1 over 2 Pi -infinity to infinity dt e to the i Omega t f of t = f tilde of Omega that is our Fourier transform convention.

So, it also implies that f of t = integral -infinity to infinity the Omega e to the -i Omega t f tilde for me so we stick to this convention so that these factors remain kept track of carefully. Now let us look at this equation and kind of take Fourier transforms on both sides to see what happens.

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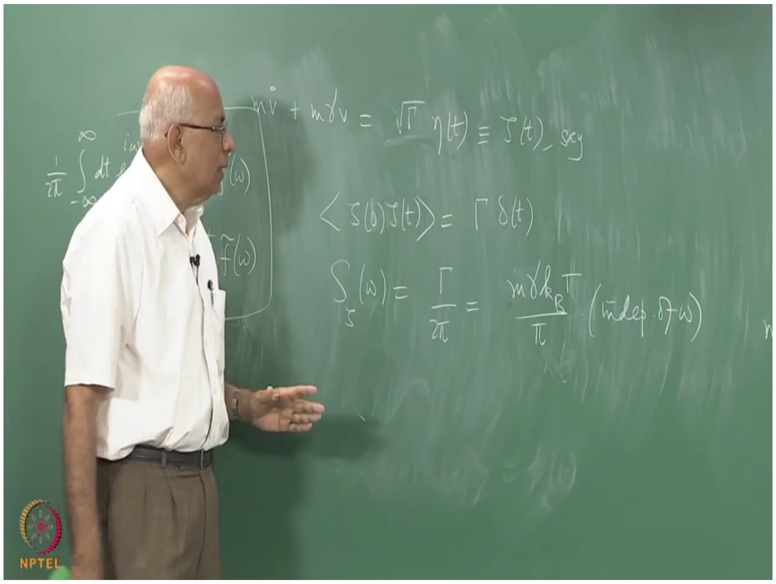

Then m times V dot well if I write V of t in this form and do V dot I pull down a -i Omega so the effect of Fourier transform is to take the derivative with respect to t and replace it by -i Omega so m times -i Omega + gamma on V tilde of Omega is = Zeta tilde of Omega or V tilde of Omega = 1 over m -i Omega + gamma Zeta tilde. The moment you have a relation of this kind in general you have one function related the Fourier transform of one the output variable related to that of the input variable through a susceptibility of this kind this is the called the dynamic mobility in this particular case.

Then the power spectra are related by taking simply the modulus squared of this susceptibility okay that is a relation which can be proved in some generality. So, let me state here without going through the details that this automatically implies that SV of Omega must be = 1 over m into -i Omega + gamma mod squared S Zeta of omega it implies that. Now all we have to do now is to put work backwards and check if this is true or not.

So, is that true we have already got we already had this guy here yes for the velocity was = this and now if I take this quantity here this is = 1 over m squared gamma squared + Omega squared SJ omega, so is this true I take a Zeta of Omega which is this and divide by m squared times gamma squared + Omega squared and I get this precisely so it checks out in this case we knew already the velocity correlation but if I did not know it I can now find it by using this relation.

We did this by a long procedure of actually writing down the distribution function for this V the solution that separates a traveler this autocorrelation actually what we did was to solve a Langevin equation and from the autocorrelation etcetera but you do not need to do that all you need to know is this relation in relation. So, in more complicated instances where you may not be able to write the explicit solution down so easily you can still write down what is the power so how are the power spectra related to each other okay.

So, this is a useful trick to write down the power spectrum of the output variable given that of the input variable. There is a name for this thing here what is this guy called engineering parlance the transfer function it is a transfer function it is exactly what it is it is the mod squared of what physicists would call the generalized susceptibility in this case the mobility okay.

So, what it is telling you is if you give me a unit applied force of frequency Omega the steady state response will also be of frequency Omega and it will be attenuated by a complex number called the generalized mobility generalized susceptibility which is this quantity. What happens if you have more than one component now things get a little more, tricky we have to be a little careful here.

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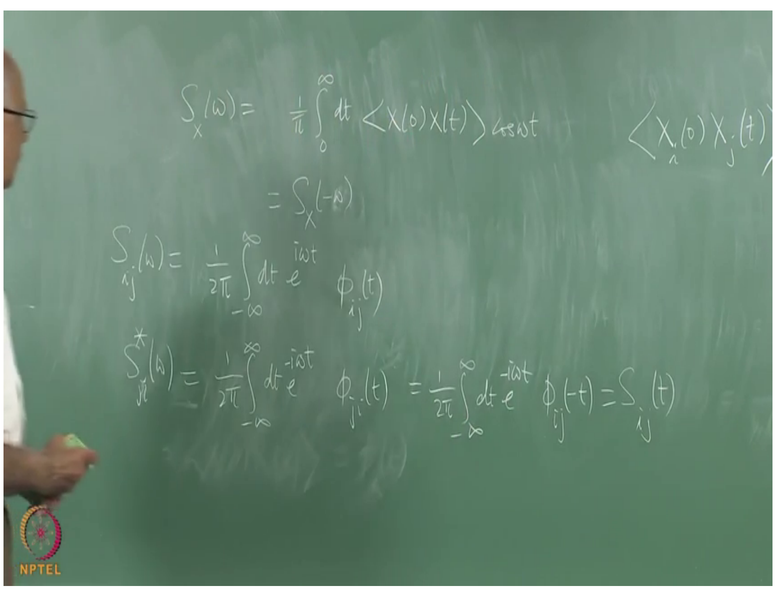

Let us look at a physical example we actually went through one where we had more than one component so there what I do is if you have got a whole lot of components of some vector process I would define Sij of Omega Oh incidentally one small property which is easy to understand we found that SX of Omega was = twice integral let us let us get all the Phi factors right was = 1 over Pi integral 0 to infinity dt X of 0 X of t cos Omega t and this is = SX of – Omega.

So, it is a symmetric function of the frequency formally that is a useful piece of information that is one of the symmetries in the problem right. Now let us look at it in the more complicated case when you have more than one component right. So, if you have a thing like Sij of Omega = 1 over 2Pi which is the -infinity to infinity dt e to the i Omega t Xi of 0 X j of t the power spectrum now becomes a matrix if i and j run from 1 to n for example it is an n by n matrix with these elements.

And there are all these cross correlations sitting here we still assume the process is stationary so that all this every one of these averages function of the time difference alone then the question is what is the corresponding property out there. But you can see by stationarity the following is true you can see that Xi of 0 Xj of t let us call this Phi ij of t defined by Phi ij of t but that must be = Xi of - t Xj of 0 by stationarity.

Because I stagger the time argument by - t on either either side but this is = Xj of 0 Xi of - t which is = Phi ji of t pardon me of Phi ji yeah mistake yeah thank you. So, we have this property here and therefore the symmetric and anti-symmetric parts of ii ij of t would be respectively even an ardent time right. So, this implies that Phi ij of t + Phi ji of t this is the symmetric part of the tensor Phi ij of t and that quantity is even in time and the odd part is odd in time okay.

There the anti symmetric part of the tensor is odd in time that follows in a straightforward way yeah what we need here is the in this place is the following so let us look at Sji of Omega = 1 over 2 Pi integral -infinity to infinity dt e to the i Omega t, so let us call this Phi ij of t can be done with it Phi ji of t but we just saw that Phi ji of t is Phi ij of - t okay and let us take the complex conjugate on both sides.

So, it is me - this guy here and a - this guy here this is real because my X is a real-valued random variable but now I change t two - t in this integration and this gives me Sij, so what is the higher dimensional counterpart of this symmetry property of s this says the power spectrum is an even function of the frequency for a single random variable for a random process a scalar random process.

In the moment you have a multi component process it says the ij component Omega sorry function of Omega it says the ijth component of that tensor is = Sji Omega star, so what does it say about this tensor or Sji of Omega this matrix it is a Hermitian matrix right.

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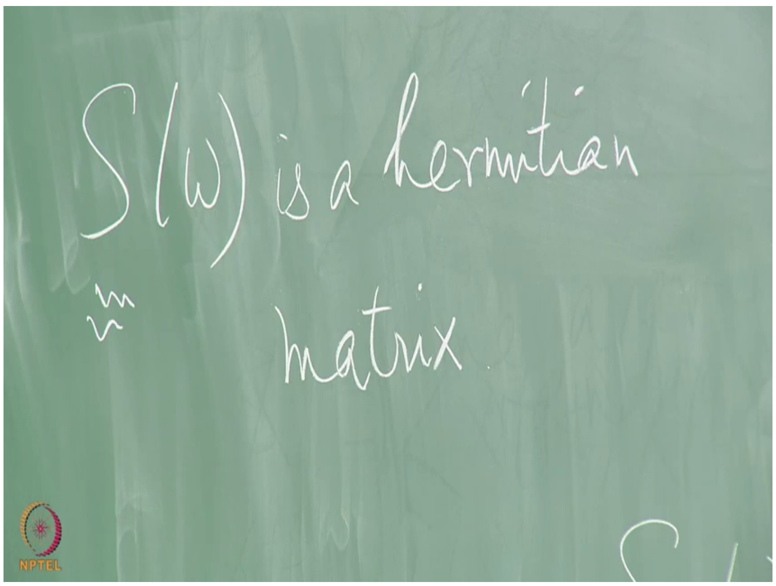

So, this matrix S whose elements are Sij of Omega is a Hermitian matrix and we can write a generalization now of the Wiener Khinchin theorem in this case. So, if you have a month we have done this already in one example let us let us take a look at that instance right.

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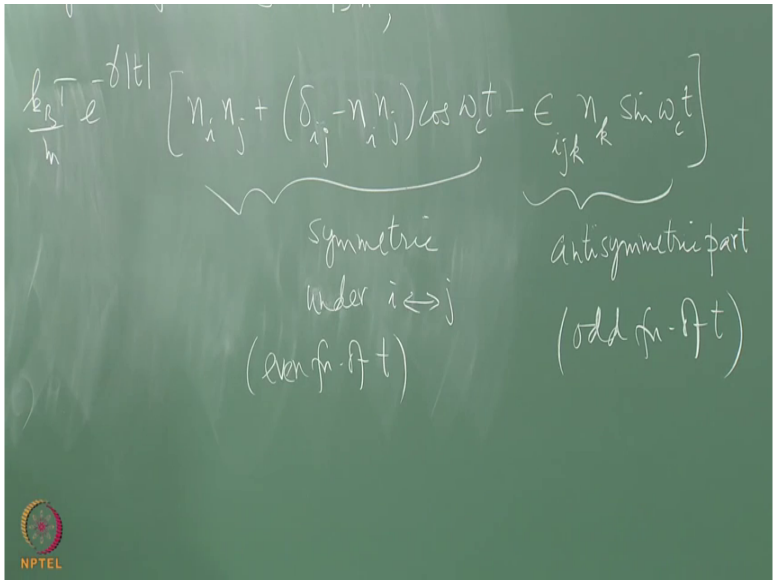

So, again going to the example of particle in a magnetic field remember that we computed for a particle in a magnetic field B = B times some unit vector in n direction we found the following we found that Vi of 0 Vj of t this correlation function by ij of t we had an explicit expression for this quantity here right this was = what it was k Boltzmann T over m that is always sitting there e to the -gamma modulus t sitting there to multiplied by;

If you recall there was a portion that depended on ni nj and a portion which depended on delta from the Kronecker delta and then there was an anti-symmetric portion. So, this was ni nj + Delta ij - ni nj cos omega the Cyclotron frequency times t - epsilon ijk nk times sine, so this is the symmetric part in i and j symmetric under ij interchange and this is the anti symmetric part.

Now what about the time reversal properties of these quantities we already saw what is going to happen we saw that Phi IJ the portion the symmetric part of this tensor must be an even function of time and the anti symmetric part must be an odd function of time right. But that is exactly what is happening so this is symmetric or even function of time and that is an even function of t and this guy is an odd function.

In the diffusion tensor this portion I stated did not make it the odd portion did not appear at all we did not actually derive that formula for the diffusion tensor but I made it as a statement I said that this portion does not contribute to the diffusion tensor at all it is only Phi ij + Phi ij of t + Phi ij of - t integrated from 0 to infinity which was = proportional to the diffusion coefficient. But the odd part remains I mean it is sitting there and so on and it will contribute to the power spectrum and to the mobility and so on.

I have not talked about this maybe I will this will contribute to the so called Hall mobility so there is the contribution which is not the usual current but the Hall current and that portion will make a contribution to it we have not looked at this in great detail we have not done we have not talked about the general and the linear response aspect of this particle in a magnetic field but this also has a physical significance it is not it is not useless okay.

So, if you give me a general process I can write down using the Wiener Khinchin theorem I can write down the power spectrum of this process and then it gives me a great deal of physical information. In particular what it does is this relation that it is = the Fourier transform of the correlation function this quantity here is in fact the response function in linear response theory. So, when you apply an external stimulus and you ask what the response of the system is like it gets proportional to this thing here.

Now what linear response theory does is essentially is in the context of statistical mechanics it is first order time dependent perturbation theory together with the statistics that the statistical mechanics classical or quantum that you need and that is essentially what it is so this autocorrelation this is a cross correlation function here is in the absence of the external perturbation and that measures the response under the external perturbation to leading order in the perturbation in external force okay.

So, that is the sort of gist of linear response theory in some sense and this is something they have not specifically talked about if time permits we will come back and make a few comments about linear response theory. But I thought that one should know this because the reason is that what we are going to do is to go on to use this power spectrum the concept of the power spectrum to look at what kind of power spectra are generated by different kinds of noise.

We already have one statement that white noise will correspond to a flat power spectrum and the kind of response we had the Langevin of our particle for example has a Lorentzian power spectrum goes with high frequencies like 1 over Omega squared by the way you have used to this in another language.

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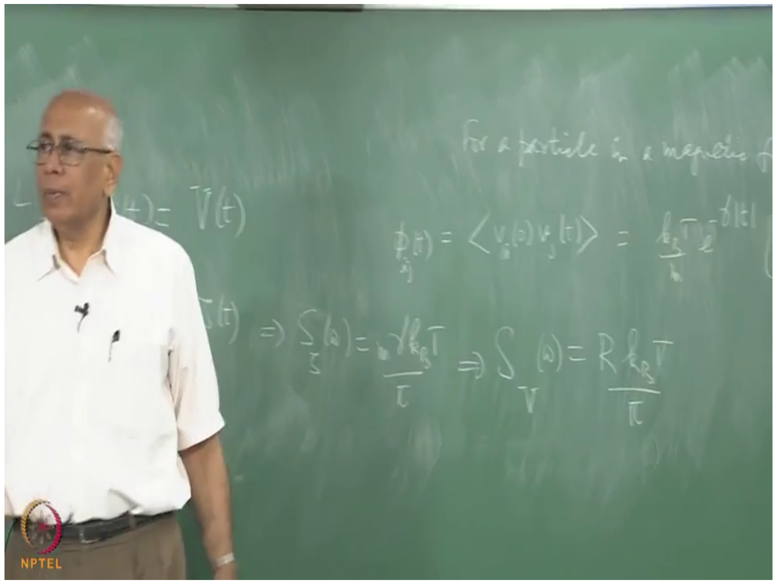

So, let me write that down point out that is exactly the same thing that we are talking about if you look at resistor R at account at some finite temperature then of course there is Brownian motion of the electrons and that leads to a instantaneous voltage across the ends of this resistor and then there is a fluctuating current. So, one could ask what is the power spectrum of the noise like what is the power spectrum of the current like and so on.

This is called Johnson noise if you measure the power spectrum of this voltage and there is a relation called the Nyquist relation which tells you what it is it tells you it is essentially proportional to the resistance and it is proportional to the absolute temperature which is why I would like to lower the temperature to reduce this noise here and that comes about very easily because this resistor is always got a self inductance.

So, it is like effectively an inductance and a resistance in parallel in which case this L so you have L di over dt + R of t = the voltage we applied or spontaneous we do not care what right in this fashion. Then this is the same as our problem was m dv over dt + m gamma V of t was =

Zeta of t and then we found that in this problem S Zeta of S Zeta of Omega was = what was it you have to tell me the factors now.

This is some gamma kt m gamma k Boltzmann T over Pi or something like that yeah with the 2Pi over Pi yeah. So, the correspondence between these two guys is that in this electromechanical analogy is m is this and m gamma is the equivalent of the resistance R that is the correspondence between the two. So, this immediately tells us that the this thing will also imply that s the voltage of Omega = m gamma which is R k Boltzmann T oh that is the form in which you are familiar with it in terms of Johnson noise right.

No almost what form are you familiar with pardon me same from 4Rkt for the factor 4 actually yeah.

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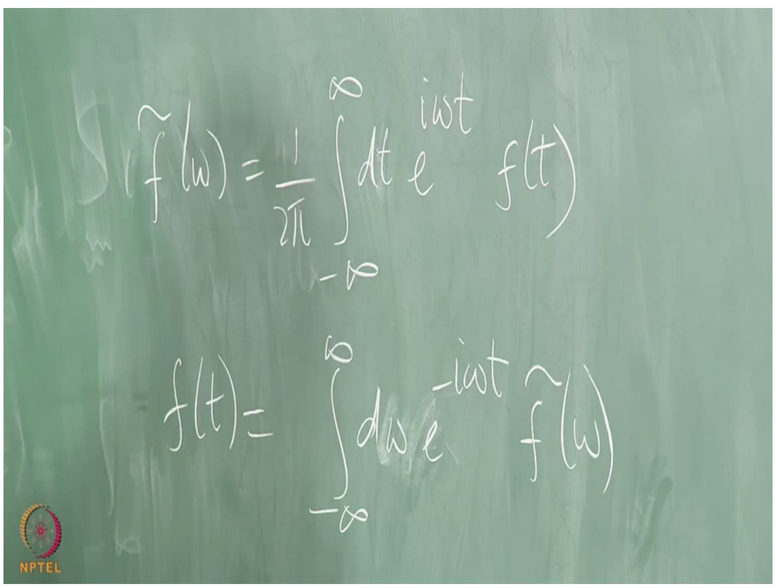

What happens is the following our Fourier transform convention was to say that f tilde of Omega was 1 over 2 pi integral -infinity to infinity dt e to the i Omega t f of t and correspondingly f of t was = integral -infinity to infinity d Omega e to the -i Omega t but the electrical engineers use a convention in which the 2Pi factor sits here 1 over 2 Pi and not there so an extra 2 Pi factor they also define the power spectrum as twice the Fourier transform.

So, there is a 4Pi factor which t multiplies this whole thing so for them this is not true it is the multiplied by 4 Pi this is = 4R this is surely familiar right that is the form in which it is written in textbooks, so this factor 4Pi is there I mean it is it is really there it has to do with the convention teachers + the fact that you defined it as 4 times the for return right twice the Fourier transform but I just chose the simplest convention.

And I chose this purely as a matter of convention there is no because this is the one that is most convenient in the usual formalism of linear response theory where you have a one sided Fourier transform with a + sign here for the generalized mobility okay the susceptibility and if you use that convention then this generalized susceptibility has no singularities in the upper half plane in omega and has singularities only in the lower half plane.

So, it was to ensure that that I needed a + sign here and the 2Pi was a matter of convention because it corresponds with what is used when you go from spatial Fourier transforms from the X to k, so I just wanted to keep that whatever it is you do you have to stick to one convention. So, this is the usual Johnson noise or whatever there is also other kinds of noise like short noise, semiconductor noise of various kinds so we will talk a little bit about that subsequently okay. So, let me stop here today.