Physical Applications of Stochastic Processes
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Lecture-18
Diffusion of a charged particle in a magnetic field

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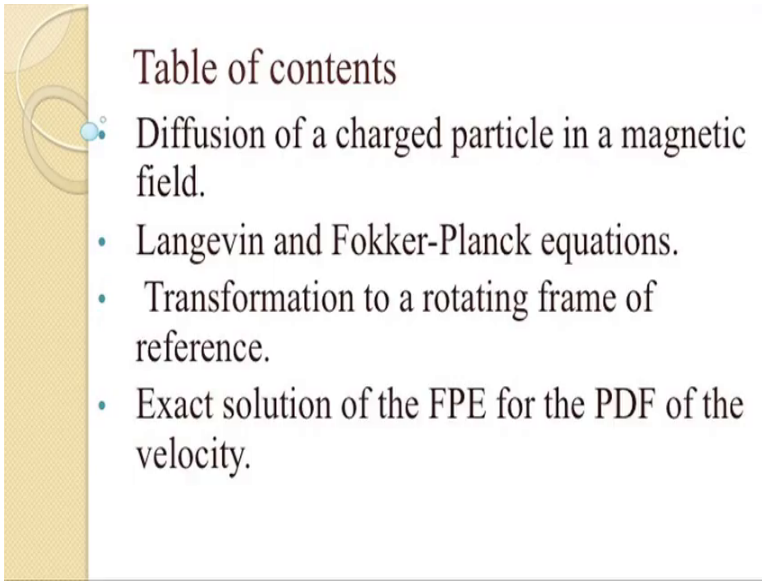

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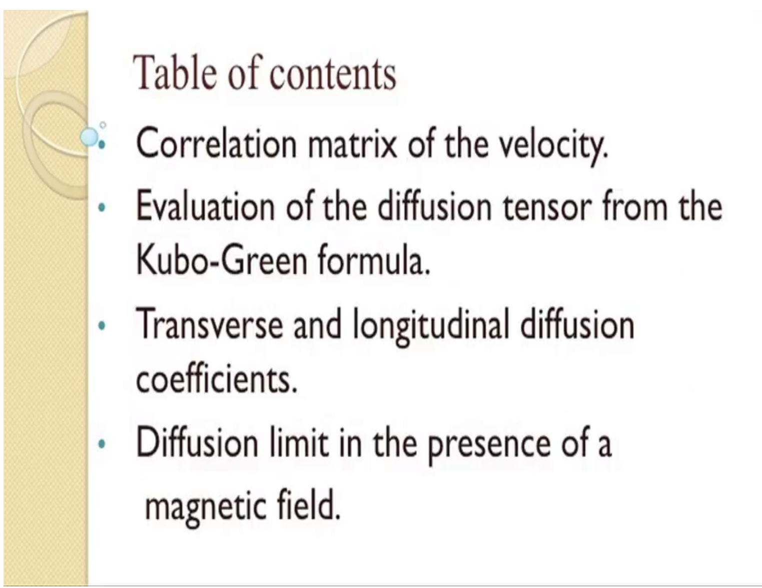

All right today we will try to work out the problem of the diffusion of a free particle in a fluid in the presence of an external magnetic field.

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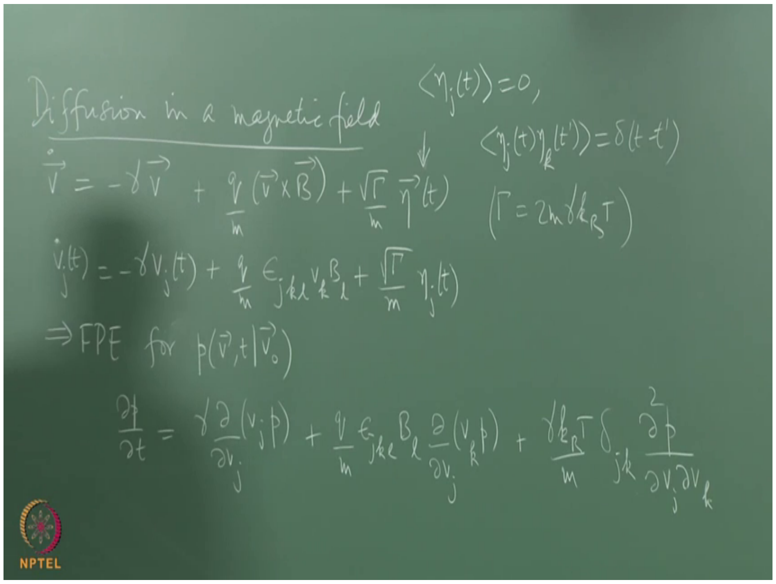

So, this is diffusion in a magnetic field I will assume a field is constant in space and time and there is a magnetic field B and we have a particle of charge q and mass m moving in this field as usual the particle is imagined to be inside a heat bath at some fixed temperature t and the question is what is the diffusion look like what does the diffusion coefficient look like in this case. It is a simple exercise because as we know the Lorentz force on the particle due to the magnetic field is linear in the velocity.

And as soon as we have a linear problem the Langevin equation can be solved explicitly. So, let us write it out step by step we look at the velocity distribution as well. Let us start by saying that the particle has a velocity V and V dot is = - gamma times V that is the usual friction and we put in + q over m that is the charge over the mass times V cross B some constant magnetic field B is applied and then there is a usual noise term which as usual we write as root gamma over m Eta of t and this is Gaussian white noise.

So, its properties are any component Eta j of t as 0 value average and a Eta j of t Eta k of t prime is Delta t – t, so it is the usual Gaussian white noise and this is the extra term out here okay in component form we need that. So, let is write it out V j dot of t = - gamma V j of t + q over m and we need the j component of this cross product which of course is epsilon jk l Vk Bl + square root of gamma over m Eta j of t for each Cartesian component we have this and the components are mixed up with each other because of the magnetic field okay.

Now as always we are going to say gamma is 2 m gamma k Boltzmann T that is the fluctuation dissipation relation that does not change and the first thing we got to ask is what do we expect? We expect that when you have this magnetic field on the Maxwellian distribution of velocities of the particle in thermal equilibrium is not disturbed. Because the magnetic field does not do any work on these particles it does not accelerate them so as to change the kinetic energy.

Merely changes the direction of the velocity for every particle but it does not do anything else. So, physically I expect that the equilibrium distribution will be Maxwellian will continue to Maxwellian value. However the Fokker-Planck equation which is the conditional density equation satisfied by the conditional probability density of the velocity will of course show you there show a presence of this field it will certainly have an effect. But asymptotical as t tends to infinity I expect it will tend again to the Maxwellian distribution. So we look at the velocity Fokker-Planck equation.

We could also look at the phase space Fokker-Planck equation the full one in r and V because we know how to do that we know how to write down the Fokker-Planck equation for the phase space density Rho as a function of r, V and t in the presence of a linear force of this kind so we will do that as well and then we try to compute what is the distribution etc. So, corresponding to this we can write down the Fokker-Planck equation immediately at once.

So, if you write it in this language for example this will imply the Fokker-Planck equation for p of Vt given some V naught so given some initial velocity V naught we can write down what the Fokker-Planck equation for this is and that is going to read Delta p over Delta t = you know - sign there so it is gamma times Delta over Delta Vj Vj p that is this part is taken care off + you can write it in a number of ways.

But here is the most convenient way of writing this + q over m epsilon jk l Bl that is a constant so that comes out and then Delta over Delta V j of Vkp in that fashion + half this guy so the square of it divided by 2 and the square this factor will go away and then this gets squared so it is gamma kt over m K Boltzmann T over m Delta jk B2 p over Delta V j this is essentially the del squared operator as you can see in velocity space.

So, that is the Fokker-Planck equation satisfied by the conditional density here for a given velocity. So, the initial condition on this is just a delta function 3 dimensional Delta function and V = V naught.

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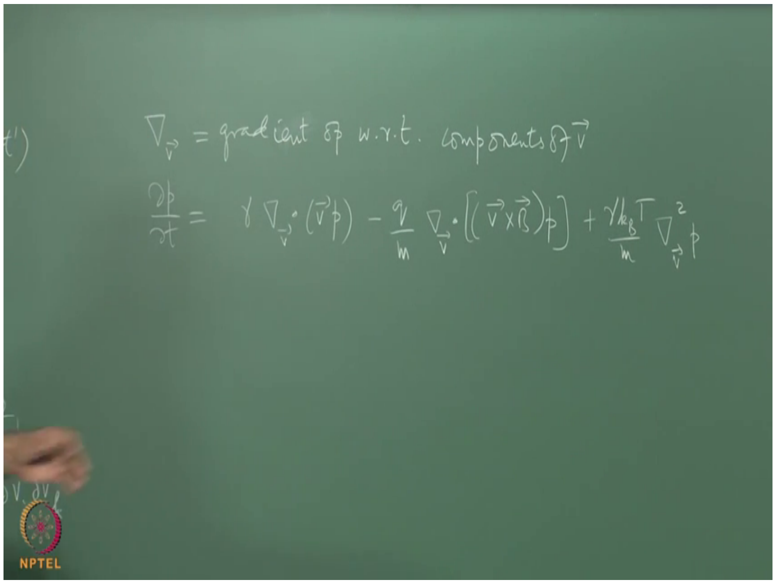

Now we can write this in vector form as well so let us do that let us put del V = gradient operator with respect to velocity components, so just for notation let us call that del V otherwise I write del it is could be with respect to position as well so let us just call it del of V and then what does this say it says Delta p over delta t = gamma times Del V dot Vp it is just the divergence here as you can see because it is the same j.

And then this guy here gives you - sorry you have got to be careful with the signs this was a + here so this equation has a - the drift comes with a - f of x if you remember so - q over m Omega we could write this down directly as it is. So, it is Del V dot V cross B times p, so it is the divergence of that that is the drift part of it + gamma k Boltzmann T over m Del B squared on p. So, that is the equation satisfied by that is the Fokker-Planck equation okay.

The task is to solve this equation subject to this initial condition but there were a little bit of simplification it is convenient as you can see I have taken the magnetic field to be in some arbitrary direction it is convenient to do that rather than specialize to the Z direction or anything like that but you can see the structure of this whole thing coming out more interestingly if you keep in general direction.

So, let us say that the magnetic field B = B times some unit vector n in some arbitrary direction right then what you have here is qB over m.

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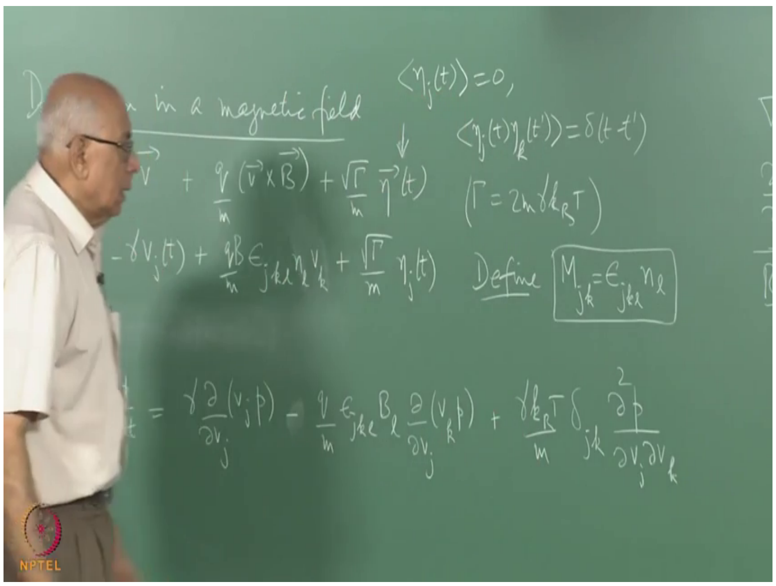

So, let us simplify this equation a little bit in fact let us simplify this equation so for Bl I am going to write B times n sub l and then I put a B here n sub l Vk and let us define a matrix m. So, let us define a matrix m jk to be = epsilon j kl nl and a matrix whose elements jk are given by epsilon jk l nl it is clear that this is an anti symmetric matrix because the epsilon symbol is totally anti-symmetric in any two indices.

So, that is one fact we need to know right away and m jk = - m okay it is anti symmetric then if you look at this for instance or in that language it does not matter which way you look at it this is going to have two derivatives one of which is going to be Delta Vk over Delta Vj that is Delta jk times p but Delta jk with epsilon jk contracted will give you 0 because this is symmetric and that anti symmetric and the other term is vk times Delta P over Delta Vj nd that of course survives right.

So, this means that you can simplify this a little bit and write this as del V on p dotted with V cross B because the del V dot V cross B is 0 identically right. So, divergence of a vector times a scalar is a scalar times the divergence of the vector + the vector dotted with the gradient of the scale and the first term vanishes in this case. So, we could also write this as V cross B dotted with Delta V, so a little simplification.

Now what does this do this thing is what we called m jk, so let us call this m jk and q B over m is a direct physical significance it is the cyclotron frequency for a charged particle of charge q and mass m in a field of magnitude B. So, let us call this Omega cyclotron, so out there we have Omega C okay. Now we could either try to solve this equation directly or we write this Fokker-Planck equation down and try to solve.

This does not matter either way we would like to get at the velocity correlation function and after that we try to use the Kubo formula to find the diffusion constant. We will also find the diffusion constant by another method going to the high friction limit and seeing whether we get the same answer or not this is what we want to check out. First before anything else this thing here what does it do.

Once you have once you have a magnetic field applied in some constant direction and you have some velocity to start with V naught and I apply this field what happens to this V naught is that it starts processing around the direction of n the unit vector n right. So, this is all that happens I mean if you look at without any noise without a damping without an external without anything except the external field just a free particle and it satisfies an equation of this kind V dot = Omega C M this is a matrix times V that is what this equation is this is = that.

Well I have written as V dot is q over M V cross B in this language okay. Now what is the solution to this equation for a given initial condition? So, the initial condition is V naught, so what happens is that if you have this as the unit vector n and you have a velocity V naught here you know that at time progresses this V naught processes in a cone around this direction of n. This means that the component of V along the field does not change whereas the component the perpendicular components the transverse components they perform circular motion with a phase difference of Pi over 2 right.

So, Vx and Vy exchange roles they move in a circle and you have this processional motion. So, what is the solution to this guy here the formal solution to this thing will imply that V of t = e to the M Omega C t acting on V naught that is the formal solution to this guy right and what we have M is a rotation matrix as we will see in a minute and we want to exponentiate it right and the angle changes at a constant rate Omega C.

So, what we need is a finite rotation formula in this case many ways of writing this down I am sure you are familiar with this from familiar what happens if you have a given vector r and then you have some direction unit Direction n and you make a rotation about this direction rotate the coordinate system about this point through an angle side.

You get a finite rotation formula yeah it is actually the generator this guy is like a rotation matrix through an angle Omega Ct as you can see right. Now of course you need to do this you need to compute this quantity but that is not very hard to do computing this quantity is not hard to do because let us do that as a side exercise.

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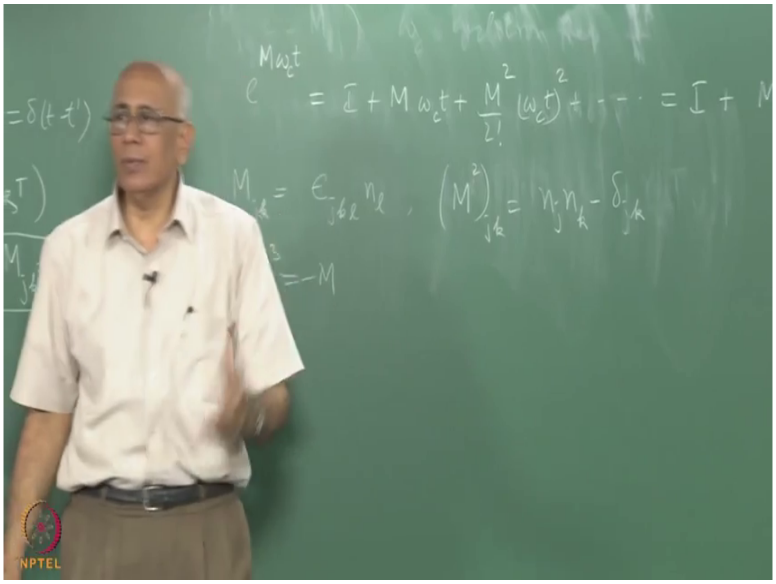

So, e to the M Omega CT = I + M Omega CT + M squared over 2 factorial Omega Ct whole squared dot, dot, dot etc. Now what is it about M that strikes you immediately because M Squared is going to be interesting M has elements jk = epsilon jk l nl where this is a component of a unit vector and what is M Squared jk = well you got to put two of these epsilon and then use the fact that this is a unit vector etcetera and simplified.

Using this formula for what happens when you take two epsilon and contract one index you get a product of Delta functions etc then Kronecker deltas right. So, in this case this turns out to be = Mj nk - Delta okay and what does mq become well you put another n here, you put another n here and use this fact here and I am sorry not ije change and k becomes = - m okay, itself check this out so, M 4 is - M Squared and so on.

So, therefore the exponential collapses completely and you can write down a formula for this guy. So, this thing here is = if I exponentiate it fully it is I + M sine Omega Ct + M Squared 1 - cosine Omega C because you see the M Squared guy starts with Omega C squared over 2 factorial so it is you have got to put that one in separately. And it is of course cost cosine has - t squared there is got a + here so it is 1 – cos.

And therefore you can write this down explicitly I do not fully what this thing is so it says this guy by the way if I change this + or - all that happens is t goes to - t so this becomes + or - well that is an interesting little exercise to play with this and it will reproduce for you the finite rotation formula.

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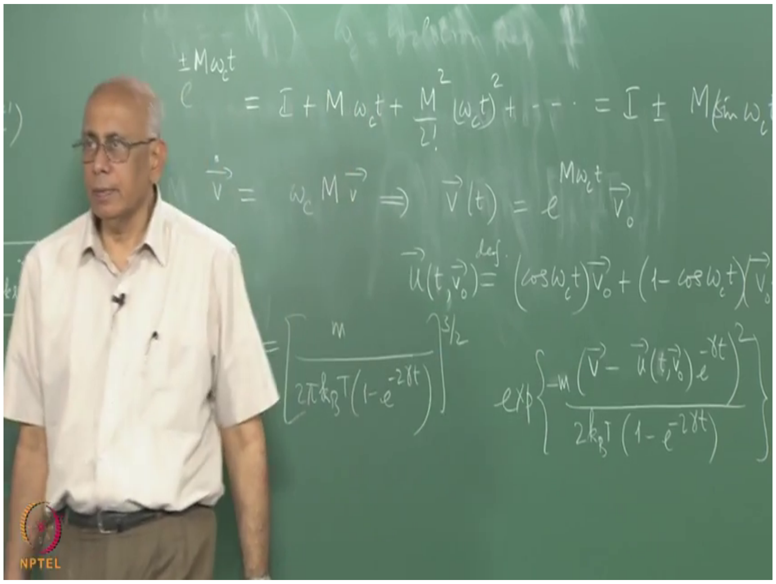

So, let us write that down let us see what happens the formula in fact is that if you have a unit vector n and I take a certain vector A and rotate it about this direction n rotate the coordinate system through an angle Psi then a goes to A prime which is precisely A to the M Psi acting on this original vector A, so it is going to be = A cos Psi + 1 - cos Psi times n dot A unit vector in the direction n.

So, if I rotate this there are only 3 possibilities one of them is its along the original vector A the new vector has a component along A it has a component along in and a component perpendicular to these two guys right. So, this + sin Psi times A cross this is called the finite rotation formula can establish this for the coordinates and after that of course a vector is a quantity which transforms exactly like the coordinates and therefore any vector would transform in this fashion.

So in fact we can write down the answer now to what the solution to the Fokker-Planck equation is explicitly. Because I have already said we have given physical argument to show that the energy does not change at all that happens is that the velocity starts processing every velocity processes yeah. Pardon me good question so I am rotating the coordinate system about this point a lot about the direction of the unit vector n.

So, if the original coordinate system had an x axis like that it now has a thing like this and the amount of rotation in that plane is Psi the angle with which you have rotated to go to another point on the tip of this cone. In our problem this rotation is happening all the time the precession. So, Psi is replaced by Omega Ct so what it is saying is that if you just had a magnetic field and you looked at just that original equation.

So, the equation is V dot = Omega C M acting on V this will imply that V at any time t would be = a to the M Omega C t acting on V naught and that can be written down this is = cos Omega Ct times V naught + 1 - cos Omega Ct times V naught dot n times n + sine Omega Ct times we know cross n, so that is the explicit solution in general to what happens to any initial velocity V naught at time t this is what it does okay. Let us give this a name let us call this u as a function of time and of course the initial velocity V naught. So, let us define u to be this, the magnitude of u is the same as a magnitude of V naught nothing happens it just processes.

Now what do you think will be the solution to this guy we can do this painfully but we can write this down on physical grounds Vt you know what do you think this will be in the absence of the magnetic field this is the on t Nolan Bach distribution and the on t Nolan Bach distribution has the variance goes like 1 - e to the - 2 gamma t and the mean value decays to 0 like e to the - gamma t so the original thing distribution was e to the - V - V naught e to the - gamma t whole squared.

Now instead of V naught you are going to have u of t comma V naught that is all that will happen so we can write this solution down it is going to be = M over 2 pi k Boltzmann t 1 - e to the - 2 gamma t the whole thing to the power 3 halves because you have got 3 components and then this 3 dimensional Gaussian e to the exponential of - V - u of t ,V naught that is u vector e to the - gamma t whole squared divided by of course there is n - M over 2k Boltzmann T 1 - e to the - 2 gamma t.

So, that is the solution to the Fokker-Planck equation all that happens in the magnetic field is that this V naught gets rotated keeps rotating and the damping is exactly as before that happens because the Langevin equation has a - gamma V on the right hand side it dumps it out. And the portion V cross B this portion the linear drift but it is a reversible part this is the reversible dip this is a reversible it due to dissipation and of course that also represents the effect of dissipation.

So, you can see from this you can check backwards that it satisfies the Fokker-Planck equation but you can see from this there are t tends to infinity this goes to the Maxwellian this part goes away it is M over 2 pi K t to the 3 halves e to the - MV squared over 2 kt, so it certainly goes to the Maxwellian distribution as t tends to infinity that is the equilibrium distribution the Maxwell in 3 dimensional Maxwellian distribution.

Our task is slightly different we would not actually find out what happens to the diffusion how does it diffuse right for that we already have a formula. We have a formula which says that the integral of the velocity correlation from 0 to infinity is in fact the diffusion coefficient. We explicitly showed that by looking at a long time behavior of the mean square displacement. Now we have indices to worry about so we got to be a little more cautious.

So, let us see we can do that and we do not have a diffusion coefficient we have a diffusion tensor now because the coefficient here when you write the diffusion equation down since we have 3 components we are going to have a diffusion tensor in this business. So, let us see what this is, well let us first find what the correlation velocity correlation is. So, we will go back and use the fact that we know the velocity is a stationary random process here. So, since we know that we can do this little heuristically.

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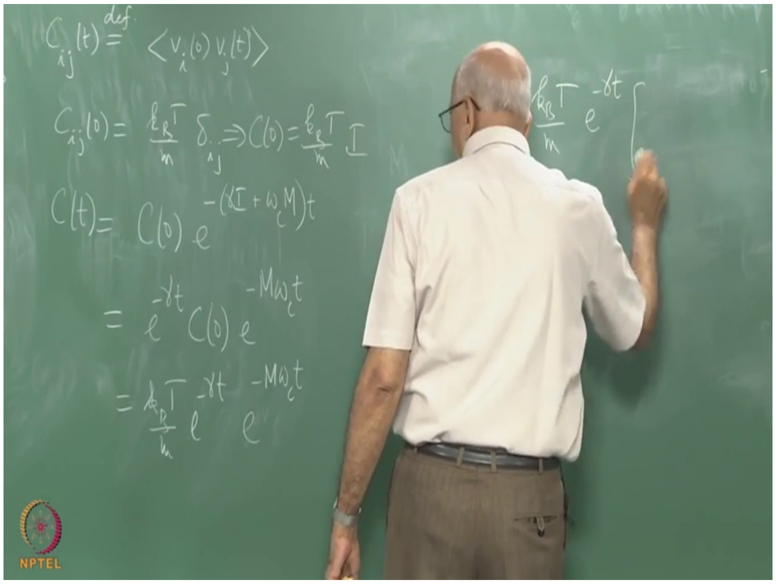

Let us write Vj dot of t = - gamma V j of t + and then there was an Omega CM and if I recall right it was Mjk Vk of t and then there was a portion which was + square root of gamma over m Eta j of t that was the Langevin equation right we would like to find out what this quantity is we are seeking C ij of t define this to be = Vi of 0 Vj of t this is the quantity we want. This angular average is the full average in equilibrium.

Now what is this quantity = what Cij of 0 = what is this guy = so it is the correlation between the Cartesian component ith component at a given time 0 with the jth component at the same time no independent different Cartesian components are independent of each other this correlation is 0 in equilibrium. But when the two are equal it is = kt over m right. So, we know that this guy = kBT over m Delta ij for each component its kBT over m.

For each i = j it is = kt over m just the mean square velocity any given component. So, let us try to compute that what we need to do is to multiply this on the left by Vi of 0 and then take averages times this whole thing and then take averages. So, the left hand side says d over dt of C ij of t = - gamma C ij of t + Omega C Mjk Cik of t this is a constant matrix and then Ci, Vi of 0 Vk of t is Cik at time t + the average value + square root of gamma n.

The average value of Vi of 0 with Eta j of t what is this = that is 0 because of causality it says the for sent for any t greater than 0 cannot dictate what the velocity was is the velocity at an earlier time is not dependent on the force at a later time even at equal times this correlation is 0 it is the acceleration that is correlated to the force right, so, this vanishes there. Now computing this for t greater than 0 because that is really what we need for the diffusion coefficient.

When you do it for t less than 0 we can compute it you have got to be a little careful you got to be really careful because remember that without this magnetic field the formula I bought for the velocity correlation was e to the - gamma modulus t and now there are going to be terms depending on a magnetic field. So, when you take t to - t you have to be a little cautious because although this dissipation term will be e to the - gamma modulus t.

The fact is that as t goes to - t you should also be careful to reverse the sign of the magnetic field okay. So, for time reversal operation you when you have an external magnetic field time reversal is taking t to - t and the applied field B - B and the physical reason for that of course is that if this field is imagined to be from due to some electromagnets some electric charges moving if you reverse time the current flows in the opposite direction the field changes sign okay.

So, the time reversal property of B is actually B goes to - B unlike E okay. so, this term is 0 now this looks almost like a matrix multiplication except that you have a little problem there is a jk and an ik but of course we know that M jk is - M kj. So, we can write this = - gamma C ij of t + Omega C C ik, we put a - sign and let us write this properly Mk, so if I call C the matrix I have d over dt C d over dt of the matrix C of t this is a matrix.

Now = - C of t times gamma i + Omega C M on the right hand side because it is C ik and kj and what is the solution to this guy this implies that C of t = e to the power - sorry it is the other way C of 0 with e to the - gamma i + Omega CM and some acting on the right so I better be careful keep it there okay. Of course this is also = e to the - unit matrix commutes with everything so I can just bring it out so it is = e to the - gamma t V of 0 e to the - Omega C M - M Omega C and that is it.

But what is C of 0 yeah so this will of course imply that C of 0 = k Boltzmann T over m times the identity matrix Delta ij is just the component of the identity matrix. So, this becomes = k

Boltzmann T over m e to the - gamma t and all we got to do now is find e to the - M Omega C but we did that already you have already done it sitting here. So, we are in good shape we actually have an expression for the correlation.

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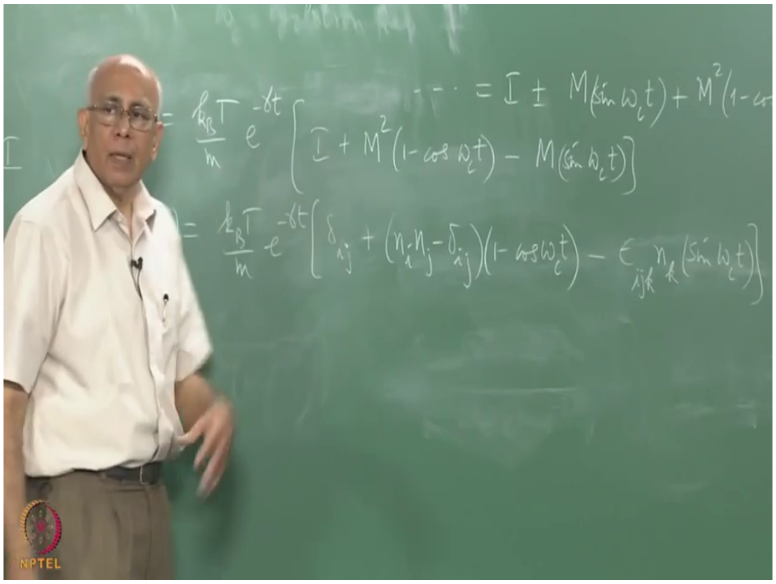

So, this says that C of t is = e to the - kBT over m e to the - gamma t times I + M Squared 1 - cos Omega Ct - because we got a - MC t M sine Omega Ct, so we have the matrix explicitly the correlation matrix and what we need is the ijth component of it, let us say C ij of t = k Boltzmann T over m e to the - gamma t what is the ijth component of I the Kronecker Delta of course Kronecker Delta Delta ij + n squared remember was ni nj - Delta ij 1 - cos Omega Ct speaking this guy here - and we call this fellow was epsilon we want ijth component.

So it is ijk nk sign Omega Ct, so we have an explicit formula for the velocity correlation. Remember that if you had a free particle in the absence of the magnetic field then the different Cartesian components did not get mixed up as a function of time and they were uncorrelated they started uncorrelated they remained uncompleted. Now because of the magnetic field it is scrambling things up so you have got all kinds of mixtures out here.

What we need to compute is the integral of this quantity over t from 0 to infinity but there is a little change in the formula here got to be little cautious here the COPO formula for the diffusion constant is now a formula for the diffusion tensor.

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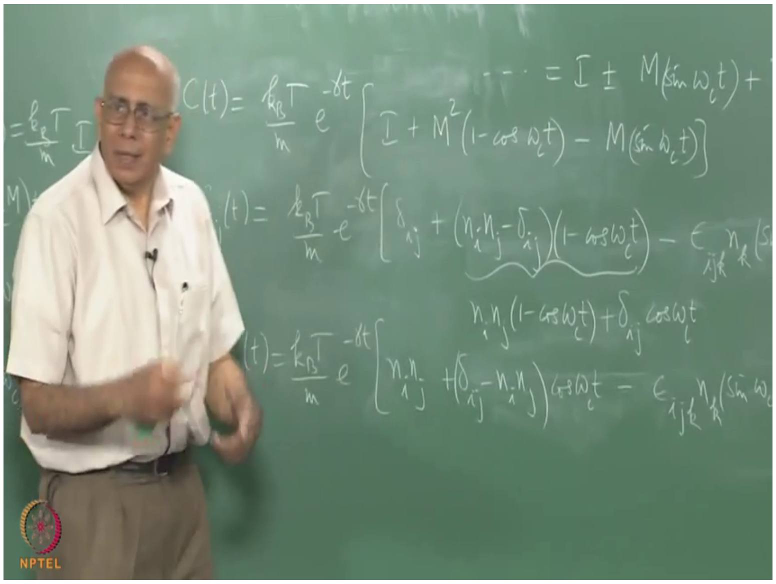

And it turns out that D which was = integral 0 to infinity dt V of 0 V of t originally. Now the correct formula is Dij the coefficient because it is a tensor now, diffusion tensor this is = dt C ij of t + Cji of t with a half it is the symmetric part of this coefficient of this tensor which turns out to be the diffusion coefficient okay. We can prove this without much difficulty I have not done linear response theory explicitly we have not talked about the COPO formula.

And how to derive it for the multi component case but in a couple of minutes we will try to corroborate that this is indeed true. So, I will write down the diffusion constant directly by looking at the high friction limit. So, what we need actually is not the integral of this but the integral the symmetric part of it right. So, this says Dij = the symmetric part this part is symmetric this part is symmetric this part is anti symmetric ni, nj and so when I take the symmetric part it vanishes it does not contribute.

And there is a factor half I can get rid of that half by this twice dividing by 2, so this immediately becomes k Boltzmann T over M by the way a little bit of simplification here itself let us write this whole thing as = k Boltzmann T or m p to the - gamma t and then there is an ni nj, let us write it like this, this thing here is ni nj 1 - cos Omega Ct + Delta ij cos because delta ij times 1 and this cancels out yeah so we have a small simplification here in this fashion.

So, let us write this Cij of = k Boltzmann T over M e to the - gamma t times we rearrange terms a little bit ni nj + Delta ij - ni nj cos Omega Ct - this other guy epsilon ijk nk sine now let us check if that is correct yeah that is correct that is right yeah. Now you will recognize that this is the longitudinal part and that is the transverse part of this tensor and that is the anti symmetric part the last term.

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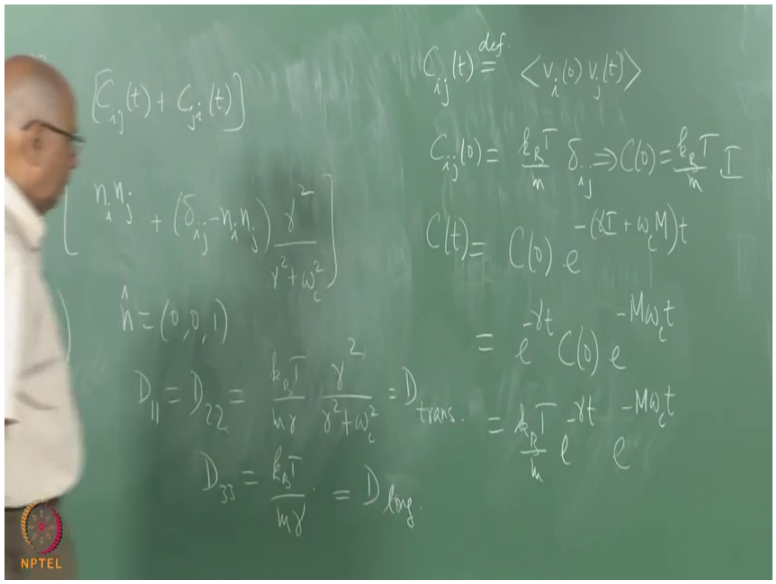

So, if I compute now the diffusion coefficient this guy here is = K Boltzmann t or M e to the - or sorry got integrate over that times I have an ni nj and I integrate e to the - gamma t that is just a 1 over gamma + there is a Delta ij - ni nj times the integral of e to the - gamma t cos Omega Ct and that is of course gamma divided by gamma squared + Omega C squared that is it.

And the other part last part vanishes because it is anti symmetric. This is of course the longitudinal part and that is the transverse part and you can see the two are slightly different because let us pull out a kt over m gamma and then it is this + that so you see at once that in the transverse direction this remember that in the original problem D was = kt over m gamma is followed by consistency in the diffusion limit we already saw this.

This is the Einstein Surveillant formula and that is sure there in the longitudinal direction but in the transverse direction it is modulated by this factor which is less than 1. So, this shows exactly how the fact that you have this cyclotron precession is inhibiting the diffusion constant it is still diffusive even in the transverse directions but it is a smaller diffusion constant instead of being a constant it is dropping off as a function of Omega C when Omega C becomes very large it will of course go to 0.

So, you can easily see if you if you took a special case if you took n to be = 001 so you have a field in the Z direction okay then it is immediately clear that D becomes diagonal and you have D11 = D22 so you put n1 = n2 = 0 and then only this term contributes, so you have kBt M gamma times gamma squared over whereas D33 then you get the 1 cancels out here and you get just this portion which is kBt okay.

So, this is = D transverse this is = the longitude, so it shows this very simple problem exactly how the diffusion is inhibited it is still diffuser which means square displacement still diverges but the fact that this particle is constantly pulled back into a circle in the transverse direction lowers your effective diffusion constant by this amount okay. This is the simplest model no of course you have a lot of other complications in a real plasma you have many more complexities.

But this tells you already how this thing happens the simplest instance okay. Can we see this directly from the Langevin equation yes because what you need to do is this and I am going to leave details as an exercise what you need to do is the following let us go back to the original equation the Langevin equation itself and write it in the following way.

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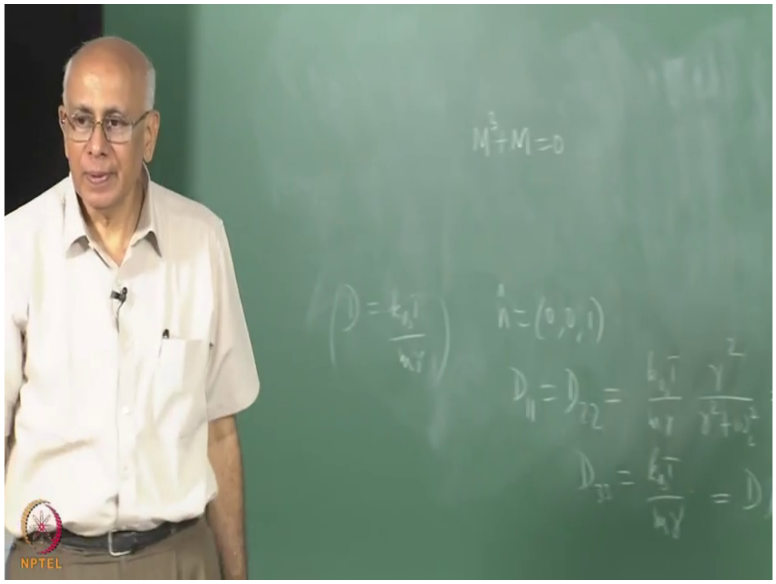

We have V dot = - gamma V + q over m V cross p + root gamma M Eta of t in this fashion right and we wrote this in a somewhat simpler form right we wrote this as = - gamma times the unit matrix + + the matrix Omega CM acting on V, we wrote it in this form right. Now in the high friction limit the inertia term is negligible, so you can throw that out and then you have a thing which says gamma i - Omega Cn acting on V which is r dot is = square root of gamma over m Eta of t where this is Gaussian white noise.

So, this is the high friction limit or diffusion limit if you like but you can rewrite this, this implies that r dot = gamma i - Omega C M inverse acting of times square root of gamma over m acting on Eta of t does this inverse exist got to be a little careful here to see if this thing exists what are the Eigen values of M yourself because we know what M is the M ij is epsilon ijk nk where nk is the component of a unit vector right.

So, we can find the Eigen values of this M very easily what do you think are the Eigen values of M its we had a property for M remember that remember that M cube + M was = 0 right. That is the characteristic polynomial of M, so lambda cubed + lambda is 0 right what are the Eigen values of M, 0 and then + or - I so 0 is certainly an Eigen value of M and therefore the inverse of M does not exist we do not care we want the inverse of gamma i - some number times M.

So, you can stay away from the Eigen values depending on what this gamma is in fact it is easy to see that this inverse exists we will find it explicitly okay. So, there is no problem in that existing but we got an equation which looks like X dot = something or the other some constant whatever matrix this is acting on white noise.

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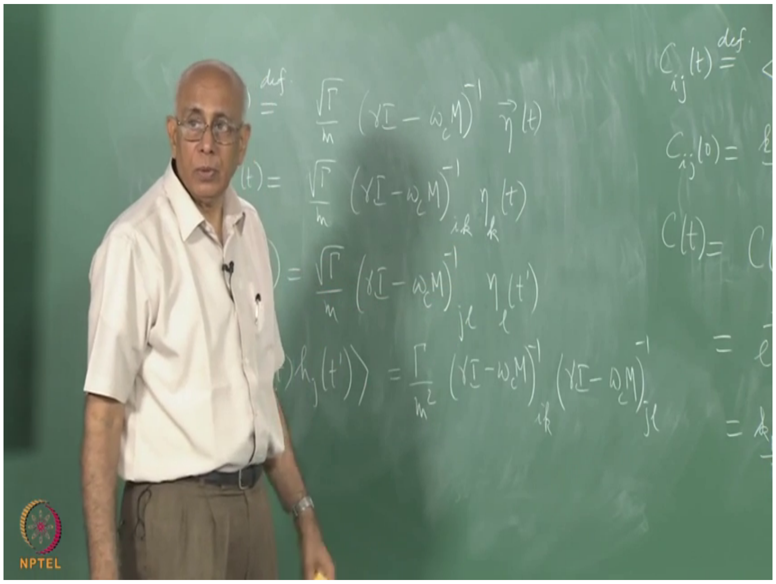

So, remember our original equation we got X dot = square root of 2D times Eta and we had one said huh this immediately implies that the probability density of this x satisfies DD2p over the x2 that was my identification of the diffusion constant. All we needed was white noise and the integral of white noise is diffusion of course here stationary white noise this guy is stationary this is a constant independent of t, so this whole noise is still a stationary noise.

It is still Delta correlated it because you can see what this noise is like so let us call h of t let us define this to be = all this garbage square root of gamma over M gamma I + - Omega CM inverse this whole thing acting on this Eta of t, now what is hi of t = its = gamma M gamma n - Omega CM inverse ik Eta k and what is hj of t prime this is = square root of gamma over M gamma i - Omega CM inverse jl acting on Eta l of t prime.

So, we can now find out what is hi of t hj of p prime and take the average that is the noise correlation right that is = gamma over M Squared gamma i - Omega CM inverse ik gamma i -

Omega CM inverse jl the correlation of Eta k over Eta l with Eta k of t with Eta l of t Prime what is that = that is a Kronecker Delta kl times the Delta function Delta of t - t prime.

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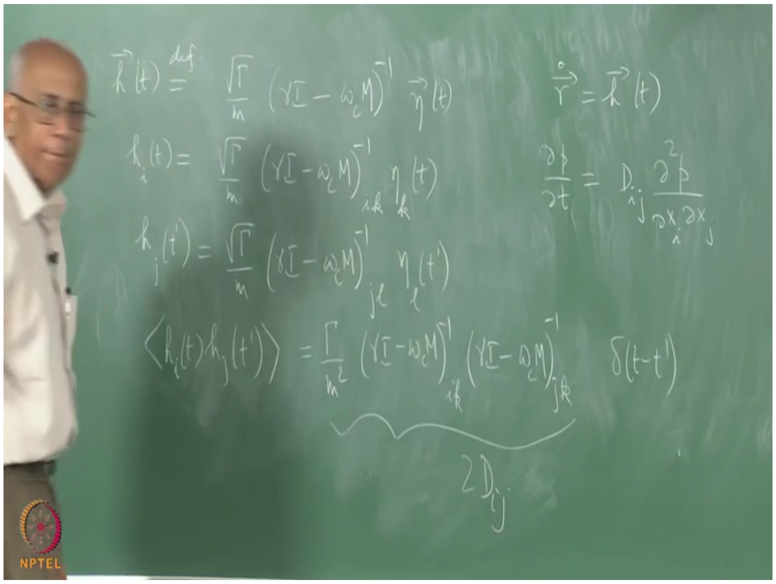

So, this guy is going to have a Kronecker Delta KL Delta of t - t prime right and of course you can finish off this immediately because this jl, kl you can get rid of this and make this a jk so it is still Delta correlated this is still a noise and it is going to be square root of 2d ij times a delta function when I do this for r dot I am going to get a square root of 2 ij whatever it is. So, this is what we need since we have proved that r dot = h of t component by component right.

This will tell you immediately that this noise is like square root of 2Dij for each component acting on the appropriate component of h on that side, on the right hand side. So, this will immediately imply d2d dp over dt = D ij, so you can identify what this Dij is from this guy here. So, this is what I had like you to do, this is 2Dij remember k is cancelled out, so this whole thing, so you can identify what all you have to do is to find this inverse in that inverse.

How are you going to do that how are you going to find these inverses well once again we know that if gamma is sufficiently large which is what you have to do it is a high friction limit then of course you can do a binomial expansion of this in powers of M and use the fact again that M cubed is n - M so it will again terminate once again okay. If you do not like that find the Laplace transform because if you find the Laplace transform of e to the power.

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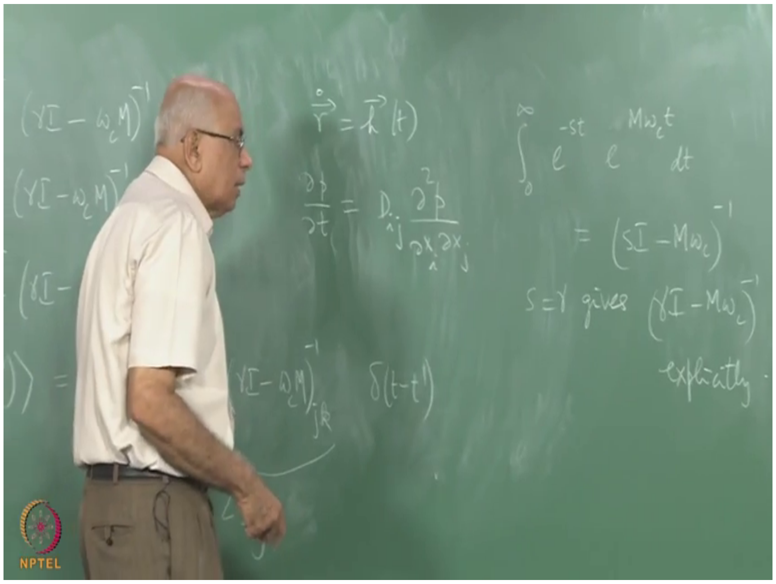

So, e to the power M Omega Ct and you find its Laplace transform it is e to the - st times this guy find the Laplace transform of M Omega Ct 0 to infinity dt this is = si - M Omega C inverse formerly this is true but we already have a formula for this guy in terms of sines, cosines and so on. So, put that in and do those simple integrals and you are going to get something which depends on M something is depends on M squared that is = this guy and now set s = gamma and you have this inverse.

So, there are many tricks for finding out the inverse you do not have to work extra to do the inverse it is already there then put that here put that here whatever expressions you get and that will and identify that with 2Dij read of dij it corroborates the fact that in the diffusion limit you can directly from the fact that the position that the velocity is Delta correlated you can actually find out with some coefficient the coefficient is essentially the diffusion coefficient.

We can compute what the D tensor is in this case but we can do this in several ways we also have you can also compute nor the phase space density completely so you can find out why we found out P of V , t for a given V not you can also find out what Rho of r V and t given are not and V naught is explicitly and that is not very hard to do either because once you have this expression for the exponential of this M matrix directly then everything follows automatically okay.

So, in a sense this completes what happens in the simplest instance of a particle moving in a magnetic field does exactly what we expected on physical grounds. So, let me stop here today and we will come back to noise next time.