Physical Applications of Stochastic Processes

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Lecture - 10
Birth-and-Death Processes

Right so I said last time that we would look at some physical example, simple physical examples of birth-and-death processes, so we will do that today. But to set the stage let me remind you of what the basic equations were.

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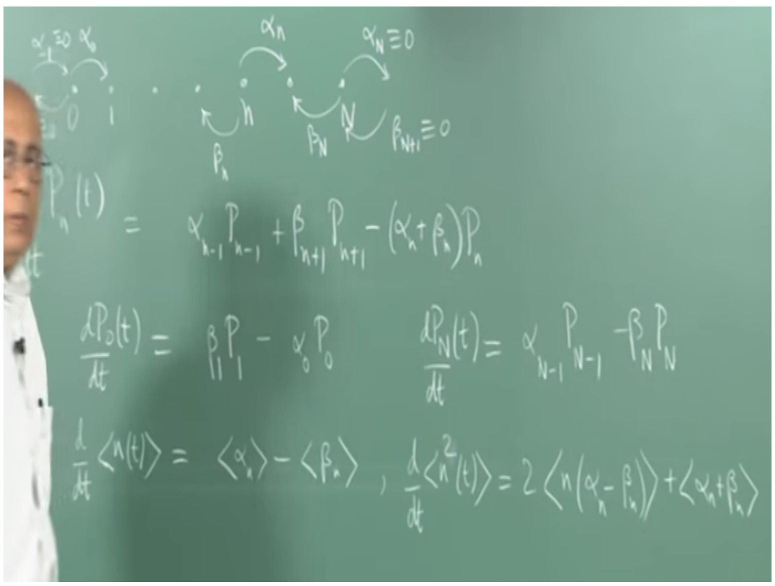

So we have a random variable n which labels states for instance and takes on integer values and I said there were 3 possibilities either n runs over all integers or over non-negative integers or over a finite range of integers okay. We will look at these cases separately. But this n has a probability P sub n at any time t such that the rate equation for this probability density for a probability is governed by some gain terms and some loss terms.

And the gain terms were just for pictorial convenience we put this on a lattice and this was site n for state n and you could move to the right with rate alpha n or to the left with a rate beta sub n which are functions of n in general and then the rate equations for this probability were alpha n - 1 P n - 1 + beta n + 1 P n + 1 - alpha n + beta n times P n. Those were the rate equations okay.

Now of course if you have boundaries then you have special equations at the boundaries. For instance suppose you have a case where this thing stops say at the point 0, the site 0, this is 1 etc. and this is some state N. Then of course you do have a rate of jump up there which is alpha not but there is no corresponding rate here. So this means that beta not is identically 0 on this side and there is no - 1 site. Therefore formally this too is not there.

Alpha n - 1 is 0 also identically. Similarly out here it is clear that there is a rate beta N going to the left. Of course there is a rate alpha N - 1 going to the right but there is no rate alpha N and this is identically 0 and there is no rate coming in so beta N + 1 is identically 0 which means that at the ends if you have 2 end points 0 and N you have to write special rate equations and these equations are dP not of t equal to and in this case it is clear that sorry alpha - 1 is identically 0. There is no state - 1 okay.

So this would be if I put n = 0, this would be a 0 here but this one is perfectly alright. So this is beta 1 P 1 minus and since alpha since beta not is 0 this is alpha not P 0. That is the equation when N = 0 on this side okay. Similarly at the other end if I put little n equal to capital N you have dPN over dt of t this must be equal to well alpha n - 1 and that is perfectly alright.

So it is flowing in from N - 1 P N - 1 but then if I put capital N beta of n plus 1 that is identically 0, this term is absent and then correspondingly this term here alpha n is 0. So minus beta N P N. These are the special equations at the boundary points okay. So when you write these rate equations down at the end points because of these boundary conditions, you have to put these 2 equations in and then solve the problem okay.

Of course one or the other boundary may be missing, may go all the way to infinity we do not care in which case this would be the general equation okay. Now of great interest as I mentioned would be the steady state or stationary solution to this. On the left hand sides the time derivative is 0 identically and then you ask what is the stationary distribution out here?

That is of great interest but before that let me also recall to you something else that we deduced which is that with given general functions alphas and betas you still can discover that d over dt of the average value n of t, this quantity was equal to, we found a formula for this and if I recall right it is equal to alpha n - beta n. Just check if that is correct. This is I believe what we found the average value.

Similarly d over dt of n square of t was equal to twice the average value of n times alpha n - beta n + alpha n + beta n okay. Those 2 equations are valid in general for arbitrary alpha n's and beta n's. Now the statement I made was that in the special case in which alpha n is a linear function of n and so is beta n then it is easy to see from here that this right hand side here involves the average of n once again.

Nothing more than that and similarly this thing here involves the average of n and n square and therefore you have a closed set of equations and you can solve them at whatever initial condition. Suppose you start in the state n 0, n not for instance, you would say this P n is a delta function at n equal to n not, that is your initial condition and then you can solve these equations. In that case n of t would be just n 0 to start with and you can write down closed solutions for these equations even if you cannot find the full distribution itself you can still solve for this.

Incidentally, if alphas and betas are linear functions in n it is not very hard to solve these equations explicitly. If they are constants then of course the solution is more or less on the lines of what we had earlier, namely we had for n running over all the integers we had the random walk, the biased random walk problem in general. But there are other variations of it depending on whether you have finite range or infinite range and so on okay.

So this much we had earlier. Now let us look at some specific instances of this whole thing. One problem we can do right away is to ask what happens if I have a semi-infinite range and I look at the random walk problem and to make it interesting for instance you could say this is a charged particle and there is an electric field in one direction which causes it preferentially to jump in one direction than the other or to look at the problem of sedimentation, you have a column of fluid and you have particles moving up and down like in the atmosphere here.
And then the question is how does the density distribute itself the actual physical density of particles as a function of the height under gravity for instance. We know that the answer is if you

have a column of fluid which is in equilibrium and at constant temperature we know that the density increases exponentially as you come downwards or decreases exponentially as you go upwards. This is the famous barometric distribution. So let us see how that comes about here directly. You can see that from the random walk problem.

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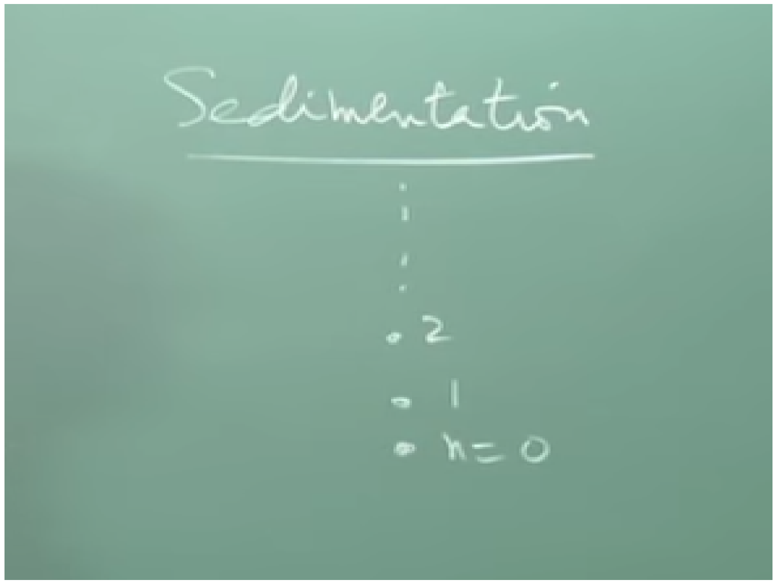

So if you like it is the problem of sedimentation okay and in this case let us put the lattice running upwards for instance. So here is n = 0 and then 1, 2, and so on upwards and I would like to know what does this look like. Now of course if I make this lattice constant go to 0 and take suitable limits I actually get a continuum diffusion model but we will try to see what the random walk model itself says explicitly okay.

Before that, we can do something even more general and that is the following. Let us assume that you have this n running from 0 to infinity say or 0 to capital N we do not care. So it is a either a semi-infinite or a finite range and let us try to find out what is the equilibrium distribution whether we can actually write this out explicitly or not. The question is can I take this set of equations and write out an explicit formula for the stationary distribution. So let us do that first.

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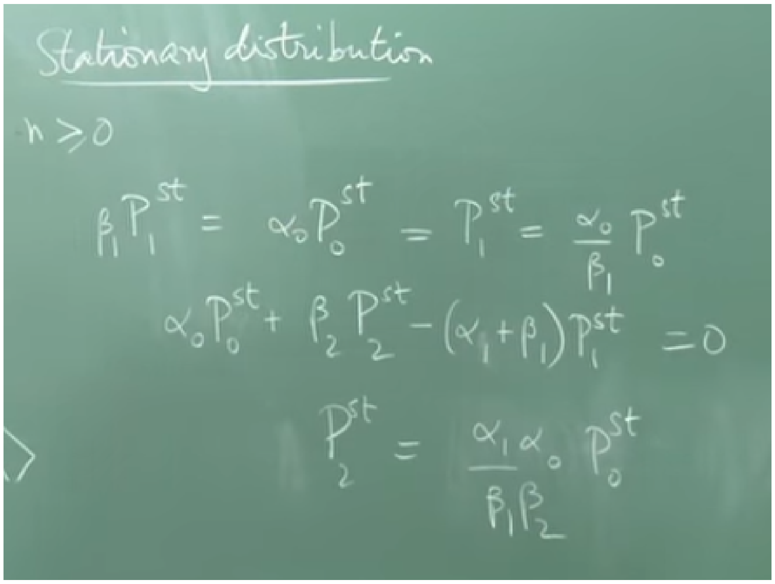

Stationary, and we are going to consider the case n greater than equal to 0. Could be infinite, could be finite, go up to some capital N, we do not care. For the stationary distribution we need to put these all these time derivatives equal to 0. That is the stationary distribution. So let us look at this last this equation here. This immediately says that P beta 1 t 1 stationary equal to alpha not P not stationary because this is 0 okay.

So it immediately says P 1 stationary equal to alpha not over beta 1 P not stationary and then we go to the next equation and what is that say. It says if I set n = 1 here, it says alpha not P not plus beta 2 P 2 - alpha not + alpha 1 + beta 1 P 1 all of which are supposed to be stationary equal to 0 right. So we get P 2 stationary with a beta 2 here equal to alpha 1 + beta 1 P 1 stationary - alpha not P not stationary.

And for P 1 let us just write alpha not over beta 1 P not stationary. So that comes out - 1 P not stationary and what does that give us. The beta 1 cancels so it is alpha 1 over beta 1. This should be an alpha not ah I took out an alpha not so there is an alpha not P not stationary. Therefore it is this, alpha 1 alpha not over beta 1 beta 2 P not stationary. Now put that in the third one. In the equation n = 2 and so on and so forth and it is easy to see what the pattern is going to be. It’s going to be exactly what we got from this first this equation for P 2.

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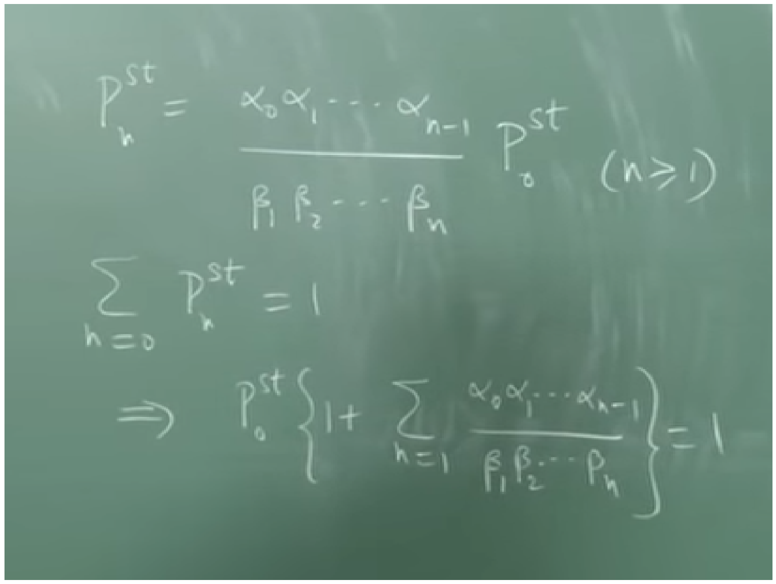

And we end up with a statement that P n stationary equal to alpha not alpha 1 dot dot dot up to alpha n - 1 over beta 1, beta 2, dot dot dot up to beta n P not stationary and this is true for n greater than equal to 1. So we are done. We need to find P not stationary and what does one do for that. Normalize, normalize the probability. So we require that summation n equal to 0 to whatever limit P n stationary should be equal to 1.

So this implies that P not stationary times 1 plus this guy so 1 plus summation n = 1 upwards alpha not alpha 1 dot, dot, dot up to n - 1 over beta 1, beta 2 up to beta n equal to 1 and the matter is over. So as long as this converges, as long as this converges remember that you have do have conditions on these alphas for this to be finite. In the case when it goes up to capital N there is no problem. This summation ends at capital N.

But in the case where it goes all the way to plus infinity, you need to have this converge. This series is converged and then you have a finite P not stationary and therefore a finite value of P n stationary immediately. Notice that this is valid. We have not made any assumption that the alphas are linear function and are a constant in n. We have not done that at all. It is still valid okay.

So we have a lot of information on this stationary distribution if it exists directly and it is some algebraic function of all the rates the alphas to the right and the betas coming to the left alone okay. Now let us look at what happens for the case of sedimentation okay. What is this sedimentation problem. It is a random walk in which under gravity you have a preferred motion to the left. So you have these molecules of air.

They are being buffeted around. We are only talking about the z coordinate now. There is a probability that they get kicked up or down etc., but the fact is that the probability of a downward transition is greater because of a constant force of gravity and in this case it is clear since the force of gravity is constant acceleration due to gravity is independent of the height it is immediately clear that the bias towards the bottom, downward direction is constant. So the beta n is actually independent of n and so is the alpha n. It comes from thermal fluctuations in general.

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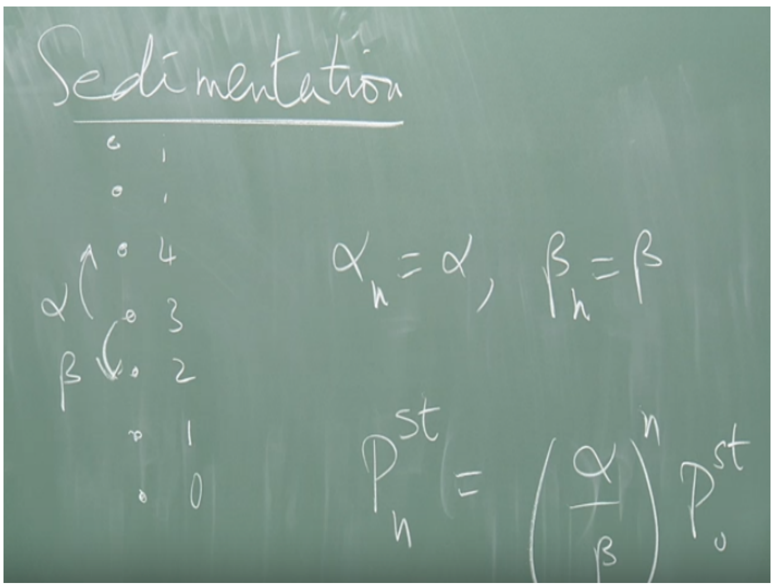

So the sedimentation problem is modeled by saying that I take a lattice and let us call this state 0, 1, 2 site 0, 1, 2, 3, 4 etc. okay and in this case alpha n equal to some alpha, beta n equal to some beta constant, independent of n such that beta is greater than alpha. There is a greater probability of jumping downwards than upwards. So while this is alpha this is beta and we have taken the boundary into account already in doing this.

We have said that there is no transition below 0. There is no site down there. So it corresponds to what are called reflecting boundary conditions. You cannot go you cannot penetrate. There is no current that is going from 0 to - 1 or anything like that. Then what does the solution look like.

What does the stationary solution look like? It says P n, now n naturally labels the site, the height above ground, this P n stationary is equal to some P not stationary but the fact is that this is equal to or we can equal to alpha not to alpha n - 1.

That is alpha over beta to the power n P not stationary and that is it okay. What kind of distribution is that? It is a geometric distribution. There is a constant alpha over beta and beta is greater than alpha okay and the normalization is trivial. Here is the normalization. This is the summation from n equal to 1 to infinity of alpha over beta to the n. So what is the normalization like?.

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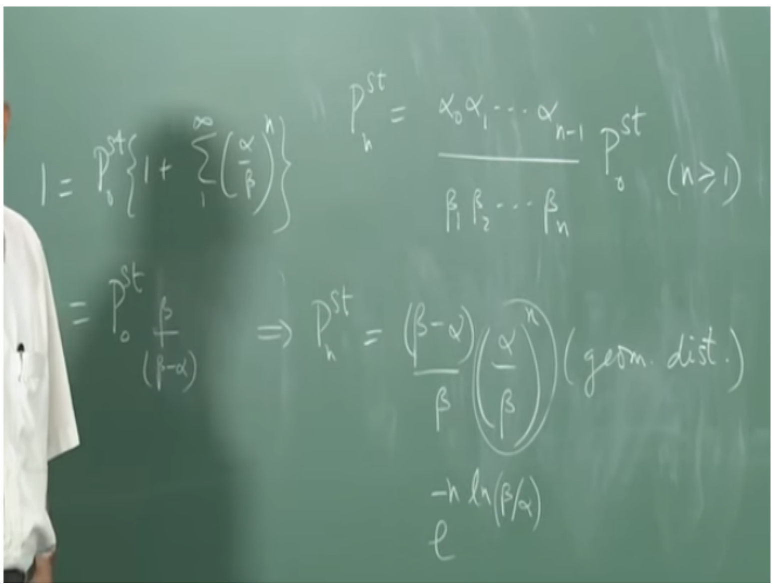

In this case P not stationary 1 + 1 to infinity of alpha over beta to the power n. 1 equal to this equal to P not stationary 1 + alpha over beta - alpha which is beta over beta - alpha. So this immediately tells us that P n stationary equal to is a geometric distribution okay. So this is actually telling you that the probability of finding yourself at a greater height is exponentially decaying as the height increase which is the same as saying that in thermal equilibrium the density is going to be an exponentially decaying function of the height okay.

That is exactly what it is because this fellow here can be written as e to the power n log alpha over beta but remember alpha is smaller than beta so the log is negative and you can write this as - n log beta over alpha and that is an exponentially decaying function of the height, n labels the height now okay, on a regular lattice. What would happen if I go to a continue what would be the parameters involved. What would what do you think would be the parameters involved.

In a physical problem where I have continuum diffusion and we will do this explicitly, write the diffusion equation down; but what do you think would be the parameters involved? These are imagine these molecules to be spheres or something like that dropping under gravity. There has to be a characteristic drift velocity due to this gravity. What would that be governed and there is a diffusion constant. That is what is kicking the molecules up and down.

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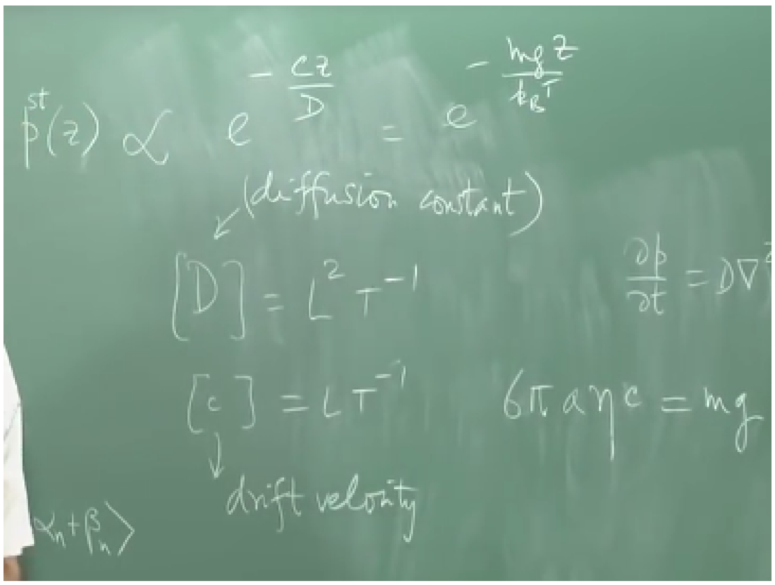

So there is certainly a diffusion constant d and what is the physical dimensions of this d? What is the physical dimensions of this diffusion constant um? Ya, remember that obeys a diffusion equation of the form delta P over delta t equal to D del square p is the diffusion equation of that kind for either the probability of density of finding a particle at some point or for the concentration in a macroscopic picture right.

Whatever it is that P cancels out on both sides. That does not play a role in the physical dimensionality of d. So what are the physical dimensions of D? Length square over time. So this guy here equal to L square t inverse. There also has to be a drift velocity. What would that depend on? It would depend on the size of the particle on the radius. What else would it depend on? There is gravity. So it will certainly depend on gravity.

What determines this drift velocity? I take a very light ball bearings or something like that and put it in oil and it drops down at terminal velocity. What determines the terminal velocity? When the viscous drag is balanced by the gravity, that is it and what is the viscous drag. There is a Stoke's formula for the viscous drag right. So clearly you have a formula which says 6 pi assuming that these guys are all spheres etc. of some radius a, a times eta that is the viscosity times the velocity right. Let us call that c the limiting velocity or the drift velocity.

That is equal to mg on this side okay. So when these 2 are balanced, you have the drift velocity c the terminal velocity c. So it depends on m, g, a, and eta the viscosity right but there is a quantity of dimensions velocity. This is your drift velocity. So what do you think the probability density is going to go like? We have got thing here in the continuum in the discrete case but now I am going to ask for P of height z and this is stationary, a height above ground.

So instead of the variable n running from 0, 1, 2, 3 upwards I have a height from the ground levels equal to 0 going upwards. What is this going to be proportional to? Where you can read it off from here? This n is going to be replaced by x or z the vertical coordinate and it is got to be a dimensionless quantity whatever is sitting in the exponent. So it has to go like e to the power minus something times z.

It has got to do that and that something cannot be time dependent and it must be a quantity of dimensions1 over length and what is the quantity of dimensions; c over d right. This famous Peclet number or whatever you call it. So cz over D. That is the barometric distribution of density in the atmosphere assuming that the whole thing is at constant temperature. But we already see that in this random walk model.

We already get an exponential decay as you go upwards and there is a boundary in the floor and you can't go below that. By the way if that is the equilibrium distribution of the height it immediately tells you here is a case where the mean is not equal to where the probability peaks. The mean height of the atmosphere the mean height is certainly not equal to the ground because we would all choke otherwise if everything is concentrated at 0 right.

So you do have an atmosphere. All the molecules do not come and sit down there. What is that due to, thermal fluctuations. It is due to fluctuations about the about the most probable value or the mean or whatever. So it tells immediately that fluctuations play a huge role, a very important. Otherwise, this density is monotonically decreasing with z as z increases and the most probable value is z = 0 okay but everything is not sitting there.

There is a finite value at which the mean exist. What would that depend on? What do you think that depends on? Well it looks like c has a viscosity sitting here but there is another relation called the fluctuation dissipation relation which also depends on the viscosity, the diffusion coefficient also depends on the viscosity and the viscosity cancels out there and what does equilibrium thermodynamics tell you.

It says that if you have an energy level epsilon, the relative probability of finding that epsilon is proportional to e to the minus epsilon over k t right. What is epsilon for a particle that is at a height z above ground, mgz. So the whole thing goes like e to the - mgz over k Boltzmann T okay right. So the characteristic, the characteristic length scale in this problem for our atmosphere is kT over mg and let us see how big that is to see if this is coming out roughly right or not. Otherwise the whole calculation is meaningless.

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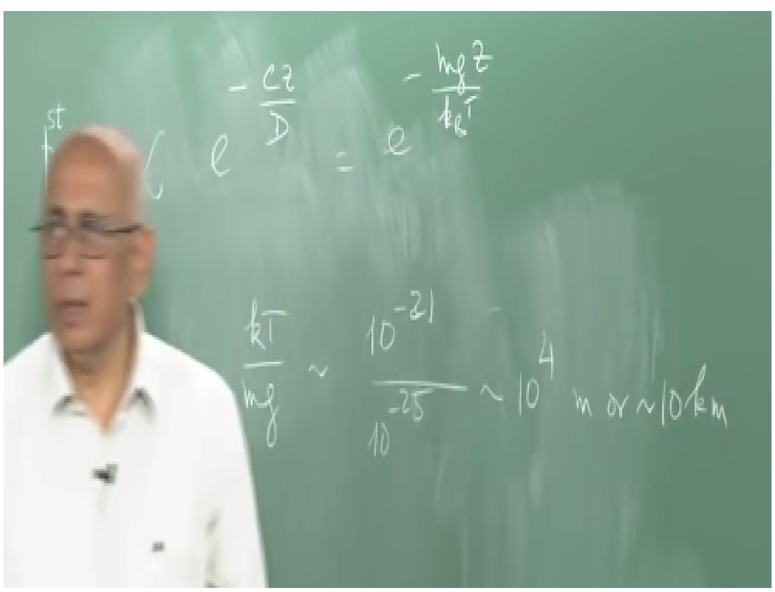

So kT over mg how big is kT? What is how big is k? 10 to the - 23. So let us work in those units. So 10 to the - 23. This is 10 to the 2 say; T in absolute temperature of the order of 300 Kelvin. So 10 to the 2/m what is m? Mass of a molecule, nitrogen molecule for instance. How big is that? Well the mass of an electron is 10 to the - 30 kilograms. The mass of a proton is 10 to the - 27 kilograms.

And we got in a nitrogen molecule you have whatever be the molecular weight. How much is that 28 or something like that. So multiply by another 10. So 10 to the - 26 and then gravity 10, of the order of 10. So this is 10 to the - 25. This is 10 to the - 21, 10 to the 4; 10 to the 4 what, meters, 10 kilometers. Absolutely right. Bang on. That is indeed the more or less the extent of the atmosphere okay.

So very simple considerations where it tells you that is all it can be with this gravity and this kind of atmosphere these kind of gas and so on and these ambient temperature property this is all it can be right. It’s the right ballpark magnitude. So this is how the barometric distribution arises. A little bit later we will solve the diffusion equation with the boundary at the floor, on the floor okay and you will see that is exactly the stationary solution. Okay let us look at a slightly more complicated exercise and this would be when you have some dependence on n.

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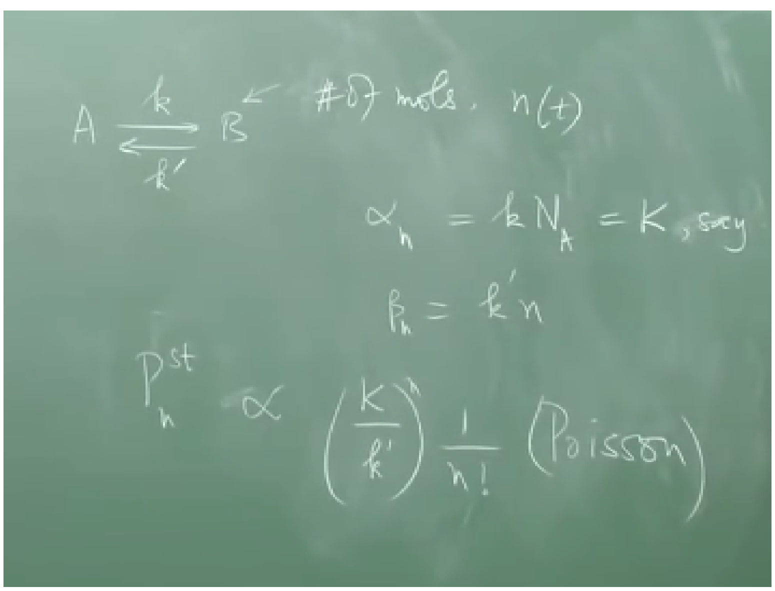

So let us look at a chemical reaction in which a species A goes to B and let us suppose B also comes back to A with some rates and I don't know what notation used for rates but the standard one is k and this is k prime. So you have molecule A going to B and B coming back to A and the product you are interested in the product B and let us suppose this reactions, an extremely simple reaction.

We will assume there are some enzymes or whatever it is promoting catalyst promoting this reaction and let us assume in the simplest case that you have a situation where you got a huge reservoir of A and you are trying to extract B from it okay and that the reaction, the depletion of A is insignificant. So we have such a large reservoir of A that the number A practically remains constant okay.

Then the rate equation for the population of this guy, number of molecules is n. It's a random variable, a function of t and we'd like to know what the stationary distribution is, what is the average number etc. etc., in this case. You already can compute the average number but we need a model for this whole business. So in this case let us assume that each molecule, each molecule B there is a certain rate at which it is being formed.

So this problem alpha n which increases the value of n is proportional to k times that is the reaction rate times the population of A but that is some huge number. So let us write it this is equal to k times N A equal to some constant say. It's not changing significantly. On the other hand beta n is k prime and proportional to the number B. Each of them has a probability or rate k prime of decaying so this is beta sub n okay. What happens now?

We can write down the rate equation for the change probability P n. But let us look at what the stationary state looks like in this case. So P stationary P n stationary apart from some normalization factor is going to be proportional to alpha not alpha 1 etc., etc. up to alpha sub n - 1. That is just equal to K raised to the power n because there are n of these factors divided by beta 1, beta 2 up to beta n.

That is equal to k prime to the power n, n! sorry k prime to the n because each of them is there times n!. So this whole thing is proportional to K over k prime to the power n 1 over n! What kind of distribution is that? It's a Poisson distribution. In the steady state therefore the population of n satisfies a Poisson distribution where the average value depends on these constants, the rate constants okay.

So here is an instance where you can actually get a very powerful result from fairly simple considerations in this fashion. let us look at a case where there is dependence on both variables. Let us look at the example I talked about earlier which has got to do with radiation.

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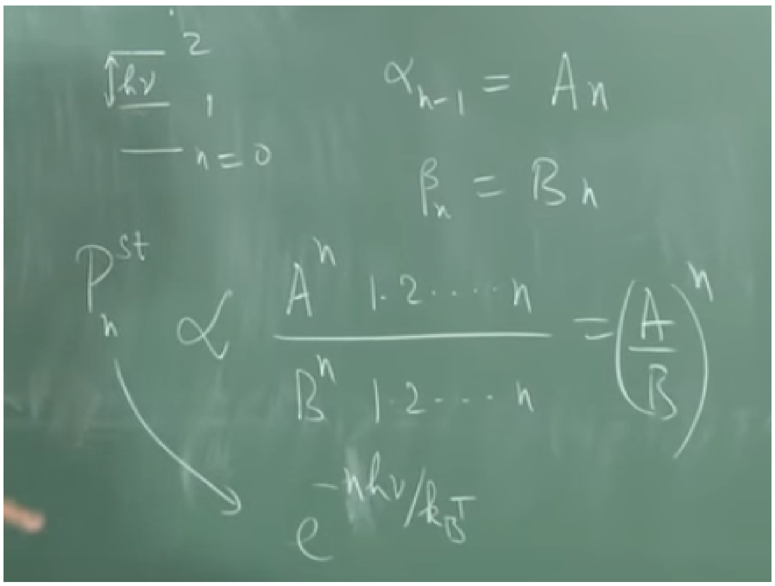

So the assumption is that you have a quantum harmonic oscillator n = 0, 1, 2, etc., etc. and you have it in a radiation field, interacting with the radiation field with exactly the right frequency. So there is this distance is h nu in energy let us say. Then the questions is what are the rates? Now, the phenomenon is as follows. You shine light on it, if an oscillator is in this state it can absorb a quantum of light and go up there.

On the other hand if it is in this state on in any exited state it can emit a photon and come down here. But there is 2 kinds of emission. There's spontaneous emission and stimulated emission. You have to include both these guys. So the model that does it is to say that alpha n - 1 is proportional to some A times n and when you actually compute the quantum mechanical process by which this absorption takes place you have what is called a dipole approximation, a matrix element which gives you the probability for this process to occur.

And that leads to a rate of jump which goes like this, a transition rate which is proportion to n, alpha n - 1. Similarly, it turns out that beta n equal to some B n on this side okay and now we ask what is the stationary distribution going to be like in this case. So what is P stationary going to look like P n stationary is proportional to or equal to in this case. It is alpha not, alpha 1, etc., etc. all the way up to n minus 1 alpha n - 1.

So it is a times a to the power n 1 into 2 up to n/beta 1, beta 2 up to n. So this is also equal to B to the n 1 into 2 up to n is A over B to the power n proportional. Now, if the oscillator is in the state n okay it is energy is n times h nu and what is the relative probability. In thermal equilibrium, when the system is in thermal equilibrium the radiation field as well as the oscillator what is P n proportion to then, the stationary distribution proportion to.

This fellow has to be proportional to e to the minus beta h nu n h nu over k Boltzmann T. That is the canonical ensemble right. So A over B has to be proportional to has to be e to the - n a, h nu over kt. This is the starting point of Einstein's derivation of Planck's law nearly okay. So you put in all the other factors etc. and you end up with Planck distribution out here. But notice in this case it was not constants and yet because of this special feature here this factor is canceled out and you still got this geometric distribution.

We have already seen that the number distribution in thermal light is indeed a geometric distribution. We already saw that using both statistics etc. I wrote it down explicitly okay. This essentially is the same thing. Now let us look at a case where you have a genuine birth or death problem model. So let us look at, so we looked at this radiation, let us look at population model, the simplest population model and a genuine birth or death problem.

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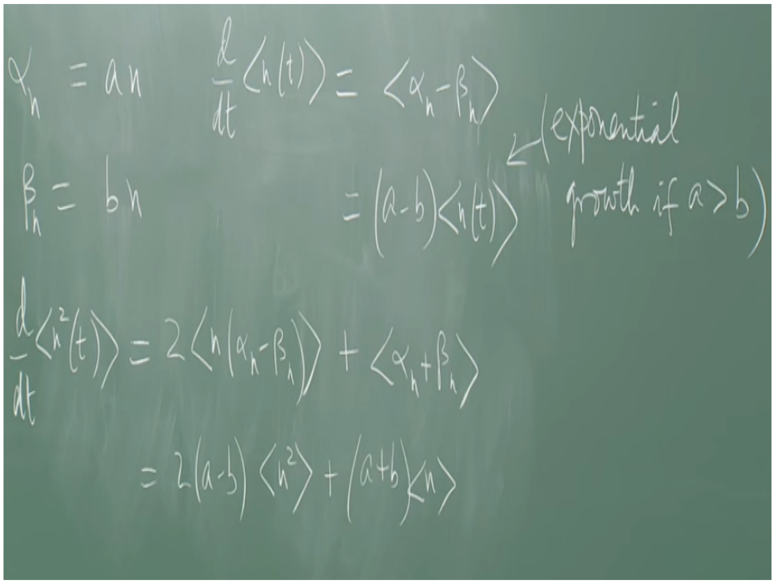

So again n starts from 0 and goes upwards okay. The rate equation is something we wrote down already but we need to make a model for the growth of this population. So let us say that alpha n this is going to be the rate at which n increases to n plus 1. So let us suppose that each make the simplest model that is all births are independent events of each other and we make a model which is completely trivial in a sense the 0th order if you like to say that each individual has a certain rate in which the individual can give rise to one more project okay, some constant rate.

So alpha n is obviously proportional to n where a is some constant and similarly each individual has a rate at which the person dies the probability of dying the rate transition probability so beta n equal to b times n. That is the total death rate and this is the total birth date here okay and using this we can now write down what the stationary distribution will look like etc. if it exist but let us look at what the average does.

Remember that the equation for the average was d over dt n of t equal to alpha n minus beta n okay. So this is equal to a minus b n of t. What does that tell you? It says this is going to be exponential growth of population if the birth rate is greater than the death rate finished. This was the famous Malthusian prediction of the exploding population okay in this simplest of models, exponentially fast.

One could of course hope that the variation that the variance will do something, will help us a little bit let us try and see what happens to the variance in this case. So let us look at the equation for d over dt n square. This was equal to twice the expectation value of n times alpha n - beta n + alpha n + beta n and what does that give us twice and again this is a minus b. So the prediction here is exponential growth if a is greater than b. So let us see what this tells us.

Oh incidentally, keeping track of the average value of the population only tells you the difference between the death rate and the birth rate or the birth rate and the death rate. It does not tell you individually what is the death rate and the birth rate that is important okay. So this is a - b and then expectation n square plus a + b expectation n in this side. So let us compute what the variance does.

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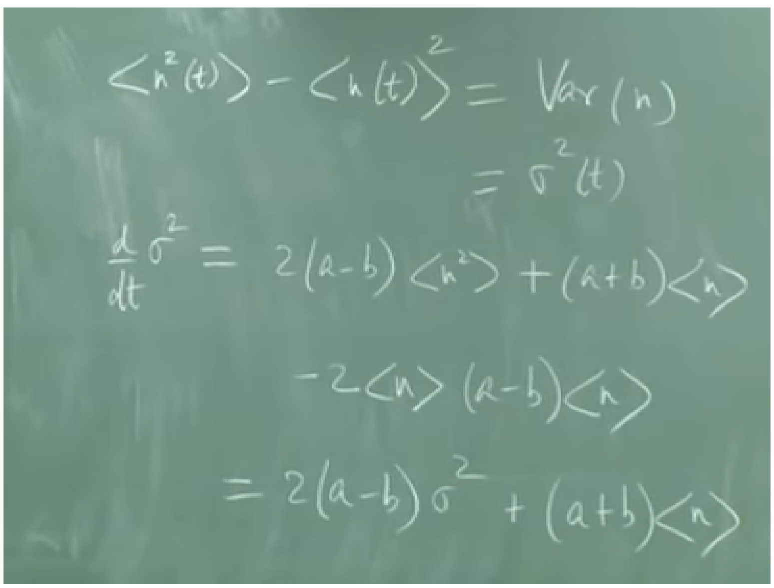

D over dt let us put n square of t - n of t whole square equal to this is the variance of n let us give it some symbol sigma square a standard symbol okay. So d over dt sigma square equal to this fellow 2 into a - b n square plus a + b n d n square over dt and then we got to subtract the time derivative of this so that is equal to minus twice n times dn over dt but dn over dt is a minus b n of t n itself. And this factor comes out.

So this equal to twice a - b n square - n average square that is sigma square once again. So sigma square plus a + b n. That too is increasing exponentially because this guy already is if a is bigger than b and this is a positive term here and this guy is sitting on the right hand side so this two will explode immediately. So this is not helping us any this simplest of models.

It is clear there is a huge scatter but at the same time the whole thing is the average is going exponentially fast. Now we can put in all kinds of various mitigating factors and improve this model and so on and so forth but the basic problem of exponential growth happens as soon as the birth rate is increasing greater than the death rate for whatever reason okay. One can look at other models which have competition which have different competing species etc., etc.

But the fact is that in the most elementary instance this is what is going to happen here. It is worth noting that if you want to measure the birth rate and death rate separately you need to measure both the variance as well as the average number because this is going to depend on this guy here and so you can get information on both a and b separately once you do this okay. There are lot of other models. There are a huge number of models which we can write down.

In general as I said if the system is linear in n then the problem is actually solvable explicitly and in the we have restricted ourselves to these one-step processes in which case this transition matrix is a tridiagonal matrix and there are lots of special tricks to take take care of this system the tridiagonal matrix. For one thing it is not symmetric to start with but you can actually make it symmetric by a simple trick.

And then you can go some distance in writing down approximate solutions etc. but again I want to emphasize that when it is linear the problem can actually be explicitly solved completely although the coefficients are not constants but depend on n you can still solve these matrixes okay. Now there are further delicate questions as to the kind of boundary conditions you put the nature of the boundary conditions whether they are absorbing or reflecting and so on.

We will come back to this but not in the context of a birth or death model but in the context of continue model of diffusion. We write the diffusion equation and look at it in the context in that context we look at absorbing boundaries reflecting boundaries and so on. So when you have extinction for example you would have what is called an absorbing boundary and that leads to its own interesting features.

So we will look at that by and by very shortly when we talk about continue. So I am not going to go further with these discrete models random walks but we will now talk about continuous Markov processes where we are going to define once again probability densities and conditional probability densities and use some physical examples to illustrate what happens in such cases.

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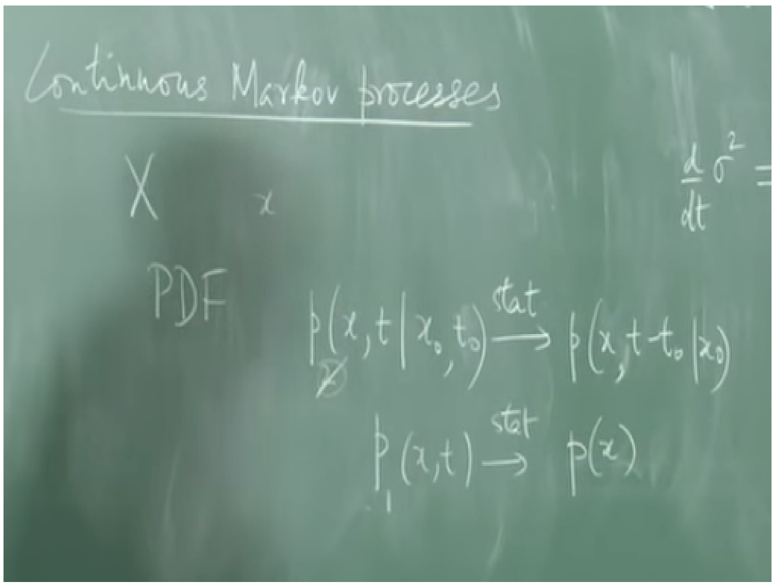

So we start off by saying that the processes we are going to look at Continuous Markov processes and as usual I will denote by X the random variable and by x curly x the set of values in a sample space assumed with a for the most general case to run from minus infinity to infinity. We will look a little later at cases where this x is a vector for instance position and velocity or position and momentum etc. but for the moment it is just a scalar variable here.

Then exactly as we said in the case of Markov processes in which the variable was discrete we are going to define a probability density functions of this joint probability density functions and in the case of a Markov process the most important one is going to be the probability density that you have x at time t given that you had some x not at time 0 and if you look at a stationary at some time t 0 for example.

If this is stationary then this becomes p of x, t - t not x not the conditional density and we look at the one time probability which is this should really have been a P 2 but I am going to omit this subscript. It will become clear from the context which one I am talking about and then you have a P 1 of x, t and of course if it is stationary it just goes to a p(x).

And exactly as in the case of the discrete variable I said if there is sufficient degree of mixing in the system then this tends when t - t not tends to infinity this conditional density tends to this density here. Depends only on x. Does not depend on the time argument okay. As before we write a chain equation down.

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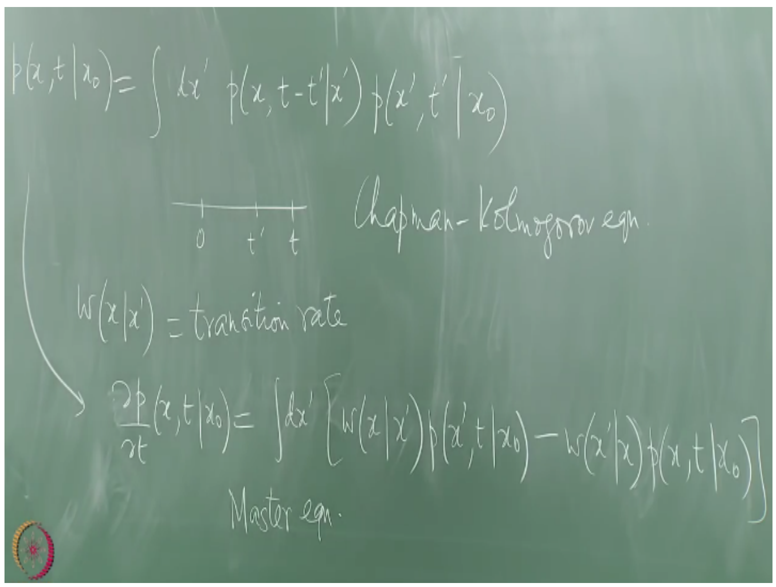

The chain equation in this case is once again a p of x, t x not equal to a summation which becomes an integration now over all intermediate values dx prime p of x t - t prime from x prime p of x prime t prime x not where and the time axis here is 0 here is t prime and here is t; any t prime in between okay. This is the Chapman Kolmogorov equation or chain equation. As before, we try to convert this into a master equation which is an integro-differential equation by defining a transition probability W of x x prime from any value x prime to a value x okay.

And then what does this equation go to? This leads to this master equation delta p over delta t of x t x not. This is equal to an integral over dx prime and then inside here you have a w of x x prime p of x prime t x not minus the lost term which in this case is x prime x t of x t x not. This is our master equation okay and the task is to try and solve this equation okay and the task is to try and solve this equation okay. We have assumed stationarity.

Otherwise this equation will have t dependences here in these transition rates even if it is Markov. We have assumed the whole thing to be stationary and if you specify these quantities now you got to specify functions here, then in principle I have an integro-differential equation which however is linear in this unknown quantity in the conditional density okay and the idea is to solve this. So we will take it from this point.

We are going to start here and see what are the possible ways in which we can solve this in various cases and then we will look at some physical examples. Let me stop here.

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