Selected Topics in Mathematical Physics
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Module - 7
Lecture - 18
Laplace Transforms (Part I)

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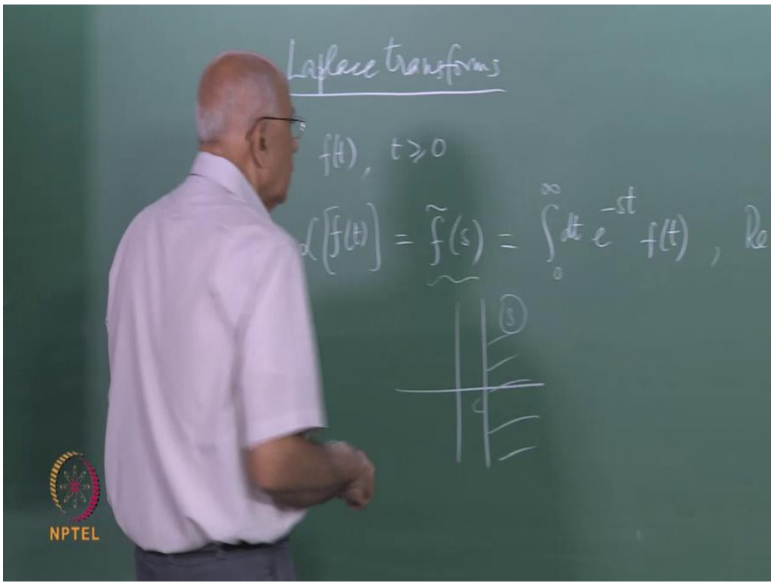

So, now let us today take up Laplace transforms which I started mentioning last time, but then now today we will look at what they are, and what the salient properties are. And I pointed out that if you have given a function f of t in the ranger t greater than equal to 0, then for instance f of t could be a response function like the ones we looked at in linear response theory. Then an integral transform of this function is extremely useful in solving say differential equations involving such functions are many, many other equivalent problems.

And the transform is defined as Laplace transform of f of t is defined as call it f tilde of s, this by definition 0 to infinity d t e to the minus s t f of t. Now, the moment I write this I need to tell you immediately in what range of s, this parameter s are variable s does this integral converge. Now, normally we look at functions f of t, which could of course go to 0 as t tends to infinity or go to some constant or maybe even diverge, but it is clear that unless this diverge is extremely badly and we will say exactly how badly it should be.

As long as the real part of s is greater than a certain value such an integral is excepted to converge, because it will kill whatever divergence this might have. Now, to start with let us write real s greater than some numbers c some positive numbers c, could even be negative if you are lucky, but in general as long as real s is bigger than a certain threshold value, you expect this integral to converge. Now, of course there are immediately lots of functions, which cannot satisfy this condition at all, for instance this f of t increase like e to the power t squared as t becomes large, then no matter how large real s is our large positive.

You are not going to be able get the integral to converge such functions do not have Laplace transforms, but a huge class of functions obviously does have Laplace transforms when we define it in this fashion. And it is immediately obvious by inspection, that this thing here in the s plane to the write of some number c, it is an analytic function. As you can see by inspection in particular, if s is real on the real axis, then the nice well defined integral if you give it an imaginary part all this does is to give you an oscillatory part, and this integral converge continues to converge.

So, by now we have enough experience to say that it is an analytic function to the write of some numbers. Now, of course the question immediately arise is, how do I continue it analytically to the left of this a line, and we will see explicitly various examples of how to do this. Now, immediately simple function f of t we could write down the Laplace transform at once.

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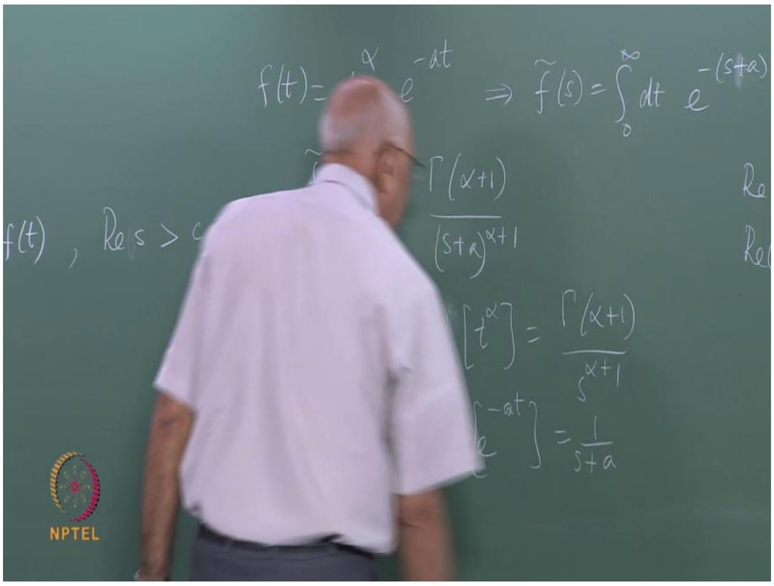

So, let us start by saying f of t equal to here is a function which t to the alpha e to the minus a t, when alpha and they are some constants, then this thing is f tilde of s equal to I put in t to the alpha here. And then this is becomes s plus a times t, I change variables to s plus a t and in some gamma function I will can see, and the answer is gamma of alpha plus 1 over s plus a to the power of alpha plus 1. Now, for what values of alpha and a would you expect this integral to converge to start with. Well, remember that what is involve is f tilde of s is 0 to infinity d t e to the minus s plus a times t, t to the power alpha.

This factor could give you trouble at the lower end, if alpha is sufficiently negative, so you would like real alpha to be greater than minus 1, otherwise that integral is not going to exist. Similarly, on this end s plus a it is clear that, as long as the real part of s plus a is greater than 0, you got convergence at infinity. So, real part of s greater than s plus a greater than 0, so you specify some a and some alpha in general some complex numbers. And as long as I keep s in that range and alpha in this range, I am All right the integral exist.

Once I have done the integral, once I have finished doing it and here is the answer, you can analytically continue this in both alpha and a, whatever complex value you please. Now, this immediately tells you as you can see there was a condition here said real s plus a must be greater than 0. So, you expect some problem when real s is equal to minus a, and indeed you can see there are s equal to minus a there is a singularity of this function theorem. And in general for complex alpha it is a branch point of some kind, so this explicit form tells you what the singularity structure is, but you keep these parameters and this range to start with do the integral, and then there is an explicit analytical continuation.

Now, many special cases come out for instance, if you set a equal to 0 this will imply that the Laplace transform, Laplace transform of this function t to the power alpha is gamma of alpha plus 1 over s to the alpha plus 1. And of course, if alpha is some integer n positive integer n, then it is n factorial over s to the n plus 1. The very important lesson comes out here, and that is the large t behavior of this quantity here is governed by this small s behavior of the transform. As you can see if t tends to infinity, and alpha is got a positive real part this thing diverges, and that divergence appears in the Laplace transform in the form of what happens near s equal to 0.

So, there is a well developed theory which tells you, how to find the asymptotic t tending to infinity behavior of f of t based on the s tending to 0 behavior of Laplace transform. And similarly how to tell the t tending to 0 behavior based on the s tending to infinity, both ways these are called abelian and turbarian theorems, we will come across some examples of it little later, but right now I want to see write from this formulae itself that the large t behavior intuitively is govern by the small s behavior, this is gone to be useful piece of information. If you put alpha equal to 0 for instance, implies Laplace transform of e to the power minus a t is 1 over s plus a.

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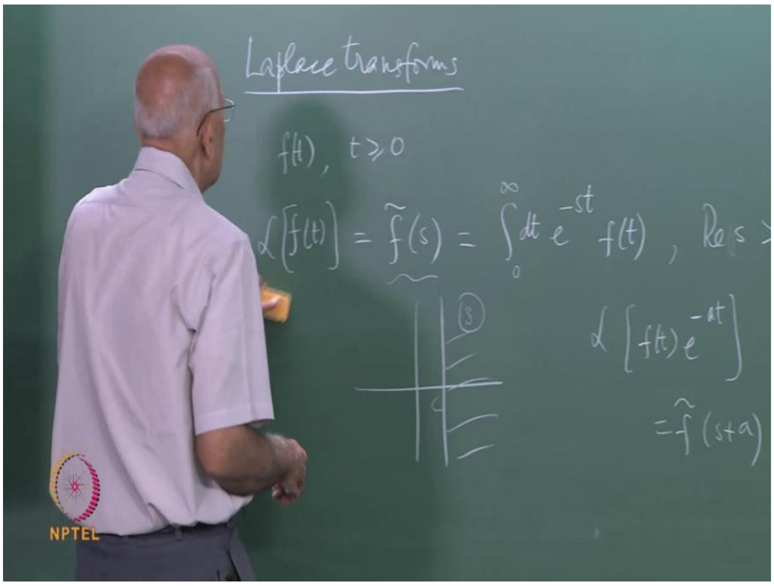

In fact as you can see if you got any function f of t, and you multiply it by an exponential like e to the minus a t are you doing to shift the Laplace transform by liberal by a. So, it follows immediately that the Laplace transform of any f of t e to the minus a t is equal to f delta of s plus a, where f delta of s is Laplace transform of f of t. So, all it does is to shift that shift s variable by e, that two is a very useful thing to remember, what about the convolution of two functions? Well, we need to know what happens to the product of two functions, what happens to the Laplace transforms?

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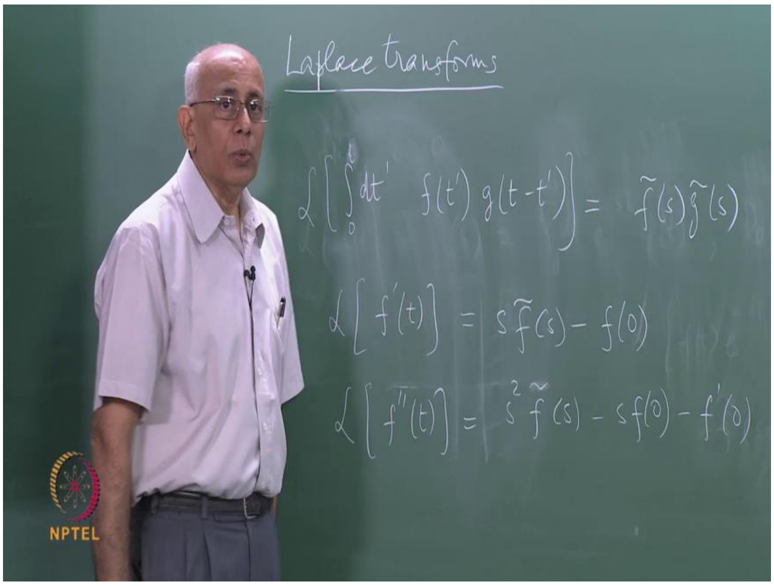

And it is a simple matter to show that the quantity like 0 to t d t prime say f of t prime and g of t minus t prime, this is called the convolution of f with g. The Laplace transform of this combination such a combination is equal to f tilde of s g tilde of s. So, convolution gets translated into multiplication, as far as a Laplace transform is concerned very useful thing, and you can now have a convolution of many functions one after the other. And then the Laplace transforms which just get multiplied in this function.

So, again a very useful theorem, notice the t the limit t here 0 to t that is the way the convolution theorem works for Laplace transforms, I urge you to establish this it is very straight forward, all you have to do is to write down the transform of each of these. And then do a little bit of algebra and you get this, what about the Laplace transforms of the derivative f dash of t, where this transfer the derivative with respective to t. Once again put this in the definition and integrate once by parts, and if you integrate by parts, then it immediately follows this is equal to s f delta of s minus a boundary terms which is f of 0, that is going to come in you integrate by parts.

The infinity part goes away because of the e to the minus s t, but the 0 contributes, and you have this. Well, you can proceed once step further, and take f double prime of t this is equal to s squared f tilde of s minus s f of 0 minus f prime of 0, these are not transform these are the values of the original function at 0, and it is derivative at t equal to 0 and so on. So, the Laplace transform of the nth derivative is s to the power n times the original Laplace transforms, and then a polynomial which involves s, and polynomial in s which involves all the initial that are, all the derivatives functions value and it is n minus 1 derivatives at t equal to 0.

So, this tells you that the operation of differentiation is being converted to the operation of multiplication by s here, and that is very useful when you want to solve differential equations. So, that is where it comes from it convert different ordinary differential equations in s. It can be converted by this trick of Laplace transforms into multiplication by s, so you get algebraic functions. And then the question is how do you invert that transform.

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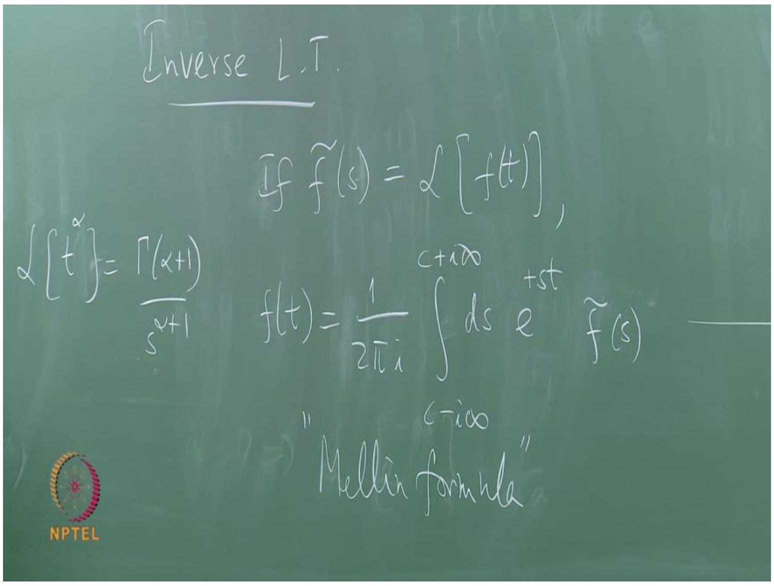

Now, the inversion of the transform involves a little more work inverse Laplace transform this is called the mellin formula. It says if f tilde of s equal to the Laplace transforms of f of t, then f of t is 1 over 2 pi i a contour integral over f tilde of s, but remember you have to be in the region of analyticity of f tilde of s. Otherwise the integral does not make sense, and that some right half plane, so this is d s e to the plus s t f tilde of or f of t f tilde of s, where the contour runs in the following way.

If this is the region of analyticity in the s plane of f tilde of s, and I said it some right half plane to the write of some point. Then the contour is the line parallel to the imaginary axis running from c minus i infinity to c plus i infinity in this fashion. And since the contour does not cross any singularities or anything like that, and we function is analytic you can deform this contour and you can move it up and down this side, but you cannot do is bring it is to the left here, because you are not guaranteed that f tilde of s is analytic there.

And in general, of course f tilde of s will not be analytic, we saw even for the function t to the alpha there was in or e to the for this function. The Laplace transform of t to the alpha was equal to gamma of alpha plus 1 over s to the alpha plus 1. So, we saw that in this case, the region of analyticity is real as strictly positive, and at s equal to 0 there is a singularity. So, in that case, so in fact of branch point this is the branch point here and there is cut running to the left here.

You certainly cannot be the contour down to this side not in it is original form any way. So, this thing here is called the mellin formula, and the question is how we going to use it, how will be going to use this to evaluate the inverse transform. Now, let us look at the simple example, let us look at this case where we had let us about the simplest case.

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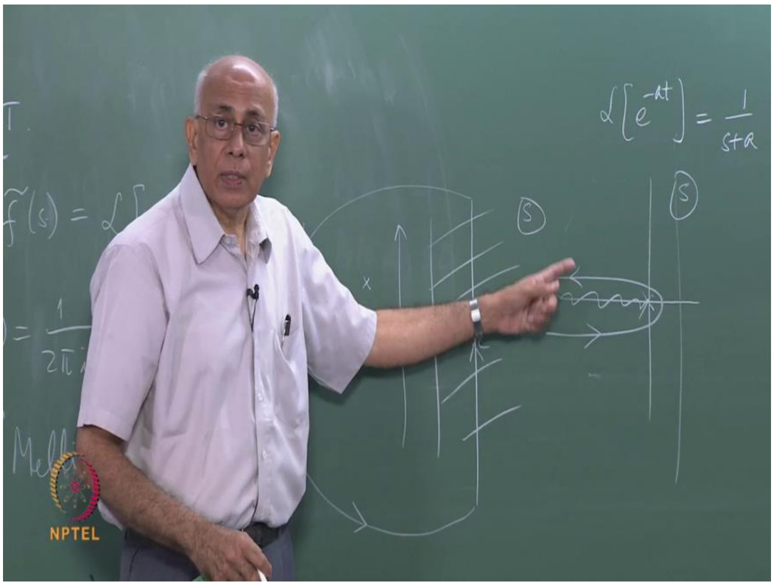

The Laplace transform of e to the minus a t was 1 over s plus a. So, that function has a pole at s equal to minus a, somewhere that pole is to some point we do not care where this a is pole at this point here. How would you, now evaluate this line integral with the pole here. Well, you try to use the residue theorem; the way to do this is of course in this case we can move the contour to the left up to this point. You can move it all the way to the left provided you pick up the contribution from this pole here, but another way of doing this would be to say All right.

Let me attach and infinite semi circle of this kind, and if this function vanishes sufficiently rapidly at infinity on this side. Then of course you are in good shape, it means that you can evaluate this integral this contribution becoming 0. Now, notice that this circle runs over real s negative, that is what we have done note here the left hand side, and this the plus sign here as the posted the original Laplace transform, which was either the minus s t was the kernel.

Now, the kernel is e to the plus s t. So, this contribution is guaranteed to vanish provided this function is good enough, because of this factor here. And it is immediately clear that if f tilde of s is any rational function of s ratio of two polynomials. It is got a whole lot of poles at various points in some left half plain then this trick is going to work, because all we have to do is to close this contour. And now, you can shrink it and pick up the residues from all the poles. So, it is immediately obvious that is as soon as you have a rational function of s as f tilde of s, the inversion is very trivial.

All you have to do is to close this and that is the end of it. Now, when you have more complicated functions more complicate singularities then it is not so trivial, for instance in this simple case this function here itself. In order to produce this function back for you from this contour integral, I have to put in here 1 over s to the power alpha alpha plus 1, and then what is it that I can do? Well, the best you can do is there is a branch cut in the s plane in this function, the original contour was like this and I can fold it over and make it like this.

And if I know the discontinuity of this function 1 over s to the alpha plus 1 across this cut, I can convert it provided the integral exist in to a line integral from there to in minus infinity, and then try to do this integral. So, the moment you have branch points, it is a little trickier to find the inverse transform, but if you have just poles as would be the case of a rational function then it is very straight forward and do this.

So, the lots of examples and some of them are given to you in the exercises, so much for the inversion. Let us look at a couple of problems, which we going to be talking about Dassel functions a little bit, because the accurate various places. But before I do that, let me given example of second order differential equation, which you can solve with Laplace transform. And this is the famous simple harmonic oscillator problem, the damped oscillator or the series L C R circuit for which in terms of the charge q in the circuit.

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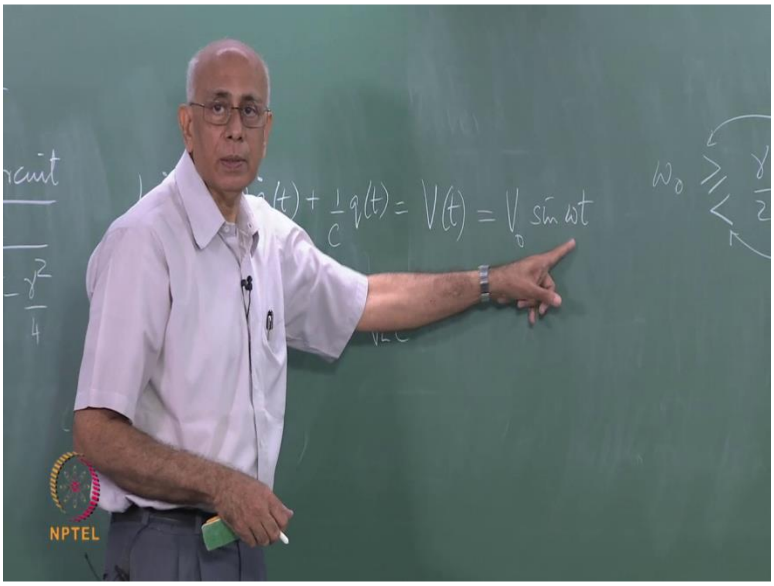

So, L C R series circuit the equation looks like L q double dot of t plus R q dot t plus 1 over c q of t is the applied voltage on the right hand side. And let us say it is sinusoidal is voltage, so some V naught sin omega t. So, that is the second order differential equation governing the charge on the capacitor, instantaneous charge in a series L C R circuit. Now, of course the first thing to do is to divide through this by this L, and then identify the fact that you have R over L equal to the time constant of an L R circuit.

And in the absence of damping thus a natural frequency for the time circuit L C circuit, which is 1 over square root of L C. So, it is can mean n to express everything in terms of these 2 parameters. And then as you know once you have this is switch it on, then this circuit depending on by the it is under damp over damp or critically damp will reach a steady state in which the current itself. And the charge will also oscillate with the frequency omega, but the force in frequency omega, but there is a transient and you can compute what the transient is explicitly.

So, I leave you to do that as an exercise all you have to do now is to use Laplace transforms. And then this becomes s squared times something or the other, and this you can s, and this it f tilde itself collect them together and invert this transform, because it is a rational function. And you have three cases, you have omega naught greater than equal to less than half gamma, they correspond to over damped under damped, critically damped and over damped. So, this is that and this that and the equalities is the critically damped case.

So, you can do solve the problem for any one of these cases, and then the others can be read out by analytic continuation. Now, what normally happens is that in the under damped case, you have an effective frequency which is omega squared minus gamma squared over 4 the square root, and in the under damped case this is real number down here and inside the square root. And therefore, you get oscillations which are damped with e to the minus half gamma t, whatever times this frequency function at this frequency that will be the transient part.

And then there is a steady state part, which will have frequency omega here. And you use to doing this by calling them the particular integral and the complementary function. So, what is the purpose of using this complementary function for and inhomogeneous equation, an equation like this an inhomogeneous equation the right hand side is non zero. So, you know the general solution is linear combination of particular integral and the complementary function, why do you add this complementary function?

It is a solution to the homogenous equation, you add as a certain amount of it, why do you do that at all? Apart from the fact that it is most general solution, but how much of do you add, what decides? How much of this critical complementary function you add to the particular integral, what you need to make the solution unique initial condition yeah. So, the purpose of adding the complementary function in a suitable at mixture is to ensure that the initial conditions are satisfy, that is what decides it.

For any inhomogeneous equation, so I urge you to do this do this in the over damped case, and then in the under damped case, and then continue analytically to the over damped case. Typically what will happen is at this quantity here, this square root will become pure imaginary in one case, and then trigonometric functions will go over in to hyperbolic functions. So, it is a nice excess little exercise to try it out explicitly. Now, let us look at some more interesting problems in this very, very standard problem.

I have in mind the problem from probability theory from the theory of random processes, because that is where be use a lot of this. And it involves what are call Bessel functions, this particular problems and we will talk about involves Bessel functions, so you are familiar no doubt with original definition of the simplest Bessel function of all. The Bessel functions of the first kind of order n are something like that.

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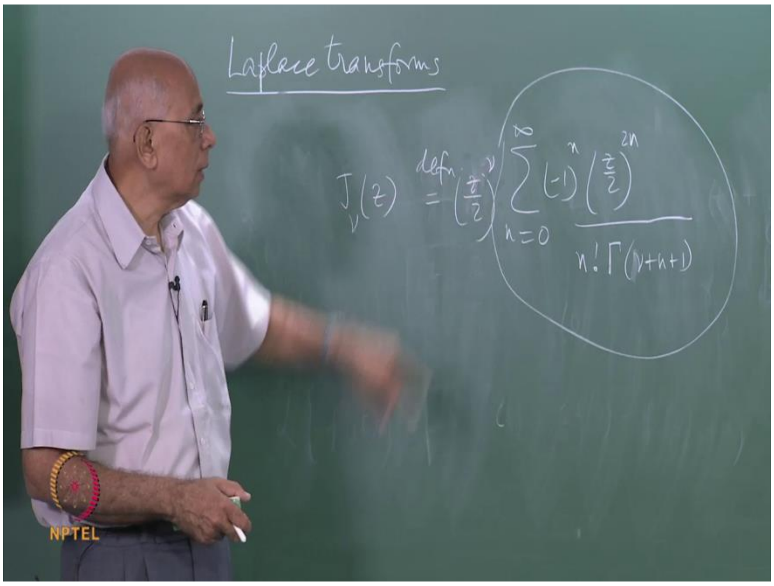

Now, what does it is look like, I like to define it in the following way. First of all I define it as a function of complex variable always, and the order is new that could be complex as well. Let us define it, I define it in the following way summation n equal to 0 to infinity z over 2 to the power nu plus 2 n divided by n factorial, and then I gamma of nu plus n plus 1 minus 1 to the power n. If you like that is my definition, you could also define it as the regular solution of Bessel's differential equation, and that is how you probably come across it first time.

But this is a good definition nu is any complex number you like, in general it is a complex numbers, but when you specify that it is an integer for instance if you put nu equal to positive integer n, then it terms out the J nu J n and J minus n are not linearly independent of each other. And you have J n as one solution, and you have the normal function as the other linearly independence solution.

But for arbitrary nu this is once solution here, and J minus nu is also independent solution, what can you say about the analytic properties of this in z, what is sort of function do you expected to be. Do you think this series has the finite radius of convergence and infinite radius of convergence, that is the powers series and z squared sitting of that. The fact that you have z to the power nu comes out integral, so this definitely z to the nu can take out z over 2 to the power nu, and then it is this. This part for arbitrary nu has a branch point at nu equal to 0 and at infinity.

So, let us leave that out, what can you say about this series? What would be the radius of convergence infinity yeah, this is the 2 factorial thus the factorial here and this another gamma function here. So, this is certainly this portion is an entire function, and it nu is an integer the whole thing is an entire function, the leading behavior as in z goes to 0 is proportional to z to the power nu. So, that is a nice entire function of z that is the Bessel function of the first kind of order nu. Now, what about it is Laplace transforms, well we want this for a function of t for instance, so let us look at for instance.

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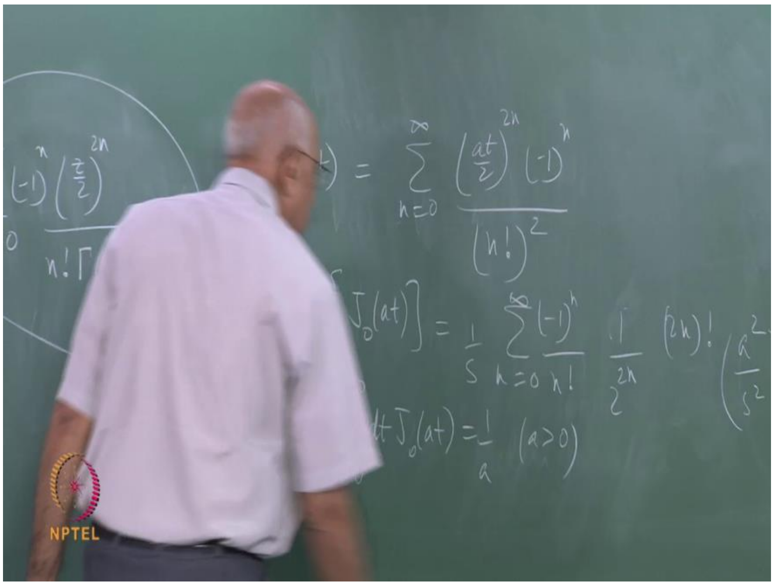

Let us look at J 0 of a t varies some constant, what is this equal to this is summation n equal to 0 infinity a t over 2 minus 1 to the power n this to the power 2 n divided by n factorial factor twice, because nu is 0 squared out here. We can find it is Laplace transform all you have to do is to remember the Laplace transform of this function. So, the Laplace transform of J 0 of a t this is equal to summation n equal to 0 to infinity minus 1 to the power n n factorial, and then a to the 2 n over 2 to the power 2 n.

And the Laplace transforms of t to the power 2 n, but we just saw what the Laplace transform of t to the power n to the factorial was. So, this is a 2 n factorial divided by S to the power 2 n plus 1. So, let us take out one factor of S in this 1 over S, and let us write this is as a squared over S squared to the power n. And the fact that you got a 2 n factorial on top n factorial n factorial terms stays, so just very strongly this is some binomial expansion.

So, I urge you to prove that this is equal to 1 over square root of s square plus a square, it is a very well known result. And we follow directly from here to checks this out. So, unlike the earlier cases the Laplace transform of a Bessel function is not a rational function, it is got the branch points at s is equal to plus or minus i a it has branch point in the square root branch points this places. Similarly, you could write down the Laplace transform of J nu in general it is got branch point in s. So, it involved the square root combination and so on and so forth.

By the way there is an interesting integral which can be done immediately, and that is if you put a equal to 0, if you put s equal to 0 then you get 1 over a. So, there is an well known integral which says 0 to infinity J 0 a t equal to 1 over a positive a. So, you can also use, once you get a Laplace transform you can put in various values and get various integral of special functions also, but that is the very well known result.

So, now let us turn to the physical problem, I have mind to the process I have in mind is part of a class random process called mark of process. And in the simplest tense sense which you going to look at today, this process is a random variable has only discrete integer values.

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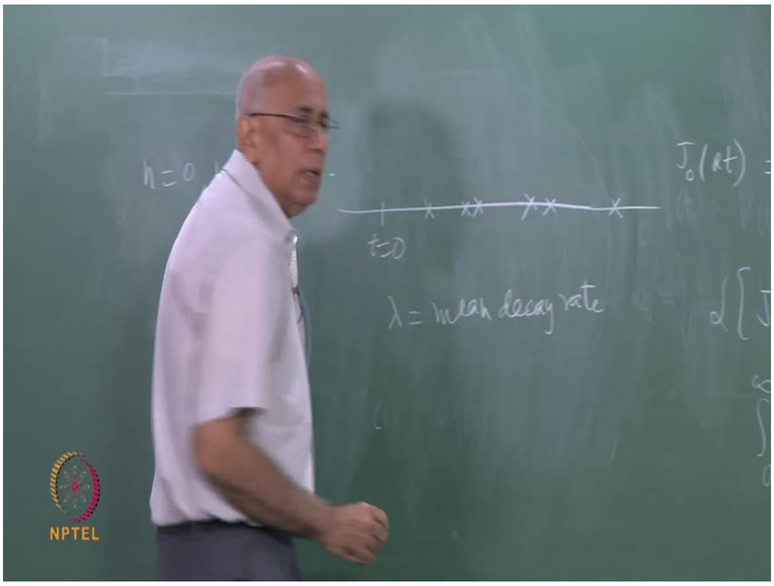

In fact, this called pass on process and the random variable concerned takes on values n equal to 0, 1, 2 etcetera. So, all the non negative integers those are going to be the value, this n changes randomly with time by prescribed rule, and the sample space is 0, 1, 2, 3 etcetera. And the problem we have in mind is that of radioactive decay. So, you have this huge sample of some radioactive substance, and you have a counter and you start measuring how many decays occur.

So, you clock every time there is a decay of nucleus that occurs, it clicks in your counter at t equal to 0 you start of fresh that the huge sample. And then you will like the thing start decaying, as you know radioactive decay proceeds at random we cannot say which nucleus is gone to decay when it proceeds at random with some average rate. So, on the time axis this is my t equal to 0, I put across every time there is a decay. So, one occurs here, another occurs here, quick succession you have 2 of them and then a gap, and then another one and so on.

And the interval between this decay events is random completely random, these decay are independent of each other. So, the sample insufficiently big, that different parts of it different nuclei decay quite independent of each other. There is a certain average rate of decays and that is called that lambda equal to mean decay rate, in the sense that it will give me a sufficient small interval of time d t. The probability that one decay occurs in this time interval is lambda d t, and the probability that no decay occurs at all is 1 minus lambda d t, then the question asked is the following.

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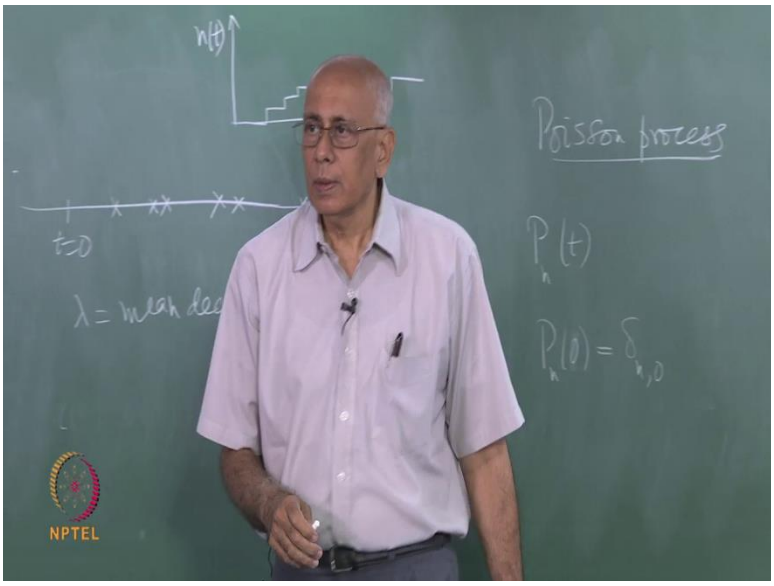

So, this thing is called a Poisson process, and we seeking the probability P n of t that in a time interval t from the time, I started the clock exactly n decays have occur n events are occurred. Now, it is clear that this n is an increase non in decreasing function of t, now the words n can only increase as time goes along, and each time it increases by 1. If you drew this n, if you plotted this n as the function of t, you would end up it is some kind of as function of t at typical n of t. A typical graph of this would be nothing happens for while, and then there is a decay.

And then it is quickly followed by two more decays, and then nothing happens for a long time and then this decay and so. So, it increases by 1 each time, the question is what is P n of t? Well, the immediately know the P n of 0 is equal to delta of n comma 0 t equal to 0 no decays of occurs, so P 0 of 0 is unity and P n of 0 0 for all n greater than 1 no decays have occur. Now, however we go to find this P n of t, we assumed that this decays are independent of each other they completely statistically uncorrelated, and this is certain mean rate of decay.

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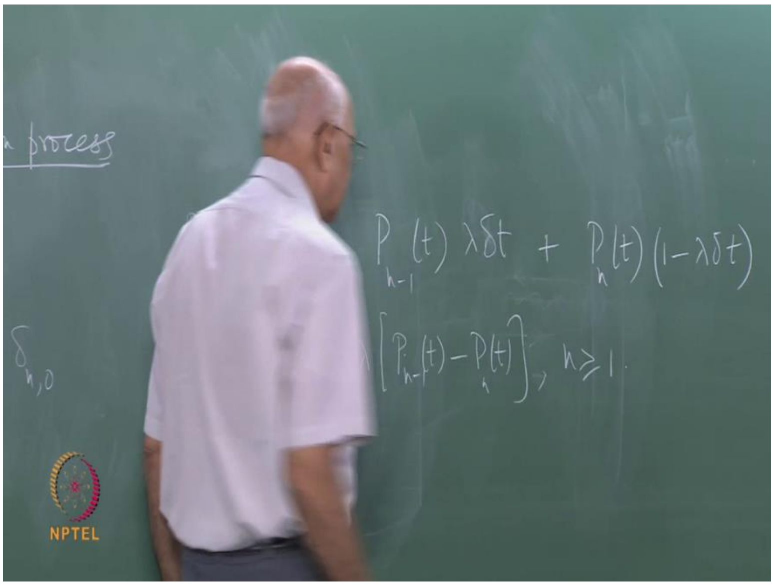

This is what part of what is called of mark of process, so we ask what is P n of t plus delta t equal to, what could this possibility. Well, at time t plus delta t if you want the probability that exactly n decays of occurred, then one possibility is that I time t n minus 1 decays have occurred, and in interval d t 1 more decay has occur. So, that probability is P n minus 1 of t multiplied by lambda d t lambda delta t. So, this is the probability that one decay occurs in time interval delta t in this infinite decimal interval, and that is the probability that n minus 1 one of them of already occurred at time t. And the only other possibility is that you have P n of t n have already occur delta n t, and then nothing happens in the time interval between t and t plus delta t.

And the probability for that is 1 minus lambda delta t that is it. This is the only possibilities the two independent mutually exclusive possibilities, in therefore you add the probabilities for each of this. And in each case it is a compound probability, you need n minus 1 one of them have to occurred till time t that is P n minus 1 t, and then multiplied by the probability that one occurs in time delta t, and similarly for the other time. Now, move this P n of t to the left divide by delta t and you take the limit, and we immediately get d P n d t equal to lambda times t minus 1 minus P n of t which is true for n greater than equal to 1 yeah.

Student: (Refer Time: 36:12)

Good question why could to have a two decays in time delta t, the answer is if you have a finite rate mean rate lambda, then the idea is that it is possible for you to choose at delta t sufficiently small that only one decay occurs in it. In other words this process what is called of finite transition rate.

Student: (Refer Time: 36:36)

That is necessary yeah.

Student: (Refer Time: 36:38)

Pardon me.

Student: (Refer Time: 36:40)

Yeah the whole point is that they are assumption is that you can choose your delta t small enough, that there is no simultaneous to decays possible, that is an assumption and so on. And it is justified them we gone to go to the limit in which delta t goes to 0 some other factor, but this is only true this equation is only true for n greater than equal to 1, for n equal to 0 you need a different equation, because what happens now?

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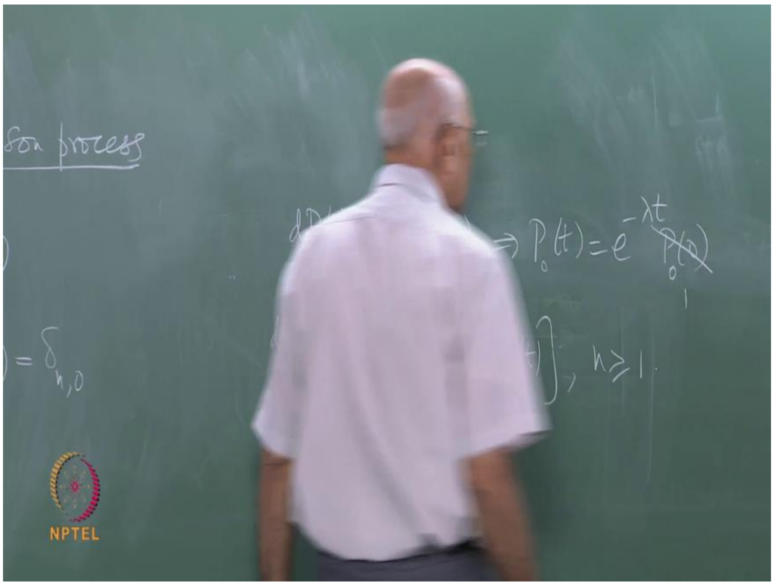

Is the rate at which d P 0 over d t changes is minus lambda P 0 of t, because from 0 you can only go up to 1 you cannot go to minus 1. So, you had no decay at all, and then all that can happen is at there is a decay at some stage, but that equation immediately can be solved. And this implies that P 0 of t is e to the minus lambda t times P 0 of 0, but that is unity that is one, because of this. So, we have a set of couple differential equations, the very simple equations the linear, but they coupled equation and we need to solve it. And what do you in such cases, when you a set of recursion relations of this kind, you define a generating function. So, let us define generating function.

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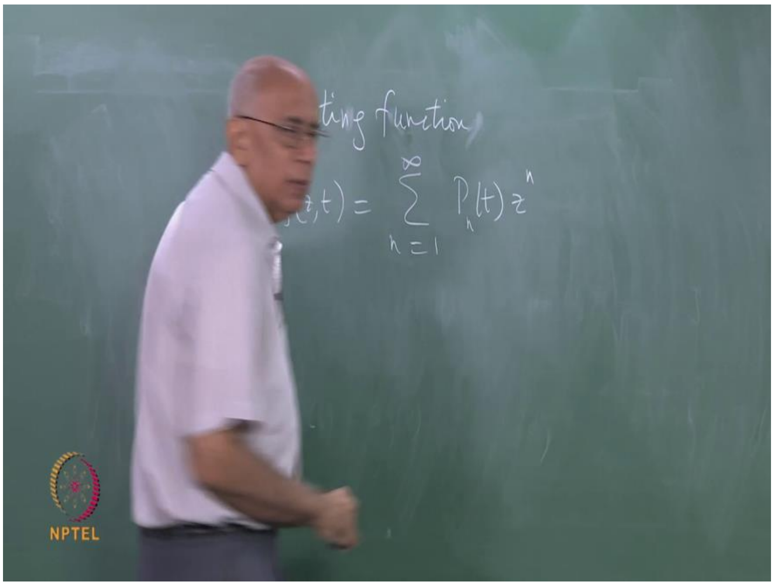

I mean you can solve this set of equation much more trivially, but let us do it by the generating function, because you going to use it in other cases. Therefore, z comma t is defined the summation n equal to 1 to infinity P n of t z power n. So, the idea is I am going to right a differential equations for f solve this differential equation. And then write out express f of z explicitly, and pick out the coefficient of z to the n in it is power series about z equal to 0 and call that P n of t that is what it is.

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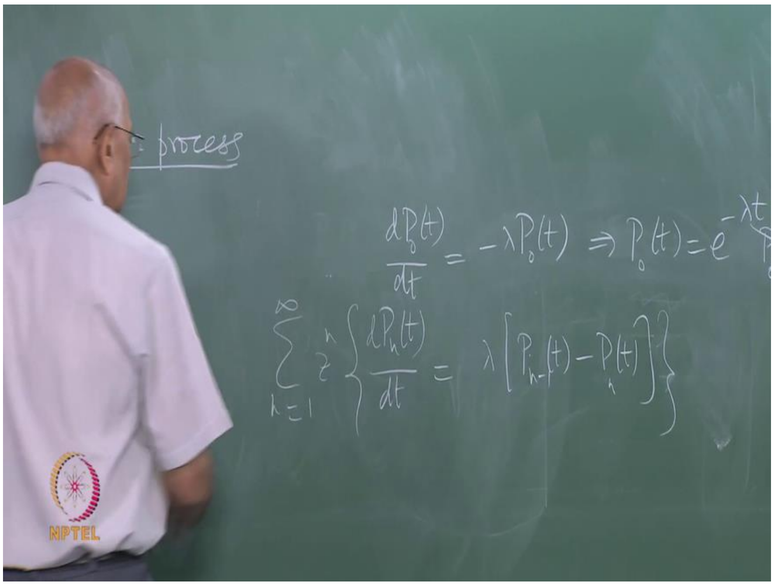

Now, what should I do, well I odd to multiply this equation on both sides by z to the power n of describe, and then sum from n equal to 1 to infinity.

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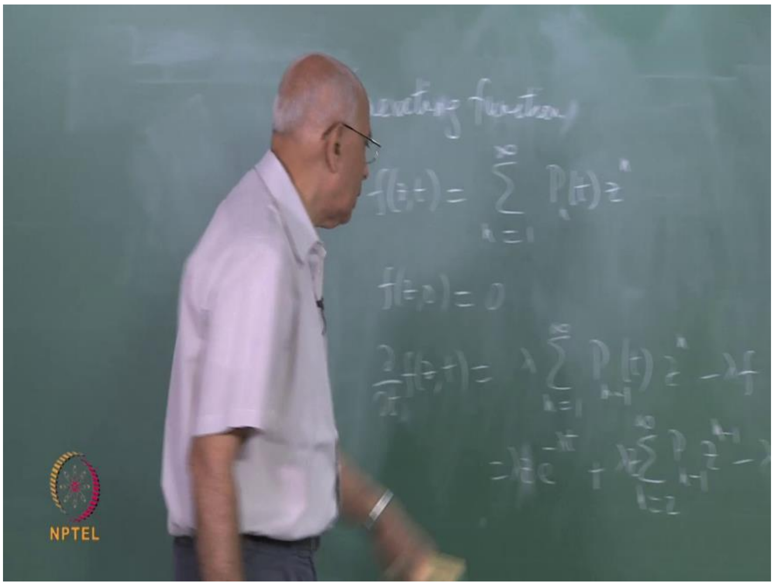

By the way what is the boundary condition on this function f of z comma 0 the initial condition, what should be the initial condition? Remember this starts from 1 0, because all these probability are 0 at t equal to 0 except P naught of t. P naught of t, it is not does not appear in the generating function. So, this is equal to 0 that is the initial condition, so what is this tell me.

Well the left hand side is delta over delta t f of z comma t, and that is equal to on the right hand side you have this term lambda times P n minus 1 of t multiplied by z to the power n. So, lambda summation n equal to 1 to infinity P n minus 1 of t z to the power n in this summation, minus lambda times f. As I am going to simply multiplied that by z to the power n, and some from 1 to infinity and what is this give me, this equal to lambda times at pull out n equal to 0 term n equal to one term here, this gives me P 0 that is e to the minus lambda t and then there is z.

So, this lambda z, and then the next term is plus lambda summation n equal to 2 to infinity P n minus 1 z to the power n minus lambda f. So, this term let us call this n minus 1 and multiplied by z. And this guy run from 2 to infinity, so I will call n minus 1 equal to n prime or something like that. So, n prime will run from 1 to infinity, and this is just f once again.

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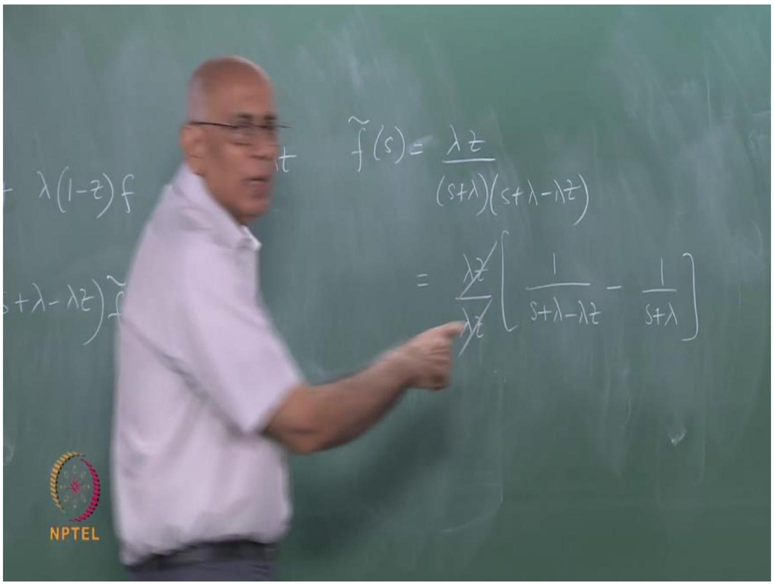

So, it is says delta f over delta t equal to minus lambda plus lambda into 1 minus z f equal to lambda z e to minus lambda t. We can solve that equation it is inhomogeneous equation first order standard form, but let us do it using Laplace transforms just for the. So, I am going to call the Laplace transform of this with respect to t, I am going to call it f delta. And then I get S f tilde of z comma S plus lambda times 1 minus z tilde of S. So, let us put the whole thing together S plus lambda minus lambda z f tilde of S equal to lambda z over S plus lambda, because the Laplace transforms of e to the minus lambda t is 1 over S plus lambda.

So, that gives me something I can invert, it says f delta of S equal to lambda z over S plus lambda times S plus lambda minus lambda z. And the obvious thing to do is to write this as partial fractions 1 over S plus lambda is write this as 1 over S plus lambda minus lambda z minus 1 over is it. And then if I do that I get 1 over lambda z, so this is 1. And this obliging will cancels out, and just gives me these two rational functions. So, let immediately tells me, what f of z comma t is equal to it is the inverse Laplace transform of this here.

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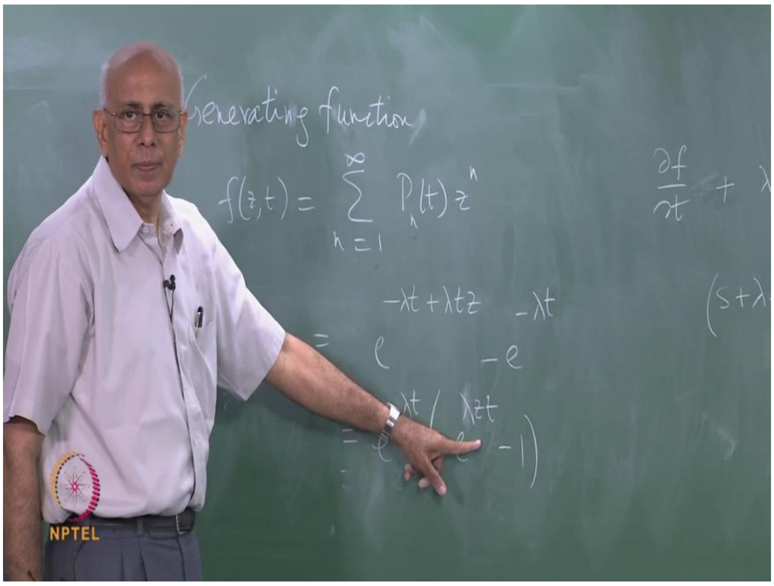

So, it is equal to e to the power minus lambda t plus lambda t z minus e to the minus lambda t, which is e to the minus lambda t comes out, and then e to the lambda at t minus 1, that is it. Check that the boundary condition is right, because you put t equal to 0 this vanishes as it. And all we need for P n of t is to find the co efficient z to the n in this and that is trivial, because it is immediately sitting right here. So, what is the final answer?

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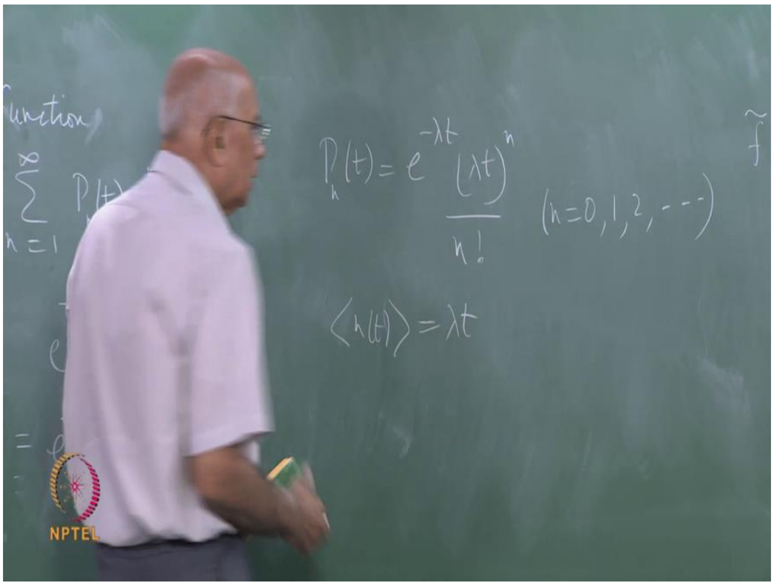

P n of t equal to e to the minus lambda t lambda t to the power n n factorial, and incidentally it is true for 0 as well, because we already know P naught is e to the minus lambda t, that is the pass on process. Now, you know this pass on distribution, because certain average and what is the average value? lambda t as it should be, what is the average number of decays that occur in time interval t lambda t, because lambda is the average rate of decay as it should be. So, this here is a pass on distribution with lambda t as the average. So, if you like you could write n of t equal to lambda t, and all the other properties of the pass on distribution of well know to you.

The variance is equal to the mean, the standard deviation is equal to the square root of the mean, the relative fluctuation goes like 1 over the square root of the mean and so on. In fact, for the pass on process, all the cumulants are also equal to the mean lambda t, that is what a simple radioactive decay does. This is an example of what is called birth process, because this random variable n can be cannot be cannot decrease, as t increases it must either stay where it was or it must go up, it cannot go down. So, it is just a pure birth process.