Selected Topoics in Mathemetical Physiscs
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Module - 5
Lecture - 13
Mobius Transformsations

So, let me a recapitulate quickly, what we already learnt about mobius transformations. And this where a 4 classes.

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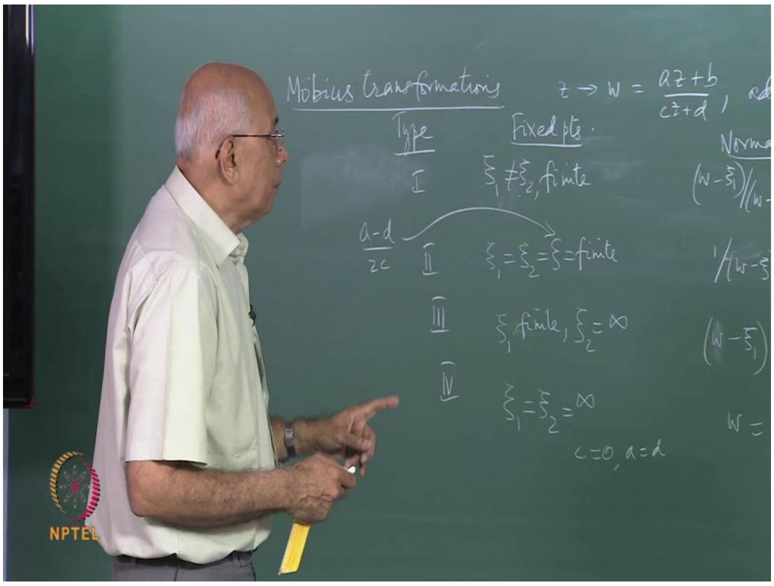

That is what we had comm of to last time. So, mobius transformations, in which a complex number z goes to w, which is a z plus b over c z plus p and we v a great, but, we can always said ad minus bc is equal to 1. Now we classified the transformations into 4, accordsing to the way in which the fixed point behaved. So, we have 2 fixed points and we call them zay 1 in 2 and they where a minus d puls or minus square roote of a plus d whole square minus 4 over 2 c. Now, when a plus d is pluse or minus 2, then off course a 2 fixed points go inside at the point a minus d over 2 c; provided c is not 0 and then c is forther 0 then you have a leniar transformations.

So, we found 4 ways of classify this transformations. And types 1 was, so lets write down fixed points and type 1 had zay 1 2 distinct and finite, whereas type 2 and zay 1 equal to zay 2 equal to zay also 5 nite. Type 3 had zay 1 equ 5 nite zay 2 fenta off to infinity. And type 4 at zay 1 equal to zay 2 equal to infinity, both for at a infinity. These are the 4 types that we distinguished. Now we also introduced idea of a normal form for all thees transformations. In other words this the not most convenient way of writing the transformation.

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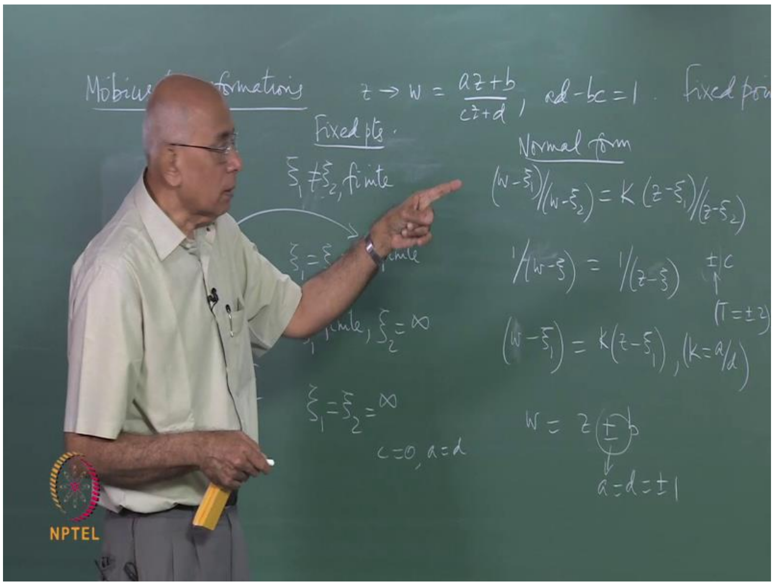

But, so, called nornmal form. So, let me write the fixed 1 up here. The normal form in this particular case was of the form; w minus zay 1 over w minus zay 2 was equal to multiplied k and zay minus i 1 zay minus z and k was the multiplier. And this was expresible entairly interms of the trace of this matrix of this co effecient a b c d. So, this was simply equal to e minus square roote of the square minus 4, where t plus square roote of t square minus 4, where t is a plus b. We used fact that, ab minus bc is 1 everywhere.

The normal form then had this form and as you could see, you could it rate this any number of times in all that happends is k becoms k to the forword n, after emitration for the n‘th it rate minus i 1 to over the nth is rate manies zay 2. So, that is extrimly convenien to write it in this form. I also pointed out that, it was valued even 1 n was a negetive venteture. So, you could back words and dothe imbures transformation and it rate that any number of times. And this would be exactly the same as before, accept this should have be ratio negative powers. Notice this is slide a sign I am big vit herence some sence. What i calls zay 1 and what i calls zay 2 is upto me have called it according to this was zay 1 and that was zay 2 and then case deffined in this fraction, but, clearly what you could do is invite that 2 call this the minus sign call that is zay 1 call that is zay 2. And them what all that will happed is a k will becom on over k.

So, it is real is the pare k and 1 over k, that is importent, that placed all in indipendent of choice a which 1 is zay 1 and which 1 is zay 2. The normal form in the this case where slightly is diffrent. As soon has you have the 2 coinside, it inflyes that a plus a hole square this 4 write your ... And then the fixed point becoms zay is becoms a minus d over 2 c, this point here. And then you can also see that, if t squared is 4, then k equal to 1 automatically. So, the multiplier becoms 1, therefore in this normal form, you wont have a multifyied. And what happends now is 1 over dumaine the zay becoms 1 over z minus zay. And then in plus or minus c and plus or minus depends on t is plus or minus 2.

So, t is plus 2 get a plus c t is a minus 2 get a minus. And remeber that this sign in this case, was equal to a minus b over the c. So, that is a normal form for the second kind. And once again you can it write it any number of times an all that will the happened is that this c will go on get ting a added 2, it will become end time see after an it rations. In this case, zay 1 is 5 nite zay 2 goes to infenity, this happeneds when c is 0, when c 0 this transformation becoms a lenier transformation. And then there are 2 fixed point, 1 of which is i can infenity and they otheres that, some find it value and the work that value of we did that last time.

The normal form now is simply z minus zay 1 k times, w minus. And the multiflied k is a over d and zay 1 is some finite value which we can figure out in this. So, this is what the normal looks like, it is just the lenier transformation in this case plus multiflyied plus shift. And finally, in this case here what happeneds is; you not only have c equal to 0, but, you have also have a equal to d, that is only possiblity. And since a d minus bc must be 1 it is say a squared is 1 or a is plus or minus 1, as it is he as, it is has it is d. And the normal form of this case some playence w equal to z plus or minus b.And this plus or minus is according as a equal to b equal to plus or minus 1 and this is your shift.

So, this is what the 4 types of this called type; this is 1, this is 2, this is 3 this is 4. So, all away transformation is classified to characteristic. The great advantage of the normal form is that, you can interate the move back both, you can composed different transformation. So, also what much more is then its original, but, it is an entirely this 1.

Now the question is we saw last time that, if you took the most handle case, may be ... I solve this for the eninterate w, after eninteration of it and we got this wright hand side to be some to be transformation, in which k to the power n appear in the numerator and non numerator. And then I ask you that most case bigger than 1, in has entance to infinity, all points move toward 1 or the 2 fixed points.

The other 1 is fell and if k was less than 1 in modules, it was reverse the roles of these to per reversed. So, that helps us see what the roles of this multified in each time. The 1 more is reason why the normal form is a such great signifacant. Now the whole of the all of this is guided by the fant that, k determines essentially what happends to the floor, this multified. And there are 3 possible cases.

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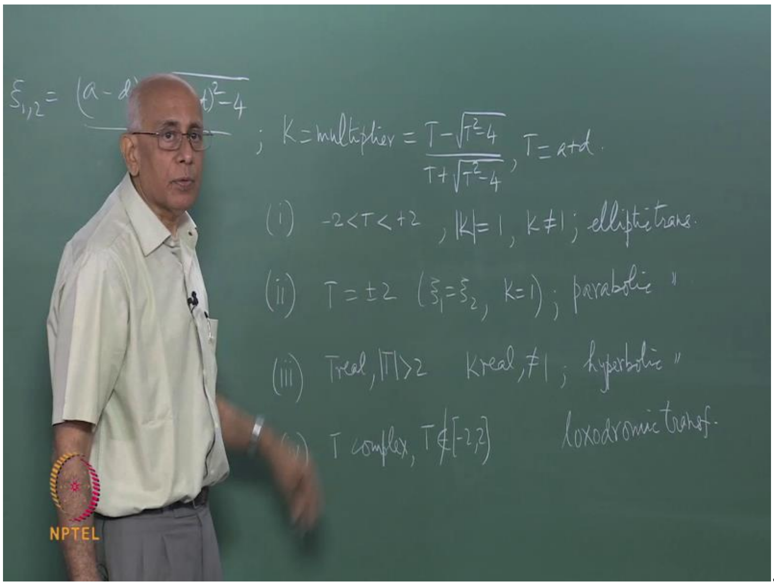

The 4 the possible cases. First case is when minus 2 less than t is less then plus 2. So, t is real, abcd at complexed corner concept general. But, if a plus t is real and its lie between minus 2 and plus 2, then its clear that,this k has unit modeles, because its a form a minus ib over a plus ib for a and b are real numbers. The modules of each of them is same; square root the a squared and b squared. So, this immediately implies k if mod k equal to 1, k itself is not equal to 1 because this thing is not 0 about here. It strictly between minus 2 or plus 2. So, mod k equal to 1, k is not equal to 1. Only in modules in these numbers is equal to 1, but its not 1 and then itself have whats called an elliptitrans transformations.

The second case is; when t equal to plus or minus 2, real and plus or minus 2. Now the 2 fix points go inside and k equal to 1. This transformation called a parabolic. The third case is; when real mod t greater than 2. So, this solve the real access if a t, but, outside minus 2 the interval minus 2 to the plus 2. What happends then, this is a numbers, this numbers suddently a fraction of some kind its a real number of some kind. So, mod k; k is a real number and this not equal to unitly. This thing is called a hyperbolic. These words come from well, its not have to see where a come from. They actually its come from the nature of the floor around the fix points in the complex plain or there is once fear

The fourth case is t complex, t not an element of minus 2. So, its not 1 the real access, its some complex numbers and this as the here this called a loxodromic transformation. This is a special case of the loxodromic transformation, when the maximum part of and of t vanishes and its it on the realises, but not the interval minus 2 to plus 2. So, using this this is the starting points of the actual classifications, in depoendent of some fix point there are some cased that, you can see k is equal to 1 this is the 2 fixed points go inside and solved. So, for, but, others and that, that is a normal used for normal forms, but, this is useful for analysing the floor, in the actual forms in the complex.

So, I want to go for the end of this, because these whole not you can do, except that I introduce the concept of this the icomentric circle and pointed out that, certail circle plus c is that plus d this case if that equal to 0, this wright this side.

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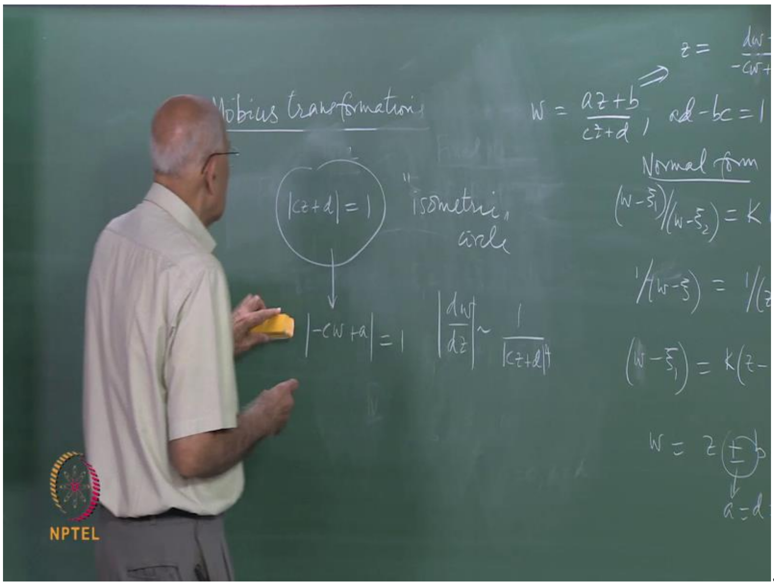

So, mod c z plus d equal to 1 for the isometric analysis. The reason we called it an isometric circle, must because lengths on this circle do not get changed, they do not get strech or contracted under the ... This circle is maped just as we known that every circle is maped on some other circle the mobius transformsations, this circle maped on some other circle. It is actually maped on isomatric circle of the inverse transformations, this is a simple excercise proof. And what was the first transformation to this? w equal to this, this things immediately applied the z was equal to dz minus b core minus cz plus a. Again ad minus bc equal to 1 still, by means w, I used that simbles.

So, this is maped on to this circles, is maped under transformation to the circle minus cw plus a mod equal to 1. You should not make a mistake of assuming that, the centre of the circle is maped to the centre of this basis. Where the centre of the circle is maped to? At infinitry, it is maped to infinity, because you put z equal to minus d over cz comes infinity right. So, this centre of this circle is maped onto infinity. Where is a centre of a these circle on to a inverse map? Infinity once again right.

So, the infinity in the z plain is maped to the point a over cw plain and minus c over d in the z plain is maped to infinity in the w plain. But, this circle is maped on to this. And what happens is in the zacobians of this transformation, dw over d zee of this think here is propotional to 1 over cz plus b into the power 4. So, what happends is that, if this is less then 1 which is intirier of the cercle, then under the maping lence expand.

So, the inside of the this circle is a maped on to the extirier of the circle and vica varsa,. But, on this circle lenthed on change, there on strech at all remain unchanged. And lot of further information about mobius transfamation makes use of this isomatric circle, the idea of the isomatric circle. We not go to do very much more with it, but, I will try to give some excersises based on this.

Now the next thing we said was the retransformation form a group. And that is very easy to see once you have the normal form, because it is clear itteration just keeps multiplying this we still have again more and more a mobius transformations once again. The composition of the any 2 is again and mobius transformations. Every transformations as a inverse there all invertible, because the determinentle is equal to 1 and so on.

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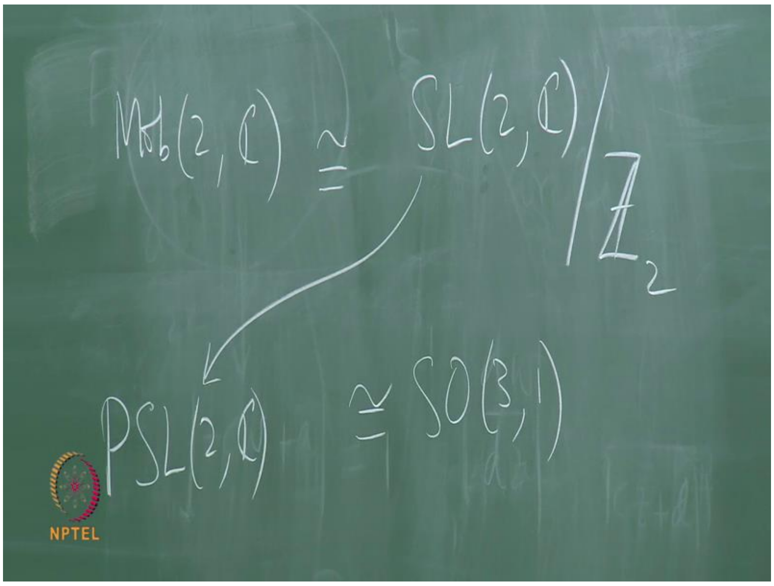

So, they formal group, but, they group is not the group of mobius. The group a mobius transformations on the complex number is isomorphic. So, morfic not to the group to see, which is the group of uni moduler, that is determinenet 1 2 by 2 matrecys with complex entry's. It is not quite that group, but, it is this group cotionted with the cyclic group of order 2, because we no that this transformation does not change, if abcd the all off them changes sign.

So, there are 2 metricys in sl 2 see, wich correspond to the same transformations and they differ only bias sign. That is taken into account by saying that this is modulo of z 2 upto a sign. This 2 2 1 homo morfism between sl 2 c and this group mobius a group here.

Now terns out this group has other interesting properties. This group is also what is call the universal covering group, for the homo genias lorence group in 3 plus 1 space time a ditementions. And that is group is also called a para meter space, which is w connected. So, once again sl 2 c over z 2 is isomorfic to the homogenic lorence group. So, this is also iso morfic group, so 3 commoder.

Therefore, the 1 to 1 correspondence between the group of projective transformations of a complex veriable or mobius transformations and the group of homogenias lorence transformations and 3 plus 1 dimentions space time, which can be exploited, but we not goin going to that, but, that is sort of mathematicle relationship, which is extrimly used from. So, this is the general mobius group. But, we could asked the question are there any sub groups of his group, which when the subset of transformations, which then serves form group among themselves? And the answer is yes.

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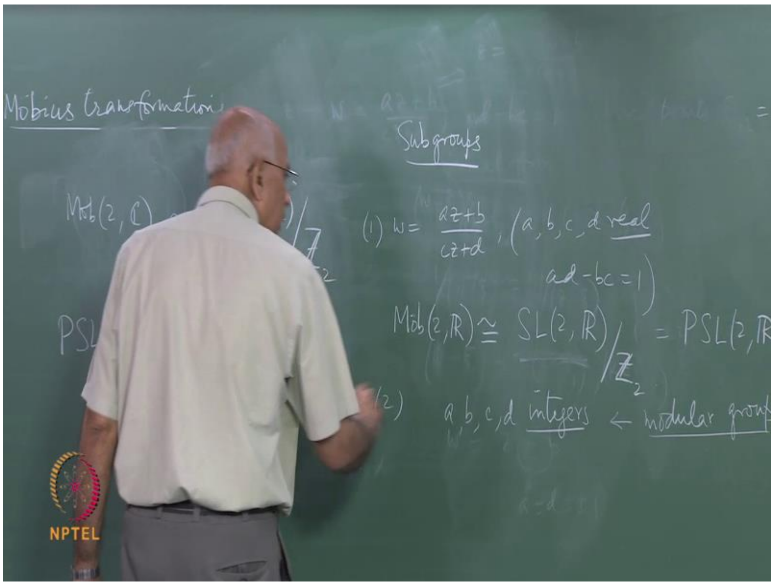

This set of transformations, let us write down the set of transformations; a w is equal to az plus b ocz plus b abcd real and ad minus bc. This set of transformations also forms a group. So it is interesting that remember z is a complex number veriable and w is a complex veriable, this combination here where abcd are real, you multiplyid 2 such real metricses, you get 1 real matrics and so on. So, they 2 form a group among themselves. Once again you can change the sign of abcd and the transformation the not change.

So, this sub group is called the mobius group on 2, on the reals and this iso morfic the special lenior group on the real, co tiented with z. By the by, there is a fancy name for this think here, this this gay here is called the projective special leniar group 2 c,it is writen in this fation to show that, it is upto a sign is goes both. Both matrics of projected on to the same entity. So, this is also a a equal to ps a, more compactive way write in the same think.

The any other groups of this kind will interesting line of s we terns out and is not hart to veryfy that if, abcd are intigers, then they are formal group of formal among themselves. So, your restric abcd to in intigers and that forms a group in itself. This group has very importent role to play in mathematics. This is called modular group. So, we identyfyed servel of them, is a 1 of them is this, real and then asecond 1 is where we follows are intiger.

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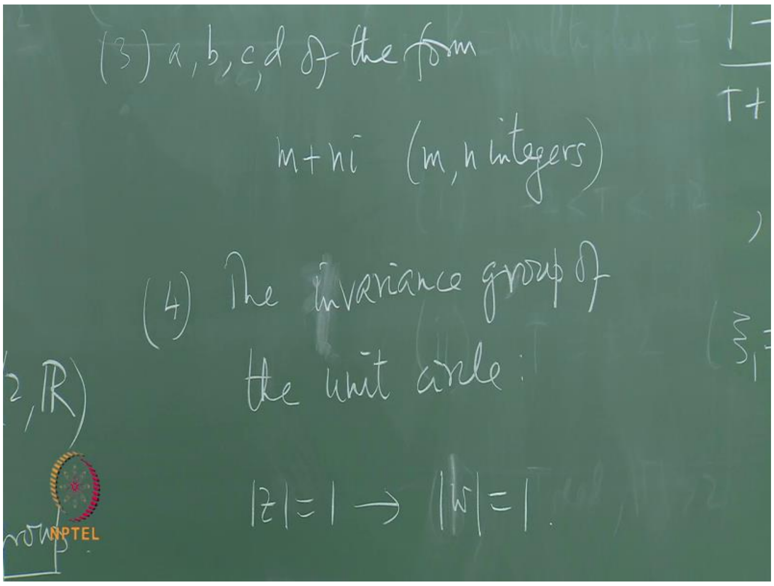

Similarly, you could have abcd of the form, of what a called goves in intigers, namely; ther of the form m plus ni mn intiger. This a called goves in intigers, where a you have complex number invites, the real part in the imaginery part of both of the intigers. They true form of group. So, all transformations in which abcd are restricted to goves in intigers, they form of group among themselves. There are several others very intresting groups for instents; sub groups. For instense you have 1, which is the the group the sub group and the group, the inverience of the unit cercle. Another words you asked the questin, what is that is set of amobius transformations, which takes a mode z equal to 1 to mode w equal to 1? So, under the map, you want modsey equal to 1, should map on to mode w equal to 1.

So, the unit circle in the z plain should mape on to the unit circle in the w plain or in the immonsphear, the equater should mape to the equations in this case. Well it reques a little bit a work to show that, this actual is a group by it is self. What you have to do is to say; all right z goes to a z plus b over is c z plus b. So, you impose this condition, given that mode z equal to 1 and ask what is the condition on the coefficients abcd, such that mode w is also equal to 1 and this mape and then you discover after little bit a worked. Then electrices exacise.

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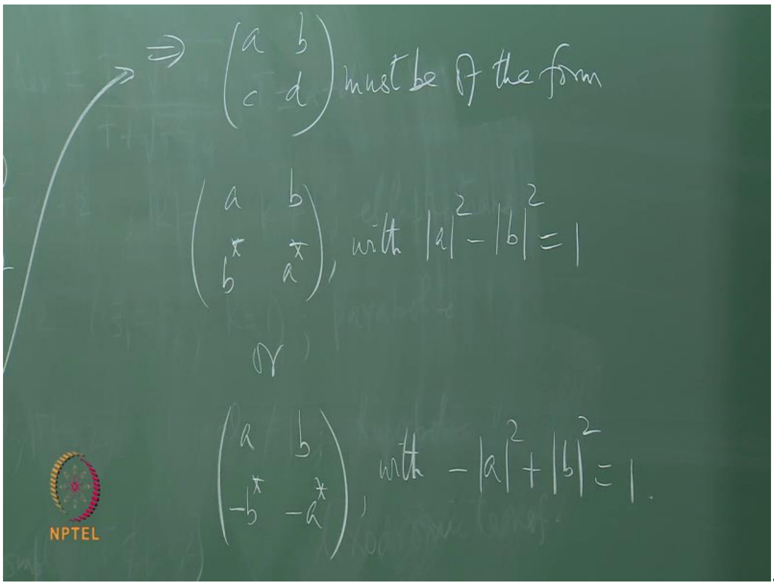

You discovered that this will imply that, the matrics abcd must be of the form either a b b star a star, with mode a square minus mode b square is equal to 1, 1 the diterminent always to be equal to be 1. So, it should be a of this form, then your garentyed 1 is conditions seticefyed, the mode z equal to 1 is mape to mode w equal to 1. Or, so let us call this 1 a, 1 set of metrisis or it should other form ab minus b star minus a star, with minus smole u square plus equal to to 1.

So, both thees up a meter. And once again you can change the sign of abcd complitly, all 4 coefficient and the transformation will not changer. But, the set of matrisys 2 by 2 matrisys, with saticefy this condition al this condition is the interest in it is self. It is very closly lenth to the set of metrisys, which give you the unitery metrisys 2 by 2 unitery matrisys. Does any 1 know the most general form of ta 2 by 2 matrics which is unitery?

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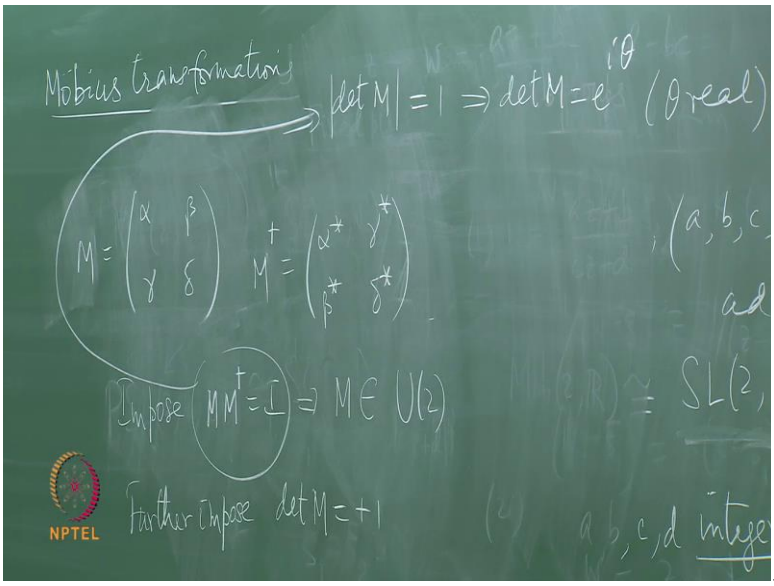

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So, you want start with the matrics alpha, beta, gyama delta. And let us this called this matrics u and you dager there for is alfa star, delta star, delta stare and delta stare, this vertion. And now imposse u u dager equal to i. So, multiply 2 and say it must be the unit the matrics. That is the unitery matrics. Now this is count give you conditions on this coefficients. And you can find out what is most general form of a 2 by 2 matrics, which is unitery. I live this your a exacise, will going to comback to this, because this is very importent is a conformation. You should be able to write down the most general 2 by 2 unitery matrics. After that will imposs the condition that they diterminents shoul also be equal 1. By the way, this will immply that you is an that this matrics, let me call it m for the movement. This will immply that, m is an element of the group you to and we group of 2 by 2 matrisys, with complex entries which i unitery. They form of groups among them selves and it is called unitery group be u 2 n you noted by u 2.

So, I want to find the most general form of this u 2. Insidently how many parameter's are sitting here? The 8 parameters, 8 real parameters, because each of thees number is a complex number in general. There 8 parameters and when I say something is unitery, when I immpose this condition, I compute this matrics and I equated to unit matrics. So, that a 4 conditions and the 4 elaments. So, there are 8 parameters in there are 4 conditoins among them. So, how many indipendent parameter would you accept? 4

indipendent parameters.

So, I want to find thous parameters. It is clear that this fellows cannot al the independent of each other. You could choose alfa and there are to the independent and then you must show the gyama and delta's you determind by or some stet's think. So, try this out. Now, ferther immposs determinent m equal to plus 1. When you do taht, you not only immpose the condition of uniterity, but, you also set the determinent is equal to 1. And now you get a ferther condition. By the way whats a determinents of this matrics? what is what is this tell you about the determinent of m? What is it tell you this very importent what is it tell you about what is this conditions tell you about the determinent of m? It is says modules determinent and equal to 1 this are modules, but, thees are complex entries. So, what it is say about determentnt. It is a pure face factore a fete as real.

So, you should before into the saying; this simply the employes that a at determinent and this plus 1 or mynas 1 no. Just says is a surface factors, could be plus 1, could be minus 1, but, it is a surface factor. So, that is needed in order to derived, what to the conditions may to have here, remember this condition. Now should ferther inposse determinent time is plus 1; that means, data must be is equal to 0. Now how many parameters you think there are? 3 independent parameters, because you have got 4 conditions in the elements, in the new got determinent equal to 1. So, there are 3 independent parameters.

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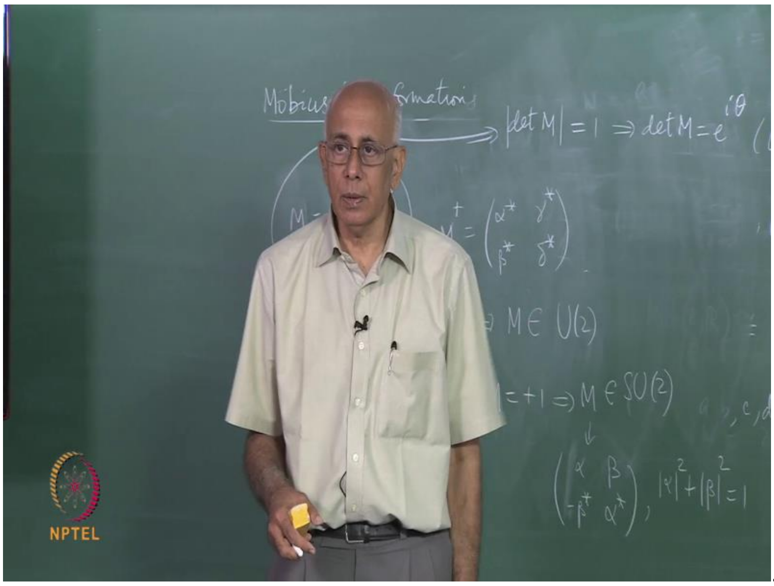

And this would employee that, m is an element of this special unitery group into dimentions s u 2. As you now s u 2 plus fundamental role in quenter machanics in spine, easpecially in spine how and it is fundamental to physics. We will say about little more about this. And the question is what is the most general matrics in s u 2? It terns out the most general element in m must way of the form are data star of the data star whith the minus 1. And the deteminent must be 1 off course, so mode alfa square, the smole bite square is equal to 1.

The most general unitery unimoduler matrics, you can write down 2 by 2 matrics, involves 2 arbetric complex number, saticefying the condition that the square of the moduly and upto 1 square. So, how many indipendent parameters are there? 3 indipendent para meters, because if you call alfa 1, alfa 2, the real in the maxinery parts of alfa and similerly for bieta. Then this say's alfa 1 square plus alfa 2 square plus deta 1 square plus deta 2 square this 1 which menas 1 of them determind by the others and the by the all 3 others. What kind of para meter space is this? s 2 then is not s 2, it is not a s 2, because what you have the parameters space in, we will they attenction to the parameter space is the verius group. I will comm back to this in a talk about rotation groups.

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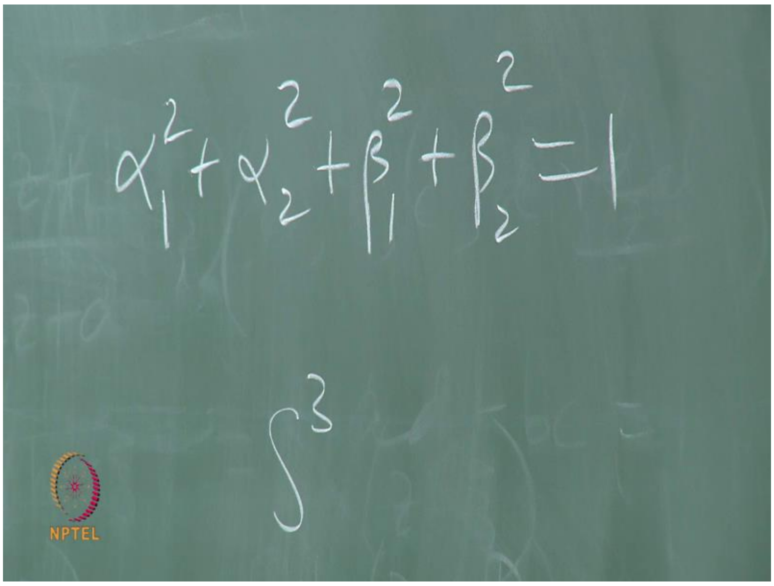

You have alfa 1 square plus alfa 2 square plus deta 1 square plus deta 2 square is equal to 1, it is 4 diameter space, alfa 1 alfa 2 deta 1 deta 2 of a real numbers, so they will live in are 4, but, then the saticefy this instence. This is the surface of highpers fear in 4 dimentions. So, 3 dimentions object and this is s 3, the 3's fear. What is the 1's fear? It is just a circle. So, 1 dimentional object is just topiligically equavellen to a circle. The 2's fear is the femilier surface of the glowth introduced in, but, a 2 dimention objects. So, mathematisions write s 2.

The factor it is in better then 3 uclition dimentions is insidental, so 2 dimential surface, this is s 3. We cannot cansive of thees is like taking 3 dimentional space in putting all the point set infinity in 1 point, is order soing it together. We can do that for a panket, but, do unit in 3 dimentions for the 3 dimention space is not the truvial and whatever it is, this is the parameters space of a s u 2. It is happends to simply connected, in the sents that, you can go from any point to any other point on this space continuesly r quise, withoutly in this space. So, it is connected and it simply connected in the sents that, every shows path on this space, can be shunk to a point, distorder to a point, without ever live in this space.

So, space where all the every close path can be complitly shunk to a point is called a simply connected space. Is s 2 simply connected? Yes. On the surface of this glow by, I can put a rubber band and shunk it without living this space. Is s 1 simply connected? No.

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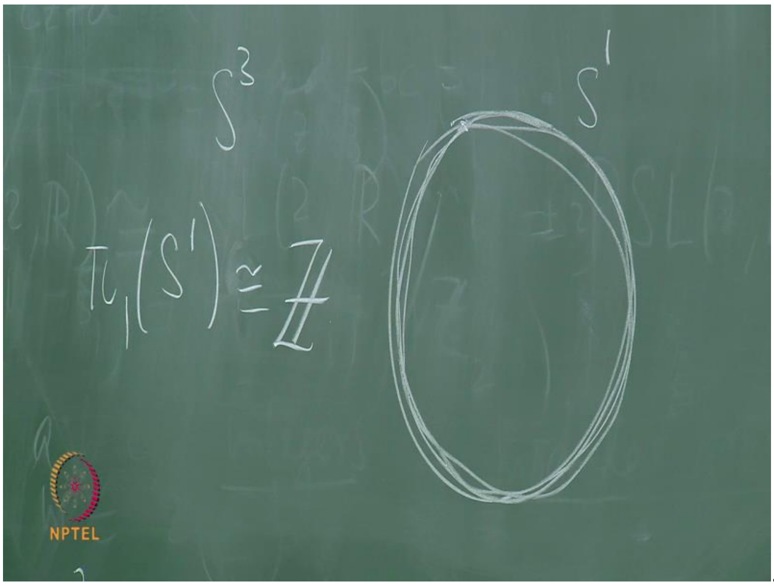

It is not, because it is clear that they this is 1 And you cannot the live this space. So, any close path would main it started point you go there and you come back. That is a close path and you can cerclly shink it was point. You could go all the way round, not quite meet the first point and come back and you can shil shink it your point. But, the movement you go all way round here and come back to starting point, it is like fitting a rubber band on the rem of it is cycle weel, then short of cutting in, this no way in which you can shink it opned it. So, that is a path which cannot be shunked a point.

Now you can do that twice and that can never be change your path will goes around once. So, you have a another path, which is distinct from thous which you can shink your point and thous which go around once and then you can do it 3 times 4 times and you can do it in the oposite direction; minus 1 times minus 2 times is gon. So, each time dippending on how many times go round, this as space in the possitive sence or the negetive sence. You have wynding number associated with the each such path. And all thees paths can be clssifyied into equalents clasess, which are lable by this wyding number.

So, 2 pass which can be diffounding to each other, bellong to the same class and they correspond to the same wyding number. Now the set of thees wynding numbers themselves form a group under addition, they intigers, they form a group under addition. So, the technical way of saying it is that, a fundamental homotopic group; namely the group of equavelents clasess of close paths on this space of s 1, happends to be the set of intigers, the group of intigers under addition. And it is a writen in a sort of evacated fation, it is writen as 5 1 of s 1, is isomorfic to this set of inclusions. We will, this as role to play and you will talk about to this little later. But, s 3 is simply connected. So, what on says is 5 1 of s 3 is a tribual group. In other word, it has only 1 element, every path can be shunk to a points.

So, there is only 1 equalvelents class. When we have 1 group with 1 element, it is called a tribual group and mathemetitience like to write to it as just 0, is just a notetion it is with there are no it is only 1 elements. So, this space is simply connected, the parameter space of su 2 which as going to play very veery fundamental gloup. Let me make a statement here, but, whenever I talk about a covering group of a cetain group, like in the case of the mobius transformation. I said sl 2 see was a covering group and modulos z to you got to mobius group that, sl 2 see is a simply connected group.

So, the covering group is simply connected obiously and that has consicquences to. We will get back up to this. Now the question is what a thees thinks? This is not quite the same as that, this is slide diffrence here between this sort of think and that kind of matrics that, because here we had alfa, bita, minus bita star alfa star and you had mode alfa squared plus mode bita squared equal 1. On the other hand, here you got a a star, b b star with mode a squared minus mode b squared equal to 1. So, you see that is little minus sign here has immidietly changed every think complitly. What is the parametere space there? Whats that equal to? What is this tell you?

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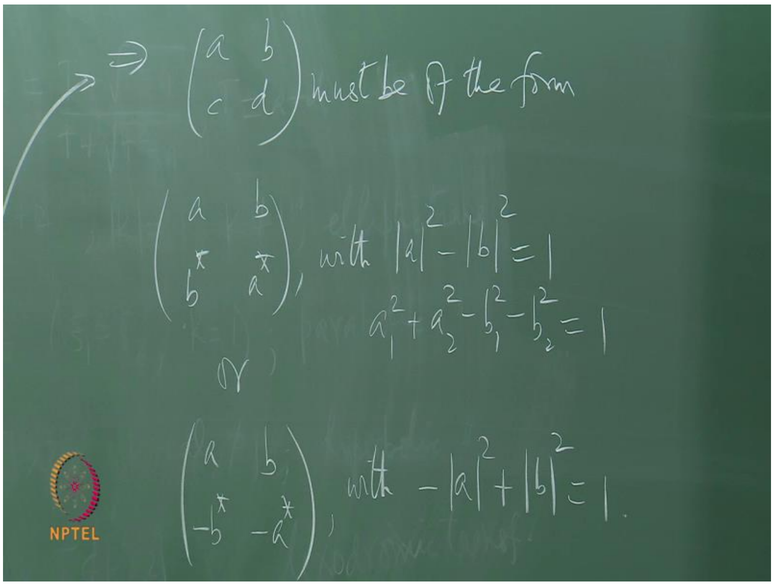

This sow that tells you that, a 1 squared plus a 2 squared minus b 1 squared minus b 2 squared equal to 1. What kind of the objectives that? It is not s 3, that is very clear, because are a minus sign there. What kind of the objective that be? It is a highper baloid.

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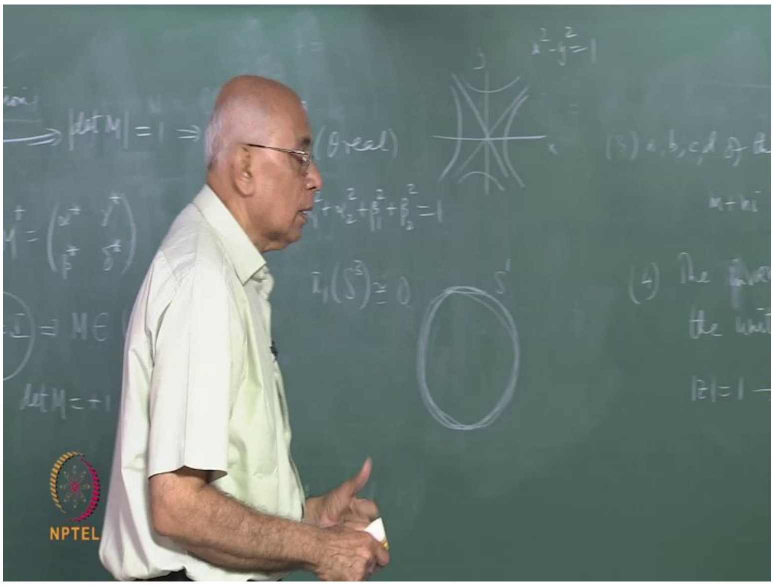

You now that x squared plus y squared equal to own it is a circle, but, x squared minus y squared equal to 1 is a highper balon, 2 branches of highper balon. So, if you had on the plain, if you had s squared minus y squared equal to 1 and you ploted x here and y here, it is thees 2 fellows. What if you had y squared minus x squared equal to 1? it is off course thees 2 fellows.And the big difference between highper balon and the circles is that, a circles is compective if point an it is boundry, but a highper bala goes of to infinity.

So, thees think, thees para meter space is unbounded. They can become a arbertry large, they can you make a 2 very large, b 1 very large and still have the diffrence very finnity. So, theese are non compact, this is theese parameter space is groups are set to be non compact, whereas a group here is in compact, in plays a very very significent role here.

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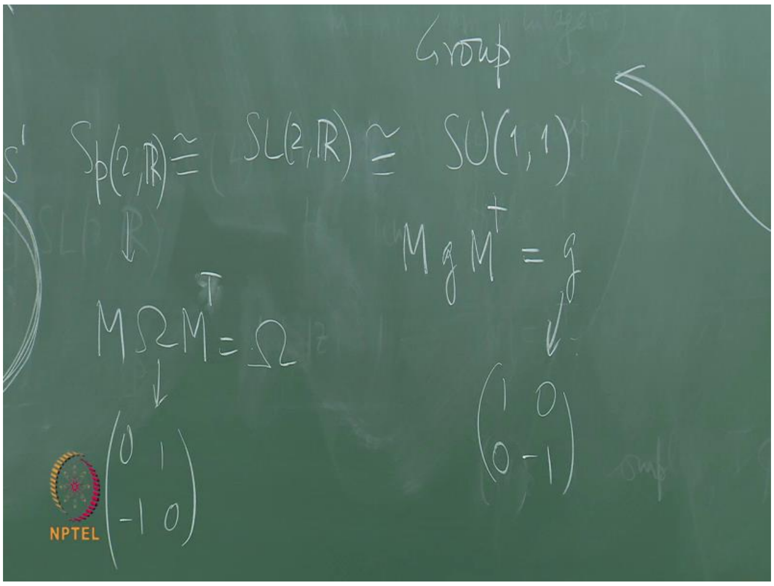

Now, this group here is the group. This set of matricys; they form the group su 1 come a 1 because of that minus sign. So, it is the set of matrisys, which do not saticefy unitarity condition, but, we saticefy a condition like mg m diagran equal to zee and zee is this matric matrics minus 1. So, I live you to very 5 that, the most general 2 by 2 matrics, which saticefys this condition with this ma matrics zee here, is either of this form or of this form, is called to be 1 of this 2.

So, theese are like 2 difference parts of the highper bala. As you can see 1 of them correspondence something like this and the other correspondce to something like that, but, they are non compact, theese space is are non compact. And this is called and indefinet unitery group. The reason is that, this suffing here can change sign, this quantity can change something, it is not a not all all the sign all the quen all the terms do not appeared with the same plus sign between the minus sign and between. That makes it non compact. This group assue 1 1, by the way, the group your talking about of mobius transformations off course is not su 1 1 it self, because as you no for every su 1 1 matrics, you can change the signs of all the terms and do it is have still the same mobius transformation right.

So, again you have that modulos at 2 business, but at the group that is ... So, that the matrics group that, is of the interest here for theese mobius, for theese mobius transformation which lives the unit circled inverient, they are related to su 1 1 is 2 plus 2

1. Now you su 1 1 itself has other interesting properties, this group happends to be isomorfic interestingly to sl 2 come a r. So, the set of 2 by 2 matricsys, with unit determinents with real entries is in 1 to 1 correspondents with thous matrisys. And it is also isomorfic to the group of canonicale transformations, with 1 degree of freedome.

So, all matrisys which satisfy a certain simplecting conditoins. So, this cantain al 2 by a 2 matrisys with real entries such that, n omega n transfort is equal to n, where this matrics omega is 0 1 minus 1 0 is a called simplectic matrisys and I talk a little bit more about it when we do group thiory, because all the canonical transformation of a systeme with an degrees of a freedam hamilton in system, will form a group called simplective group sp n cup 2 n come a r an of that this is the case n equal to 1. So, happends that, when n equal to 1, you hav all theese wonderfull relations, there all iso mofric future.

So, again exacise for you which is to show that, they group of the unit cercle the group that, lives unit cercle in verient, would contain matrisys of this kind or this kind and nothing els is possible. And ferther that the most general matrics, which saticefy this conditions is is again the samething. So, you just a pice of valuibra, which we have to do. But, i suggest to do this first beacause, it is will tell you how to go about it in the most general case. Other a any other sub group, it is will that lots of other sub groups, but, here is and interesting 1.

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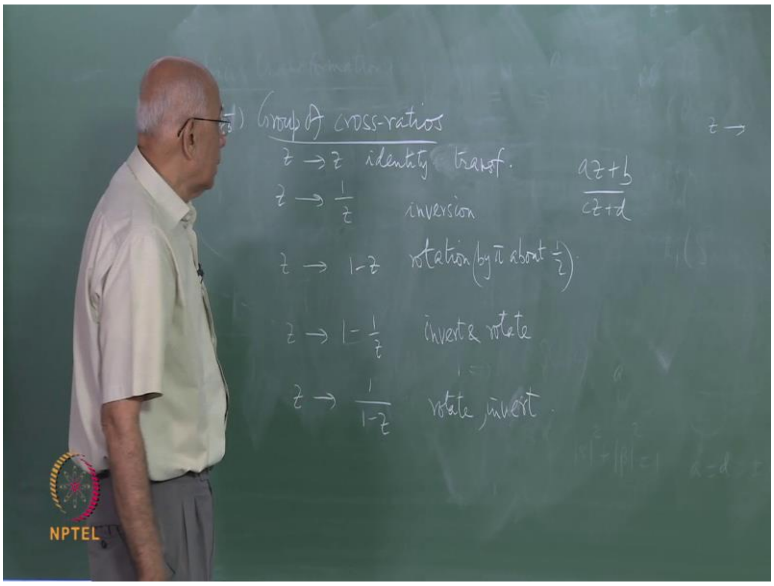

This is called the group of crossed ratio and not sure of it is the other names. So, how many you have the done about 3 or 4, this is probaly 4 or the fifth 1 is in the fifth 1. You had the goves in intigers to which is the follwing. Suppose the z goes to 1 over z, that is in the invertion right, z goes to 1 minus z, what kind of transformation is this? Well it is say's about the point z equal to half, you rotat by pai and z goes to 1 minus z and the point half remains in varient, as you can see. So, it is a rotation about half, but, since you also changing z to minus z is a minus sign there, it is a rotation by pai. So, this is in inversion and is the rotation. You combined this and you can have z goes to 1 minus 1 over z.

So, you first invered, then you rotate. You could do the other thing. So, you could rotate and invered. This side goes to ... what is could you do?

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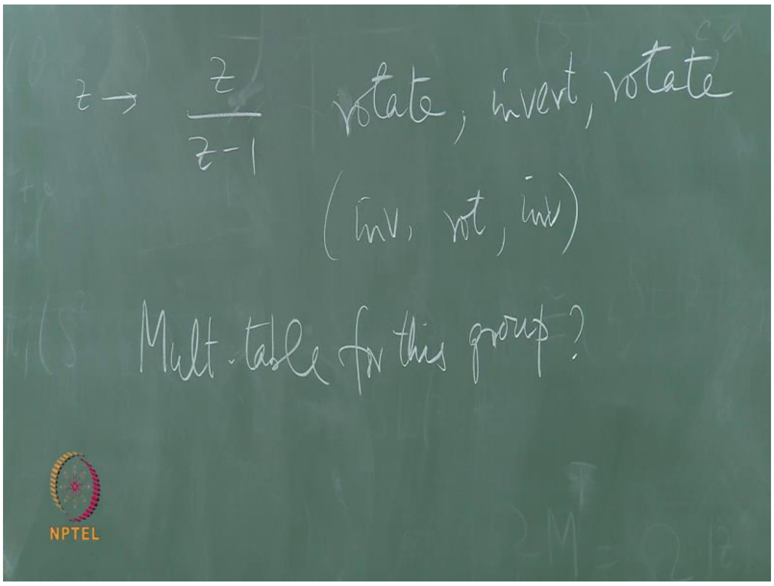

You could do z goes to ... So, after you do this. So, you rotate invered and you rotate again and then you get 1 minus this stuff and what to do that do. So, they will give you z and the minus 1 you cancel. So, that to becoms to z goes to z was it minus 1. So, this say's rotate invered rotate. You could do the other way, you could invered rotate invered. Now what will that do? Well if you invered, you get this you rotate, you get this and then invered to get back to back current. So, that is not to anything, you where you could also just well have written this as; invered rotate invered. And off course, you show something as a is a group, you need the identity elements. So, z goes to z is the identity tronsformation. And it is not hard to veryfy that theese form a group by themselves and they mobius transformations special casess.

So, for instent in this case, d is y is a it should compare it, with what happeneds to az plus b over cz plus d. It is clear that, in this case you said well here abcd's truvial in this case, nothing happends accept that, all the this is 0 that is equal to this, you cancel and 0 exectra. So, what is the identical correspond to? It say's c is 0 and then a over d is 1 b over d is 1 and we put ad minus bc also, he could 1. So, that is a tribual case, but, here this would mean that a is 0 and d is 0 and so on, for every 1 of this transformations, but, is a tribual think to check that, they form of group among themselves. This is a fine 8 order group a 6 element.

So, you should write down for theese 6 elements, the group multiplication table. They composition of 2 more will give you 1 more exectra. So, had like you to write to this down. Is this an abelion group? Do this fellows come you to the each other? Setling not, you can see that they producessed the different risults if, you do it in different orders. So, it is not in abilion group, it is to not all together it tribual group. It is a group of order 6, it has a lot of a interesting property's it is. And it is called a group of a cross ratio's for a reason, I want you to discovere, we will playing with this thinks here. Remember under the mobius transformation, cross ratio's do not change, they remaining the unaltarate.

So, there is a simple reason why this is called the group of cross ratio's and I want to discoverds this. For the several other prtoerties a of theese transformation, but, be live that to the a live that to the a exacises right, there some practise understanding this and then, if neccesory will come back and discorse thous properties once again. But, let me summerise finally, by saying what is the significant's of the theese transformations. Mobius transformations the most general possible transformations, which a 1 to 1 mapes a very immanced fear a there immanced fear, there inverte to be ther 1 to 1, this a most general form.

If you did this keeping the pointed infinity fixed, then you just get lenior transformations, but, the movement you allowe the pointed infinity to mape the some other point, you get this interesting group of transformations. And, because it is called all theese connections will be the other's group in so on of very great intereste in many many contacts. And, because it is an maping by in analitic function, which confirmed. Circles are maped into circles under this transformation. This is certain circle which is not streched or contracted and then there are verias possibilities for is streching a fixed points in the flow around theese points in the so around so forth. So, those are some of the interesting properties of the obius transformations. Now 1 point which I forgot to make about the inverience group of the unit circle.

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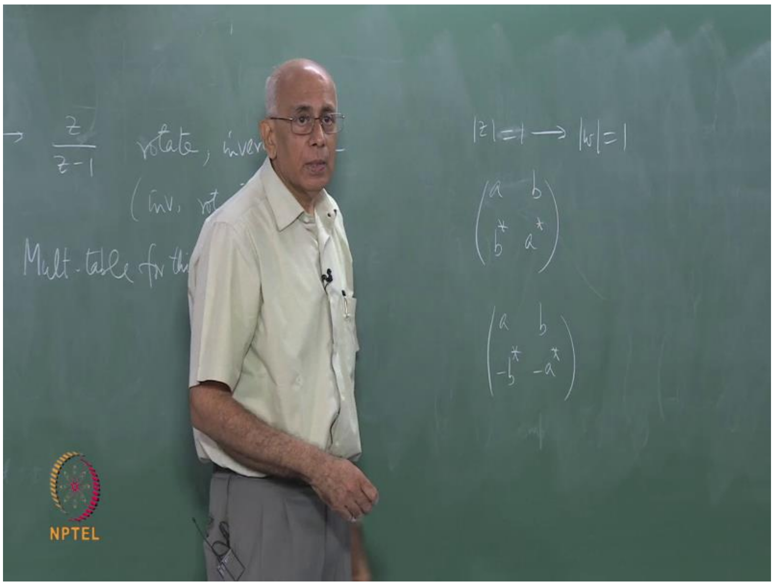

The unit circle is that, the transformations of the form a b b star a star as suppose to transformations of form a ab and minus b star minus a star is the follwing. Under both theese transformations, remember that mode z equal to 1 is a maped down to mode w equal to 1. But, you could asked what is the main difference between theese 2 transformations? They are 2 parts of highper bala, 2 different branches right. Under this transformations, point inside the unit circle an z and point maped to point inside the unit circle and w. And thous out side a maped to thous out side. That is something else you can veryfy easyly.

On the other hand, under this set of transformations, point inside the unit circle here mode z less then 1, are maped to mode w greater then 1 outside the unit circle and wisevarsa. So, that is the different between this set and this set of the matrics. And this set of matrisys, there are 2 thiss this joint set of matrisys together, they constitute the su 1 1 2. And when you interms of mobius transformations, this property of mape in they unit circle to it is self, keeping the intirier and changed is restricted to 1 of this. And exchanging the intirier an and the extirier in this this set of transformations.Theese are all easily very find and you should do. So, write down valuegible as quite simple. So, once you show that, this is the most general form of a matrics, which satisfy this condition, then it is easy to come of a, to dirive theese conclutions as well.