Select/Special Topics in 'Theory of Atomic Collisions and Spectroscopy'
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Lecture 10
Resonant Condition in the lth Partial Wave

Greetings, we considered in the collision phenomena the breaking of an incident plane mono energetic wave into partial waves and we discussed that usually partial waves with the smallest angular momentum quantum number is l = 0, 1, etcetera, are quite sufficient. And in most cases in fact l= 0 alone is good enough. Because higher partial waves with higher angular momentum they do not penetrate into the collision region.

There is a centrifugal barrier there is so many you know all the related physics is what we have discussed. Nevertheless if you remember the expression for the tangent of the phase shifts you have a certain condition which I will be referring to as the resonant condition and from the expression of the tangent of the phase shift which I will recall once again.

There is a certain resonant condition and depending on this resonant condition it may be necessary to consider some of the other partial waves. So, when you are doing S wave scattering it might be necessary to take into account P wave scattering and may be even higher partial waves that possibility need not be ruled out but it turns out that P waves are sufficient in those cases.
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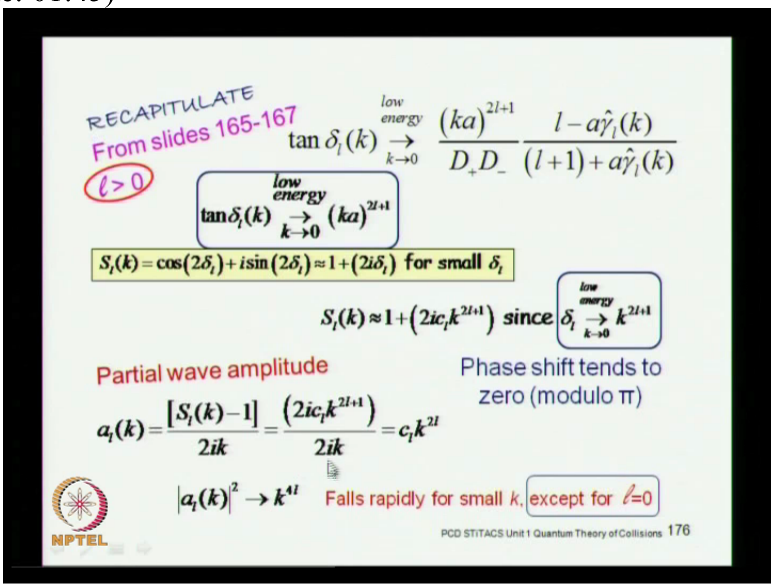

So, let us go back to this expression for the tangent of the phase shift in the lth partial wave and I bring back this expression from the previous class we had. From this was the discussion on slide number 165 through 167. And we had an expression for the tangent of the phase shift and we saw that in the low energy limit. So, this is the low energy scattering that we are now focusing our attention on.

And in this regime you have got an expression in which the phase shift goes as 2l + 1 at power of k okay h cross square k square by 2m is the energy. So, this is going as k to the 2l + 1 and then there are other factors this is for l greater than 0. So, the phase shift goes as 2l + 1 is power of the momentum and notice that at small energy the phase shifts will be very small because it goes as the tangent goes as k to the power 2l + 1.

And tan theta and theta and sine theta they are all nearly equal to each other as theta tends to 0 right. So, as k tends to 0, tan theta is very close to theta, so the phase shift itself goes as k to the power 2l + 1. And the S matrix element which is cos 2delta - i sine delta, so this becomes nearly = 1 +2i delta okay because of delta being small. And the small value of delta is k to the 2l + 1.

So, the S matrix becomes 1+2i delta, delta is proportional to k, 2l to the 1 if that proportionality is c, you have got the S matrix given as 1 + 2 ic k to the 2l +1. Now notice that the phase shift tends to 0 as k tends to 0 and this would be modulo pi as such. And if you look at the partial wave amplitude which is S - 1 over 2 ik then S -1 this 1 will cancel and then you have got this 2ick to the 2l + 1 by 2ik.

So, one of these powers are in 2l + 1 gets cancelled by the power of k in the denominator and you have got the partial wave amplitude go which goes as k to the 2l. And of course it falls very rapidly if you take the modulas square, it falls very rapidly except of course for l = 0 for l = 0 we have to go through a different relationship.

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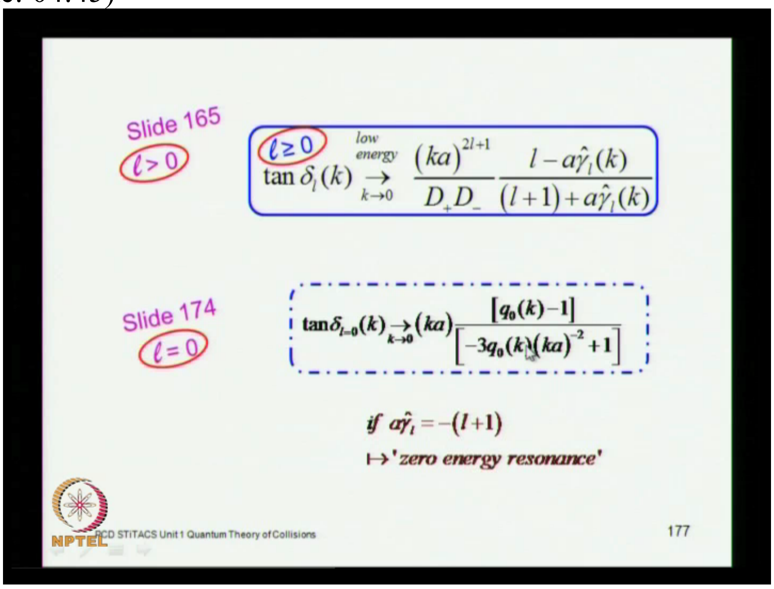

So, this is what you have for l greater than 0. For l = 0 this also we obtain this expression in the previous class that the tangent of phase shift for l = 0 is given by this expression okay. Now this expression looks a little bit different from the expression for l greater than 0 but we will show that both of these expressions are really equivalent and the relationship over here in the upper expression is valid not only for l greater than 0 but also for l = 0.

So, that is a general expression, so we will show the equivalence of this form also to the one for l greater than 0, so that we can use the general expression. And then notice that all these considerations are valid and we talk about the energy dependence in terms of you know k going to the 2l + 1 except when this denominator l + 1 + a gamma.

This itself goes to 0 okay, if this denominator is to go to 0 for some region. So, a gamma when this is = - of l + 1 when this happens then you will have a zero energy resonance as k tends to 0 okay. So, these are the resonant conditions that we will also discuss today.

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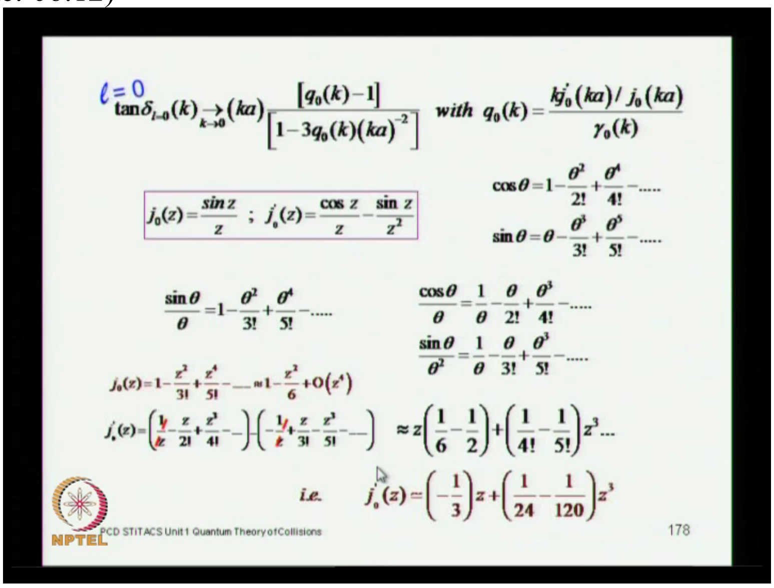

So, for l = 0 this is the general form and q0 is defined by this ratio and you need the Bessel function but you have got the explicit analytical form for the Bessel functions for l = 0. So, you have got the Bessel function and this is the derivative of the Bessel function you need it over here because this is kj prime by j right. So, this is j0 this is the derivative with respect to z and you have got the sine and the cosine functions.

You are looking at the low energy limit theta going to zero because theta is your angle ka, so as k tends to 0 ka tends to 0 that is angle theta goes going to 0. So, you look at the power series expansions of sine and cosine functions and then focus on the terms which are important as theta goes to zero.

So, you also need a sine theta by theta over here this will give you the Bessel function and then you also need the cos theta by theta and the sine theta by theta square. So, these expansions are also given here, so if you put all of these terms together in your Bessel functions you find that the Bessel function for l = 0 goes as 1 - z squared over 6 + terms of the order of z to the 4 okay.

Now z prime is the difference of these two quantities so you take this expression and take this expression notice that this 1 over theta and -1 over theta terms are there right. So, they will cancel this 1 over z and this -1 over z will cancel. And now you have the term the leading term goes as z to the power 1.

It is fact, it is multiplication factors are 1 over root 2 - 1 over root 2 and + 1 over root 3. So, it is 1 over 6 - 1 over 2 at + there is a term in z cube okay. So, putting all of these terms together you get the derivative of the Bessel function which goes as - one third z + 1 over 24 - 1 over 120 times z cube.

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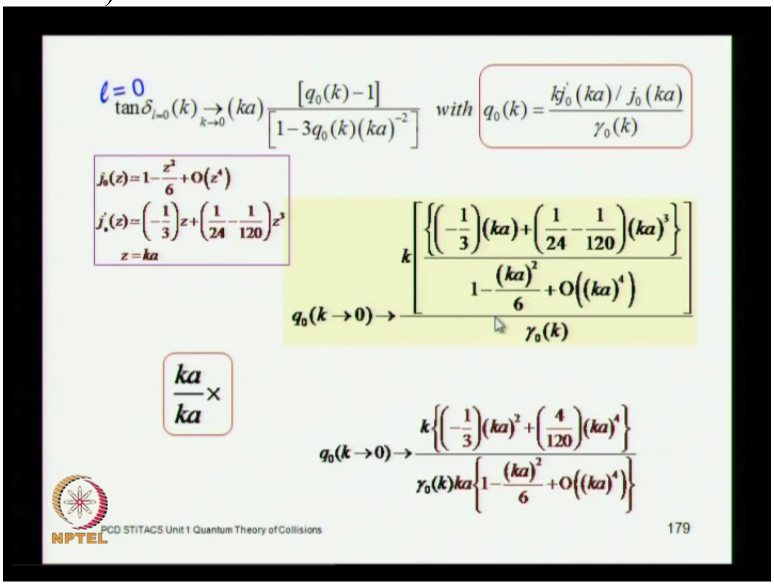

So, you have these Bessel functions and the derivative now in this expression you need q0, so q0 is k times j0 prime by j0, j0 prime by j0 you get from here, this is j0 prime which is one third z but z is ka, so I have written -1 over third one third ka + 124 - 120 z cube, z cube is ka cube. Then you divide it by this 1 - z square over 6 + terms of the order of z to the 4 which is ka to the 4 right. And then the whole thing is divided by gamma 0k which is here.

So, if you multiply and divide this expression by ka then in the numerator in place of this ka you get ka square and in this term which is of the order of ka cube you get ka to the power 4 because you are multiplied by ka and the denominator also you multiply ka. So, you have got gamma 0k multiplied this ka.

And this remaining factor which is 1 - ka square by 6 + terms of the order of 4th power of k. Now notice that the k cancels, so you can get a little bit of simplification

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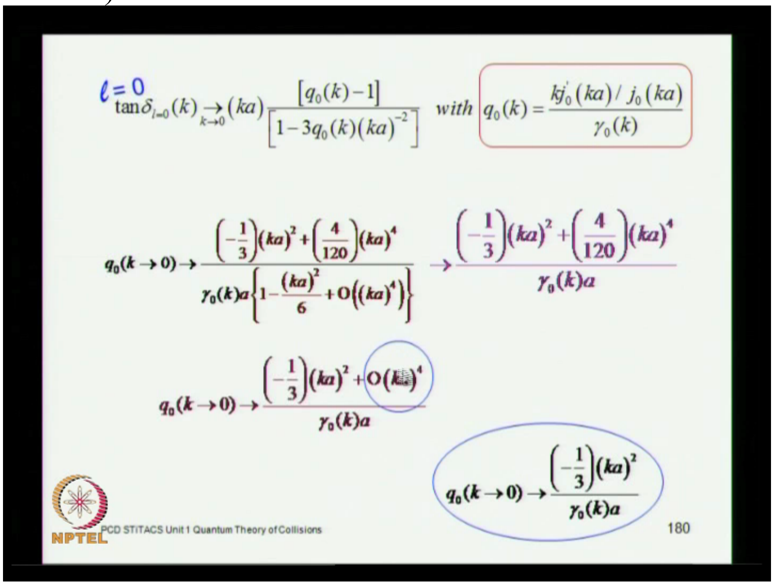

And after cancelling the k this is what you get. And if you look at the low energy limit of this expression then in the denominator you have got one which is a fairly large number compared to powers of k which tend to zero okay. So, the leading term here is one, so you have got gamma 0ka in the denominator. And typically in unless you have to really include the higher order term you can take you can ignore this term the term in fourth power of k.

And you get an approximate expression for q0 which is – 1 third ka square over gamma 0a. And this is the term this is the expression that you can use. This term we have ignored at that at the moment but we may invoke it when we invoke it when we consider resonance conditions because under resonant conditions it may become necessary to take into account the next higher order terms.

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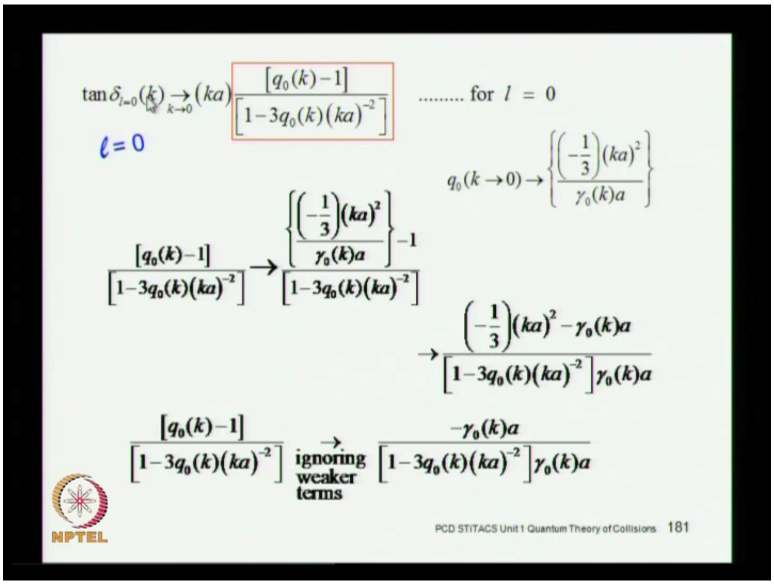

So, ignoring the fourth power in k for the time being you have got q0 and approximate expression for q0 in the limit k going to 0 and this expression for q0 you can put in this box because q0 appears over here as well as over here. So, you have q0 - 1 which is this is q0 - 1 divided by 1 - 3 times q0 ka to the – 2. And if you simplify this multiplied by gamma 0a, so gamma 0a multiplying this - 1 will give you - gamma 0a.

But then you will get a gamma 0a in the denominator which appears over here. So, if you now ignore weaker terms okay in higher powers of k as k tends to 0 your expression here is basically - gamma 0a in the numerator ignoring this term compared to this. And then you have 1-3qk to the ka to the - 2 times gamma 0a. Now this is only the term in this red box and you have to multiply it by ka to get the tan delta.

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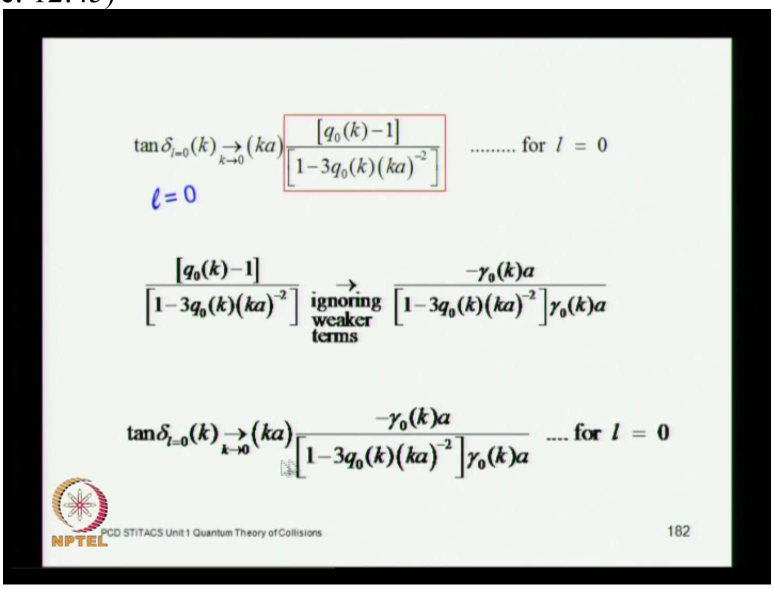

So, this is the term in the red box which we have determined and if you multiply it by ka you get the tangent of the phase shift for l = 0.

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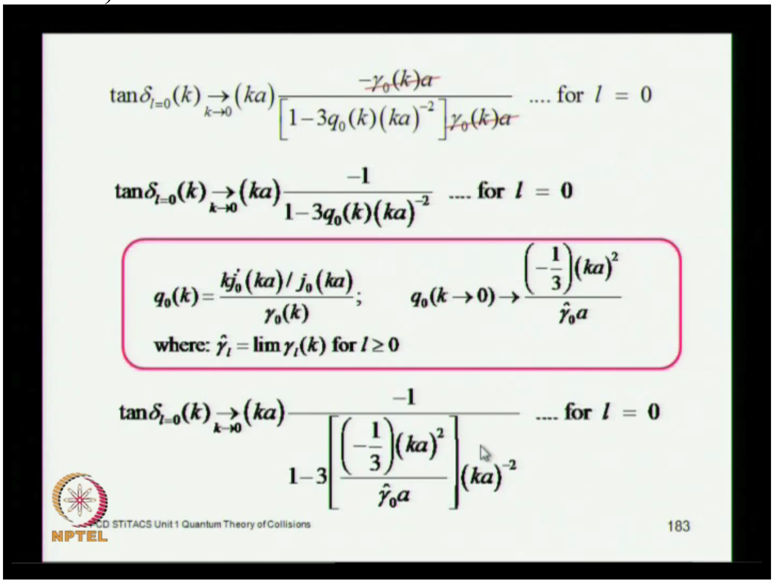

Now so far so good gamma 0a cancels, so you can forget about it okay, now look at this term and it is going to simplify to the form that we have used for higher values of l as well. So, in the low energy limit the value of gamma as k tends to 0 is represented by gamma with a hat on the top or gamma carrot right.

This is the low energy k tending to zero limit of gamma. So, you have got the q0 in the low energy limit and you can put this expression for q0 over here and you get ka times - 1 divided by 1 - 3 times q0, q0 is this expression here which is – 1 third ka square over gamma carrot a which is what you have over here and this ka to the - 2 comes here. So, that is your term for tangent of the phase shift for l = zero for the S wave scattering.

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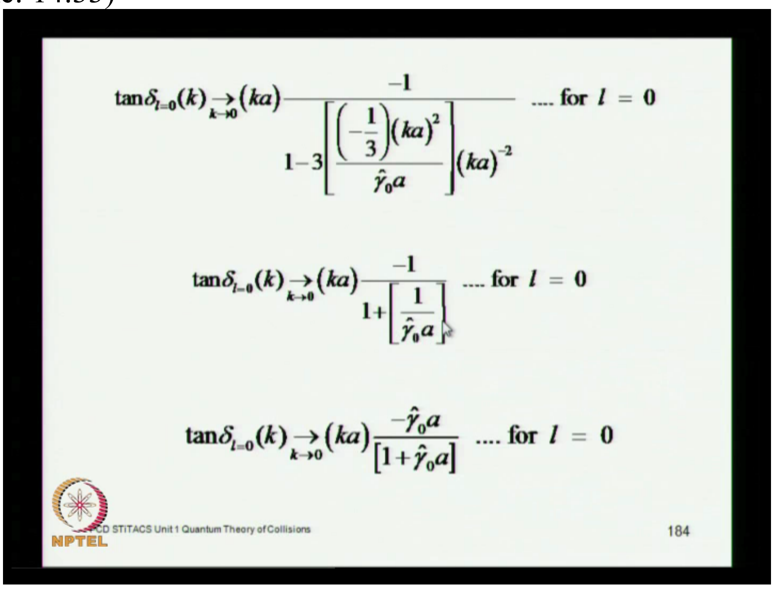

Now we are interested in the resonant condition okay, so I have only simplified this because you have got a -3 multiplying this -1 over 3, so that will give you + 1 here right. So, you have got 1 and then you have got a + 1 over here, you have got a gamma 0a, this is ka to the power 2, this is ka to the power of – 2. So, those two terms cancel each other, so this is rather simple expression now right.

So, having simplified this gamma 0a can be taken at the top, so this goes as - gamma 0a and then you have got 1 + gamma 0 in the denominator.
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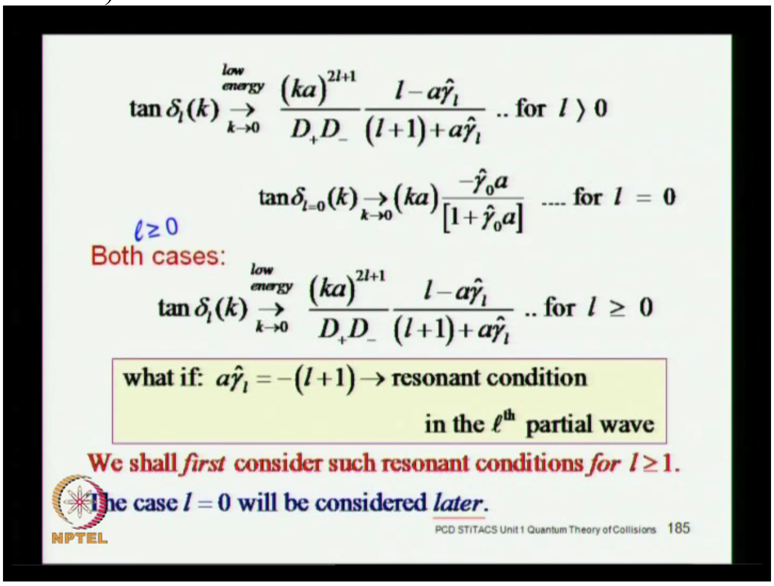

Now this is the expression you had for l greater than 0 the expression that you now have after moving this gamma 0a in the numerator is this, and notice that these two forms are completely equivalent. Because in both cases you can write the same general expression and this is what I had indicated earlier that even the tangent for l = 0 can be written in the same general form and this expression is valid not just for l greater than 0 but also for l = 0.

Because for l = 0, this l is 0 this l + 1 is 1 and then you get 1 + gamma 0a right. So, it is a general expression for both l = 0, l as well as for l greater than 0. And now we consider the resonant condition in the lth partial wave whether it is l= 0, l = 1 or whatever right.(Question time: 16:05- not Audible) Yeah we have taken the leading terms so we first consider the effect of the leading terms and higher order terms will be invoked if and when necessary.

And this should not be the generalised equation it is the general expression for the background cross sections the resonant terms, other terms means cross sections of higher partial waves is already ignorable that is the reason we are taking the leading terms okay. We are trying to see if higher partial waves need to be considered in the resonant region okay.

If at all they need to be considered it would be in the resonant region. And what is their importance in the resonant region that is the question we are raising here. So, we asked what would happen in the resonant condition if there is a residence in the lth partial wave if this denominator goes to 0 if - of l + 1 is = a gamma l what would happen.

So, that is the question that we take up and we will first address this question for l either = 1 or higher than 1, l = 0 is a special case which we will consider a little later. So, first we focus on either resonances in P waves or in higher waves resonances in S waves we will consider separately.

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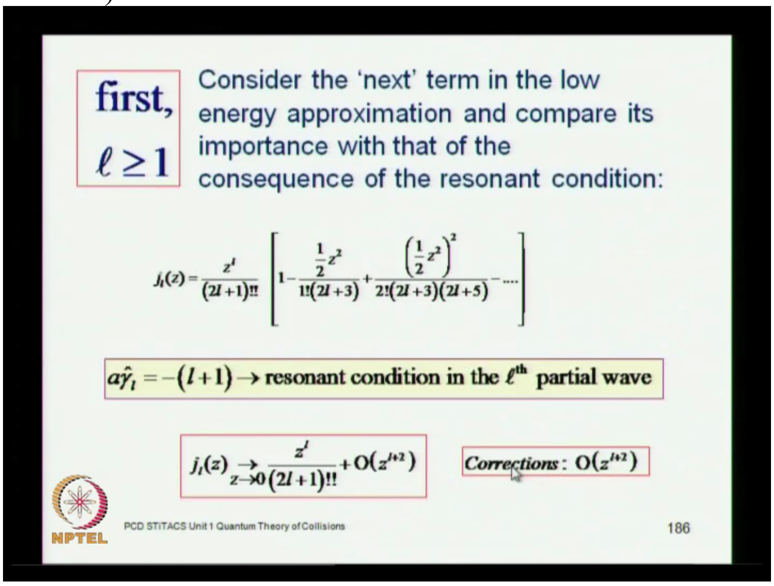

So, l = 1 is the P wave, so l = 1 or higher these are the terms that we first consider. Now this is the general form of the Bessel function in power series as k tends to 0, z is ka you are interested in this resonant condition. And we have taken the leading term which goes as z to the power l in the Bessel function right.

If at all you have to consider corrections the most important correction you will have to consider first is this term which goes as z to the power 2. So, the corrections will be of the order of z to the power l into z to the power 2 which is terms of the order of z to the power l + 2. So, these are the sections which you will have to consider okay that is the order of magnitude there are other constants but not to worry about it.

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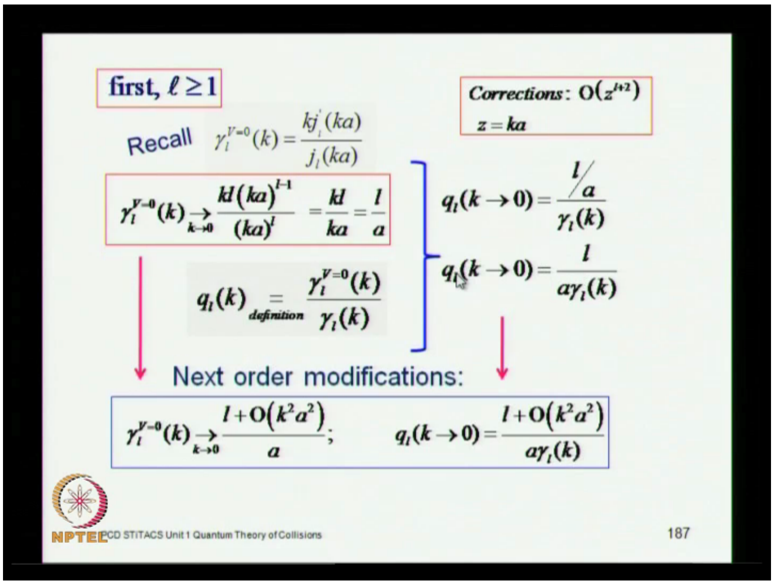

So now consider gamma and if there is no potential if v = 0, then gamma we can get explicitly its low energy limit k going to 0 goes as ka to the power l - 1 over ka to the power l. So, that gives you l over a, because there is an additional k here right. So, you get l over a and this is the definition of q, so using these two together you find the low energy limit of the factor q as l over a divided by gamma right.

So, that is the limit limiting value for the factor q and if you have to take into consideration the higher order terms or the most important corrections to this you will have to consider l over a but in addition to that you will have terms in z square or which is the square of ka or k squared a squared okay.

So, that is the order of magnitude of the corrections that you have to take into account. The corresponding correction in q will be l over a, and then you will have terms in k square a square.

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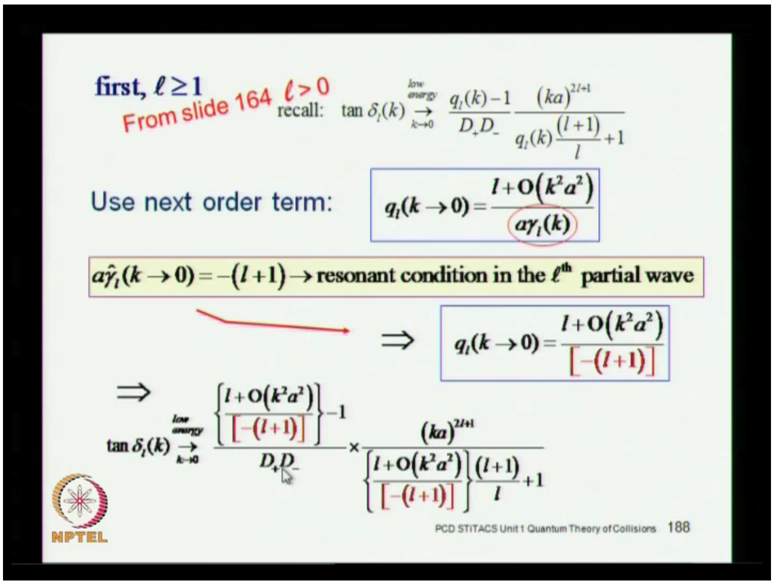

Now let us see what this leads us to so this is the expression for the tangent of the phase shift. The next order term q will have to be corrected to the next order which is k square a square correction. And we want to consider this under the resonant condition when a gamma = - of l +1 right that is the resonance condition.

So, when that happens this a gamma can be replaced by - of l + 1 for this consideration. So, that gives you this expression for q and this is the expression for q which will go in the expression for the tangent of the phase shift. So, this is the expression for the tangent of the phase shift in the resonant conditions when you have taken into account the next order correction right.

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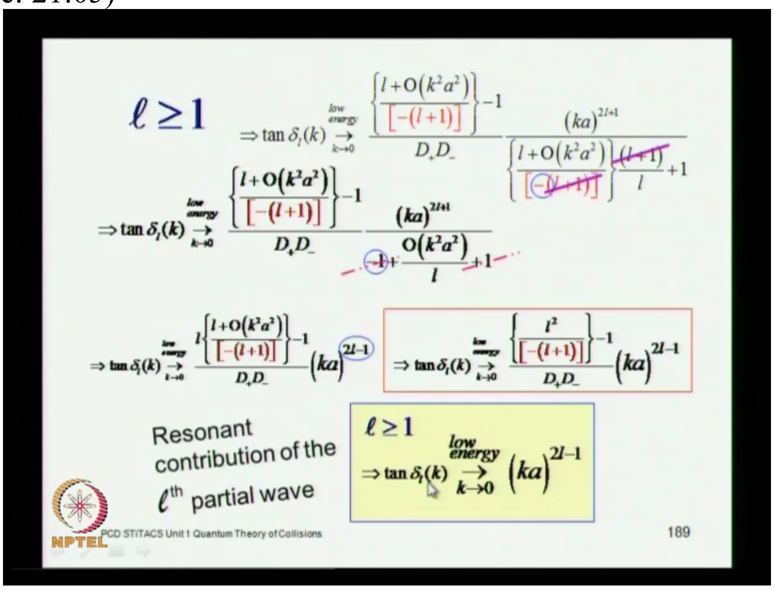

So, this is for l = 1 or l greater than 1, l = 0 we have agreed to discuss separately and now notice that this l + 1 cancels, this l + 1 in the denominator, this l would cancel this l but there is a - sign over here, so that gives you - 1 okay. And the remaining terms you have got this term of the order of k square a square divided by this l and there is a 1 here okay. So, now you can cancel this - 1 with this 1 and that gives you a simplified expression.

You have got l and this factor here coming from q and then you have got a - 1 here and then k to the power 2l – 1. Why -1 because you have a 2l + 1 over here divided by the terms of the order k square a square right. So, you get ka to the power 2l – 1, so this l has been taken into the numerator it is sitting over here right.

So, this is how the phase shift goes as 2l to the – 1. Now the phase shift goes as 2l to the – 1. Then the low energy behaviour for this the resonant contribution will go as ka to the power 2l -1 okay that is the dominant contribution.

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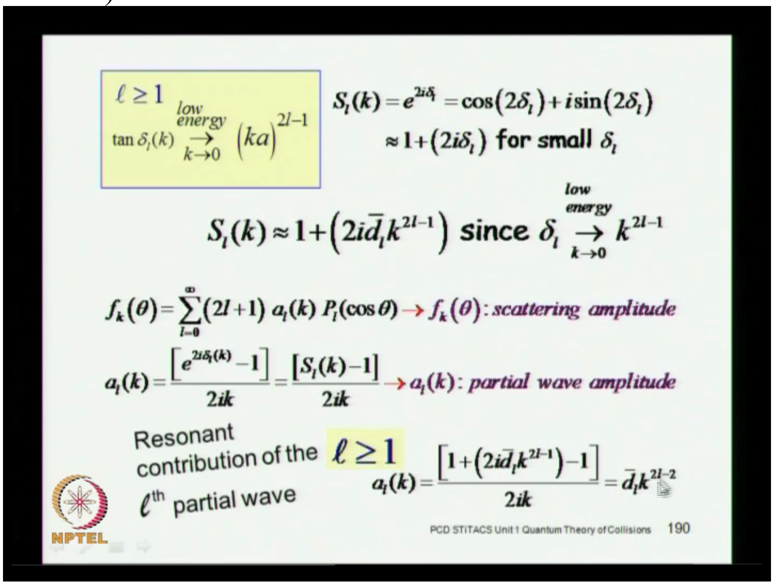

Now what about the S matrix element now the phase shifts are small, so the S matrix will be 1 + 2i times the phase shift. But the phase shift goes as to 2lk to the 2l – 1. So, it is proportional to k to the 2l - 1 that proportionality I have written as d bar for the lth partial wave okay. Whatever that proportionality is you can put in all the other constants of that.

So, the S matrix element goes as 1 + 2id k to the 2l-1 and the partial wave amplitude will be given by S - 1 over 2ik, so when you take S – 1, this 1 will cancel, so there is a 1 here and there is a - 1 here. And then you are left with 2idk to the 2l + 2l – 1divided by 2ik, so the 2i's cancel you are left with dk to the 2l – 1.

But there is a k in the denominator so you get k to the 2l-2 right now that is a very nice result because you find that the partial wave amplitude goes as k to the power 2l - 2.

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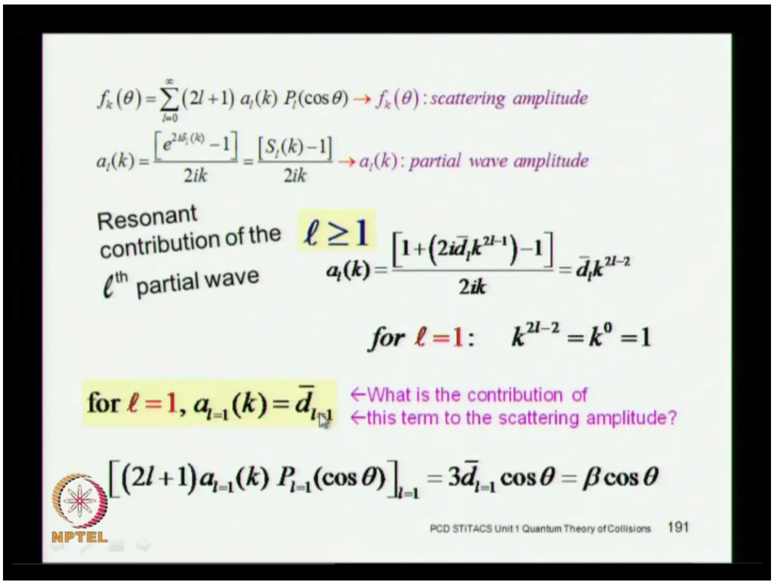

And it will give you a corresponding expression in the scattering amplitude it will make a contribution to the scattering amplitude. So, this is the resonant contribution which goes as 2l - 2, let us ask what is its value for l = 1. For l = 1, k to the 2l- 2 becomes constant right. So, for l = 1 it does not vanish it becomes constant and therefore you cannot ignore it. So, the resonant contribution from the P-wave cannot be ignored.

If you are considering just S wave scattering, if you have such a condition that the resonance condition is satisfied. So, this does not vanish, so k to the 2l- 2 becomes constant and the partial wave amplitude becomes d bar for l = 1 and its contribution to the scattering amplitude will be given by this general expression. So, you have got 2l + 1 with l = 1, so 2l + 1 with l = 1 will give you 3 times this amplitude which is d bar.

So, 3 times d bar + Pl cos theta for l = 1 which is cosine theta, so the contribution of l = 1 the resonant contribution from the P wave to S wave scattering will be angle dependent it will go as cosine theta and beta is a certain parameter which will tell you how you know what is its total magnitude. So, beta will be scaled by cos theta, so this will be the angular dependence, so beta is some sort of an angular distribution asymmetry parameter.

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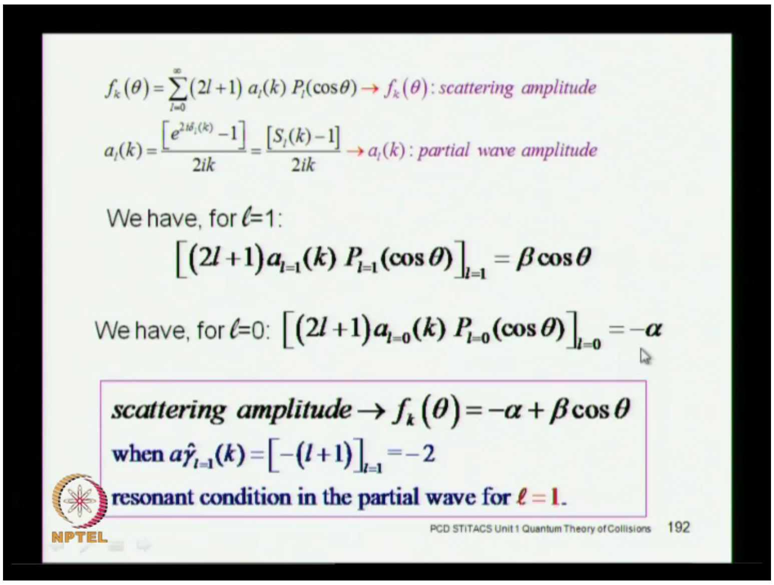

So, this is the contribution for l = 1 now you want to consider higher partial waves let us consider l = 2 but let us see what this does to the S wave scattering. So, S waves scattering for l = 0 we already know that this is equal to the negative of the scattering length right. So, we had defined the scattering length earlier.

So, this is already minus alpha and the scattering amplitude then will become - alpha + this term contribution resonant contribution from l = 1 this is the contribution from S waves, this is the contribution from P waves and the net contribution will be - alpha + beta cos theta and this is when the resonance condition is satisfied for l = 1. When it is not satisfied you got only alpha.

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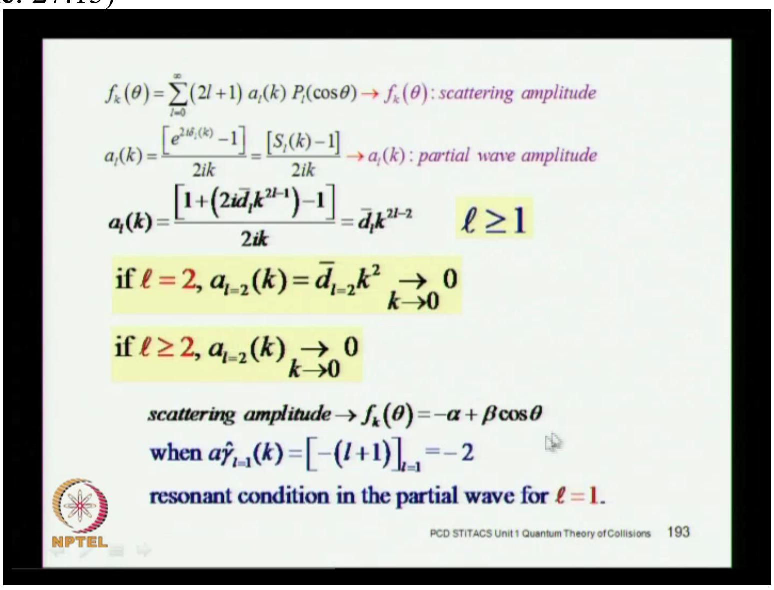

Now what if you take the next partially, so we consider the P waves now like consider the d wave l =2. Now for l = 2, you get 2 to the power 4 - 2 which is 2 right, 2l- 2 for l = 2 is 2. So, you get a quadratic dependence on k and this term happily vanishes as k goes to 0. So, it will make no contribution to the scattering amplitude if l = 2. What if l = 3, if it is anything more than 2 it will go to 0 even faster right.

Because 2l will increase in right, so instead of 2l - 2 with l = 2, you will have l = 3 so you it will be 3 into 2, 6-2 so it will go to 0 as k to the 4. So, it will go to 0 even faster, so higher partial waves all the higher partial waves really will make no contribution even if it has a resonance. So, under the resonant conditions it is important to consider only the P waves and when you have a resonant condition in the P wave for l = 1.

Then the scattering amplitude goes as minus alpha is corrected by this cosine theta term which is coming from the resonant contribution from the P waves and that is why the S wave scattering is so important in collision physics okay. Because it really gives you a major or major part at least in the low energy region, of course when you go to higher energies and so on then you have to consider higher partial wave so okay.

So right now our focus is in the low energy regime and in this region the S wave scattering is the most important and the dominant one for low-energy scattering. You do not have to consider higher partial waves even if there is a resonance the only other higher partial way that you to consider is just l = 1 even l = 2 and all you do not have to consider. But mind you this is the low energy scattering phenomenology that we are discussing right now.

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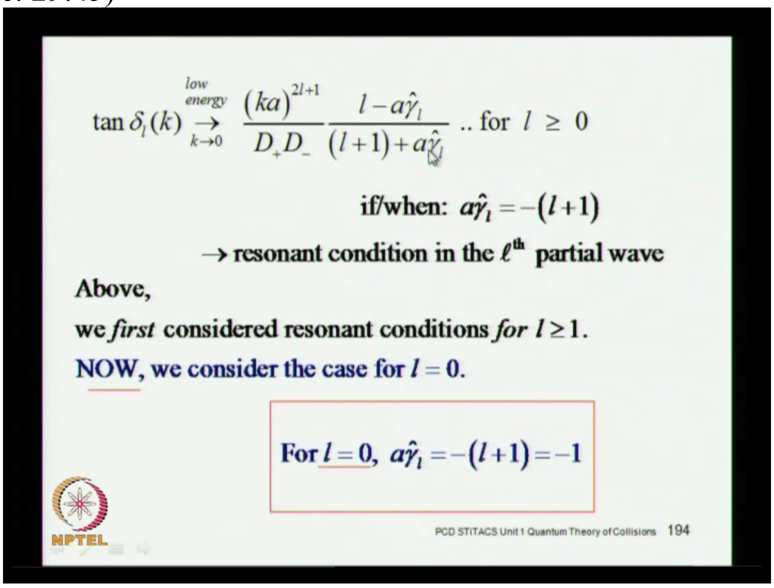

So, this is the result that we have got. This is the resonant condition in the lth partial wave or we did not consider the l =0. I said that we will consider resonance in the l = 1 and above, l = 0 is a very peculiar situation okay and a very interesting one as well. So, let us consider now the case for l = 0. Now for l = 0, this condition l + 1 = - of a gamma l is l = 0. So a gamma l becomes - 1 this is your resonant condition right.

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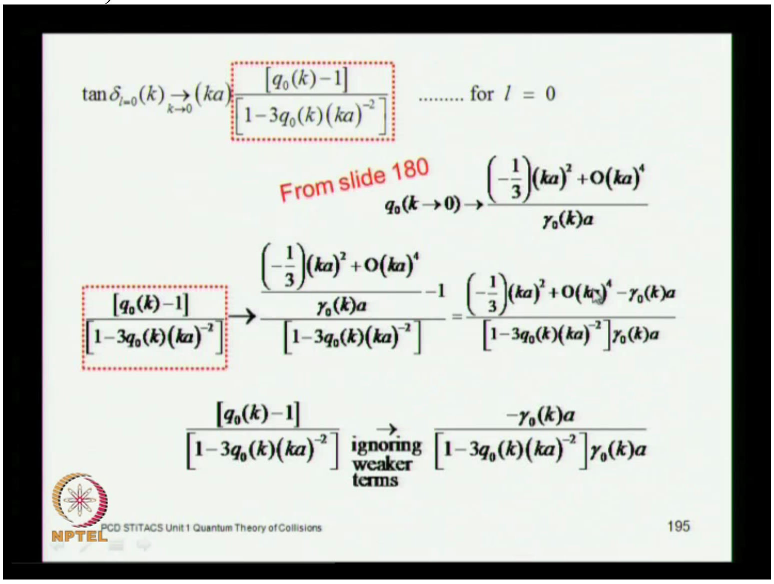

So, we will use this resonant condition in the general expression for the phase shift and for the q factor. And now we will consider these terms, so this is the term in this box in this rectangular box, so I consider this term first and later on you will multiply it by ka. So, this is the term that you had considered right, q remember we had taken the leading term which was quadratic in k. The term that we had ignored was in the fourth power of k.

And now to consider the resonance we should not ignore the next order term because if it is either more important or less important, so that comparison will have to be made because whenever you make corrections okay. All corrections of the same order must be made and it makes no sense to make a correction of a weaker order when you are ignoring term of a higher order. So, what is the role of this fourth power of k in this case.

We will have to consider so as long as you ignore the weaker terms you get q - 1 over this, this is the term in the rectangular box and the leading term over here. If you ignore the quadratic term and the fourth power term then this is the leading term is - gamma 0a in the denominator you have got this term 1-3qka to the -2 multiplied by this gamma 0a.

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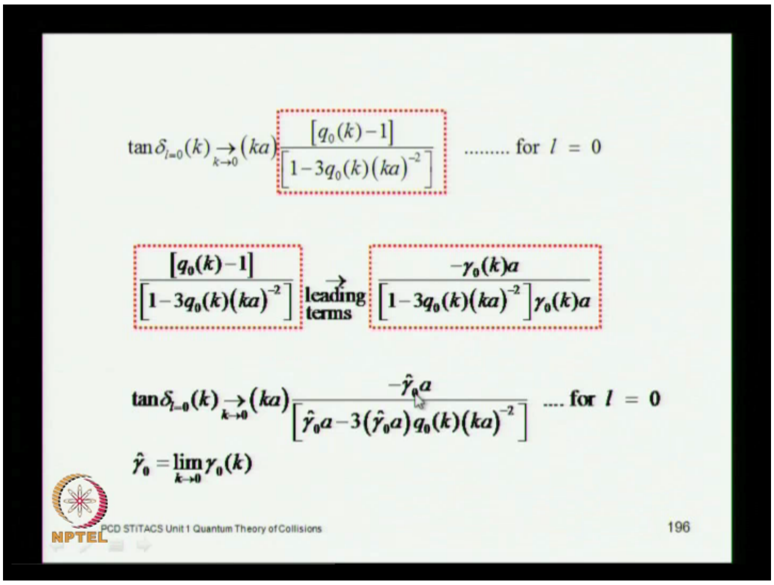

And for l = 0 is what we are considering now. So, this is the limiting value as k tends to 0 gamma which depends on the energy will take its limiting value and this limiting value is what we have denoted by gamma carrot like a gamma hat right. So, this is the limiting value for gamma, so that is the value that I need to consider not arbitrary values. And I have multiplied this gamma a to both of these terms.

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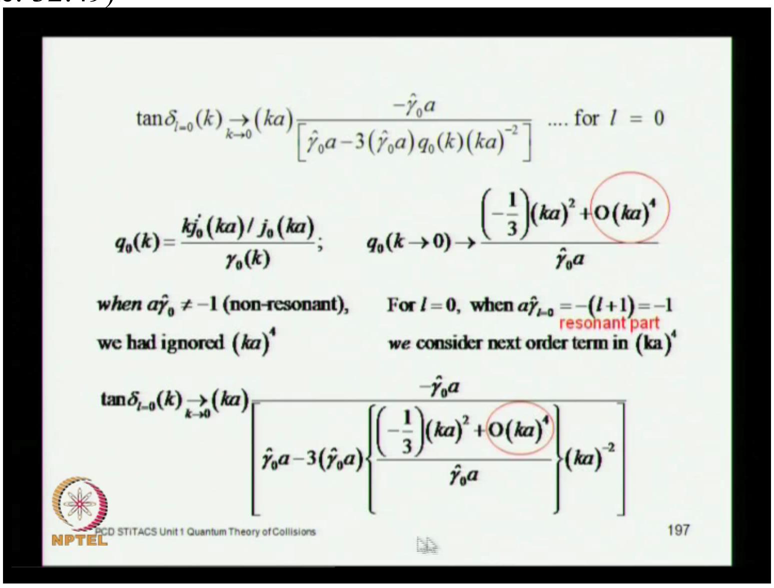

Now this term you ignoring the non-resonant region and you can consider it now in the resonant region. So, that you can make a comparison, so we consider this term this is the correction which goes in the fourth power of k. So, what is the importance of this, so now we are not going to throw it okay, so let us take into consideration this term. So, we use the value of q0 corrected for this next order term and using this q.

So, q0 is now -1 third ka to the 2 + this fourth power in ka which is here divided by this gamma 0a and then you have got this ka to the - 2 outside the bracket right. So, inside you have got the value of q0, so q0 you have now take replaced the value of q0 with the next order correction.

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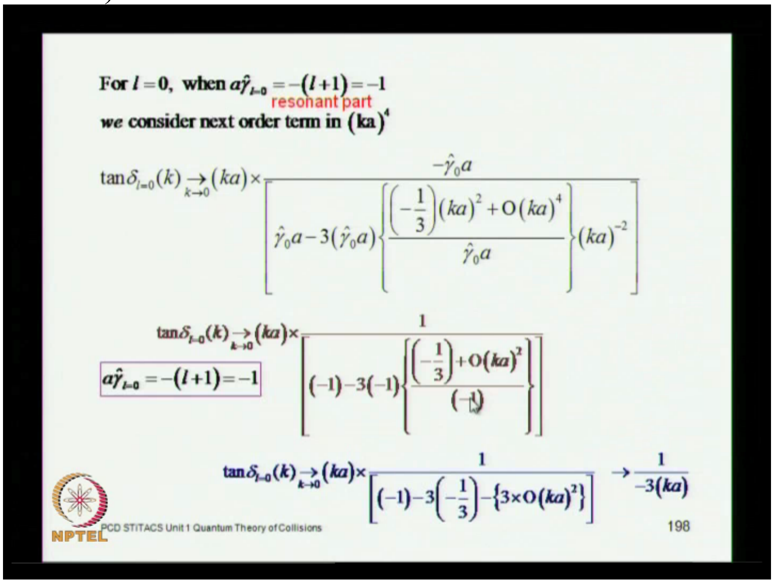

So, this will take into account the effect of the resonance in the l = 0 partial wave in the S wave scattering. So, let us simplify these terms a little bit because in the resonant condition a gamma = -1 when l = 0. So, a gamma is – 1, so - a gamma becomes + 1 in the numerator right in the denominator this a gamma becomes -1, this a gamma becomes -1, this a gamma in the denominator this becomes - 1 and you have to keep track of the - signs carefully.

And if you multiply it carefully you find that the tangent of the phase shift goes as 1 over k okay because this - 1 and this - 1 will cancel this 3 into 1 over 3 will give you 1. But then there is a - sign here and a - sign here, so there are three - signs and one over here. So, if you keep track of all the - signs you get one over thrice ka, so the tangent of the phase shift goes as 1 over k. So, what is going to happen to this phase shift as k goes to zero.

As k goes to 0 the tangent goes to infinity it blows up okay, now this is a peculiar situation okay, this is a resonant condition in S waves.

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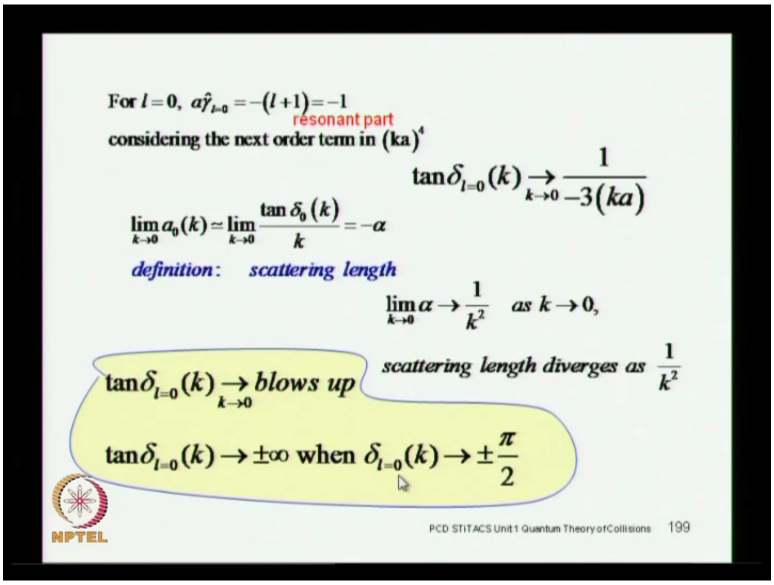

So, the tangent of the phase shift actually blows up in this case. So, what happens to the scattering length, the scattering length is this tangent divided by alpha, this is the scattering length which is -alpha. So, this gets further divided by k tan delta by k. So, it is already 1 over k, so this scattering length will go 1 over k square and as k goes to 0 this will blow up.

So, the tangent blows up and when the tangent blows up the angle itself goes to either pi by 2 or -pi by 2, it is pi by 2 modulo pi right. So, that is a very peculiar situation the phase shift becomes pi by 2.

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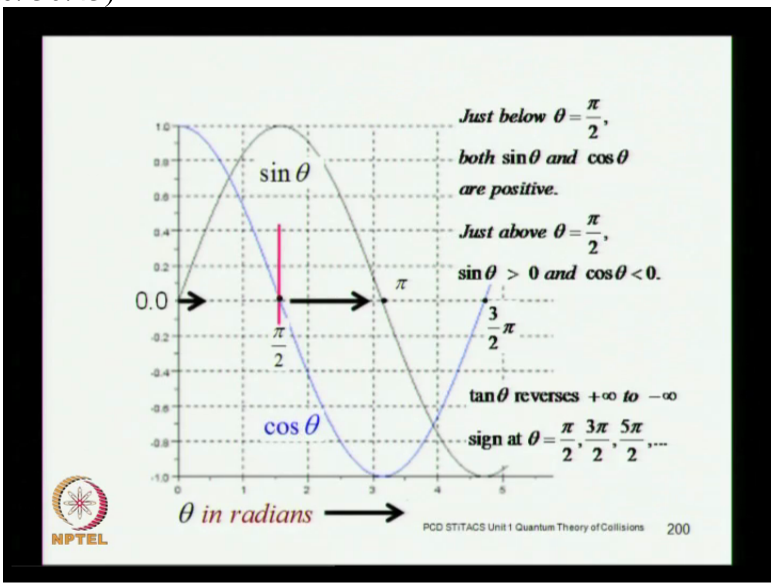

So, let us look at these tangent figures and this is a figure for the sine and the cosine figures and if you look at the first quadrant from 0 to pi by 2 then in this quadrant below theta = pi by 2 both sine and cosine functions are positive and above theta = pi by 2 sine theta is greater than 0 but cosine theta is negative which means that the tangent changes its sign and the tangent reverses from + infinity to – infinity.

Jumps from + infinity to - infinity at pi by 2 okay. There is a discontinuity there and this happens not only at pi by 2 but it will also happen at 3 pi by 2, 5 pi by 2 and so on.
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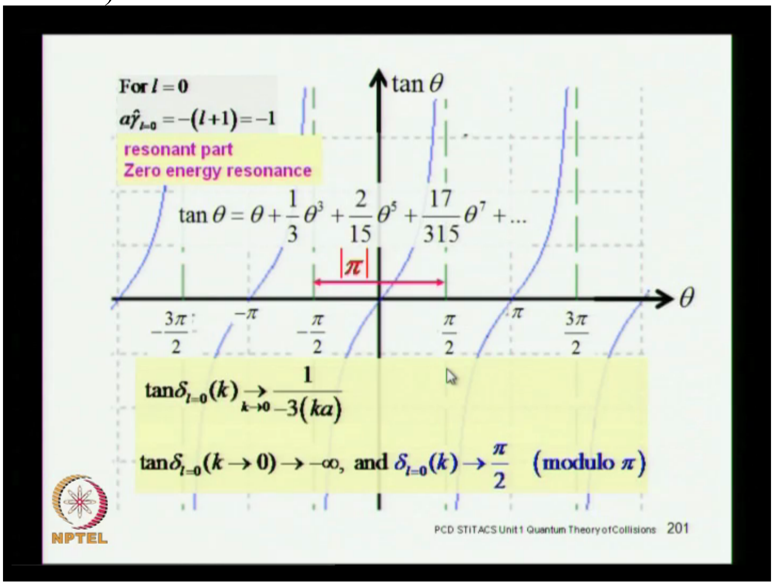

So, this is the how the tangent figure will look like and this is the resonant condition that we are examining. And the, this is the general power series expansion for tangent and it really blows up as in the neighbourhood of pi by 2, 3 pi by 2 and so on. And the phase shift itself is pi by 2 modulo pi okay, if the tangent goes to infinity the angle itself must go to pi by 2 right. Because it is at pi by 2 that the tangent blows up modulo pi of course. So, the tangent of course reverses its sign at pi by 2, 3 pi by 2 and so on.
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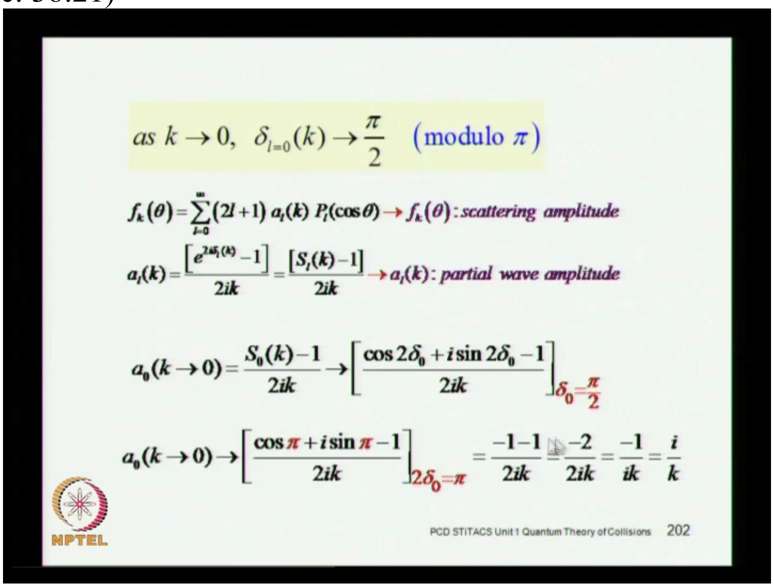

So, as k tends to 0 the phase shift goes to pi by 2 it does not go to 0. It instead of going to 0 it goes to pi by 2 in the resonance region what does it do to the scattering amplitude and the partial wave amplitude, the partial wave amplitude is S - 1 over 2 ik, so S is cos 2 delta - i sine 2 delta and this you must evaluate at delta not at small delta but at delta = pi by 2 okay.

The phase shift is no longer small, so you do not approximate sine delta = delta you do not a proximate cos delta = 1. But you put in the actual values of the sine and cosine not for delta but for 2 delta because that is what appears in the S matrix. Because you have got e to the 2i delta here right.

So, for delta = delta 0 which we have found that this phase shift is pi by 2, twice delta will be pi for delta 0 = pi by 2 twice delta will be cosine of pi cosine of pi is - 1 sine of pi is 0. So, you have got a - 1 in a - 1 which gives you - 2 in the numerator and you have got a 2ik in the denominator. See you get - 1 over ik or + i over k. That is what you get for the partial wave amplitude.

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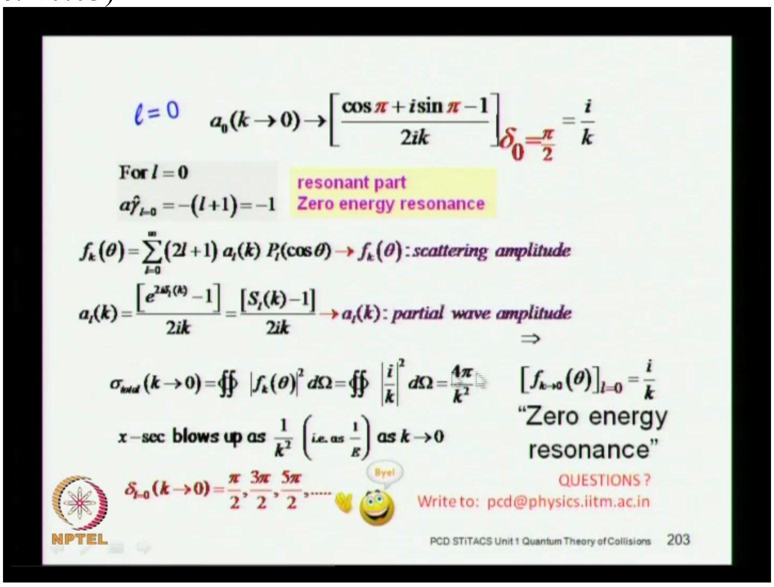

And from the partial wave amplitude you also get the scattering amplitude okay. So, the scattering amplitude goes as i over k and if the scattering amplitude goes as i over k the cross section corresponding to this will blow up as 1 over k square. So, at the resonance the cross section blows up at what right does it go, it goes as one over k square. Why does it go as one over k square because the partial wave amplitude goes as i over k okay.

So, the phase shift under this resonant conditions goes as pi by 2, 3 pi by 2, 5 pi by 2 this is a very peculiar thing happening in the S wave scattering and this phenomenology is very intimately connected to a very important theorem in collision physics namely the Levinson’s theorem which I will be discussing in the next class.

Because it will connect the scattering phase shifts with the number of bound states and you notice already that you are having a certain resonance over here and the question we are going to ask is why is it that there is a resonance okay. What is the physical reason which goes into the generation of this resonance? So, mathematically we know what happens at that resonance.

But what is the physical condition which is leading to that resonance. So, these are some of the details which will go into the discussion of the Levinson's theorem which I will take up in the next class so there is any question. For now I will be happy to take (Question not audible: 42:02) the resonances any phenomenon which occurs under a specified condition right.

Like if you have an external periodic force which is applied to another object which has got a natural internal, natural intrinsic frequency of vibration. Then if the frequency of the external force and this is exactly equal that is when you have a resonance right. So, the resonant conditions happen only at specific conditions if the external periodic force frequency is either less or more than the natural frequency.

You would move away from the resonance okay, so in our expression for the tangent of the phase shift which goes into the expression for the phase shift and then into the expression for the cross section. Because the cross section is in terms of the phase shifts right which determines this.
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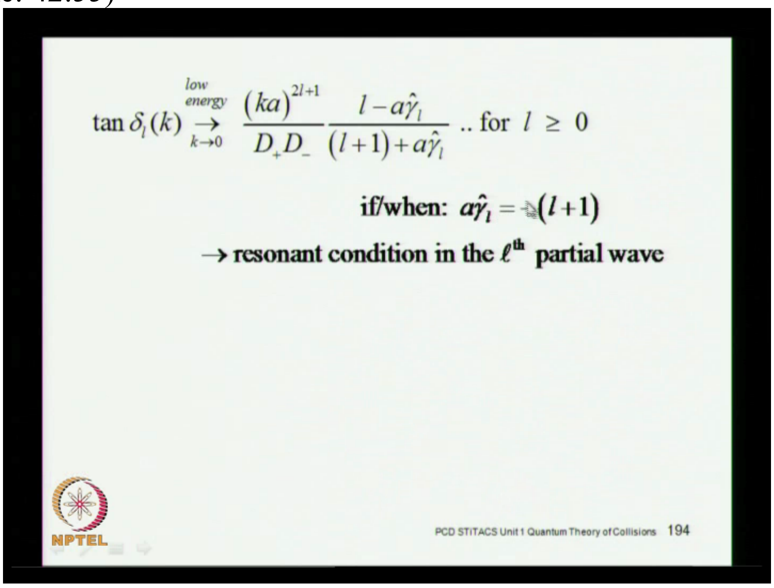

So, if you look at the general expression here notice that the phase shift goes as 2l + 1 power of k. So, as k tends to 0, it becomes diminishing less power okay. So, the phase shift will go is k to the power 2l + 1 however there is this factor which multiplies this and this factor is l + 1 + a gamma.

Now gamma is the limiting value of the ratio gamma that you have defined. Now who knows what values it will take when you multiply it by the range of the potential? When you multiply gamma which is the limiting value of gamma as k tends to 0 when you multiply this limiting value of gamma by the range of the potential which is a.

It can take a variety of values and it is like the external periodic force which can be applied at a variety of frequencies. But what if that frequency is a particular one and in this case what if the limiting value of gamma is such that when it multiplies a it becomes exactly equal to - of l + 1, if that happens that you have a resonance.

(Question time: 44:34)If I am taking alpha gamma is equal to – of l + a delta equals to zero so that thing blows up so here only I have the condition that delta is my tan delta is zero and infinity, if I take the condition that alpha gamma = - a gamma yeah a gamma alpha is the skating length okay.

If that is 0 itself, so here only the tan delta if a gamma is 0, no the denominator is right, if that is 0 here only right, so that my tan delta thing equal to infinity, so here only the delta becomes pi by 2 that is right. So, the question you are asking is how does it affect the cross section okay.

And to be able to do that you have to compare the resonant contributions from different partial waves to the leading terms. So, the leading term is the S wave scattering what we found out that the resonance under the resonance condition it is not sufficient to take into account only S wave scattering. But you need to take into account the resonant contribution from the P wave okay.

The next question you ask is you have a leading contribution from S waves but the S waves also may undergo a resonance because a gamma 0 can become = - 1. What stops it from becoming - 1 that is a resonance condition for the S waves. So, these are the conditions that we are now considering okay. So, any other question so thank you very much and we will take it from here in the next class.