Quantum Mechanics - I
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Lecture - 8
Linear Harmonic Oscillator

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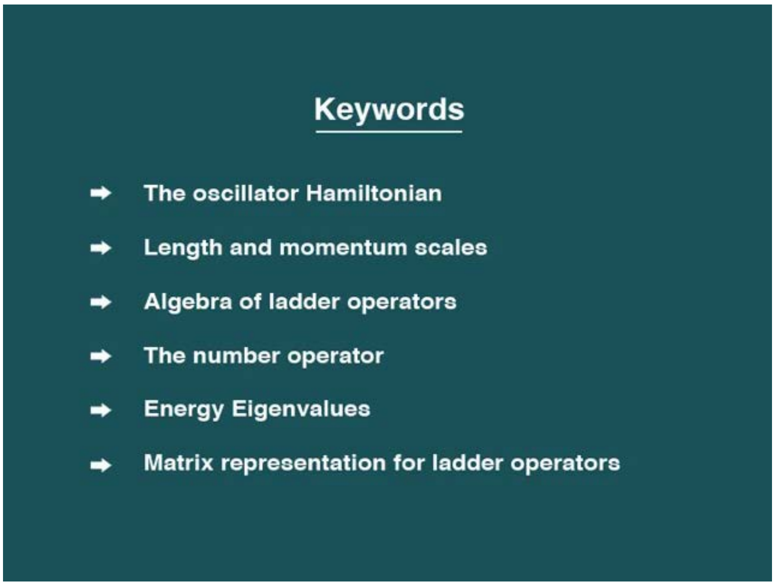

In the last lecture that I gave, I discussed the uncertainty principle. In general, uncertainty principle where we had the relation

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That the variance in the measurement of A, times the variance in the measurement of b is greater than or equal to, the right hand side had two contributions: one had the commutator of A with B it is expectation value mod square times quarter, and the other had the modulus of the expectation value of the anticommutator of A minus expectation value of A, with B minus expectation value of B.

We looked at two cases. The 1st case, we looked at was the spin system or equivalently the 2 level atom model where we computed this, in the state ket e which was an Eigen state of S Z, with Eigen value h cross by 2. We found that this quantity, was greater than or equal to a finite non zero number. That is because the spread in S y was non zero and the spread in S x was non zero. Then we looked at the 2nd example, where we computed this in the state ket e again. So, we were looking at an Eigen state of S z. So clearly, this object was 0 and therefore: the product was 0 but we also saw that the spread in S x was not 0. So that was the 2nd example that we had. The 3rd example: that I started looking at, was a linear harmonic oscillator. The quantum linear harmonic oscillator which was what we will do in some detail today.

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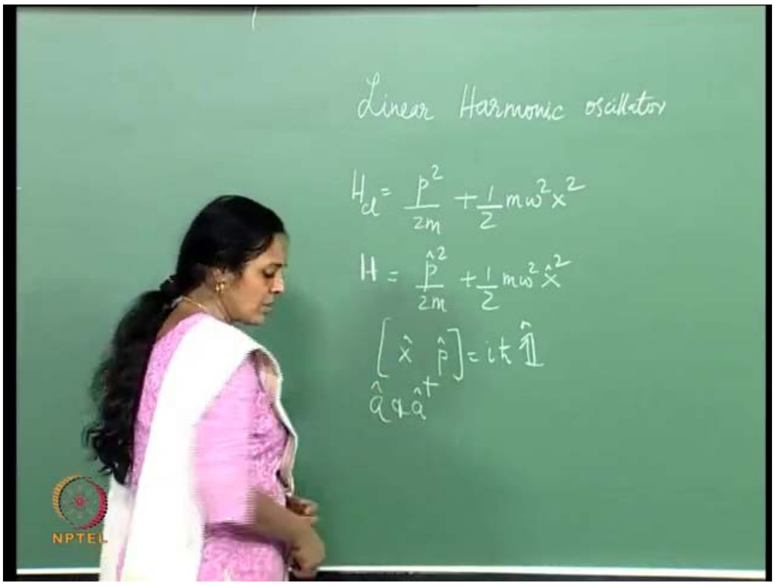

So, we will be looking at the linear harmonic oscillator, the simple harmonic oscillator. And, as I indicated yesterday: we could start from the classical Hamiltonian. By p, I mean the linear momentum corresponding to the variable x. So, the generalized coordinate is x, the generalized momentum is p. Of course, x would be a function of time and so would t be a function of time. But right now, we are not discussing dynamics at all. So, I have this oscillator with p squared by 2 m plus half m omega squared x squared as the Hamiltonian, the classical Hamiltonian.

So, the quantum Hamiltonian, which I will merely call H would mean, replace all dynamical variables by operators. And of course, in this case with the understanding with the commutator of x with p, is i h cross times the identity. The point is this: I would like to create some dimensionless operators using x and p and as I indicated yesterday, we can produce two operators a and a dagger, which are functions of the operators x and p. And today, I will put in the m omega h cross everything in the right place and define them as follows:

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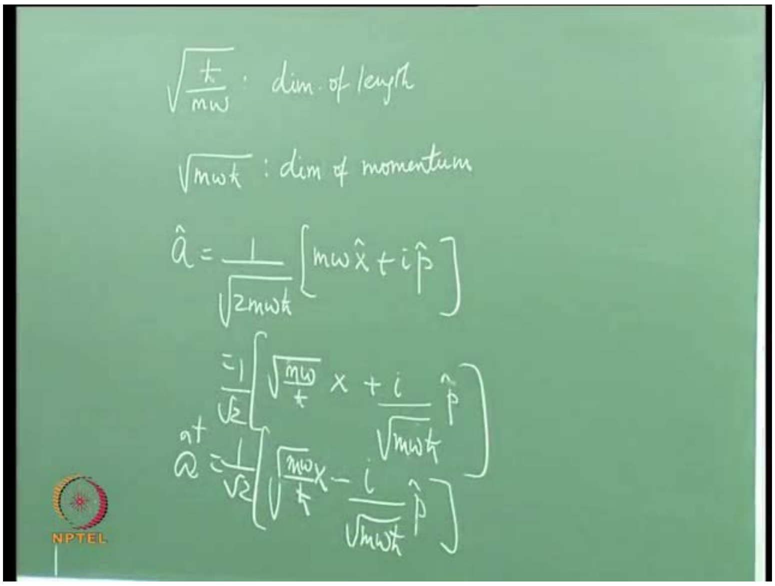

The object root of h cross by m omega has dimensions of length: that is frequency, that is mass, and that is action which is energy second. And therefore, this combination has dimensions of length, root of m omega h cross is dimensions of momentum. So, if I remember this and I write a as the combination I have removed the caps of the operators, you would normally do that. But, i believe it is good for us to just remember that a x and p are operators, even if the caps were not put on top of them. You see this object, is simply 1 by root 2, root of m omega by h cross x plus i by root of m omega h cross p.

Looking at this, it is clear that I have divided out length here, and I have divided out momentum there, to create this object a: an operator which is dimensionless, a dagger is its Hermitian conjugate. So, this is what I have.

Now, you can find the commutator of a with a dagger and that is simply one. Using, the fact that the x p commutator is i h cross times identity. The commutator of x with x is zero. The commutator of x with p is i h cross and pull out all these constants. Similarly, the commutator of p with x will give me minus i h cross, apart from these constants.

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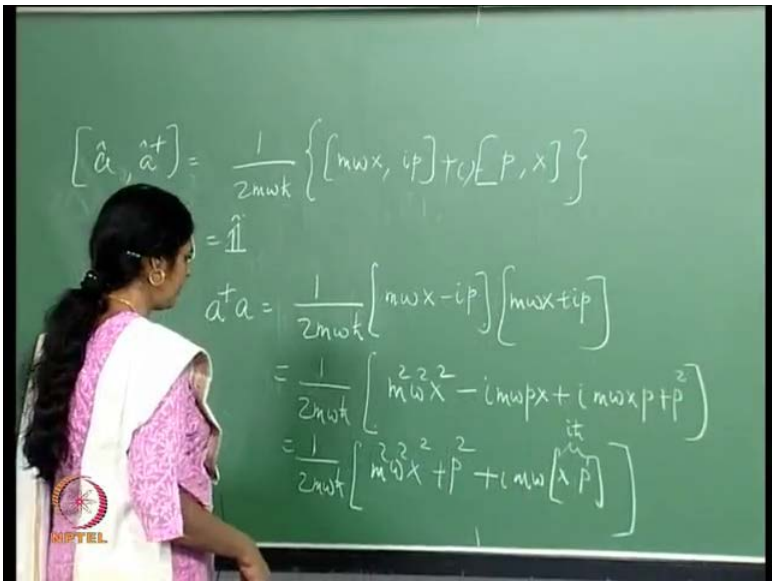

The commutator of p with p is 0 and that is how i get a a dagger is 1 by 1 I mean, the identity operator. So, I could have written commutator x p is i h cross times identity or I could have written commutator a a dagger is equal to 1. Now, given this operator we would like to recast the Hamiltonian, the oscillator Hamiltonian in terms of a and a dagger. So, let us just find out what is the object a dagger a. So, a dagger a is 1 by 2 m omega h cross. So, a dagger is m omega x minus i p and a is m omega x plus i p and that is going to be a dagger a.

So, let us work this out; one has to keep the order carefully. I have the first term m squared omega squared x squared then I have minus i m omega p x. These are operators, so I should be careful; plus i m omega x p plus p squared. Well, that is simply 1 by 2 m omega h cross, m squared omega squared x squared, plus p squared, plus i m omega commutator of x with p. But, the commutator of x with p is i h cross. And therefore, this object simplifies and we get an expression for a dagger a.

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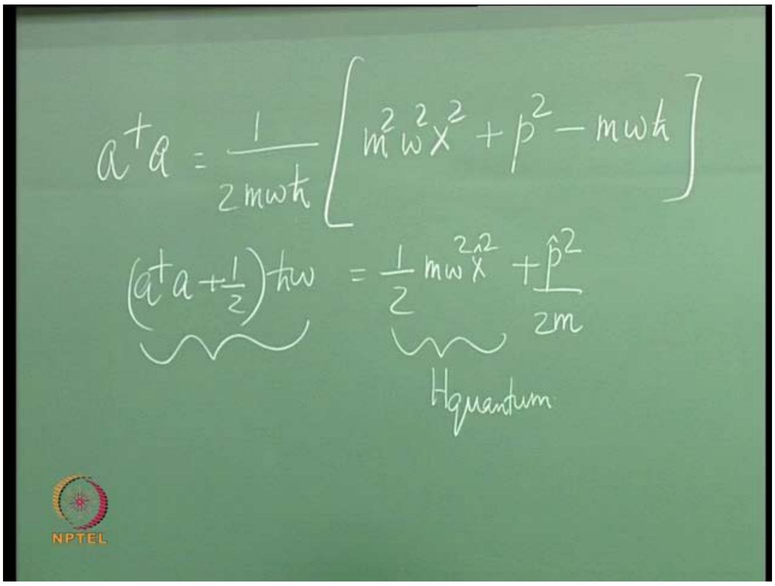

I pull out a 1 by 2 m omega h cross and then I have m squared omega squared x squared, plus p squared minus m omega h cross. Now, a and a daggers are dimensionless objects by construct. So, as you can see this is dimensionless and omega h cross has dimensionless of p squared. Ensure, I have an m omega by h cross when I combine this factor with m squared omega squared, and that has the dimensions of x squared. So, it is ok as it starts, and now we can check that a dagger a plus half, h cross omega to give me dimensions of energy is simply half m omega squared x squared plus p squared by 2 m. All I have done, is pull out this factor.

There is a minus half that comes between this and that I pulled it to this side. This is dimensionless. Multiplied it with h cross omega to give it dimensions and that is simply the rest of it which we recognized with the operator sign on top there, with the hat there. This we recognize as h quantum and so you see. I can write the Hamiltonian for the quantum oscillator as p squared by 2 m plus half m omega squared x squared or as a dagger a plus half h cross omega. So, let us see what is the physical significance of the operators a and a dagger.

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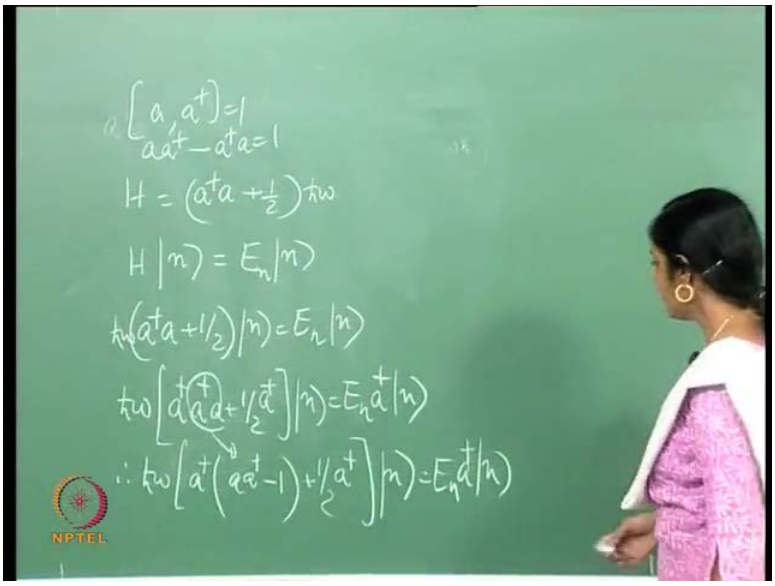

Consider, the Hamiltonian a dagger a plus half h cross omega. This is the quantum Hamiltonian. Look at Eigen states of H. We will label them by ket n where n is a label. One has to find out the range of values that n can take. So, Eigen values would be given by E sub n and these are numbers that one has to find out. I therefore, have a dagger a plus half h cross omega acting on n, gives me E sub n n.

Let us multiply this by a dagger on both sides. So, this is what I have. Well, I can do the following thing: I remember that a a dagger commutator is 1 which means, that a a dagger minus a dagger a is 1. So, I can recast a dagger a in terms of a a dagger. Therefore, I have h cross omega a dagger times a a dagger minus 1, that is the 1st term, plus half a dagger acting on ket n. That is, the left hand side is E sub n a dagger acting on ket n. I can do better, I would now say:

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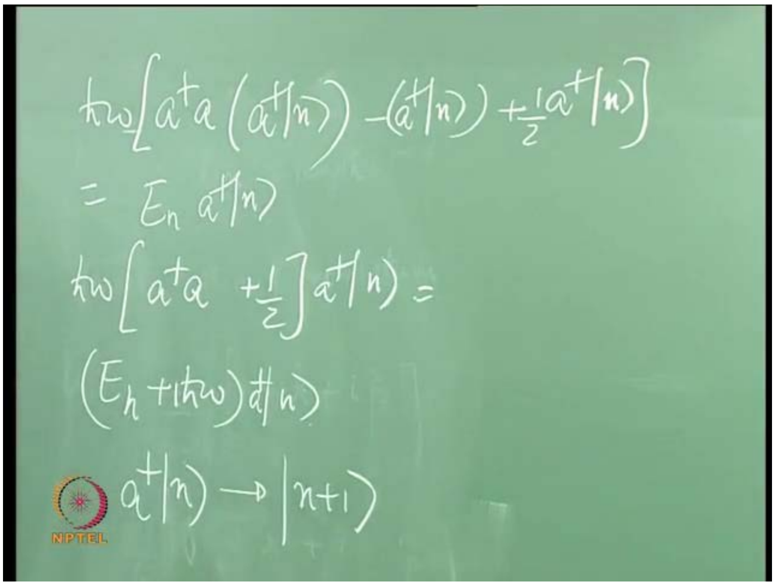

That h cross omega a dagger a from the 1st term and I have a dagger ket n, that is the 1st term, minus a dagger ket n, plus half a dagger ket n, that quantity is E n a dagger ket n. In other words, I have shown that, that object is E n plus 1 h cross omega, a dagger acting on ket n, a dagger acting on ket n is a state which is an Eigen state of the Hamiltonian, the oscillator Hamiltonian with Eigen value E n plus 1 h cross omega. So, it looks like a dagger acts on ket n to take it to a state which is E n plus 1 h cross omega. So, I would like to call that n plus 1 by way of notation because to the original energy E sub n, we have added an amount 1 h cross omega.

So, physically it looks like a dagger takes the system from a state of lower energy E sub n to a state of higher energy: E sub n plus 1 h cross omega. This is the increase in the energy. So, that is an excited state compared to the state with energy E sub n. So, repeated application of a dagger on a given state will increase the total energy by 1 h cross omega each time, taking us to different states from ket n, we go to a state ket n plus 1, ket n plus 2, ket n plus 3 and so on. I want to emphasize the fact that n is just a label. We have to find out the range of values that n will take but as of now n is merely a label. Similarly, we can find out what happens when a acts on ket n. So, let us do that.

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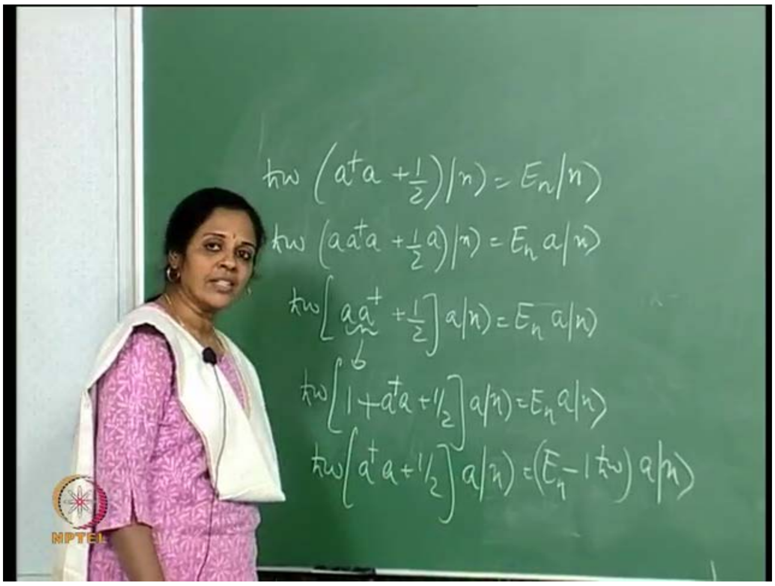

So, again we begin with the Hamiltonian and the Eigen value equation and this time multiply by a. So, a a dagger a plus half a ket n is E sub n a ket n. Well, certainly that tells us that a a dagger plus half a n is e n a n. And, this object using the commutation relation (Refer Slide Time: 11:55) we have got a a dagger there. So, a a dagger is 1 plus a dagger a. In other words, what I have shown is this. When a acts on ket n it takes it to a state which is also an Eigen state of the Hamiltonian but with the energy reduced by 1 h cross omega. So, repeated application of a on a given state ket n seems to reduce its energy each time by 1 h cross omega taking it to a new state and therefore,

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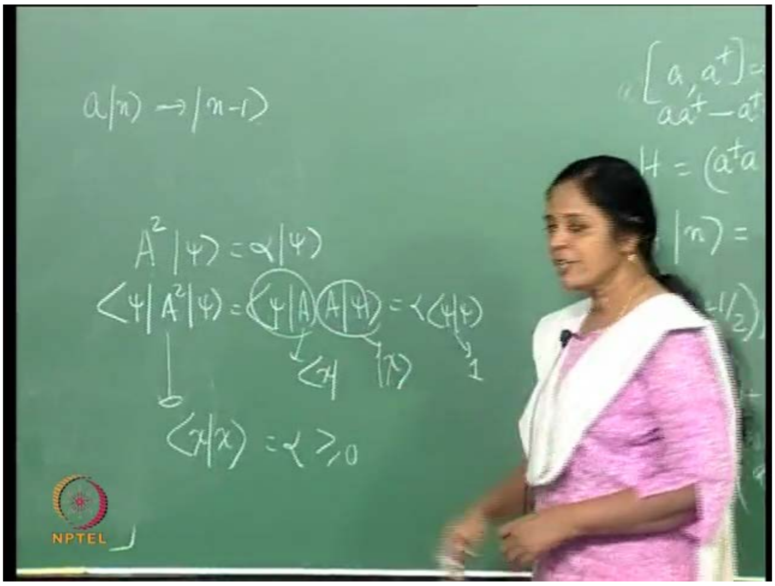

In the notation that I have used should be labeled as the state n minus 1. So, a dagger is called the raising operator and a is called the lowering operator. Because a dagger raises the energy of the original state by h cross omega taking it to a new state of the harmonic oscillator, both the original state and the new state are Eigen states of the oscillator Hamiltonian, energy Eigen states.

Similarly, a acting on a given state lowers the energy in steps of h cross omega taking us to new states: n minus 1; n minus 2 and so on, which label the states. So n is a label. So now, let us look at the following: the Hamiltonian itself when written in terms of x and p, it is clear that it is a quadratic form. There is an x square and there is a p square. Take any operator of the form a squared. Suppose psi is an Eigen state of a squared with Eigen value, I can just call it alpha for the moment alpha psi. Then psi A squared psi, this is a Hermitian operator, is psi A A psi. I could call A psi, as the state chi and therefore this is the bra.

So, this object is the inner product chi chi. On the left hand side, I have alpha psi psi and if this is normalized to 1 i have chi chi is equal to alpha. The inner product of the state chi with itself is equal to alpha. But, as we know inner products are greater than or equal to 0. And therefore, the operator of this form can have only an Eigen value which is non negative. The same thing happens in the case of the harmonic oscillator because x squared is the square of the operator x and p square is the square of the operator p.

And therefore, the Hamiltonian is a positive operator and will have an Eigen value which is non negative.So, it is clear that when the energy gets reduced by repeated application of the lowering operator a, we cannot get to a stage where the net energy becomes negative. Now, that is an important point because it tells us that finally, by repeated application of a.

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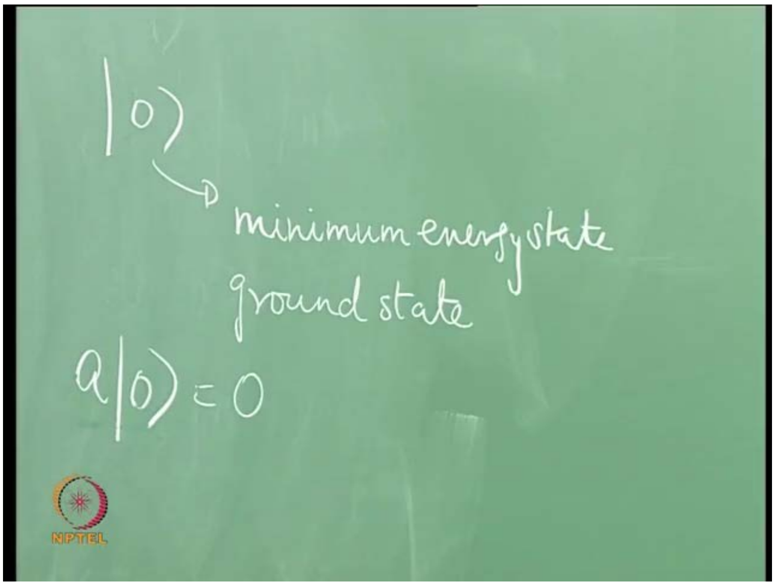

We will get to the lowest energy state which i will call, the minimum energy state or ground state. It is not the null vector. It need not be a state with energy value equal to 0. This is simply a label. So, n is equal to 0 denotes the lowest energy state. So, we get to this lowest energy state such that when I act with a on that state it should simply give me 0. It cannot take it to another state which has an energy which is negative. And therefore, the ground state is a state with positive energy or definitely non negative energy and when a acts on the ground state, it cannot take it to any other state which has got negative energy.

So, the spectrum therefore; the Eigen spectrum of the Hamiltonian has a ket zero, when n is zero. I can use a dagger and go to ket 1 which has got energy corresponding to the state ket 0 plus 1 h cross omega then n is equal to 2, which is the next state by application of a dagger once again on ket one. And that takes me to a state with energy of the ground state, plus 2 h cross omega and so on. So, it looks like the states are equally spaced.

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So, basically I have a ground state for the oscillator which is ket 0. This is the state on which a acts to give me the number 0. Then there is a 1st excited state. I went here by using a dagger. The ground state, let us say has energy E 0 then the 1st excited state has energy, E 0 plus 1 h cross omega. Then there is a 2nd excited state, went there by using a dagger this has got energy, E 0 plus 2 h cross omega. I will call that E sub 2. This is E sub 1 and so on.

There is no constraint on the upper bound. This is an infinite set of states which can be got by repeated application of a dagger on the ground state. Now, suppose I want to go down in the ladder. I would do it using an a. So, I could start with any state, apply a on it and bring it down to the next state that is the state with energy lower than that, the difference being 1 h cross omega. Now, let us look at what the energy values actually are.

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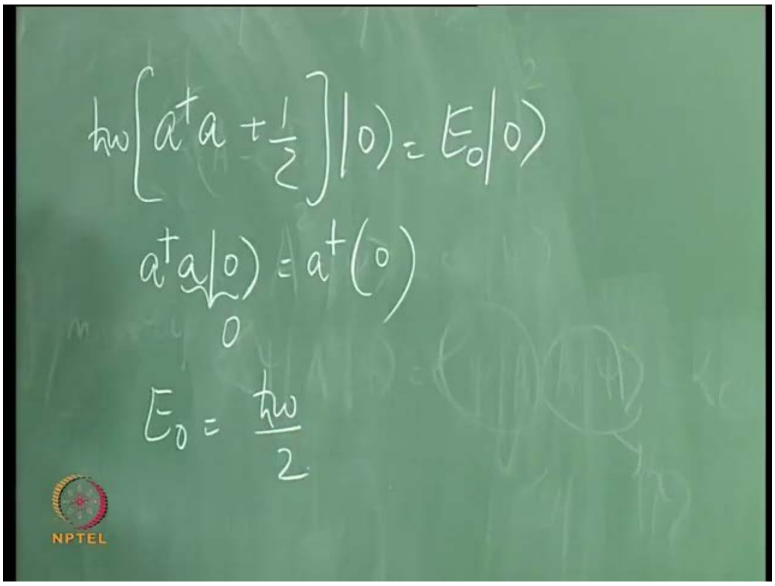

The energy values can be got using the Eigen value equation: a dagger a plus half h cross omega acting on ket 0, is e 0 ket 0. Look at the 1st term, a dagger a acting on ket 0 is, 0 because a on ket 0 is 0. And therefore that tells me, that e 0 is h cross omega by 2. I now have the complete picture.

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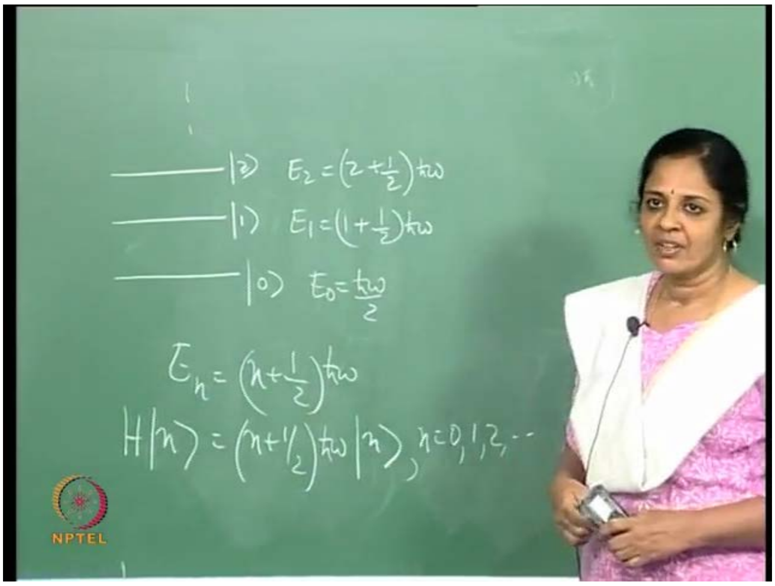

It is clear that ket 0 has energy h cross omega by 2 which I will call the 0 point energy because the lowest energy is a non 0 positive value h cross omega by 2. Then there is ket 1 with energy E 1 which is E 0 plus h cross omega, so 1 plus half h cross omega. Then, there is E 2 with energy 2 plus half h cross omega because I would have added one more increment to this and so on. And therefore, E sub n is n plus half h cross omega.

I can now write down the Eigen value equation; ket n is an Eigen state of the Hamiltonian with energy value n plus half h cross omega and n takes values: 0 1 2 3 and so on. So, this is the Eigen value equation corresponding to the harmonic oscillator. Already, we can see that in Quantum mechanics a new feature has emerged. The lowest energy is not 0. The lowest energy is half h cross omega. Further, the energy is quantized. In contrast to the spin systems that we have seen till now, here is an infinite spectrum of states. So, it is clear that we are dealing with an infinite dimensional linear vector space. It is always possible to normalize these energy Eigen states (Refer Slide Time: 23:15). We can always use the Gram Schmidt method.

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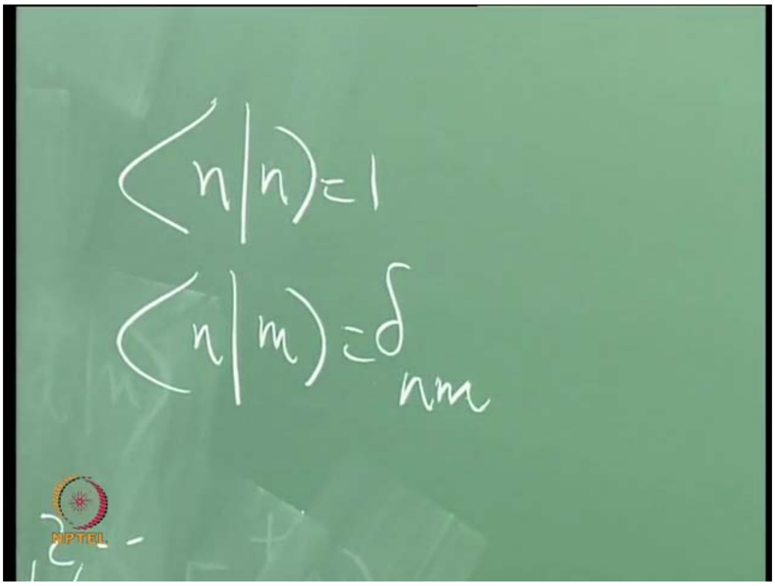

So, that in general the states are normalized to 1 and they are mutually orthogonal. Therefore, these would be the basis states with span. These are the energy Eigen states and they would be the basis states that span this infinite dimensional linear vector space, which has got an equally spaced energy spectrum. We have to do better. We have to find out what exactly is the action of a on ket n.

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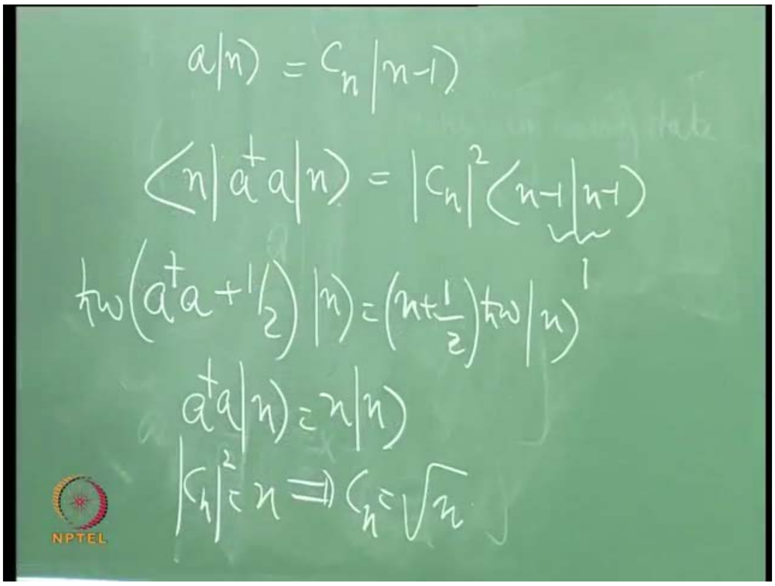

While, we know that a on ket n takes it to a state which we label as ket n plus 1. In general, this would be true. This should be n minus 1 because a is an annihilation operator. So, we need to find out what exactly is C n. I can always find out this object. This is simply mod C n squared, inner product of the state n minus 1 with itself, since the states are all normalized to unity. That is simply mod C n squared because C n is in general a complex number.

I emphasize, that a is not a Hermitian operator. Therefore it is, 1st of all this cannot represent observables, physical observables. But, linear combinations a plus a dagger, a minus a dagger and so on represent physical observables. They are Hermitian. They represent position and momentum, for instance in this case. I have a number here which is given by mod C n squared and on this side I know that, a dagger a acting on ket n is n ket n because h cross omega, a dagger a plus half acting on ket n was n plus half h cross omega ket n. Therefore, a dagger a on ket n is n ket n. And therefore, mod C n squared is equal to n. I can always choose my C n to be positive and take it to be square root of n. So, now I understand what a does to ket n.

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This is the equation that I have; a acts on ket n to give me root n ket n minus 1. Similarly, what does a dagger do, a dagger acts on ket n to take it to some C n prime ket n plus 1 where, C n prime is in general a complex number. I can always find out this object n a a dagger n but a a dagger is 1 plus a dagger a. And therefore, this object is n n plus n times n n which is n plus 1. So, I have C n prime mod squared is equal to n plus 1, which implies that I can choose my C n prime to be root of n plus one.

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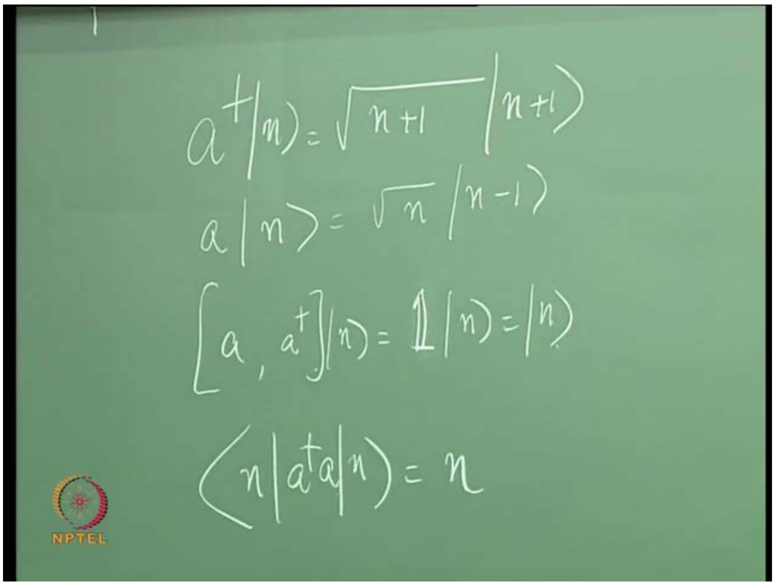

And therefore, I have the equation. This is 1 plus a dagger a n n and that is why I got an n plus 1 (Refer Slide Time: 30:04). This would have been an n here and a 1 there and therefore, I got an n plus 1.

So, I have a dagger on ket n is root of n plus 1, ket n plus 1 and a on ket n is root n ket n. I also have a a dagger acting on any of the states, ket n is like the identity operator acting on the state ket n, n minus 1 here. So, this is the algebra of operators that I have. In a sense, in the spin system if you recall, we could work with the algebra of: s x, s y and s z or equivalently s plus, s minus and s z. Here, I could work with x and p and the commutation between them or I could work with a and a dagger and the commutator which involves a a dagger and the identity operator.

Now, we wish to give matrix representation for these operators. As I have already pointed out, that is an infinite dimensional space. The ((Refer Time: 33:07)) the diagonal operator here, is this object and therefore diagonal elements are non 0 and they take values: 0, 1, 2, 3 and so on. So, it is pretty clear what a dagger a is in terms of matrices. 1st of all the column vectors,

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I could represent ket 0 as an infinite string of numbers in the column 1 and the rest of them 0, ket 1 as: 0 1 and the rest of them 0, ket 2 as: 0 0 1 and the rest of them 0 and so on.

Analogous to the spin system where we have the two component column vectors 1 0 and 0 1 in the case of the 2 level atom and in the case of the 3 level atom, we had 1 0 0; 0 1 0 and 0 0 1 as the basis vectors. Now given this, a dagger a itself is an infinite matrix which has got diagonal elements: 0 1 2 3 4 and so on, all other entries being 0. I emphasize that we are working in the basis, with the basis states which are energy Eigen states, Eigen states of the Hamiltonian and the Hamiltonian is just a dagger a plus half h cross omega. Now, what about a matrix representation for a and a dagger.

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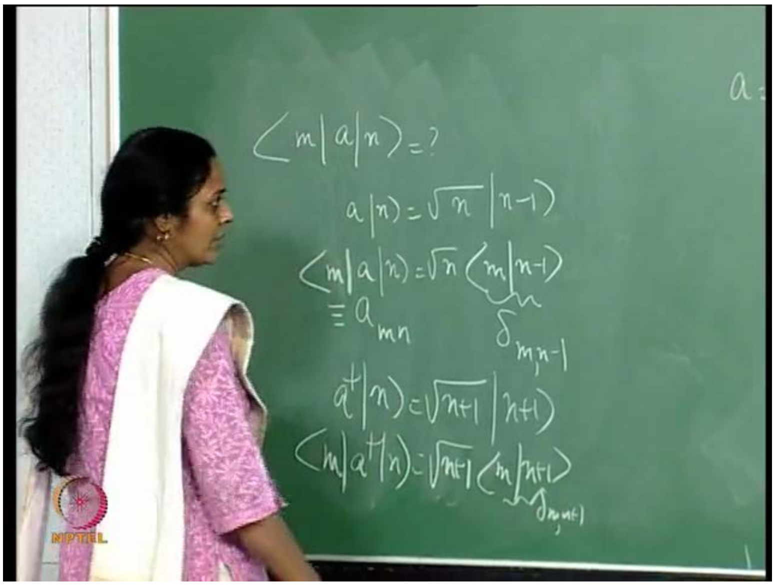

So, what is the m n-th element of the matrix a for instance. I know, that a on ket n is root of n ket n minus 1. Therefore, m a ket n is root n m n minus 1, inner product. This is just delta m n minus 1. So, it fires when m is equal to n minus 1 and is 0 otherwise. So, this would be simply replaced by the short hand notation a m n. Take the matrix a, this is the m-th row and that is the n-th column. So, this is the m n-th matrix element of the matrix a.

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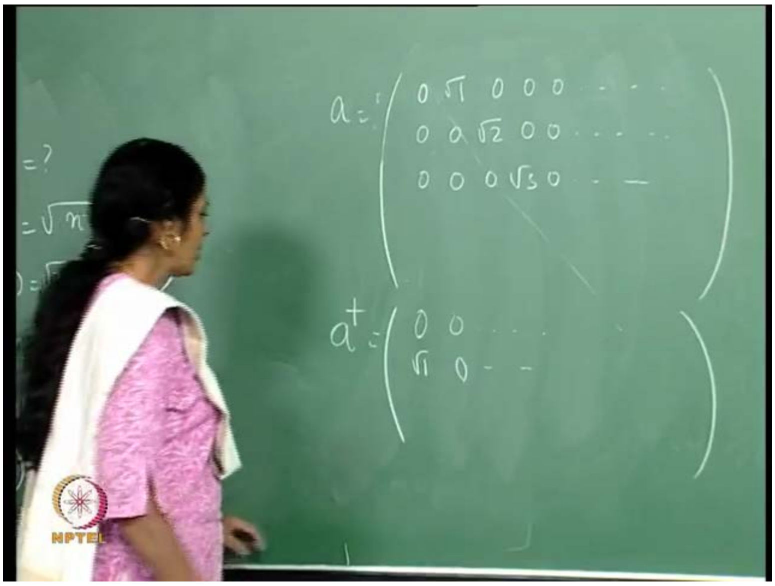

So, it is clear that this is again an infinite dimensional matrix. And, if the rows and columns are labeled: 0 1 2 3 this way and 0 1 2 3 that way it is clear that the diagonal elements are all 0 and then i have: 0 root 1 0 0 0 and so on, 0 0 root 2 0 0 and so on, 0 0 0 root 3 0 and so on. And, that is how I fill up the entire matrix and this is the matrix a, a dagger, you can easily follow this procedure. (Refer Slide Time: 34:54) Recall, that a dagger on ket n is root of n plus 1, ket n plus 1 and once more we can find out the matrix element.

This inner product gives us delta m n plus 1, the Kronecker delta m n plus 1. (Refer Slide Time: 35:48) But, I could well find out the Hermitian conjugate of a, that means interchange rows and columns and so on. Just interchange the rows and the columns and keep writing it so that this becomes an upper triangular matrix and that becomes, a lower triangle matrix. So, this is the diagonal out here and anything below the diagonal is 0 and above the diagonal I have entries. Just next to the diagonal, next to the principle diagonal all other elements are 0. Here, just below the principle diagonal I have entries, all other elements would be 0.

So, I can therefore fill out the various elements and I have matrix representations for the basis states a, a dagger and a dagger a. Infinite dimensional matrices, in contrast to the finite dimensional matrices that we discussed earlier on, when we studied the 2 level atom and the 3 level atom. I did this, in order that I may look at the uncertainty product

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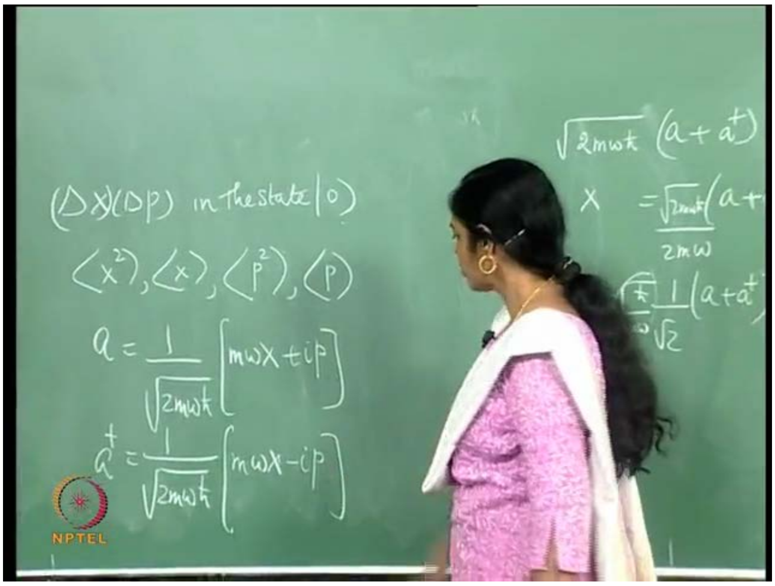

Delta x delta p in the ground state of the oscillator. All I need to do is, now calculate: expectation value x squares, expectation value x, expectation value p squared and expectation value p in this state. And then of course, I know the variance delta x squared and delta p squared. So, we can do that. Recall, that a was 1 by root of 2 m omega h cross, m omega x plus i p and a dagger was 1 by root of 2 m omega h cross, m omega x minus i p and therefore,

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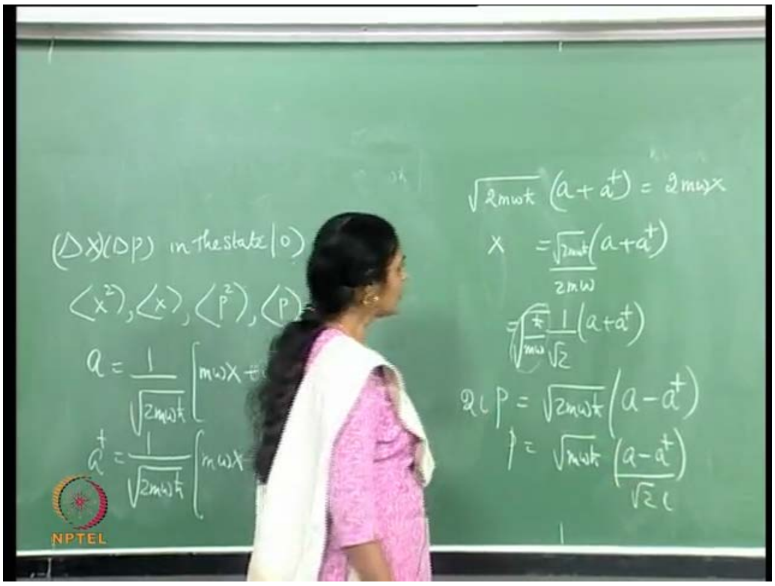

Root of 2 m omega h cross, a plus a dagger is 2 m omega x when I add both of them. So, that tells me that x is equal to a plus a dagger root of 2 m omega h cross, divided by 2 m omega. That already gives me a 1 by root 2 a plus a dagger. This multiplies root of h cross by m omega. As I pointed out earlier on: this has the dimensions of length, this is dimensionless and therefore, we get x with the correct dimensions.

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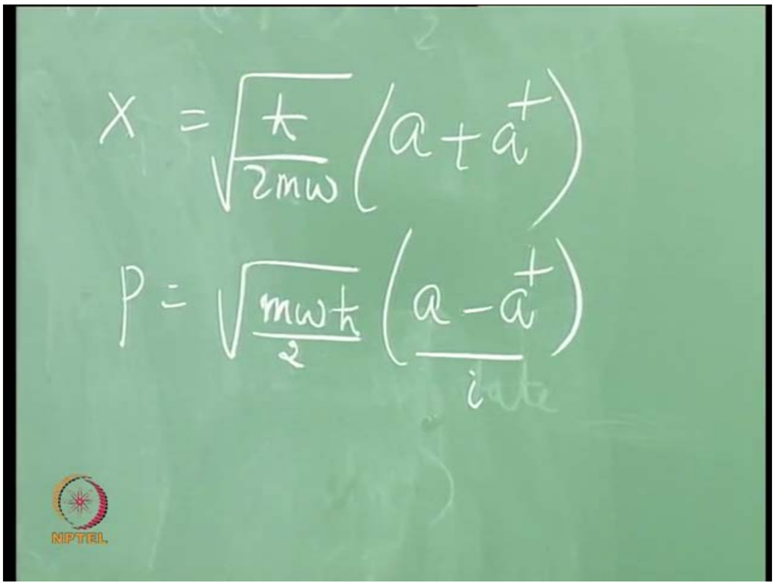

So, let me write it here. x is equal to root of h cross by 2 m omega, a plus a dagger. Similarly, I can find out p by subtracting 1 from the other. So, root of 2 m omega h cross, a minus a dagger is 2 i p. I have just subtracted one from the other (Refer Slide Time: 38:11).

(Refer Slide Time: 39:11) Therefore, p is root of m omega h cross a minus a dagger by root 2 i. This has dimensions of momentum and this is dimensionless. (Refer Slide Time: 40:12) So, we are through. The i is very important: it takes care of the commutation relation x p is equal to i h cross. So, now all I need to do is use this to calculate the expectation values that are needed to find out the variance. So, let us do that right away.

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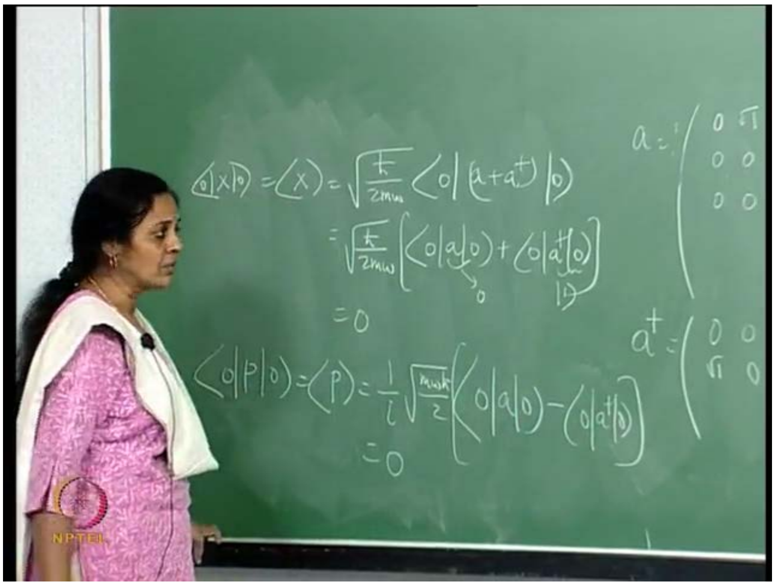

1st of all expectation value of x in this state, my short hand notation is just that. It is root of h cross by 2 m omega, expectation value of a plus a dagger in that state. So, that is the 1st term and this is the 2nd term. Now, a acting on 0 0. So this term drops out. Now, a dagger acting on 0 is root 1 ket 1 and because of orthonormality of the states, ket one is orthogonal to ket 0 in any case.

So, expectation value of x in the ground state of the oscillator is 0. In fact, it turns out that expectation value of p is also 0 in the ground state. We will see that right away because expectation value of p, which I will denote in this fashion, is apart from root of m omega h cross by 2. It is 0 a 0 expectation, minus 0 a dagger 0 and for the same reasons that I gave earlier this is also 0. So, I have expectation x is equal to 0 and expectation p is equal to 0 but the variance is not 0 because expectation x squared and expectation p squared are not 0. So, let us look at expectation x squared.

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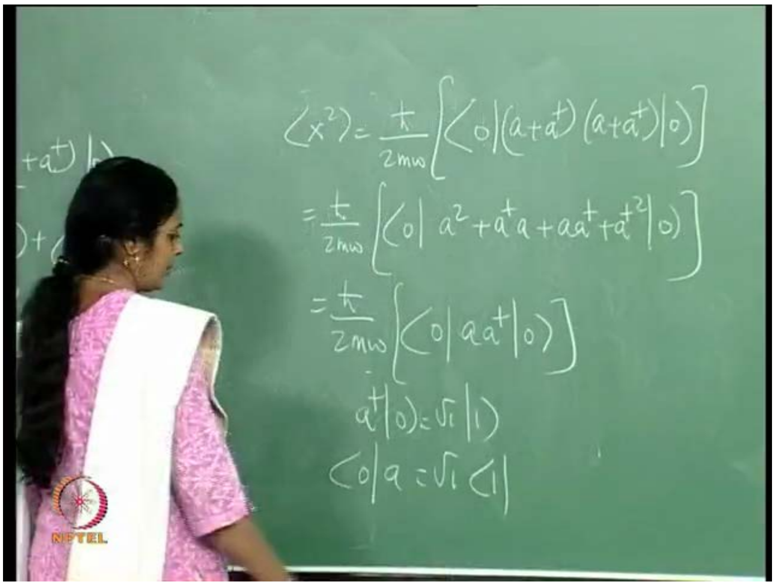

Expectation x squared as an h cross by 2 m omega, a plus a dagger the whole squared, its expectation value. I should be careful with the ordering because these are operators. That is h cross by 2 m omega. So, let us do this carefully term by term. The 1st term is an a squared in the product plus an a dagger a plus, an a a dagger, plus a dagger squared 0. Now, look at it term by term. Look at the matrix element. Look, at the expectation value of a square in this state. 1st of all a acts on 0 to give me 0.

And therefore, that does not contribute. Let us look at a dagger a, a dagger a acting on ket 0 is 0 ket 0. Simply because a on ket 0 is 0. So, that does not contribute either. The last term, a dagger square on ket 0, the 1st time you apply a dagger on ket 0, you get ket 1. The 2nd time gets you ket 2. So, you need to find the inner product of bra 0 with ket 2 and that is 0, because ket 2 is orthogonal to ket 0. The only term that contributes is out here. Now, a dagger on ket 0 is root 1 ket 1. Its Hermitian conjugate is root 1 bra 1. Therefore, I find that expectation x square in the ground state of the oscillator is out here.

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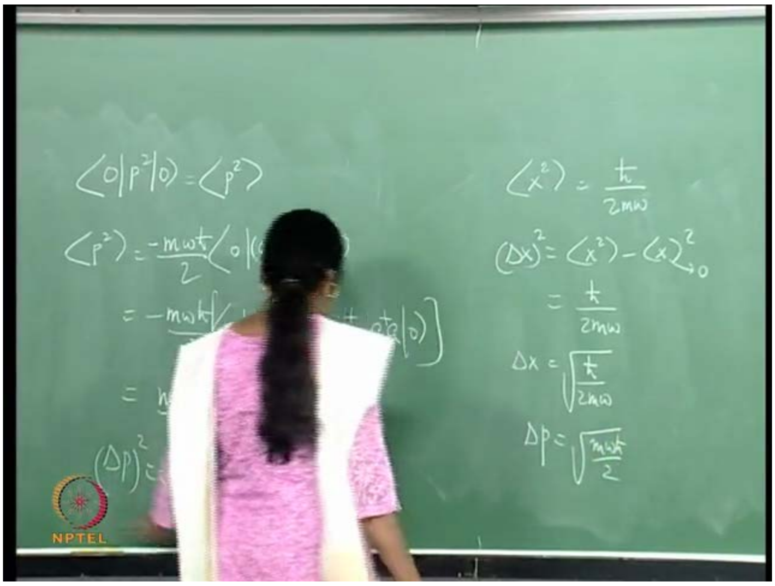

It is h cross by 2 m omega. So, delta x the whole squared is expectation x squared minus expectation x the whole squares. Now, this object was 0 and this was h cross by 2 m omega or delta x is equal to root of h cross by 2 m omega. So, there is the spread in x although the mean value is 0. There is a spread in x and that is given here. Let us look at delta p. So, once more I would do this calculation. I already know that expectation p was 0 in this state. We have seen that.

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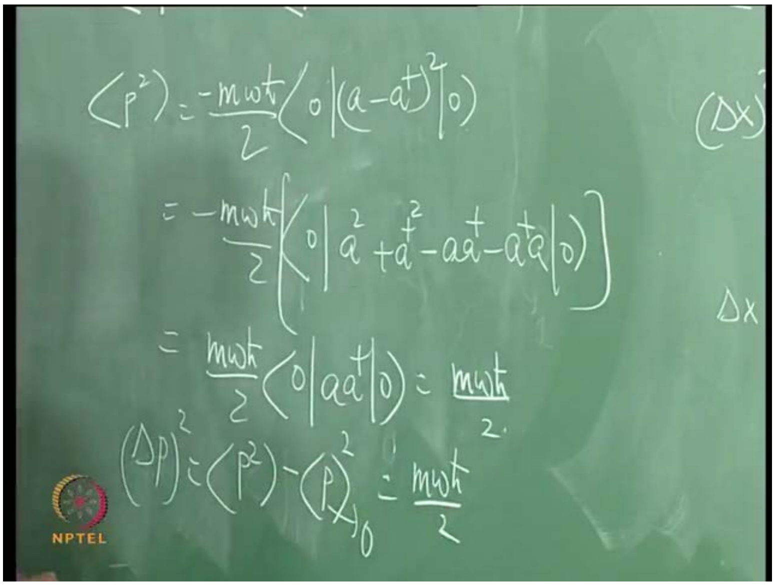

So, we need to find 0 p squared 0. I would denote this simply by this average of p squared. So, I get expectation p square is equal to m omega h cross by 2. There was an i squares which was a minus 1. From here, (Refer Slide Time: 40:12) which is what I put there. And I have an a minus a dagger, which is minus m omega h cross by 2 the whole square because I am looking at p square. That is an, a square plus a dagger square minus a a dagger minus a dagger a 0.

As, I have already argued: this does not contribute, this does not contribute and that does not contribute. The only object that contributes is here. I can always put this as plus m omega h cross by 2, expectation 0 a a dagger 0 and we just saw where that was 1. And therefore, we have this. Therefore, delta p squared is expectation p square minus expectation p the whole squared. This object was 0 and delta p squared was equal to m omega h cross by 2. So, here we are, (Refer Slide Time: 45:32) delta x is root of h cross by 2 m omega. Delta p is root of m omega h cross by 2. Therefore, the product delta x delta p in the ground state of the oscillator is very clear what it is.

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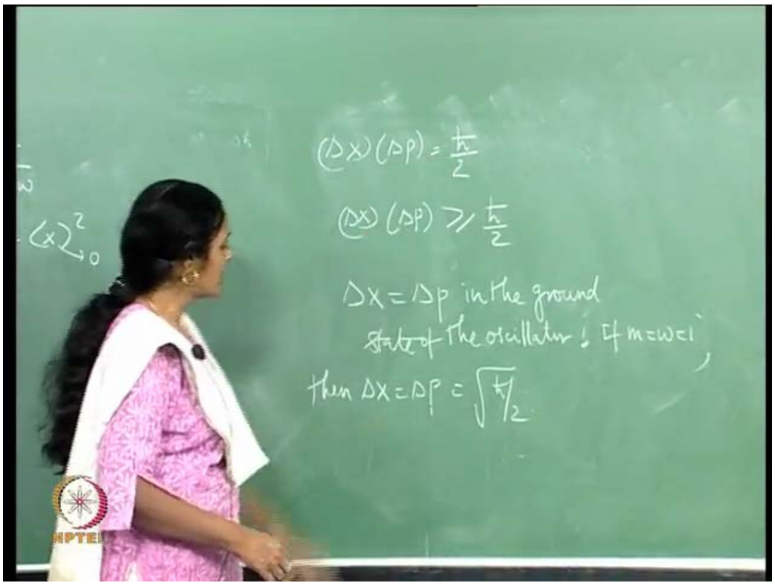

It is a minimum uncertainty state. You would recall that the uncertainty principle told us: that delta x delta p would be greater than or equal to h cross by 2. Now, here is a state, the ground state of the simple harmonic oscillator where delta x delta p is equal to h cross by 2. The equality holds. What does that mean? Does it mean that delta x is non 0 and delta p is non zero? Not necessarily, yes there are states where delta x is non 0 and delta p is non 0. Therefore, you cannot simultaneously accurately measure with arbitrarily high precision both x and p. That is certainly true in this case. On the other hand, you could be in a position Eigen state. If you are in a position Eigen state, the state is an Eigen state of the position operator.

So, delta x is 0 which means that in that state delta p is infinity, so that the product is a finite non 0 quantity. Similarly, you could be in a momentum Eigen state or if the state is a momentum Eigen state then delta p is 0 but delta x would be infinite. You would not be in a position to predict the value of x. Delta x would be infinite and delta p would be 0 in such a state and then the product is a finite quantity.

In the case, of the oscillator if n equals 1 and omega equals 1, it is clear that delta x is equal to delta p in the ground state of the oscillator. It is a minimum uncertainty state. And, delta x is equal to delta p is equal to, in natural units, if m is equal to omega equal to 1. Of course, if I set h cross is equal to 1 that just gives me a root of 1 by 2 here and a root of 1 by 2 there. If m is equal to omega is equal to 1 then delta x is equal to delta p is equal to root of h cross by 2. In natural units, h cross also would be set equal to 1. Then delta x is equal to delta p is equal to 1 by root 2 but as it stands, (Refer Slide Time: 45:32), this is the value of delta x and that is the value of delta p and the product is tells us it is a minimum uncertainty state.