Quantum Mechanics-I

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Lecture - 36
Perturbation Theory – I

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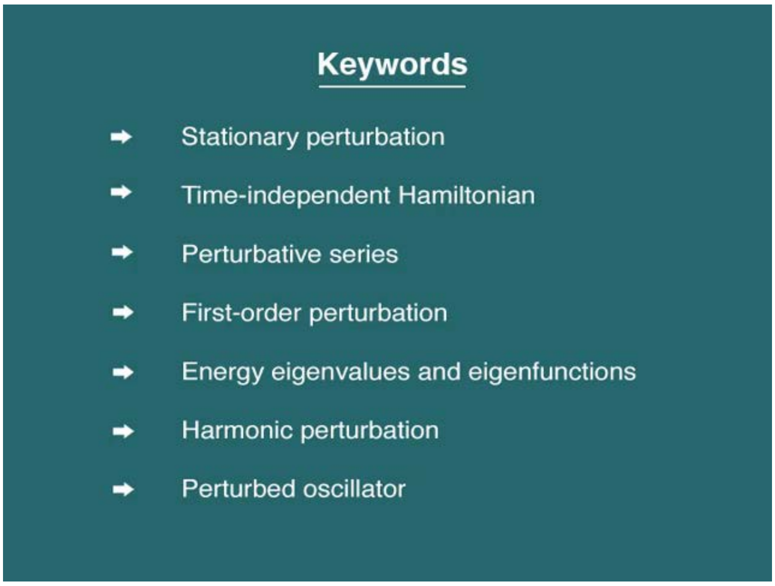

Over the last few lectures, we have looked at the dynamics of the state of a system. That means, that you subject the system to various potentials and then you look at the way the manner in which the wavefunction changes, when subject to those potentials. Several interesting things can happen. For instance, even in the case of a Harmonic oscillator potential, you know that an initial Gaussian, continues to be a Gaussian and merely oscillates, preserving its Gaussian form. So, this is the kind of statement that you could make, about a coherence state of light, which is moving in free space, because that too is modeled by Gaussian wavefunction. On the other hand, as you depart from coherence. You have seen, that the state of the system, for instance the one photon added state, does not preserve its shape or form as it moves.

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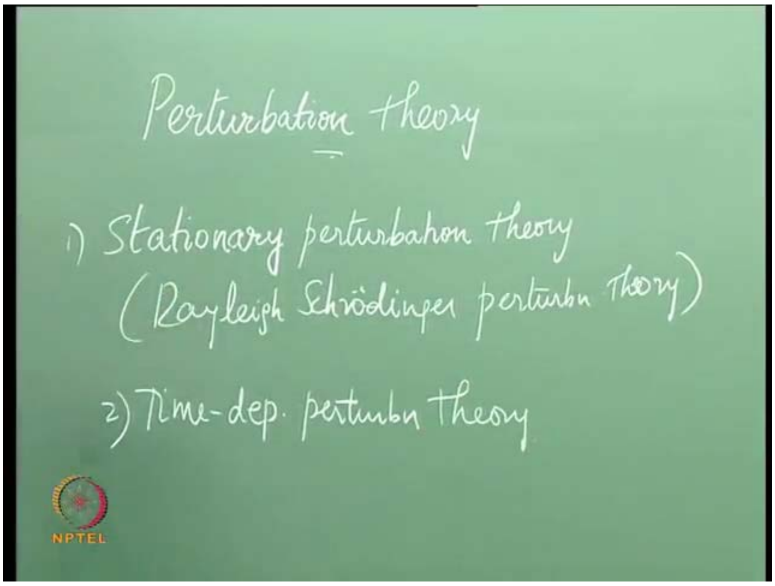

Now we will look at a new class, a new variety of problems and these come under the title perturbation theory. In other words, you look at a system on which there is an external field or force, which acts an external Hamiltonian in general. Another Hamiltonian, a perturbing Hamiltonian has to be added to the original Hamiltonian and therefore, the system is affected. Now, the external agent which acts on the physical system of concern could be a time dependent Hamiltonian or it could be guided by a time independent Hamiltonian.

So, depending on whether the external agent is modeled by a time dependent Hamiltonian or a time independent Hamiltonian, you will do stationary perturbation theory if it is time independent. So, stationary perturbation theory, also called the Rayleigh Schrodinger perturbation theory. Now, if you also have a time dependent Hamiltonian which models the external perturbation then, it is the time dependent perturbation theory.

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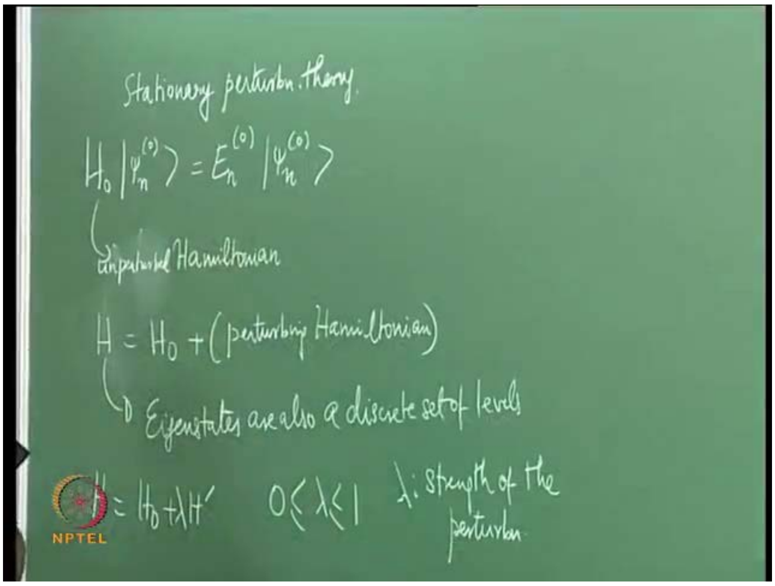

We will first be looking at stationary perturbation theory and the point here is this. Suppose, you started with the physical system who's Hamiltonian is given by H naught. This corresponds to the free system, which has not been subject to any external agent and suppose, the Eigenbasis are given by psi n's. But, I will also put a superscript there, 0 to show that this is the unperturbed Eigenbasis and this is the Eigenvalue equation.

So, for instance, if you took a free Harmonic oscillator. H naught would be p squared by a linear Harmonic oscillator, H naught is the free Hamiltonian, by free I mean, unperturbed Hamiltonian. So, let us write unperturbed Hamiltonian and this in the case of the linear Harmonic oscillator would be p square by 2 m plus half k x square, where k is m omega square. And these, would simply be the ket n's or the Fock states of the oscillator and the E n zeros, would be just n plus half h cross omega and this is the Eigenvalue equation, which you know and which you have solve for.

We have solved for this set of Eigenvalues and the Eigenfunctions using the Schrodinger formalism. We have solved for this and shown that these wave functions, that is in the position representation, they would be given by essentially the Hermite polynomials, apart from other factors. Now, once you apply an external perturbation, you have a new Hamiltonian and this Hamiltonian has an H naught plus a perturbing Hamiltonian.

When you say stationary perturbation theory, you mean the following thing. First of all we are dealing with discrete Eigenstates. So, here you have a set of discrete levels, psi 1

0, psi 2 0, psi 0 0, because n can also take the value 0 in the case of Harmonic oscillator for instance. So, you have a discrete set of levels and there is this perturbing Hamiltonian which was some external field possibly which acted on the oscillator or on the given physical system at sometime. And then it has been removed so that the system now settles down and equilibrates to a new set of discrete levels, so this one's Eigenstate are also discrete, a discrete set of levels.

So, we are doing stationary perturbation theory in contrast to time dependent perturbation theory. Now, this perturbing Hamiltonian, I could just write it as some H prime, but for the sake of book keeping, I would like to introduce a parameter lambda here and lambda can take values 0 to 1. Basically, if the perturbation is switched off, that is like saying that lambda 0 and you only have the unperturbed Hamiltonian. The system is not perturbed, lambda is 1, the perturbation is comparable to H naught. So, that is the way it is. Lambda can take any value from 0 to 1 and therefore, lambda is the strength of the perturbation. So, I write the new Hamiltonian as H naught plus lambda H prime.

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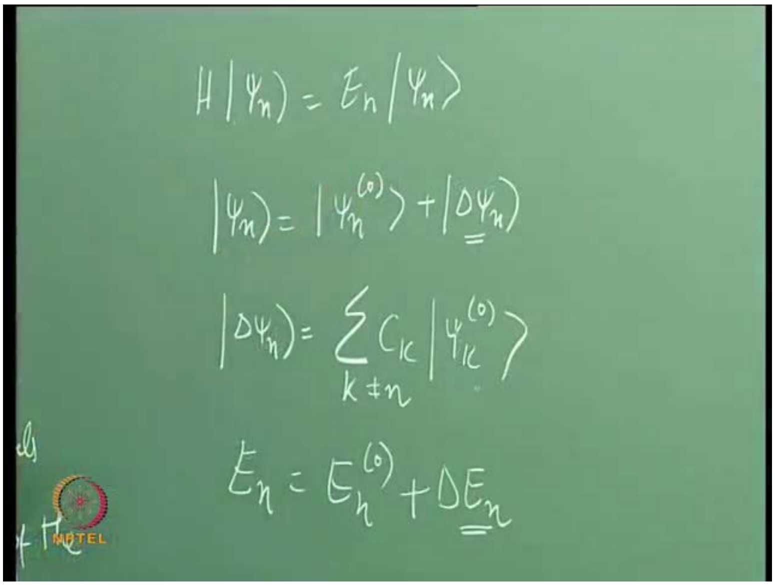

The very important feature here is the following. The Eigenstates of the new Hamiltonian let me refer to them as psi n and the new Eigenvalues as E sub n psi n. (Refer Slide Time: 03:37) The important thing is that the psi n's are also expandable in terms of this basis set psi n 0. So, basically the perturbation changes psi n 0 by some quantity. I represent this in ket notation so psi n the n-th level, new state which is an Eigenstate of the full

Hamiltonian H is the old n-th level plus some delta psi n. Now, this delta psi n can be expanded, in terms of the old basis. (Refer Slide Time: 03:37) So, the perturbation does not change the Hilbert space of the system per se in the sense, that I can retain the old basis set.

So, delta psi n can really be expanded in terms of the old basis set in this fashion, except that, I would assume that, the perturbation in this superposition does not have a non-zero coefficient corresponding to n. In other words, delta psi n does not have a contribution along the component psi n 0, does not have a component along ket psi n 0. So, this is what I have for delta psi n. Now, E n itself correspondingly, will be the old energy Eigenvalue plus an addition delta E n because of the perturbation. Now, the aim of perturbation theory is to estimate delta E n and delta psi n.

In other words, you want the new energy values and you also want the corresponding energy Eigenstates, by that I mean, the Eigenstates and the Eigenvalues of the full Hamiltonian H, which includes the free or the unperturbed Hamiltonian plus the perturbation (Refer Slide Time: 03:37). Now how do I go about doing this? The aim is to develop a perturbation series. In principle this is an infinite series and if it is possible to truncate the series somewhere, effectively if it is possible to make such an approximation. Then it is very good, because I do not have to work with an infinite series and perturbation is a very effective procedure. The perturbation series can be truncated and the series itself will be written in powers of this (Refer Slide Time: 03:37) strength lambda.

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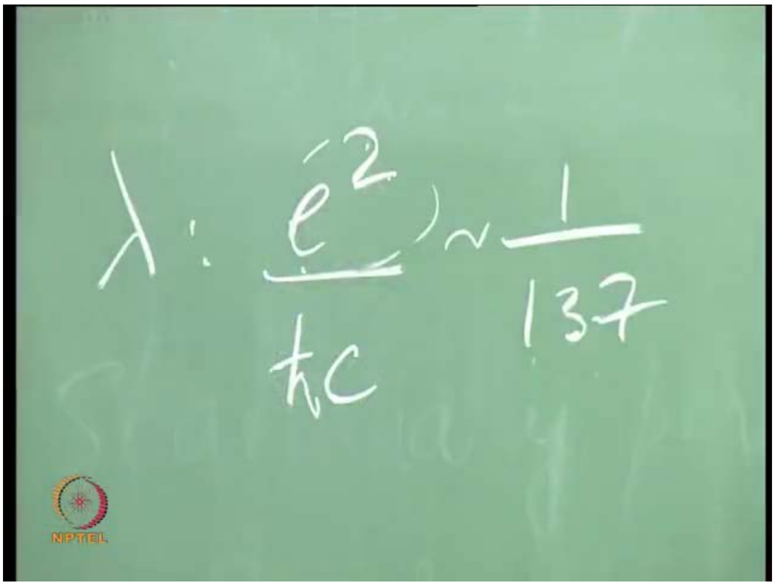

So, for instance if you are talking about electromagnetism, lambda is usually the parameter e square, where e is electric charge by h cross c and this is 1 by 137 and therefore, if you made a series in powers of lambda. The 1st term lambda to the 1, that means the 1st order in the perturbation has 1 by 137 multiplying things. The next order is 1 by 137 squared and that is much smaller than 1 by 137 and so on. So, you make an expansion in terms of the quantity whose powers, the strength of the perturbation itself is 1 by 137 in some units.

And therefore, higher terms could possibly make lesser contribution and you can stop the series at some point, you can truncate the series and find out, what the wavefunction is. (Refer Slide Time: 08:23) psi n and the corresponding E n to that order of approximation. On the other hand, it is possible that you will never be able to truncate the series. And then of course, perturbation theory itself is not particularly useful. So, we will see specific instances, where we can truncate the series and find out what the final wavefunction and the corresponding Eigenvalues are, to that approximation.

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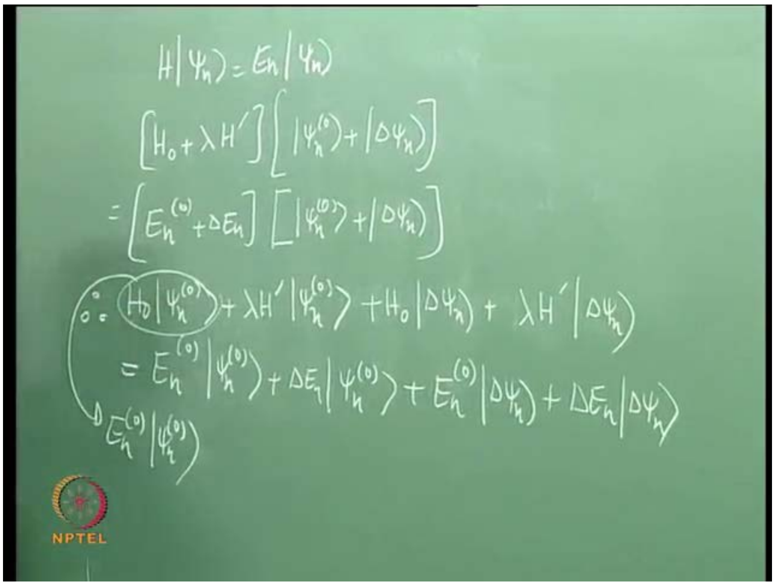

So, the 1st stage is to write H psi n equal E n psi n as H naught plus lambda H prime psi n itself is psi n 0 plus delta psi n. This quantity is equal to E n 0 plus delta E n psi n 0 plus delta psi n. So, when I expand this: I have H naught psi n 0 plus lambda H prime psi n 0 plus H naught delta psi n plus lambda H prime delta psi n is E n 0 psi n 0 plus delta E n, which is a number, psi n 0 and then of course, I have the last two terms: E n 0 delta psi n plus delta E n delta psi n. I merely expanded out things, in terms of H naught, H prime, psi n 0 delta psi n and so on. But, I know that h naught psi n 0, I know that this quantity is E n 0 psi n 0 and therefore, these two just cancel out and then what do I have?

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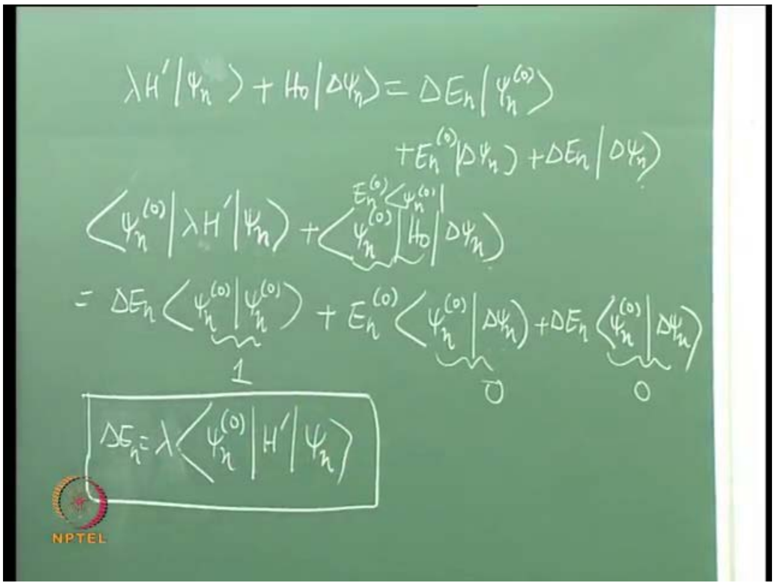

I have lambda H prime psi n 0 plus H naught delta psi n plus lambda H prime delta psi n.

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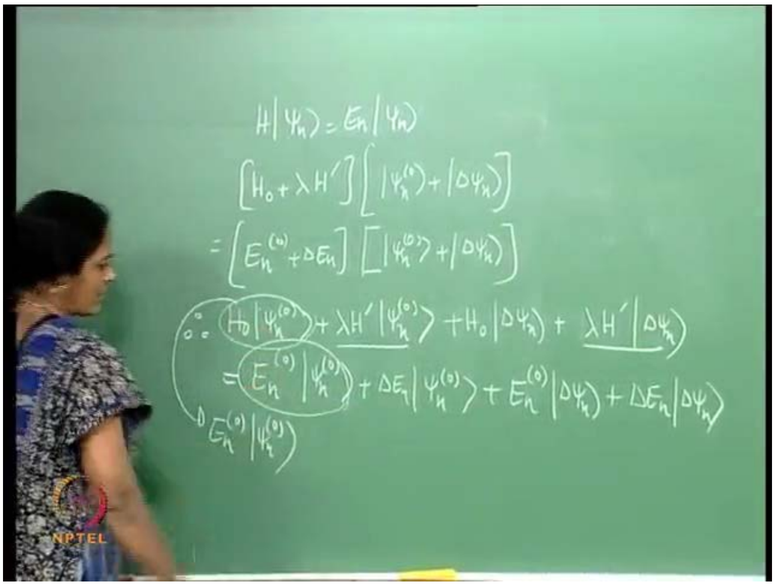

So, I might as well just clubbed the two terms and write lambda H prime psi n 0 plus delta psi n and that is just psi n plus H naught delta psi n. Notice that these two terms, just to care of each other, they balanced each other and therefore, I am left with delta E n psi n 0, out there plus E n 0 delta psi n plus delta E n delta psi n. So, this is what I have. These two terms balanced out each other. (Refer Slide Time: 15:29)

I simply combined lambda H prime ket psi n 0 plus ket delta psi n and wrote that there as the 1st term. Then I had an H naught delta psi n and on this side, I have a delta E n psi n 0 and E n 0 ket delta psi n, plus a delta E n delta psi n. My aim is to find delta E n. I need to find the extra amount and therefore, let me do this. So, I have lambda H prime it is sandwiched between ket psi n and bra psi n 0 that is my first term plus psi n 0 H naught delta psi n. That is delta E n, psi n 0 psi n 0 and that is 1, because we have chosen an orthonormal basis and therefore, this inner product is 1 plus E n 0 psi n 0 delta psi n plus delta E n psi n 0 delta psi n .

Notice the following: we said that delta psi n, our assumption was that. (Refer Slide Time: 08:23) Delta psi n is expanded in terms, is a superposition of the psi k zeros, but not psi n 0 and therefore, this is 0. Those two terms drop out and therefore, I have delta E n equals this object psi n 0 lambda H prime psi n plus psi n 0 H naught delta psi n. But, notice that H naught psi n 0 is E n 0 psi n 0 and that term therefore drops out, because delta psi n does not have a contribution along psi n 0.

So, that term also drops out and therefore, I have a very simple relation like this. Let me pull the lambda outside, H prime psi n. So, this is delta E n. In other words, the effect of the perturbation is to shift the energy value which was originally E n 0 by an amount delta E n, which is calculable and which is given in this manner. You merely have to find the following scalar quantity. You need to sandwich H prime between psi n and psi n 0.

Notice that psi n is the full wavefunction, where as psi n 0 was the unperturbed wavefunction and H prime is the perturbing Hamiltonian, lambda H prime is a perturbing Hamiltonian. So, here is a very simple way of estimating delta E n provided, I know what psi n is and as I said psi n has to be perturbatively estimated. In other words, you write psi n in the following manner as a series in powers of lambda.

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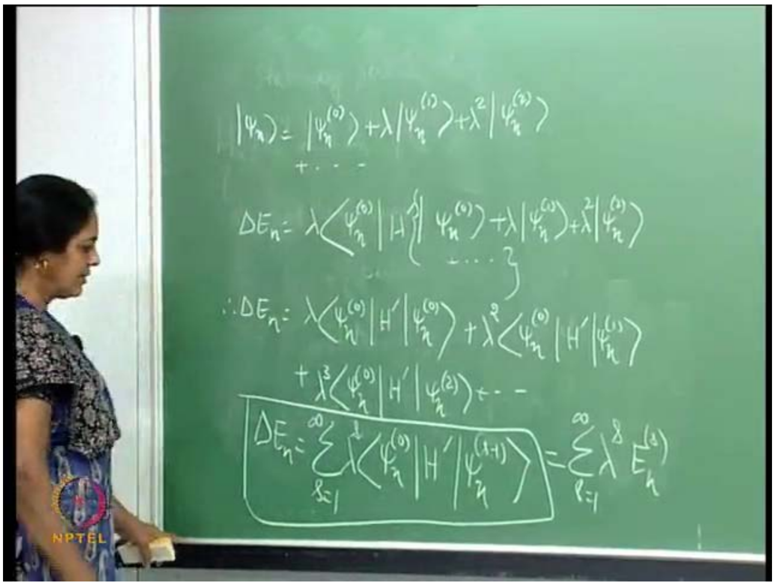

So, psi n is psi n 0, if there is no perturbation, plus lambda psi n 1. This is to 1st order in the perturbation plus lambda square psi n 2; this is to 2nd order in the perturbation and so on. So, this is the manner in which psi n is expanded and you could estimate the wavefunction perhaps, by truncating the series here. That means to 1st order in the perturbation, or truncating the series here which means, you estimate psi n to 2nd order in the perturbation and so on.

So, now when you use that, you just have delta E n is lambda, psi n 0. What I have written here? (Refer Slide Time: 15:11) H prime, which is merely the perturbing Hamiltonian, psi n 0 plus lambda psi n 1 and so on that series. So, this is what I have. So, let us see what this gives us? Therefore, delta E n is lambda psi n 0 H prime psi n 0. Suppose, you initially started with the state psi n 0, what is the probability amplitude that you land up in a final state psi n 0? So, that is what this gives you, plus lambda squared psi n 0 H prime, psi n 1 plus lambda cubed psi n 0 H prime psi n 2 and so on.

So, I can always write this in a compact form: delta E n is summation over s equals 1 to infinity, lambda to the s, psi n 0 the unperturbed wavefunction, the perturbing Hamiltonian psi n s minus 1. Now, this is a good thing for us to know because if you want to find delta E n to order lambda, that mean you set s equals 1. You merely need to know the wavefunction to a lower order psi n 0. If you wanted to estimate delta E n to the 2nd order, that is lambda squared times something. Then you need to know the wavefunction only to 1st order, because that is an s minus 1 out there.

So, basically if you know psi n 0 and psi n 1 you can estimate delta E n, to 2nd order in the perturbation. If you only know psi n 0, you can estimate delta E n to 1st order in the perturbation and so on. So, the wavefunction is something you need to know to a lower order, one order lower, in order to find out, the correction to the original energy to that order. So, that is a nice thing and therefore, I can well write this as summation s equals 1 to infinity, lambda to the s, E n s, where E n s is precisely this object.

So, you see delta E n is also expanded in powers of lambda. You expanded psi n in powers of lambda and the change delta E n also in powers of lambda. So, the total energy will be E n 0. So, that is to 1st order, to zeroth order in the perturbation you have E n 0 plus delta E n, which is written in terms of the 1st order in lambda times E n 1 and so on. So that is what you have. Now you see we can feed all these things in and find out, what is a change in energy delta e n? And what is the change in the wavefunction delta psi n?

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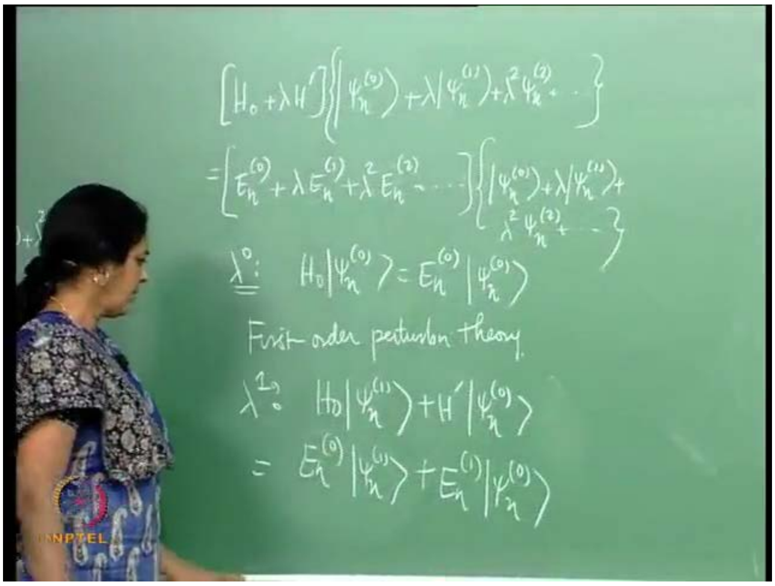

So, let us see how we go about this. So, go back to your equation and you have H naught plus lambda H prime psi n 0 plus delta psi n is lambda psi n 1 plus lambda square psi n 2 plus so on equals E n 0, which was the unperturbed energy plus lambda E n 1, from here, (Refer Slide Time: 19:55) plus lambda squared E n 2 plus so on. So, I expand that also as a series in lambda times psi n. So, I can write that as psi n 0 plus lambda psi n 1 plus ambda squared psi n 2 plus so on.

Now, let us look at zeroth order that means, only look at terms which have no lambda dependents and equate them. For order 0 what do I have? I have H naught psi n 0 on that side, because every other terms contains lambda is E n 0 psi 0. This is merely a statement that if you have the unperturbed system. Then it is guided by a Hamiltonian H naught and the basis states, the energy Eigenstates are psi n 0, the corresponding energy Eigenvalues are E n 0, say discrete set of values.

Now, if you did 1st order perturbation theory, that means you compare coefficients, to order lambda to the 1 or lambda and equate them in this equation and what do I have? I have H naught psi n 1, from here plus H prime psi n 0 and that is all I can have here, because I need to just look at terms multiplying lambda and this comes with lambda squared, lambda cubed and so on beyond that. So, that the contribution is only from these two terms.

But on this side I have E n 0. This is equal to E n 0, psi n 0. It cannot make a contribution, because you are looking at powers of lambda so its E n 0 psi n 1, out here plus E n 1 psi n 0. So, the 1st order contribution E n 1, but the wavefunction is psi n 0. So, this is what I have. I have this equation, if I work to order lambda that means 1st order perturbation theory. I can write this better, I can simply group terms and I can write.

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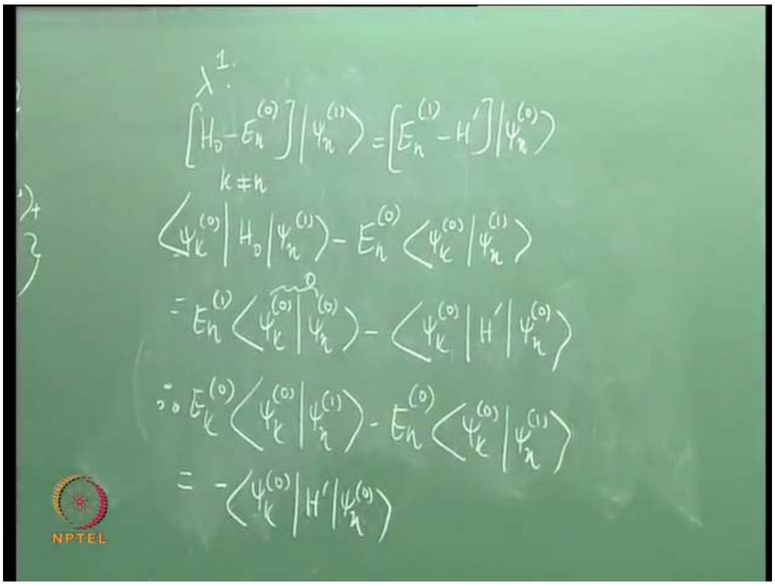

So, this is 1st order perturbation theory. H naught minus E n 0, psi n 1 equals E n 1 minus H prime psi n 0. So, we use lambda for book keeping purposes so you work to order lambda, lambda square lambda cubed, zeroth order in lambda which is the free case and so on. So, this is the equation that I have. My aim is to find out E n 1 and I have a formula for that, because we have already said (Refer Slide Time: 19:55) that E n 1 out here. Put s equals 1 so that is psi n 0 H prime psi n 0 that is E n 1 so I know E n 1. H prime is given to me, that is the perturbing Hamiltonian, I know psi n 0 that is the old energy basis with which I began and my aim is to find out psi n 1.

So, there is a very simple way of doing this. So, let us choose k not equal to n and do the following: minus E n 0 psi k 0 psi n 1 equals E n 1 psi k 0 psi n 0 minus psi k 0 H prime psi n 0. So, I have just used bra psi k 0 on this side. I know what this is. I know that H naught psi k 0 is E k 0 psi k 0 therefore, E k 0 psi k 0 psi n 1 and they can have a non-zero overlap. Remember that psi n 1 is the wavefunction to 1st order in the perturbation. So, it is the 1st order contribution to delta psi n and a very crucial input was that delta psi n did not have a contribution along psi n 0 and we have selected k not equal to n. So, delta psi n when expanded in terms of the basis set, the psi k 0's, it would have a component along psi k 0 in general, provided k is not equal to n and that is what I have used here. This is in general non-zero.

So, minus E n 0 same thing out there psi k 0, psi n 1, this is 0, because we have chosen k not equal to n and since this is the Eigenbasis of the unperturbed Hamiltonian and they are mutually orthogonal. So, this term does not contribute, but I have equals minus psi k 0, H prime psi n 0. So, you see once more what figures is nearly this matrix element of H prime, between the unperturbed state psi n 0 and psi k 0. In order to estimate things in the 1st order of the perturbation, I need to know only the wavefunction to zeroth order in the perturbation.

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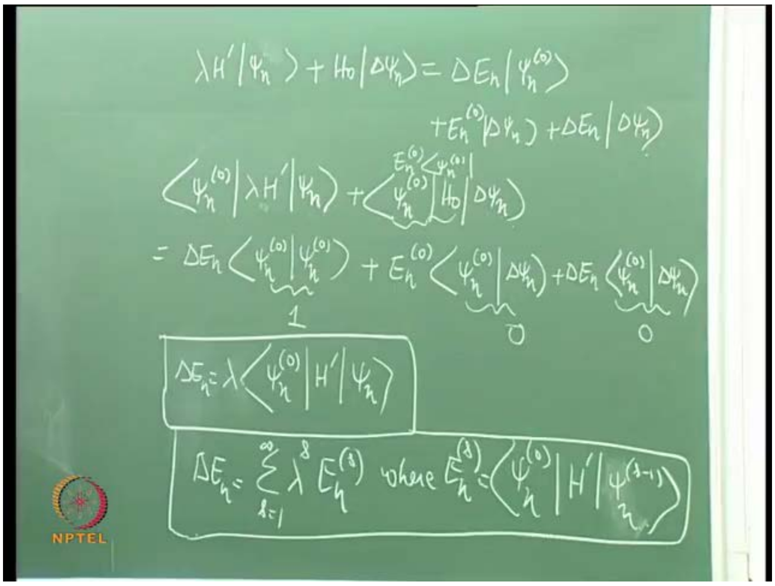

So, I need to put down delta e n. We have already shown that that is summation s is equal to 1, to infinity lambda to the s E n s where E n to any order is really this object, (Refer Slide Time: 19:55) it psi n 0 H prime psi n s minus 1. So, these are important things that we have already shown. So, that is the way it is. So, going back to this equation, (Refer Slide Time: 28:01) we are trying to estimate the wavefunction and the energy Eigenvalues to 1st order in the perturbation and we have an equation of this form and therefore, I can write.

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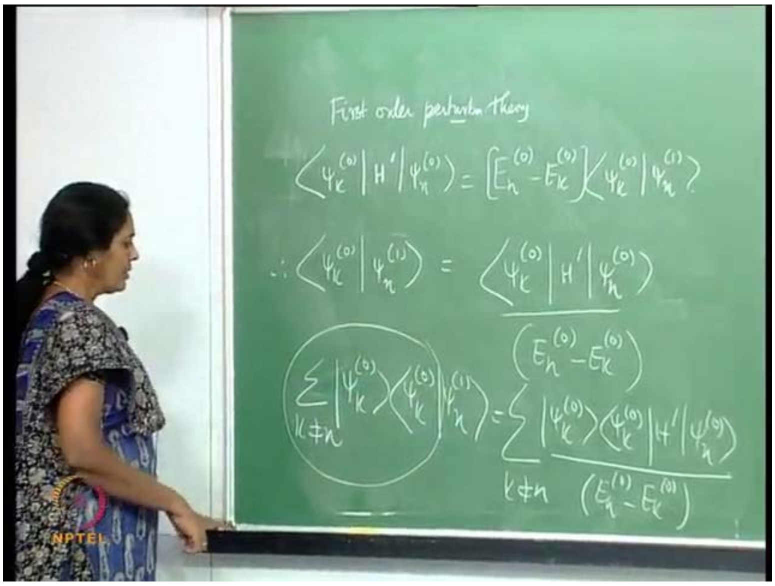

So, 1st order perturbation so I have psi k 0 H prime psi n 0 equals E n 0 minus E k 0 (Refer Slide Time: 28:01) times a non managing quantity in general, which is psi k 0 psi n 1 inner product and this is what I have. So, I can well write it in the following fashion, remember I need to find out psi n 1, this is the quantity ket psi n 1 that I need to estimate. So, take this matrix element and divide it by this number. The energy corresponding to the k-th level, the energy corresponding to the n-th level and n is not equal to k that is the way we have taken things.

So, if I want to estimate ket psi n 1, all I have to do is to do a summation k not equal to n, psi k 0 psi k 0 psi n 1. This object is summation k not equal to n, psi k 0 psi k 0 H prime psi n 0 by E n 0 minus E k 0. And this is very interesting, because since k is not equal to n and you are summing over the case, look at this part. This object should simply be equal to identity, if I had also put k equals n.

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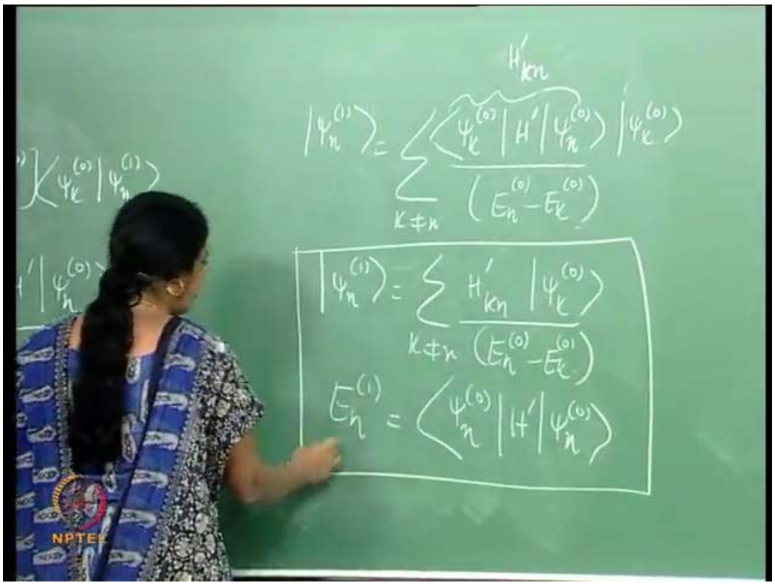

So, therefore, this reduces to identity minus the projector onto ket psi n, but because psi n 1 has no component along psi n 0, I have psi n 1 equal summation k not equal to n (Refer Slide Time: 33:03) and here unfortunately, I cannot simply do that summation, because there is an E k out here and this is a matrix element which I will call H prime n k n. But for the moment let me just write it down. Psi k 0, this is what I have, divided by E n 0 minus E k 0.

So, this summation extends all over and this is what I have. All I need to know is this matrix element which I will call H prime k n. It is the k n-th element of the matrix H prime. H prime is an operator and if you give a matrix representation, this is clearly the k n-th element of the matrix H prime and I need to know the unperturbed wave functions and I sum over all k's not equal to n and I know that this would not blow up, because n is not equal to k and this is the manner in which I find psi n 1.

So, I have psi n 1 is summation k not equal to n, H prime k n psi k 0 divided by E n 0 minus E k 0 and E n to 1st order is something that I know. (Refer Slide Time: 31:56) E n to 1st order is psi n 0, H prime psi n 0. So, you see this formalism turns out to be extremely useful, because to determine the energy values and the wavefunctions to 1st order, I only need to know the energy values and the wavefunctions to zeroth order.

So, if I now compared, if I did 2nd order perturbation theory and I took that huge expression that we had earlier and I equate terms with coefficient lambda squared. Then I will find that the aim would be to estimate psi n to 2nd order and E n to 2nd order in that case. And all I would need would be the wavefunction to zeroth order and 1st order, which I would have estimated already, the zeroth order wavefunctions I know, they are the free Hamiltonian wave functions, Eigenstates and then the 1st order wavefunction which I would have gotten from here.

Similarly, I would need to know E n 0's, that entire set and the psi n 0's. So, you see if I know the energy, zeroth order and 1st order and I know the wavefunction zeroth order and 1st order. I know the energy and the wavefunction to 2nd order and so on. So, I need to know these values to an order less than what I wish to determine and therefore, it becomes some kind of a recursion relation where I feed in the zeroth order values and get the 1st order values, feed in the 1st order values and get the 2nd order values and so on.

Now, these are the results that I get for the contribution to 1st order to the energy and to the wavefunction, because of the perturbation. So, let us use these formulae, let us try to illustrate this with a very simple example, the linear Harmonic oscillator, where I add a perturbation such that the Harmonic nature is still maintained.

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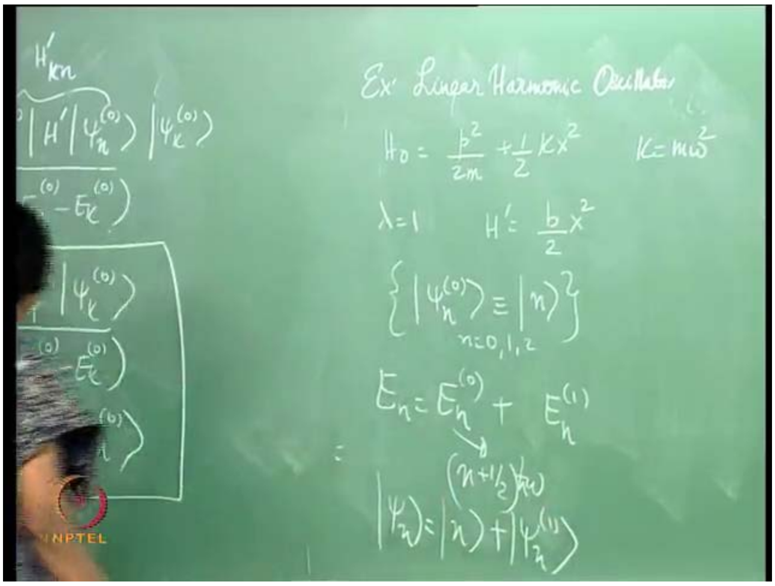

So, example, the linear Harmonic oscillator, it is the simplest example I can think of. So, you know that H naught is p squared by 2 m plus half K x squared. So, K is m omega square and I give you an H prime. Let me set lambda equals 1 then H prime is b by 0 X squared. So, since this is also quadratic in X, the Harmonic nature is going to be maintained, there is no Anharmonicity in this problem. So, this is H prime, lambda was set is equal to 1. So, I wish to find out the new wavefunction. I know the old wave functions. I know that the set psi n 0 in my notation is simply ket n, n taking value: 0 1 2 3 etcetera. These are the Eigenstates over the free Hamiltonian, over the unperturbed Hamiltonian.

So, given that, (Refer Slide Time: 35:45) let me first estimate E n 1, the 1st order contribution to the energy. Now to 1st order, the total energy will be E n 0 which I know is n plus half H cross omega plus lambda E n 1, but I have set lambda equals 1 in this problem. So, this is going to be the contribution to energy to 1st order and similarly, psi n to 1st order would be psi n 0 which is simply ket n in my usual notation plus the contribution psi n 1, which I will calculate using this. (Refer Slide Time: 35:45) All I need is the matrix element of the perturbing Hamiltonian, the K n-th matrix element of the perturbing Hamiltonian. The Hamiltonian is given to me. So, I can well do this calculation. So, this is a very simple illustrative example to show the power perturbation theory.

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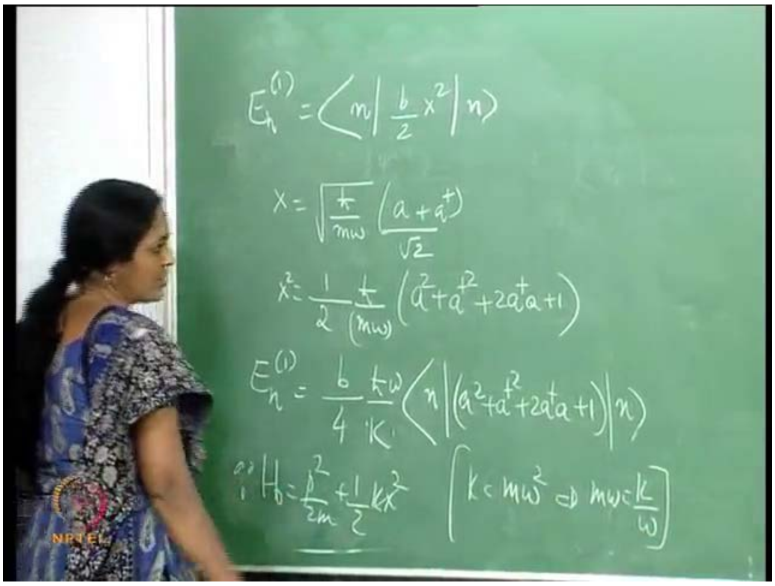

So, let us look at E n 1 in our example. This is simply n H prime n and since b is a constant I put that out and x squared has to be written and I will prefer to write it in terms of the ladder operators a and a dagger. You will recall that X is root of h cross by m omega a plus a dagger by root 2. Let me remind you that this is what provides the length scale in the problem. So, X squared was a half h cross by m omega a plus a dagger the whole squared and if you keep the ordering right, this is what it is. And therefore, E n 1 is simply b by 4 h cross by m omega n a squared plus a dagger squared plus 2 a dagger a plus 1 n.

Now, since I had written my H naught as p squared by 2 m plus half K x squared. So, let us see K is really m omega square and I seem to want m omega all the time. So, m omega is k by omega and therefore, I can write this as b by 4 h cross omega by K, because of this reason. So, let us see where this takes us. Look at this, a square is a lowering operator. So, when a square acts on n brings it down to 1st time it acts, a acts on n to take it down to ket n minus 1 and then the a again, that makes it ket n minus 2 and since they are orthogonal states, you do not get a contribution from this term which is the expectation value of a squared in the state n.

Similarly, from a dagger squared you do not get a contribution. However, you get a contribution from a dagger a because you would recall that ket n is an Eigenstate of a dagger a with Eigenvalue n.

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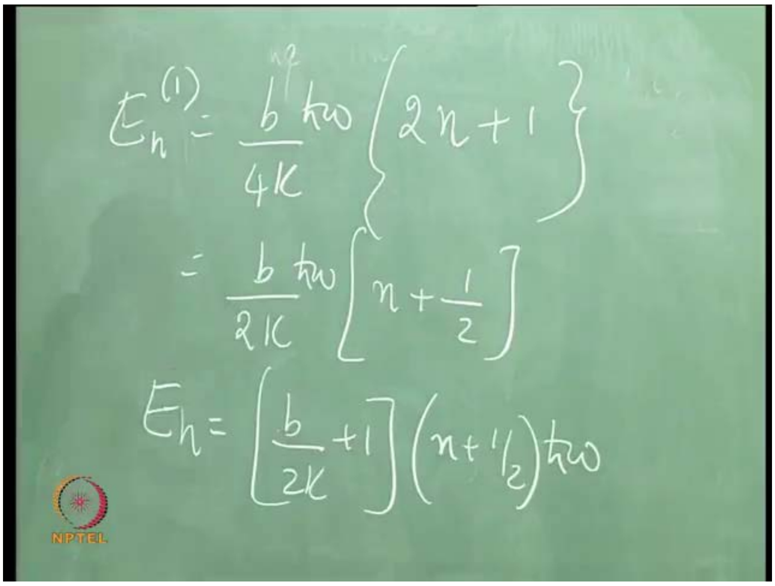

And therefore, you have E n 1 is equal to b by 4 K h cross omega, (Refer Slide Time: 43:56) twice n from here plus 1 from there. So, this can be written as b by 2 K h cross omega n plus half. So, this is what I have for E n 1 and therefore, E n to 1st order is e n 0 which is n plus half h cross omega plus E n 1. So, it is simply b by 2 K plus 1, n plus half h cross omega. So, this is what I have. So, this is my value of energy inclusive of 1st order corrections, because of the perturbation. So, now let us go ahead and estimate (Refer Slide Time: 35:45) psi n 1, this involves H prime K n psi k 0 and let us find out that matrix element.

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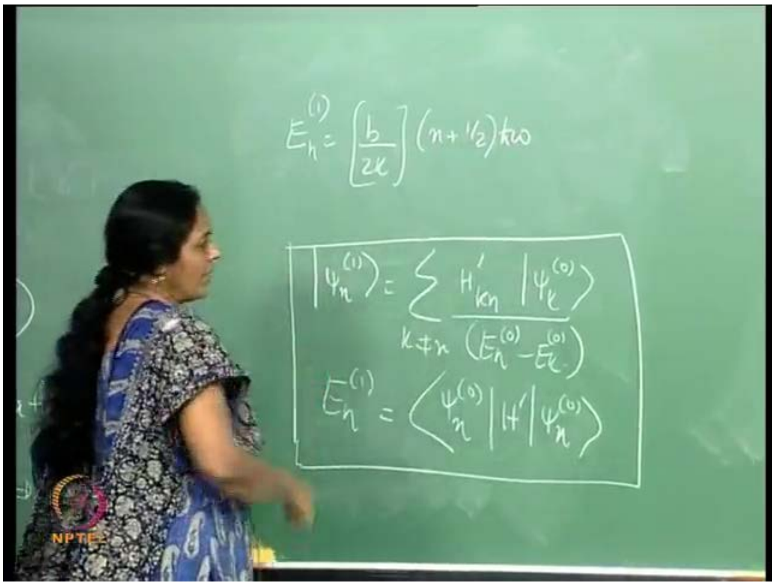

So, let me write this somewhere, for the Harmonic oscillator problem. I have found out E n so let us just say E n 1 is b by 2 k times n plus half h cross omega. This is what I have. Now, as far as psi n 1 is concerned, I need to do the following thing.

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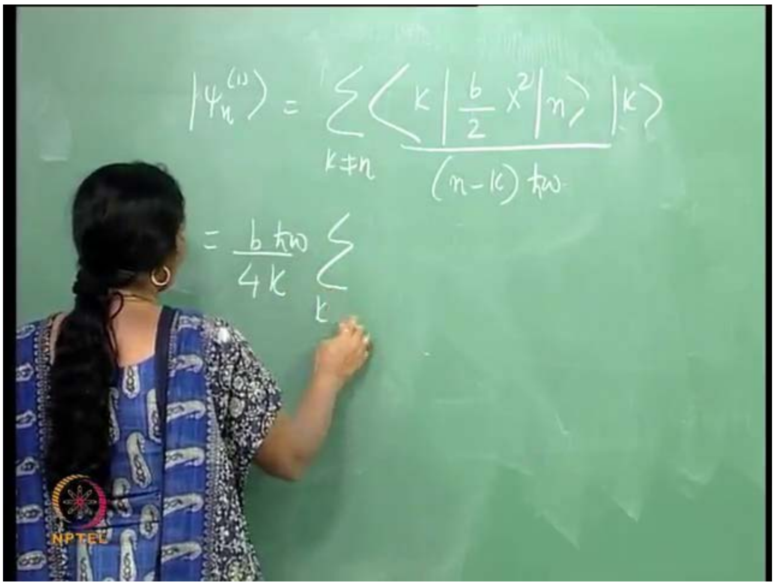

The wavefunction to 1st order in the perturbation is summation K not equal to n, I need the matrix element H prime K n, which is K b by 2 X squared n psi K 0, which is K. So, that is what I have there, (Refer Slide Time: 46:27) divided by E n 0 minus E k 0, recall that E n 0 is n plus half h cross omega and E k 0, is k plus half h cross omega. So, you just have an n minus k and k is not equal to n, h cross omega. So, this object can well be written once more as b by 4 k h cross omega, summation, this K is not be confused with that k.

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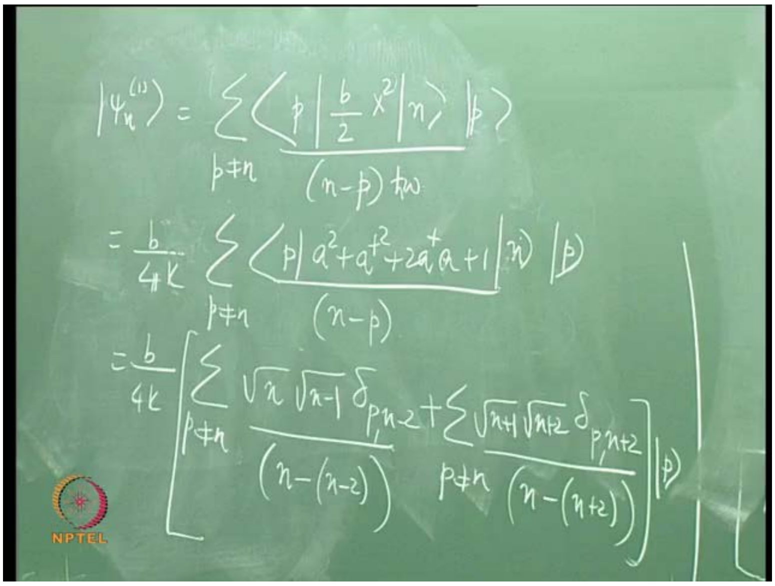

This is K, which was m omega squared. If you wish I am willing to write this, as p not equal to n in this fashion and therefore, we just have d by 4 K h cross omega. Summation over p not equal to n, this K is m omega squared. And then you have p a squared plus a dagger squared plus 2 a dagger a plus 1 n p by n minus p h cross omega. So, the h cross omega cancels out and I have a b by 4 K. Now, this is a nice thing to do because certain terms make a contribution; I have summation p not equal to n look at the 1st term. The 1st term brings out a root n ket n minus 1, root n minus 1 ket n minus 2, delta p n minus 2. The denominator is p is equal to n minus 2 that is the 1st term.

The 2nd term gives me summation p not equal to n. The 2nd term gives me a dagger on root n, gives me root n plus 1 ket n plus 1, once more gives me root n plus 2, ket n plus 2 So, that is a delta p n plus 2. The denominator is n minus n plus 2, that is the 2nd term. The 3rd term cannot make a contribution, because a dagger a on ket n gives me ket n and this gives me a delta n p, but you have to sum over all p not equal to n. Similarly, the 4th term does not make a contribution and of course, I have ket p. So, this is what I have. Now, this is easy to simplify and all I get is the following. I can remove this summation and the delta sign and therefore, the p is to be replaced by n minus 2.

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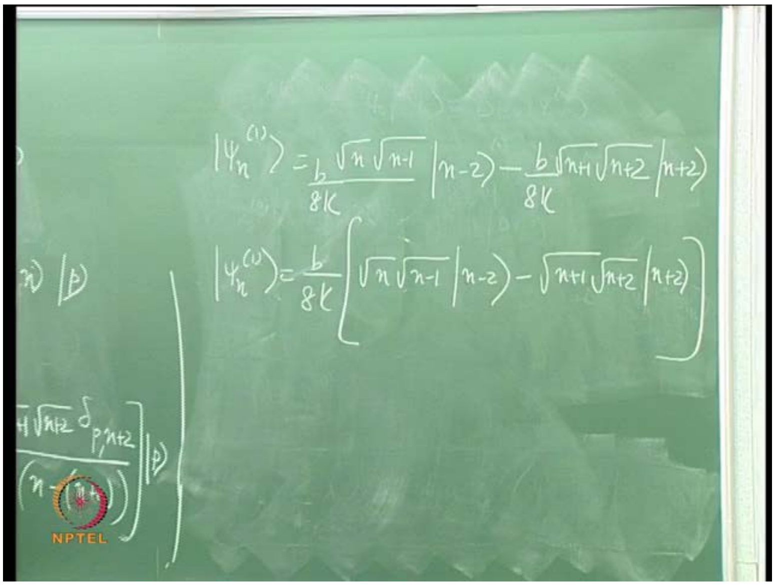

So, I have psi n 1. This is the 1st order contribution to the wavefunction, is root n root n minus 1, the denominator is a 2, (Refer Slide Time: 48:46) of course, I have a b by 4 k. So, I have a b by 8 K and since p is replaced by n minus 2, I have ket n minus 2. That is the 1st term coming from here, (Refer Slide Time: 47:53) the 2nd term has a p replaced by n plus 2 and therefore, that gives me a minus 2. So, let me put a relative negative sign, minus b by 8 K again and I have a root of n plus 1, root of n plus 2 ket n plus 2. So, this is psi n 1. So, I have found out the contribution to energy and the contribution to the wavefunction in 1st order perturbation theory, if I were looking at a Harmonic oscillator with the Harmonic perturbation, in the sense that the perturbation is also a quadratic perturbation.

So, you see the contribution to 1st order does not come from psi n 0, it does not come from ket n. It comes from ket n minus 2 and it comes from ket n plus 2, which is consistent with the fact that our delta psi n did not have a component along psi n 0. So, this is what I have in 1st order perturbation. (Refer Slide Time: 46:27) So, here is the estimate of the energy and here is the estimate of the changed wavefunction and I have just demonstrated 1st order perturbation theory to you. We will look at 2nd order perturbation theory and Harmonicity and so on in subsequent lectures.

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