Quantum Mechanics- I
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Lecture - 28
A Charged Particle in a Uniform Magnetic Field

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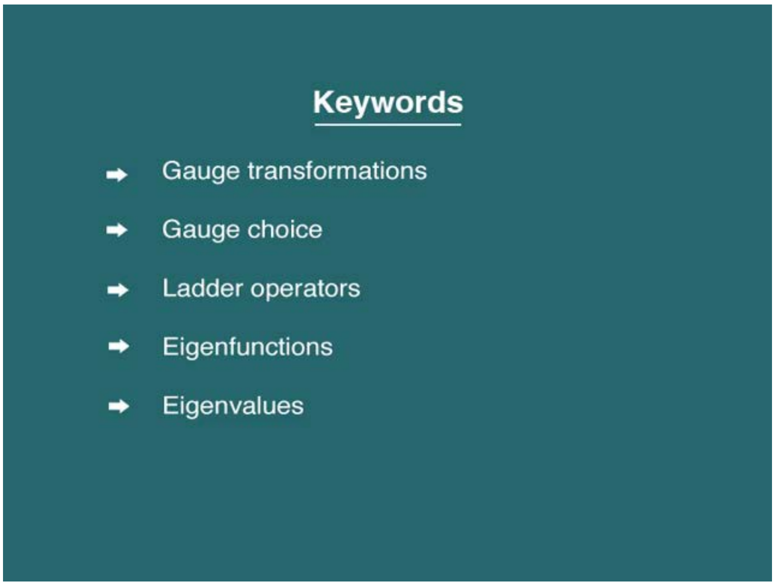

Yesterday we were talking about a charged particle, in a homogenous magnetic field and I said I will take the field along the z direction.

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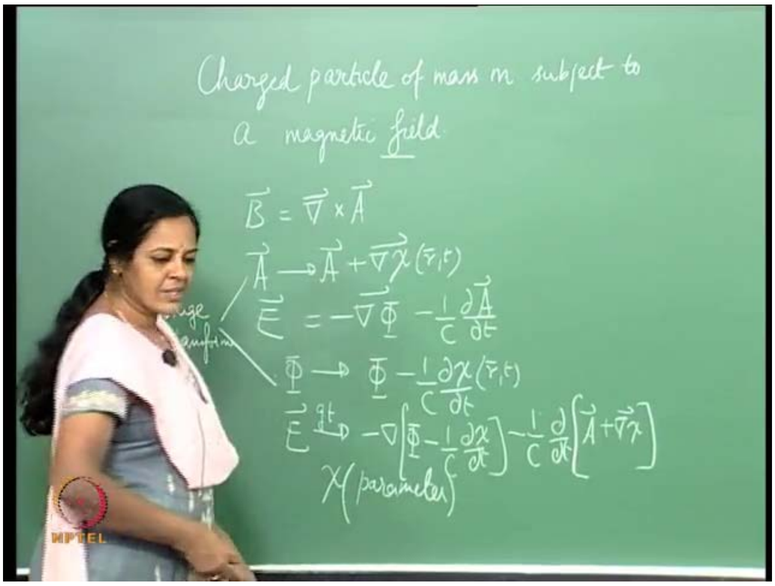

So, today we will continue with that problem, of a charged particle, of mass m, subject to a uniform magnetic field. And as I mentioned to you yesterday, the Eigen functions would involve the Eigen functions, of the oscillator type, of the linear harmonic oscillator type and also, a part of the Eigen function would correspond to that of a free particle and we will see how exactly that comes. So, first of all the magnetic field B itself, can be written in terms of the vector potential, as del cross A. A is the vector potential and I said that if I did a gauge transformation, where A changed to A plus grad chi, it simply does not change B.

So this is an example, of a gauge transformation. In fact the full set of transformations, are best understood, if we look at the electric field as well, because the electric field can be written as, minus gradient of the scalar potential phi minus 1 by C, delta A by delta t. And therefore, it is not enough if A alone made a transformation. In order to keep the electric field unchanged under the gauged transformation, phi changes correspondingly to phi prime, which is phi minus 1 by C, delta chi by delta t. The same chi, that appears there in the A equation. So, these two define the gauge transformation.

The gauge transformation is not a space time transformation, as you can see the transformation is made on the vector and the scalar potential, not on space time. So, it is an example of an internal symmetry transformation. Something that does not change space time itself, but only changes other quantities in this case A and phi. Now it is very clear, that if A changed in this manner and phi changed in that manner, E is left unchanged. Because, under the gauge transformation, E goes to minus gradient, of phi minus 1 by C, delta chi by delta t and minus 1 by C, delta by delta t.

A has changed to A plus grad chi, it is evident that here minus grad phi minus 1 by C, delta A by delta t, continues to B and this term, 1 by C gradient of delta chi by delta t, comes with a plus sign and here 1 by C, gradient of delta chi by delta t comes with a minus sign. Therefore, they cancel out and the electric field is left unchanged, under the gauge transformation. Even like the magnetic field is left unchanged under the gauge transformation. Chi is called the gauge parameter and it is clear, that any chi will do the trick. There are no particular conditions that I have used on chi except that it is a scalar and therefore, I have an infinite choice of chi's that I could use.

The only feature that I need to remember is this. That if a changes by gradient of a given specific chi, phi should change in this manner, using the same chi. Now under a gauge transformation therefore, the vector and scalar potentials change, leaving the electric and magnetic fields invariant. So, several choice of A's and phi's can be used.

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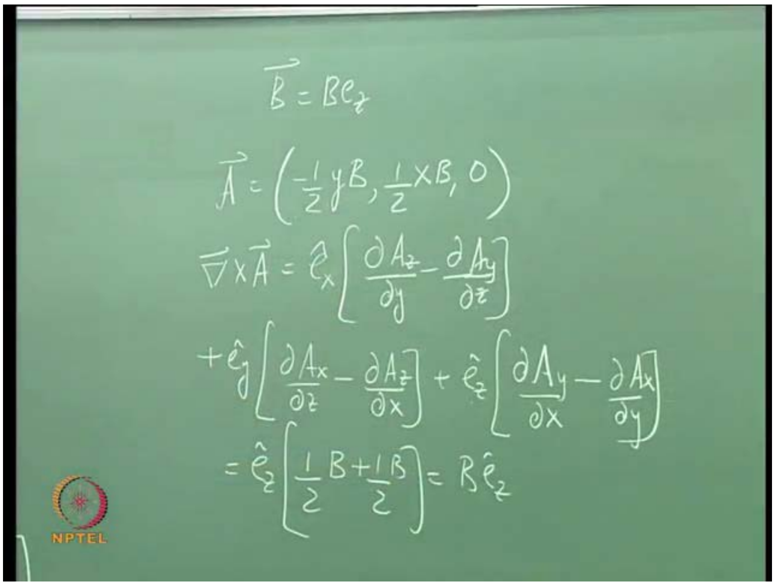

Let me look at one of them. So, A could well be minus half y B, I have selected B to B along the z axis. It is a constant here, not a function of space time. So, it is a homogeneous magnetic field and I have defined that direction to be the z axis. So, A could be this, now it is pretty clear, that if i look at dell cross A, that is simply going to be e x, in general of course, delta A z by delta y, minus delta A y by delta z, plus e y times delta A x by delta z, minus delta A z by delta x, plus e z times, delta A y by delta x, minus delta A x by delta y.

Now with this choice of the vector potential, since A z is 0, this does not contribute, A y is not a function of z. So, this does not contribute, so there is no component along the x axis. Similarly, look at this A x does not have z in it and therefore, this does not contribute and A z is 0. So, there is a no component along the y direction, however delta A y by delta x, so the only non zero term, non managing term, is along the z direction and I have a half B and then from here, I have another half B. So, that just gives me a b e z. So, this is a possible choice of A and in fact subsequently during this lecture, I will work with this choice of A. But, this is not the only choice.

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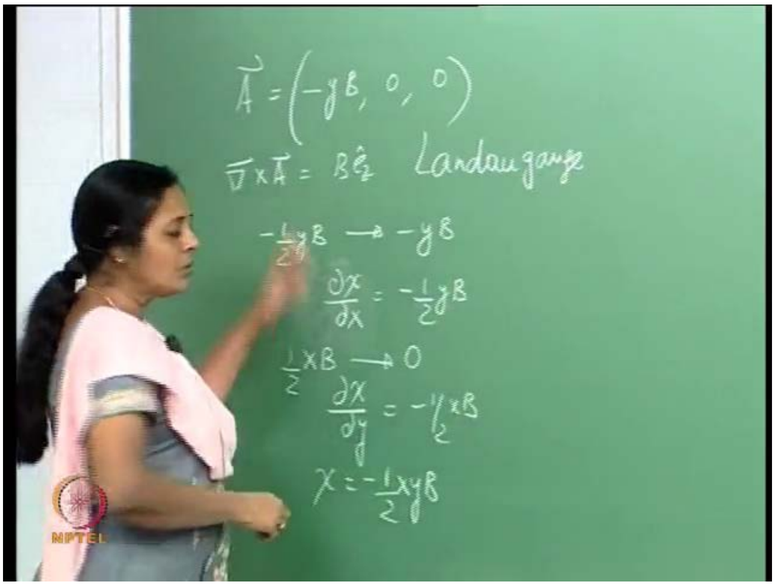

Clearly, I can have another set of components, for A. I could chose A, to be minus y B,0 0. Now, if I did this, it is clear that dell cross A, is again B e z, can be easily checked out. So it is possible to choose, this set or that set for A. But, then these 2 would differ by a gauge transformation. As you can see, originally A was minus half y B and that has gone to minus y B and therefore, grad chi, the x component would be delta chi by delta x, delta chi by delta x is minus half y B. Similarly, half x B, that was A y, went to 0. Therefore, delta chi by delta y is minus half x B and A z was 0 anyway.

So, it is possible, that I can choose chi, to be minus half x y B. That is a possibility for chi, because, surely delta chi by delta x, gives me that and delta chi by delta y gives me minus half x B. So, these 2 sets, that I have selected for A are related by a gauge transformation. It is also possible to choose A x to be 0, A y to be non zero and A z to be 0. Any one of these is a possible choice of gauge. So, when I say that, I have selected, I have chosen a gauge, it means that, I have already selected my components, A x, A y, A z and those are the components with which I am going to work.

The freedom that, I have is called gauge freedom, the freedom to choose any one of these infinite sets. Each one related to the other, by a suitable gauge transformation. In other words, by a specific choice of the parameter chi. So, in general one works with any choice of gauge, a popular gauge is a Landau gauge. Where 2 components are 0 and the 3rd component is non zero. So, that is a possible choice. But, as I have said, I would work with this choice (Refer Slide Time: 05:13) for A, in the latter half of my lecture.

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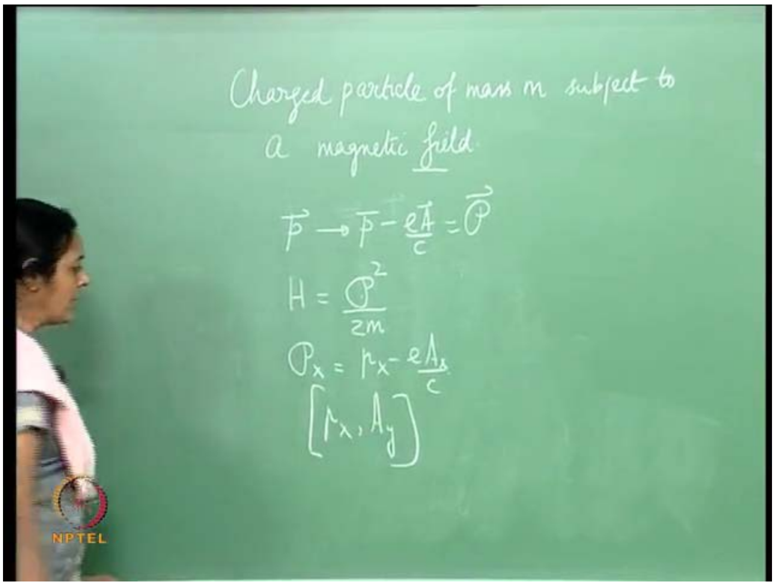

But now, let us get back with this introduction, to the charge particle in a Homogenous magnetic field and as I pointed out in the last lecture. There is a minimal coupling prescription; p goes to p minus e A by c. And this object I refer to it as script P, P. The Hamiltonian itself, therefore, was P squared by 2 m. Originally it was p squared by 2 m and that has now become script P square by 2 m. So, P x is p x minus e A x by c and so on. P y is P y minus e A y by c, P z is p z minus e A z by c. The point is the following: When I do a P squared; it obviously involves P x squared, P y squared and P z squared.

In terms of the linear momentum of the particle, it would involve the square of the linear momentum of the particle and also a square of the vector potential.

So, I have p squared and a squared in the Hamiltonian, when I expand it. There are no cubic terms, of course, there are cross terms. There would be a p dot A kind of term and an A dot p term and so on. I understand however that, I am dealing with the quadratic form I am dealing with terms, which only involve quadratics in p and A, or cross terms p dot A and so on and therefore, that rings a bell. Is it possible, to somehow choose variables, such that this problem gets mapped on to the linear Harmonic oscillator problem? Because, I know that I have dealt with the quadratic form of the Hamiltonian there and since, I know the energy Eigen values and the Eigen functions, for that problem.

Would it be possible to use that in understanding, what the energy spectrum of the charge particle in a magnetic field is? So that is the question: that we will address here. In order to do that, I first of all have to find appropriate canonically conjugate variables. You will recall that in the harmonic oscillator problem, we had the commutator, between x and p, p sub x if you wish. Because, it was a linear Harmonic oscillator, the commutator between x and p, was i h cross. So, my 1st job here is to identify two such objects, pertaining to this problem. Where, the commutator is i h cross.

Then even as I combined, x and p, I had x plus i p, barring some constants and x minus i p. Its Hermitian conjugate. Those were the ladder operators. So, here to if I can find such ladder operators, my job is done. So, my 1st problem here is to find out various commutators. So, in that spirit, I 1st wish to find the commutator, of p x with A y. Now that is an easy job. I work in the position representation because A x, A y etcetera are functions of x y and z.

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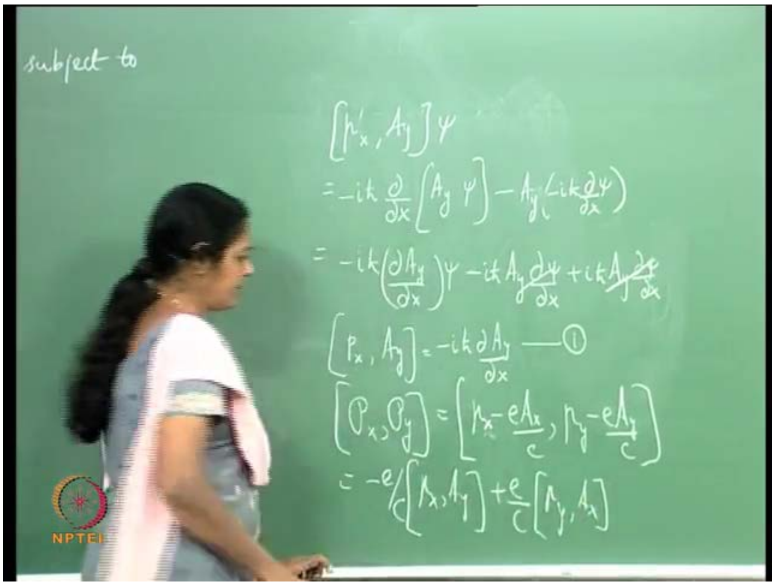

So p x A y, psi, is minus i h cross, delta by delta x, of A y psi. Right now, I do not have a choice for A y, I work with the general A. Minus of A y minus i h cross delta by delta x psi. So, this object is minus i h cross, delta A y by delta x, times psi, minus i h cross A y, delta psi by delta x, plus i h cross A y, delta psi by delta x. So, this is the same as, minus i h cross delta A y, by delta x psi and therefore, the commutator p x A y, is minus i h cross delta A y by delta x. Clearly this is a cyclic relation. In the sense that if I take p y commutator with A z, I will have minus i h cross delta A z by delta y. Similarly, p z commutator with A x and so on. So, this is the 1st thing that I have.

Now my aim since the Hamiltonian is in terms of P’s, the next step is to use this to find out the commutator, of P x with P y. Now, this would mean, the commutator of p x minus e A x by c, with p y minus e A y by c. Surely p x and p y commute with each other and I already know the commutator between p x and A y. Even as I know the commutator between p y and A x. So, this quantity is simply going to be minus e by c, commutator of p x with A y, plus e by c commutator of p y with A x. Because, a commutator of A x with p y, is minus the commutator of p y with A x.

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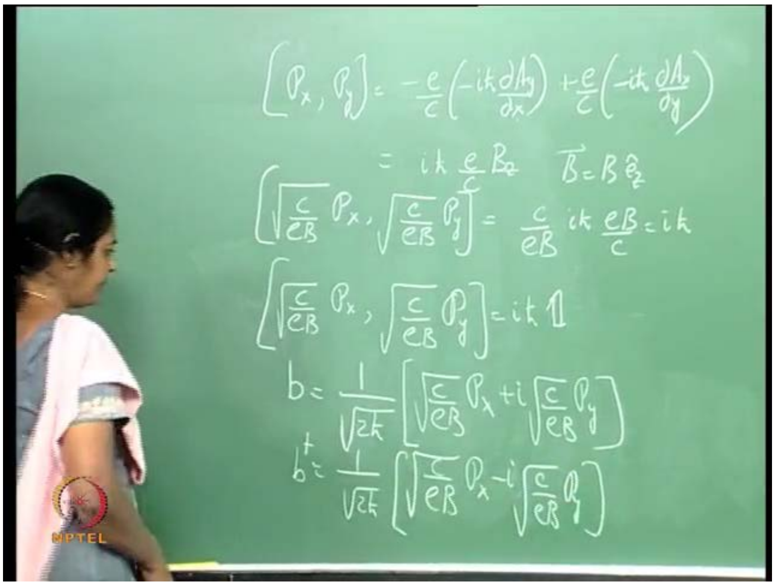

So, I can now substitute and find out what this is? I therefore, have commutator of p x with p y, is minus e by c, minus i h cross delta A y by delta x. That is the 1st term, plus e by c, minus i h cross, delta A x by delta y. So, that is the same as i h cross, e by c, I have delta A y by delta x minus delta A x by delta y and that is B z. As I said right now, I have not assumed it to be along the z direction, the magnetic field to be along the z direction. But, P x P y is i h cross e by c b z, which means that, it should be possible for me to scale things here.

I should be able to decide, I should be able to give appropriate values for scale P x and P y, in such a fashion. that I simply get, i h cross as my answer. And that is what would try to do now. Because, if I have root of c by e, let us now say that B is B e z. Therefore, I will just call this B, instead of B z. So, if I take root of c by e B P x commutative with root of c by e B P y. That is simply going to be c by e B, commutator of P x with P y, which I know is this object. There is an identity operator there but, by now we know that we do not have to put it in but it is there. And therefore, this is the commutation relation that I have, which is analogous to the commutator of x with p.

So, I have root of c by e B P x, root of c by e B P y is i h cross identity. So, my hunch was right, it should be possible now, to define ladder operators, appropriate ladder operators. I will call them b and b dagger, as linear combinations of P x and P y, which satisfy the algebra commutator b with b dagger is identity. So, I go about it this way, as in the case of the linear Harmonic oscillator. It define a b, which is 1 by root of 2 h cross. Root of c by e B P x, plus i root of c by e B P y. Therefore, b dagger is 1 by root of 2 h cross, the dagger of this the complex the Hermitian conjugate. Now, in the commutator b with b dagger therefore, it is clear what is going to happen.

So, the commutator of b with b dagger is simply identity. Because, this will pullout an overall factor, in the commutator of b with b dagger, I get an overall factor 1 by 2 h cross. There is a commutator, which is c by e B, a root of c by e B P x, with minus i root of c by e B p y and I have got that commutator here. That is an i h cross already with this minus i, it gives me a plus. Similarly, out here that gives me the same value i h cross with the plus and therefore, there is a 2, which cancels out, with the 2 that comes out as the overall coefficient.

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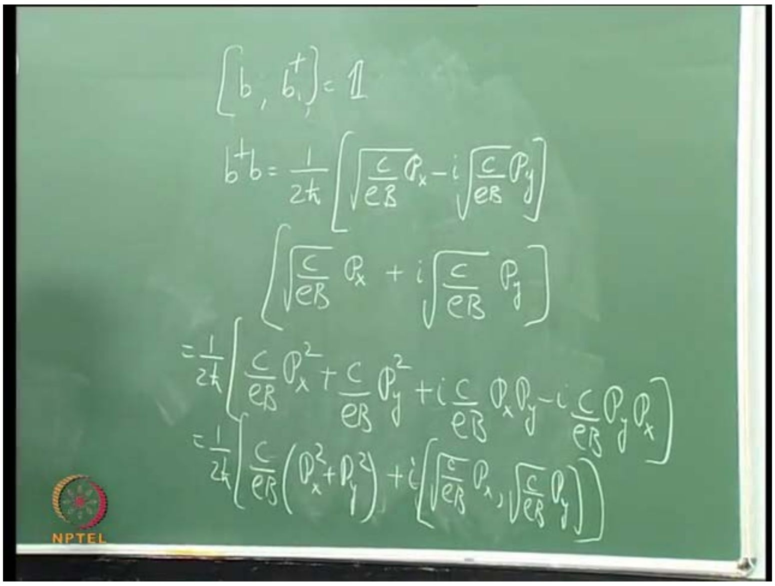

It is a trivial matter to check that b, b dagger is identity and that is good news for me. Because, now all I have to do is find out b dagger b. So, if I expand b dagger b, it is 1 by 2 h cross, root of c by e B P x, minus i root of c by e B P y. These are all P's, times root of c by e B P x, plus i root of c by e B P y. That is 1 by 2 h cross, the 1st term is just c by e B P x squared and this times that, gives me plus c by e B P y squared. Then I have cross terms.

The 1st thing is plus i c by e B P x P y and the next one is minus i c by e B P y P x. But, that is clearly a commutator. I can write this as 1 by 2 h cross, c by e B P x squared plus

P y squared, plus i commutator of root of c by e B P x, with root of c by e B P y. I already know what that commutator is we have got that here. (Refer Slide Time: 17:03) Root of c by e B P x commutator, with root of c by e B P y, is nearly i h cross. Therefore, I can substitute i h cross there. What do I get then? My final answer is quite simple. Let me write that here.

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So, I was trying to figure out, what b dagger b was and that is 1 by 2 h cross, P x squared plus P y squared. There is an overall coefficient c by e B, multiplying that and then I have plus i, times i h cross, which is minus h cross and therefore, c by 2 h cross e B P x squared plus P y squared. Suppose, I pull that out, minus e B h cross by c, equals b dagger b. I can do better, I can pull the 2 inside and, divide this by 2 and then I am through. Because, I now see, that b dagger b plus half, times e B by h cross C, is P x squared plus P y squared by 2.

What I want is P x squared plus P y squared by 2 m. And therefore, this is simply going to be, e h cross by m c, b dagger b plus half times e h cross by m c, e and h cross went up there. So e h cross by m c, is P x square plus P y square by 2 m and so it is clear that this is of the Harmonic oscillator form e h cross by m c there is a B of course. That is P x squared plus P y squared by 2 m. Apart of the Hamiltonian has therefore, been written in terms of something analogous to the Harmonic oscillator, Hamiltonian. But, remember that the total Hamiltonian was P x squared plus P y squared by 2 m, plus P z squared by 2 m.

But, P z is p z minus e A z by c but since A z is 0, P z squared by 2 m, is simply replaced by p z squared by 2 m. You should remember that this is an operator. It is not a number, this is a Hamiltonian, it is an operator p z square by 2 m. But, therefore, the Hamiltonian can be written in a very easy form, as b dagger b plus half, times e h cross by m c B, plus p z squared by 2 m and this is an easy thing for me to handle. Because, I certainly know the Eigen spectrum of this part, it is n plus half h cross omega. Where n takes values instead of h cross omega now, I will write e h cross by m c B.

This is n plus half e h cross by m c B that is going to be the Eigen value, for a given value of B, I have a set of discreet energy levels. Each labeled by the values of n, n taking values 0 1 2 3 etcetera. And this is precisely the Hamiltonian, for a free particle and I know the solution for the free particle Hamiltonian. I know the solution for the Eigen state of the free particle Hamiltonian. So which is what, I will now use.

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I have the following solution, the energy Eigen value E n, is n plus half, e h cross by m c B, n taking value 0 1 2 3 etcetera. Plus the Eigen value of the operator p z, (Refer Slide Time: 25:55) 1st of all this is a free particle Hamiltonian and therefore, I have the number p, which is the Eigen value of the operator P z. P z acting on psi z, is p psi z and that p is what figures here. Psi of z itself is a plane wave solution, e to the, i p z by h cross.

So, the linear momentum of the particle is p there is a plane wave solution; it behaves like a free particle, with the constant linear momentum. Along the z direction e to the, i p z by h cross, is the solution. And as far as psi x y is concerned, that is the Eigen function responding to this, discreet spectrum. I know that the solutions are the Harmonic oscillator solutions psi n. It is clear that psi 0, the lowest state will be an Eigen state of the operator b with the Eigen value 0, trivial Eigen value 0 psi 0. This is like the ground state of the Harmonic oscillator and therefore, I write b is psi 0 is equal to 0. And then, I create the various excited states by repeated application of b dagger.

So, I repeatedly apply b dagger n times to psi 0, that gives me psi n and for the sake of normalization, I divide by root of n factorial. So, this is what I have. So, these are the Eigen values and the Eigen functions can be given as the Eigen functions, are can be written out here. Because we will do some work with the Eigen functions now.

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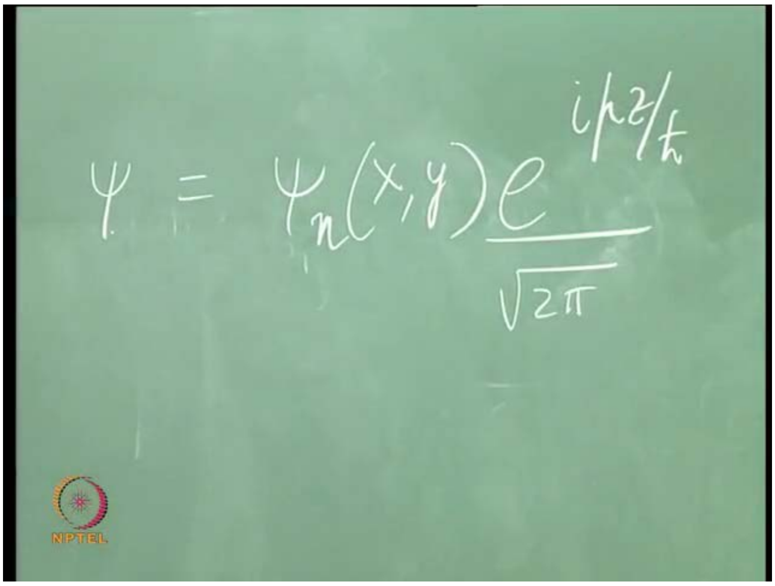

So, psi is psi of x y, which i call psi n of x y, because it is in the x y plane and along the z axis, I have e to the, i p z by h cross. It is usual to normalize that with a root 2 pi.

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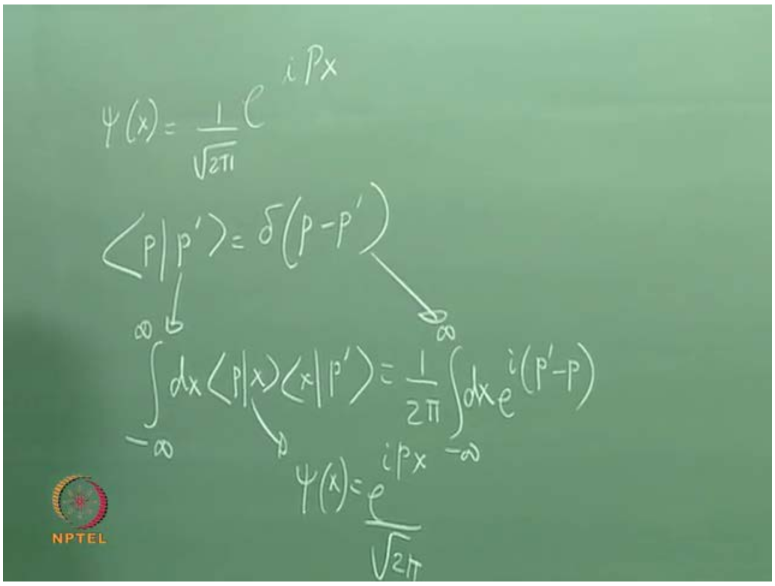

Now how come a plane wave solution, like say e to the, i p x, pick up a factor, 1 by root 2 pi, consider this. There is a simple way of understanding this, we know the following. This object is delta p minus p prime. But, of course, I can always write this suppose, I were working from minus infinity to infinity. I can put in a complete set of states, like this. Because, an integral over the allowed space, d x ket x bra x, is identity and that is what I have sandwiched here and I know that this is the wave function so this would have an e to the, i p prime x and this would have an e to the minus i p x. I have written it in the x representation.

But, I can use a form of the delta function. This can be written, in that following fashion and then when I compare the two of them it is clear, that I need to put in a 1 by 2 pi on this side. And therefore, if I have an e to the i p prime minus p x. There should be a 1 by 2 pi, it is off by 1 by 2 pi and so each of this, say psi of x, would be e to the i p x by root of 2 pi. So, that is how I get the root 2 pi there. So that is, one factor which could be put in but, quite apart from that there is the whole business, of normalizing e 2 the i p z by h cross. It is a plane wave solution.

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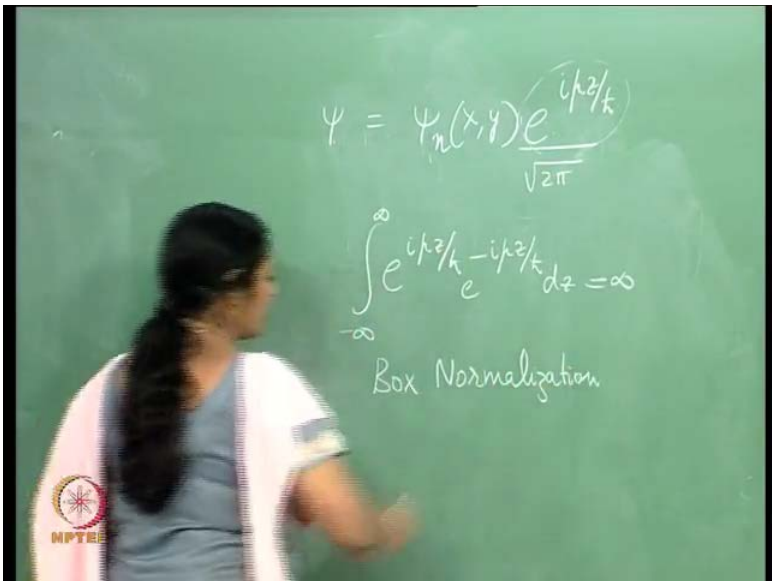

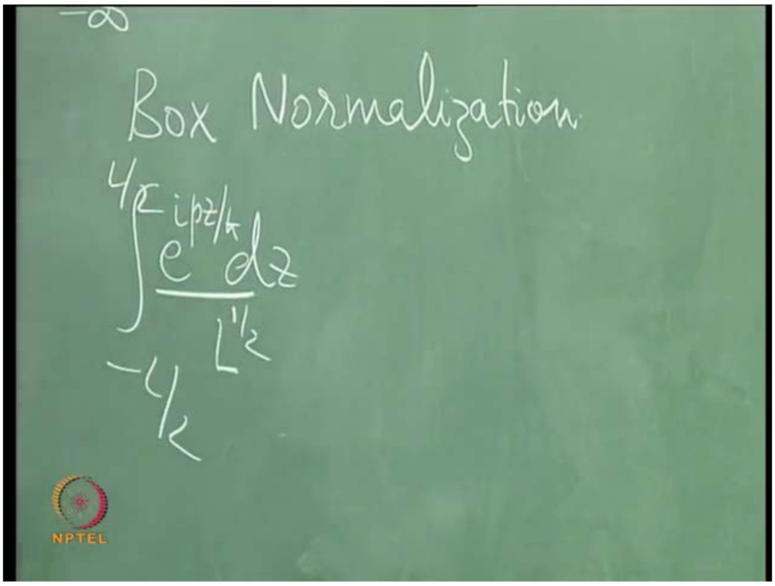

So, if you look at psi star psi, where psi e to the, i p z by h cross and z itself goes from minus infinity to infinity. You can see that this blows up. But, plane wave solutions are extremely common in quantum mechanics, the free particles is a simplest example we can think of. So, normally in order to normalize a plane wave solution of this form, e to the, i p z by h cross, you take recourse to box normalization. In one dimension for instance, you imagine that the particle is really moving freely within a box. Let us say size L. So, the box goes from minus L by 2 to L by 2. In that case, if you started with e to the i p z by h cross, by L to the power of half. It is clear that this normalizable, if this is the wave function.

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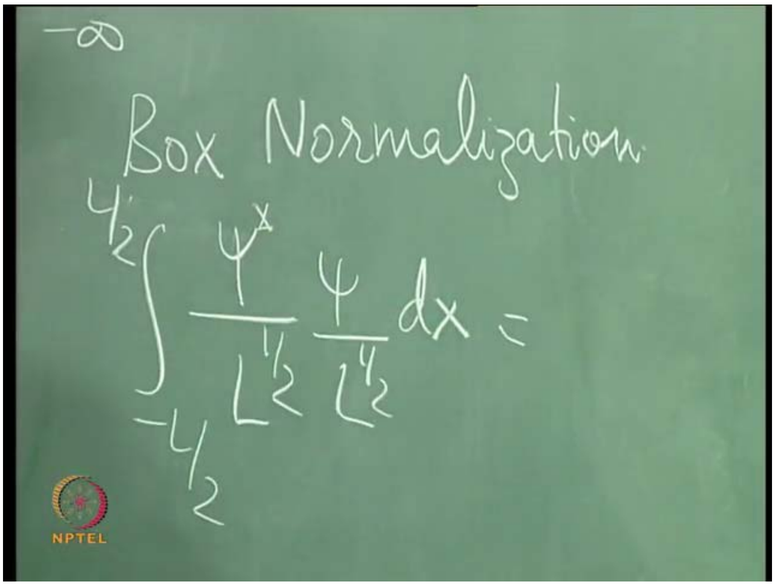

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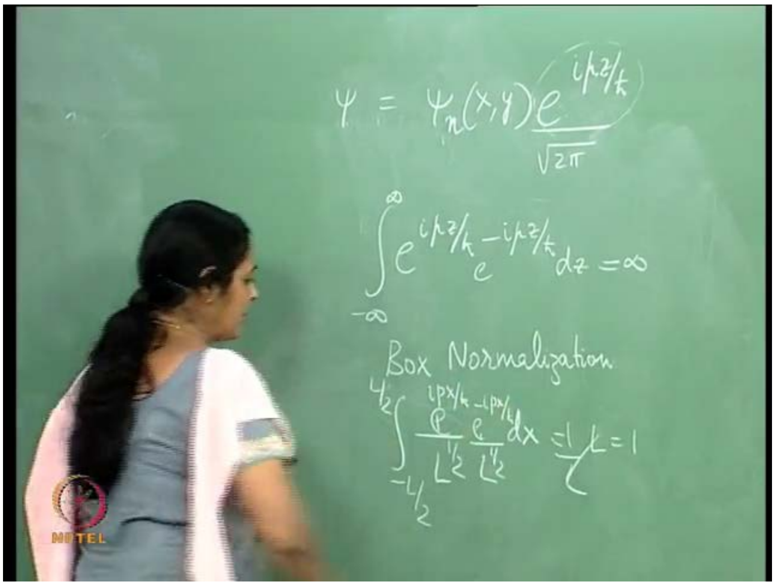

Start with some psi by L to the power of half, integral minus L by 2 to L by 2, psi star psi, d z or d x. Whichever be the direction, the axis on which the box is. Then you find that if this cancels out, if e to the i cancels with e to the minus i. You are left with the, 1 by L, because, that is a way you normalized it and d x, when integrated, gives me an L and that gives me a 1. So, really it is common in the normalization. In the box normalization to start for every dimension, you divide by root L and therefore, in 3 dimensions, the wave function is down by L to the 3 by 2.

So, L to the minus 3 by 2, times the wave function is what you could use, if you were doing an integration like this, in 3 dimensions. So, this box of course, is I have done it for 1 dimension. Otherwise x y and z go from minus L by 2 to L by 2 and in each case you pick out, an L to the minus half and therefore, the box normalization involves dividing by L to the half, dividing by L to the half, for each dimension that you look at. Then you do the problem of a free particle, inside that box. Finally, take the limit L going to infinity. I mean clearly the limit of a function is not the same as a value of the function at that point and that is precisely how the box normalization is done, in the case of a plane wave problem. But, that is as far as a normalization is concerned, the structure itself is this. This as you will recall this part (Refer Slide Time: 30:55) psi sub n of x, y would involve the Hermite polynomials.

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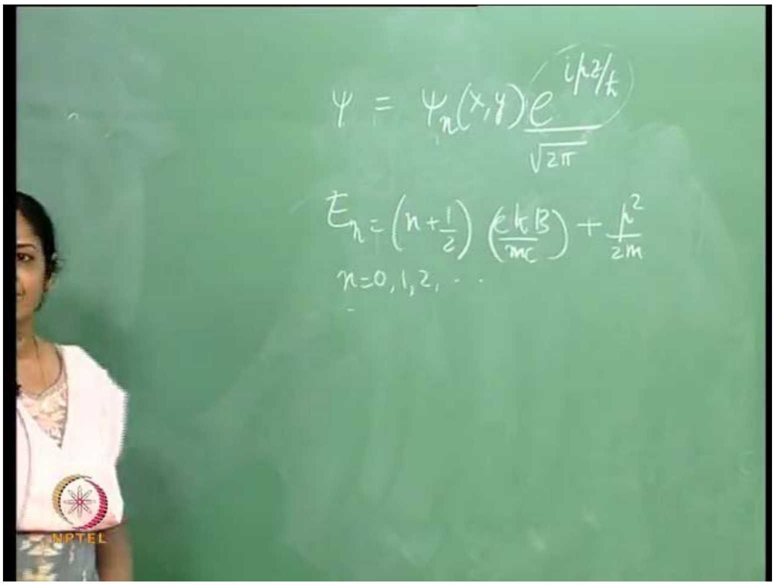

You will recall that the ground state of the Harmonic oscillator was a Gaussian function. So, I would expect a Gaussian for psi 0 of x y. I would expect Gaussian function of x and y, if n is 0. So, it is certainly worth looking at what happens to the ground state, psi 0 of x y. The energy Eigen values of course, are n plus half, e h cross B by m c, plus p squared by 2 m, n taking value 0 1 2 etcetera and those are the energy Eigen functions, I have to normalize it suitably. Now let us look at a specific choice of a. We have already said that B is along the z axis. In other words, I choose the direction of B to be the z axis and I wish to work with the choice.

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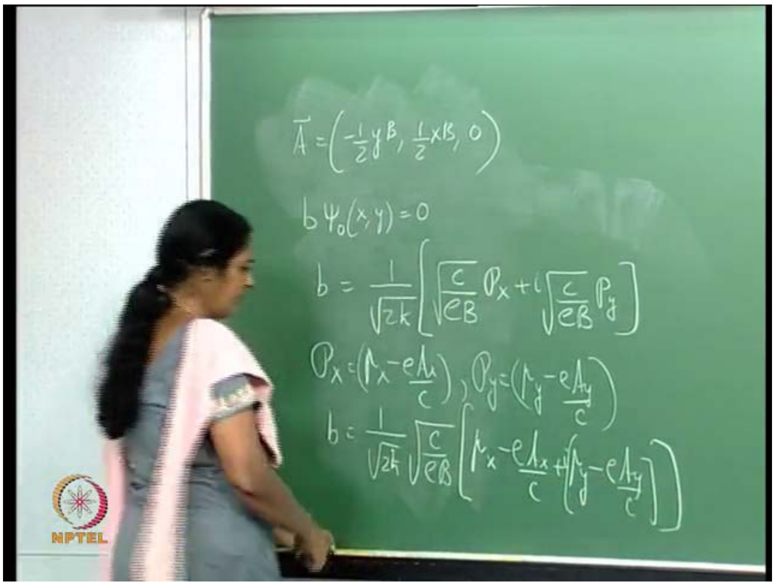

So, I am going to choose A, to be minus half y B, half x B 0. These are the components of A. Want to look at the ground state solution and look at the Gaussian form and get back the Gaussian form. So, I have b psi 0, of x y equals 0. Even to begin with, I realize the following: Since B is along the z axis, there should be a cylindrical symmetry in this problem. Even in the classical analogue, when we look at a charged particle moving in the presence of a Homogenous magnetic field, it is helical motion and therefore there is circular symmetry. There are planes, where there is a symmetry.

So, I would expect to see that kind of symmetry in my solution. I now have to write down b, which I already have, b was defined as 1 by root of 2 h cross. P x plus i p y and that object was root of C by e B P x plus i root of C by e B P y. So, this was my definition of b. Since this was the manner in which we had define b. Let us now recall, that P x is p x minus e A x by C and P y was p y minus e A y by C and substitute for a x and a y accordingly. And then you have b equals 1 by root of 2 h cross, root of C by e B, times p x minus e A x by C plus i times p y minus e A y by c.

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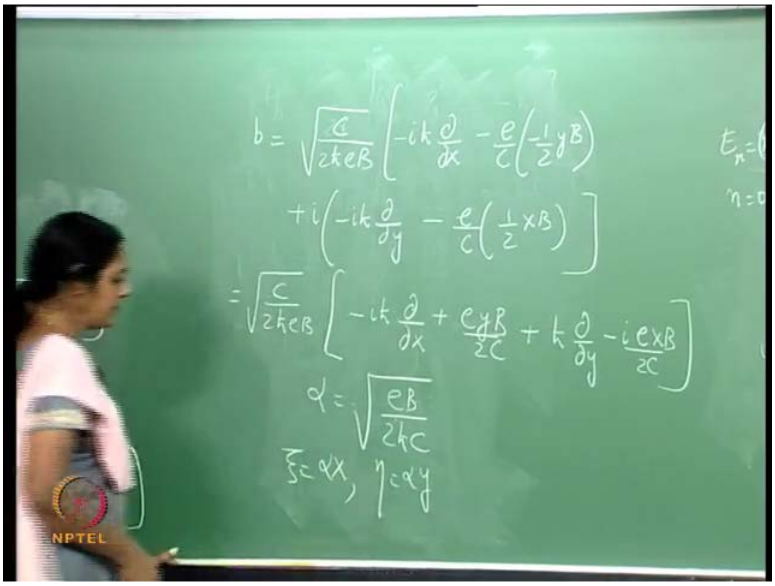

Substitute for p x, as minus i h cross delta by delta x. Similarly, for p y and use the values of A x and A y that we have selected. In other words we are using a specific gauge now and therefore, b would simply be root of C, by 2 h cross e B, times minus i h cross delta by delta x, minus e by C, a x is minus half y b. Plus i times, minus i h cross delta by delta y for p y, minus e by C, half x B for a y. So, this is what we have. This quantity, is simply root of C by 2 h cross e B, times minus i h cross delta by delta x, plus e y B by 2 c, plus h cross delta by delta y, minus i e x B by 2 c.So, this is what we have, when we expand this.

It is good to change variables and also absorb constants suitably. I define a constant lambda, a alpha, if you wish, which is root of, e B by 2 h cross c. Now if you did that, you define a variable zeta, which is alpha x and eta which is alpha y. So, that I can get rid of unnecessary constants, so you see the 1st term for instance. The x would be replaced by zeta by alpha and I will have delta by delta zeta instead of, delta by delta x and delta by delta eta, instead of delta by delta y. With this replacement, it is a simple matter, to rewrite this equation, in terms of zeta and eta.

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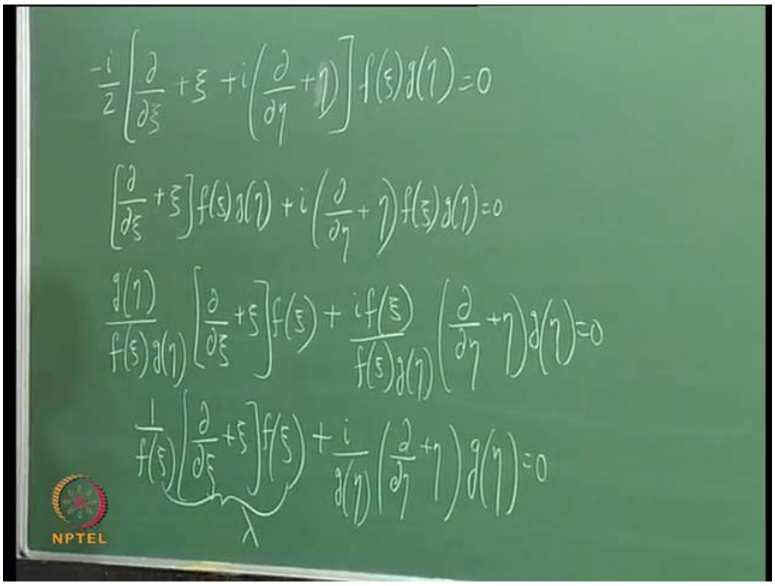

So, in terms of the variables zeta and eta, what do I get for the differential equation? The equation is simply that, b acting on the ground state is 0. It is the annihilation operator, on the ground state. So, I have, apart from a minus i by 2, delta by delta zeta plus zeta. If I recast the equation in terms zeta and eta, plus i times delta by delta eta, plus eta. This acts on the wave function which is a function of zeta and eta now and I would like to do that, in terms of a separation of variables. I will write the wave function, of as f of zeta g of eta and then this operator, acting on that wave function is 0.

So, I basically have, delta by delta zeta plus zeta, f of zeta g of eta, plus i delta by delta eta plus, that is eta not zeta, f of zeta, g of eta equals 0. As we always do in separation of variables, let me divide by f of zeta g of eta. So, basically, I have g of eta by f of zeta g of eta, delta by delta zeta, plus zeta acting on f of zeta. I have pulled the g of eta outside, because this is only an operator function of zeta and cannot act on g of eta. And similarly here, I can write this in the following manner, like I always do, when I separate variables.

So, that just tells me that 1 by f of zeta, delta by delta zeta plus zeta, f of zeta plus i by g of eta, delta by delta eta plus eta, acting on g of eta equals 0. So, this is nearly a function of zeta and this is going to give me a function of eta and if this equation has to be true. This should be equal to some separation constant, I will call that lambda. So, this is lambda and this is minus lambda and therefore, they cancel out.

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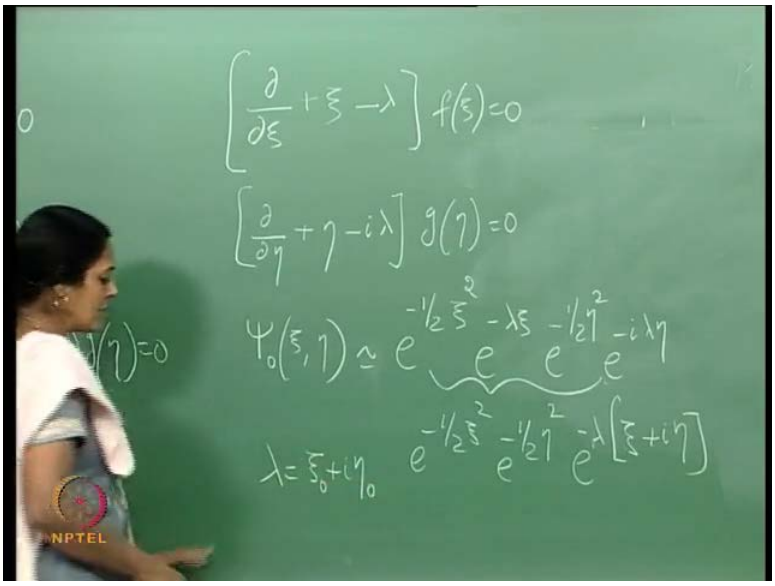

Which therefore, means in terms of the separated equations, delta by delta zeta plus zeta minus lambda, acting on f of zeta equals 0, that is my 1st equation. And similarly, delta by delta eta plus eta minus i lambda there is an i there and I have to take care of that, acting on g of eta equals 0. What is the solution? The solution is simple. Now, I have psi 0 of zeta eta and I have written the 0 there to show that, we are discussing the ground state wave function. These are Gaussians, so the solution is e to the minus half zeta squared from there, but that is not all. There is a lambda and that is going to give me an e to the minus lambda zeta, e to the minus half eta squared, e to the minus i lambda eta, that is the solution.

Apart from constants, over all constants normalization let us not worry about that, you have this essentially you have this. So, basically I am worrying about overall multiplicative constants and so on. But, this is what I have and this can be written as e to the minus half zeta squared, e to the minus half eta squared, e to the minus lambda zeta plus i eta. I will comment about this a little later but, I can also write this if I complete squares and so on, suppose I write lambda which in general can be complex. This is a well behaved function even if lambda is complex. So, if I write this as some zeta naught plus i eta naught, write it in terms of its real and imaginary parts.

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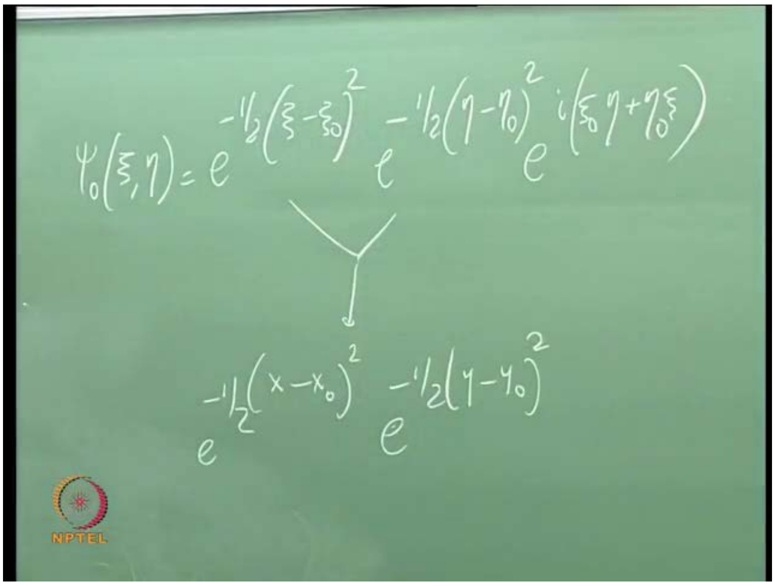

Then this psi, can be written also as, e to the minus half zeta minus zeta naught the whole squared. It is a shifted Gaussian, e to the minus half eta minus eta naught the whole squared and e to the i, zeta naught eta plus eta naught zeta. I leave it to you as a simple exercise, to put it in this form. So, I could think of it the solution could be written this way, or that way and again I have not worried about normalization overall constants and so on and so forth. So, this is what I have. Now a variety if comments can be made, on the solution first of all since, this is what comes out of the x y plane, because, you will recall that zeta and eta were functions of x and y. It would essentially involve x minus x naught the whole squared and e to the minus half y minus y naught the whole squared.

Of course as far as z is concerned, there is an e to the i p z by root 2 pi, that stays. So, the full solution has apart from all these things, it also has an e to the i p z by root 2 pi, which we have discussed earlier. So, now if you look at the x y plane, the solution only depends on the distance, x minus x naught the whole squared, that distance, from x naught comma y naught. So, there is a circular symmetry that you see in this problem, one would expect it because, I know for instance that, once I map it on to the Harmonic oscillator, as far as a ground state is concerned, it is a Gaussian.

This turns out to be shifted by x naught and y naught and the x naught and y naught came because, the separation constant was written in terms of zeta naught and eta naught. (Refer Slide Time: 46:20) which were related to x naught and y naught and then, you see the circular symmetry in the x y plane, which is a sort of vestige if you wish or very consistent if you wish, with the classical situation, where in the presence of a magnetic field you have circular trajectories. So, that is one thing, an interesting observation is the following that if you look at this term here. (Refer Slide Time: 46:20)

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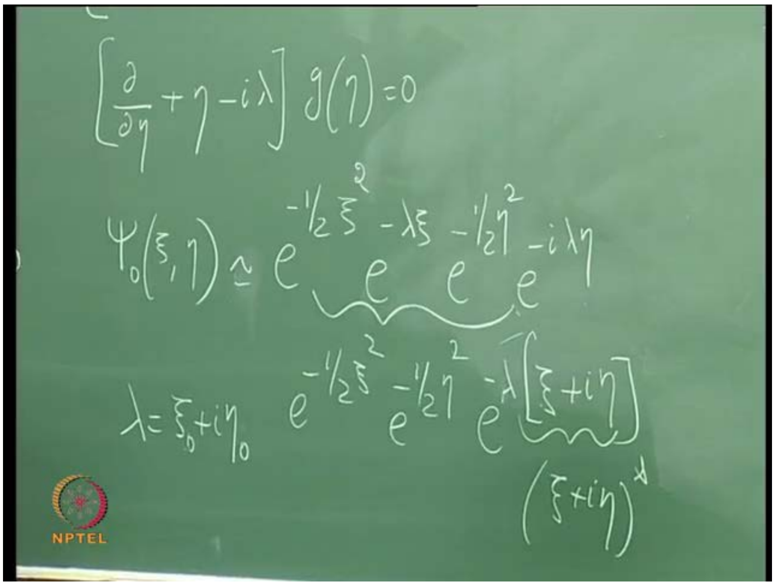

This is an exponential series and so I can expanded it in powers of this, exponential series can be expanded in the powers of zeta plus i eta to the power of s. There are an infinite number of terms and they will involve zeta plus i eta to the power of s. Where s takes values 0 1 2 3 4 and so on. And those are all linearly independent objects. I could choose any 1 of them, in the solution. And therefore, I have an infinite degeneracy, even in the ground state. So, if I take a circular section, in the x y plane, there is an infinite degeneracy. Because, for any value of S 0 or 1 or 2, I have one solution and all those solutions are linearly independent. The series is expanded in terms of this, these are called the Landau levels.

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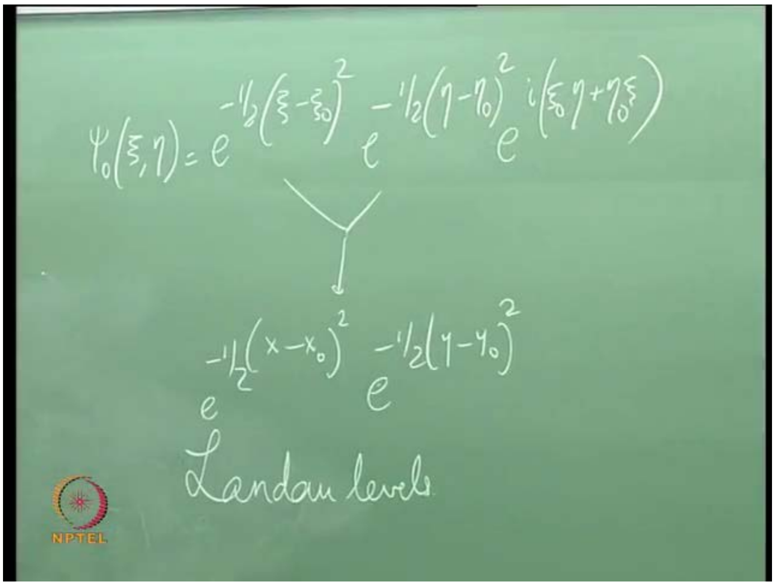

That means, that while it looks like the Harmonic oscillator solution in the x y plane, even the ground state is infinitely degenerate and the levels are called the Landau level. To summarize, we have very many interesting features, in this problem of the charged particle, in the presence of a Homogenous magnetic field. There is a discrete spectrum, the Eigen value is n, n taking values 0 1 2 3, apart from the plus half and the constants, that are analogous to h cross omega in this problem.

But, then there is also a continuum in the z axis, because p takes the momentum takes a constant value and any value at that. It is a plane wave solution, (Refer Slide Time: 52:02) infinite degeneracy even in the ground state, a circular symmetry, which is reminiscent of what one sees, in the presence of a magnetic field, in a classical context. And these are the take home lessons, because the spectrum is continuous along the z axis and discrete in the x y plane.