Quantum Mechanics - I
Prof. Dr. S. Lakshmi Bala
Department of Physics
Indian Institute of Technology, Madras

Lecture - 20
Infinite Dimensional Linear Vector Spaces

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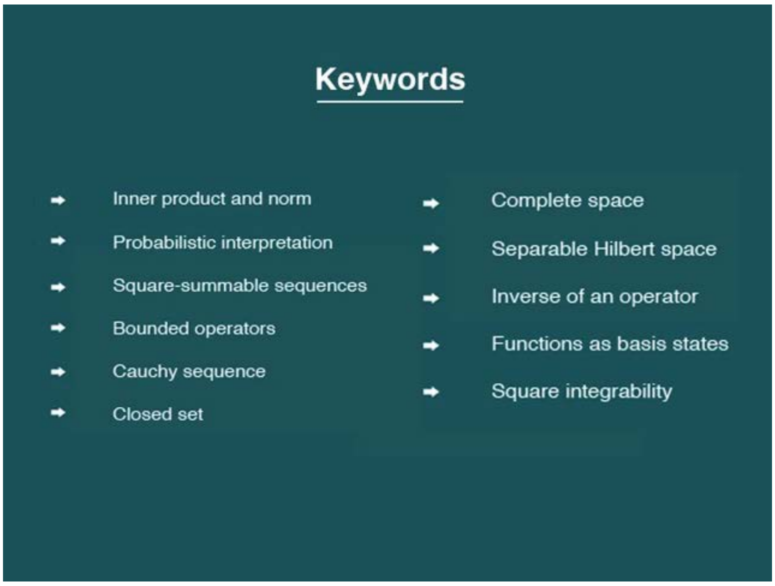

All my lectures till now, have dealt with finite dimensional linear vector spaces. We would introduce certain very important concepts for instance, the inner product structure, the probabilistic interpretation of quantum mechanics, linear operators and so on in a linear vector space. Now, many physical systems have infinite dimensional linear vector spaces associated with them. We need to carry over these concepts to infinite dimensional linear vector spaces. In the process certain concepts would undergo small modifications, but the probabilistic interpretation of quantum mechanics will continue to stay.

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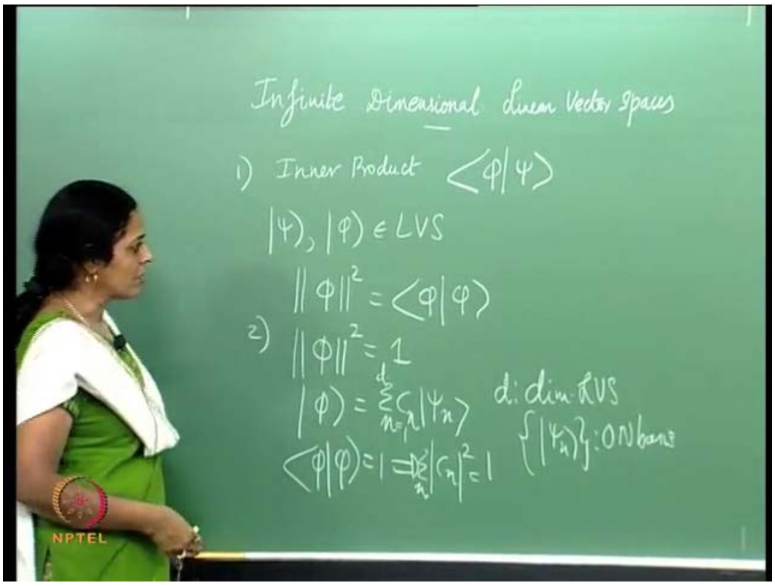

So, it would be good to now introduce crucial concepts associated with the structure of infinite dimensional linear vector spaces starting from what we already know about finite dimensional spaces. These are the ingredients of a finite dimensional linear vector space: the inner product structure denoted in the Dirac notation by bra phi ket psi, where phi and psi belong to the linear vector space. This helps us define the norm of phi, the square of the norm and that gives us a concept of length and distances between vectors if it comes to that. So, we would like to retain this property. We would like to have an inner product structure and be able to define lengths of vectors, also carry over ideas of the triangle inequality and the Cauchy Schwarz inequality to infinite dimensional linear vector spaces.

Now, this helps us normalize the object, the length of phi and in fact we would like this to be 1. This means the following that if I expand ket phi in terms of an orthonormal basis set say psi n as summation over n C n psi n, n takes values 1 to d where d is the dimension of the linear vector space and we are now looking at finite dimensional linear vector spaces. Then because psi phi is equal to 1 this implies that summation over n going all the way from 1 to d mod C n square is 1. This is a very important point. C n's could in general be complex, but they happen to be the expansion coefficients when phi is written in terms of an O N basis set psi n.

Clearly, I could have expanded phi in terms of another orthonormal basis in the same linear vector space, finite dimensional linear vector space. Then the coefficients would be different and there is a way of going from one basis to another basis through a unitary transformation which we demonstrated earlier.

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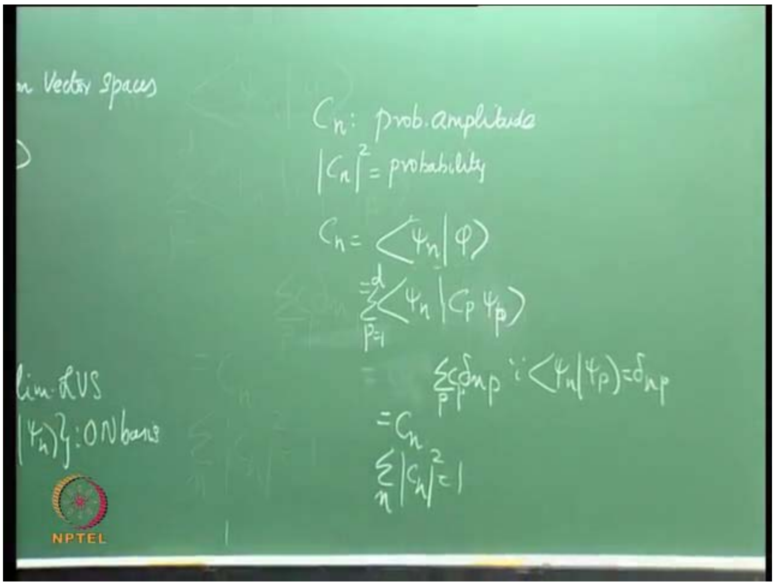

This helps us give the probabilistic interpretation of quantum mechanics because I would now interpret C n as the probability amplitude and mod C n square for a given n as the probability of what? You see C n itself is this object, because that is the same as psi n summation over, let us say P equals 1 to d, C p psi p and that gives me a delta n p. So, I basically have the summation goes and I have a C n because this gives me summation P delta n p because psi n psi p is delta n p and C p was a number which I pulled out and because of this C p delta n p and that became C n. (Refer Slide Time: 01:00)

Therefore, if I have expanded the state phi in terms of the basis set psi n, I extract the coefficient C n corresponding to the basis vector psi n by precisely this method. And that such a C n is the probability amplitude and mod C n squared is the probability of this overlap psi n with phi. The total probability is 1, and that tells me that summation over n mod C n square is equal to 1. This probabilistic interpretation is very fundamental to quantum mechanics and we would like to carry this into the structure of linear vector spaces even in finite dimensional, even in infinite dimensional linear vector spaces.

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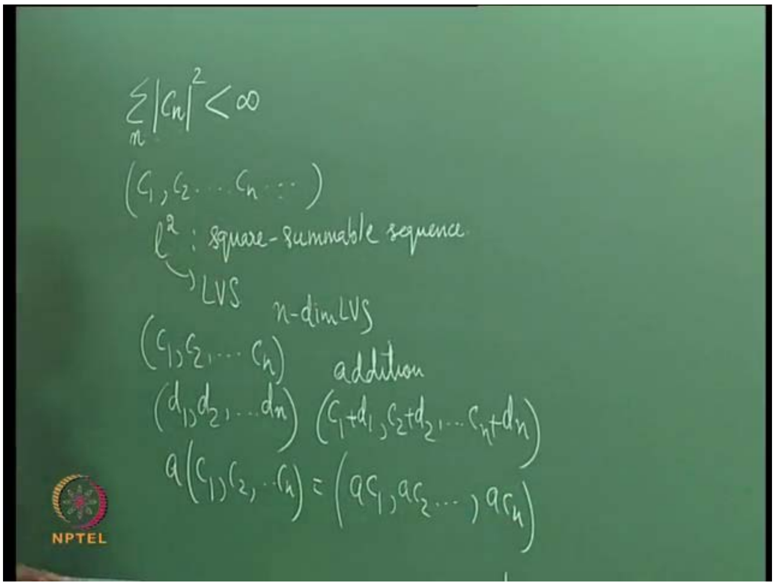

One thing is clear that, because of that we have a situation where summation over n mod C n square is less than infinity. So, here is a sequence of numbers: C 1, C 2, C n and so on. This is a finite sequence because I am dealing with a finite dimensional linear vector space and therefore, there is only a finite basis set. But this sequence satisfies this or it is a square summable sequence. Now, such a sequence belongs to a linear vector space itself. Such sequences: C 1, C 2, C n which satisfy this belong to the space l 2 of square summable sequences. l 2 is a linear vector space. The set of all such sequences are states in a linear vector space l 2 which is the space of square summable sequences and as in every linear vector space I need to define the concept of addition and scalar multiplication. That is defined point wise.

Suppose, I have a sequence: C 1, C 2, C n and suppose I were considering only a finite dimensional, n dimensional linear vector space so that I stop there and I have another sequence: d 1, d 2 to d n. Addition gives me c 1 plus d 1, c 2 plus d 2 and so on. So, it is point wise addition. Similarly, scalar multiplication is defined as a C 1, C 2, C n where a is a scalar as a C 1, a C 2, a C n. So, with addition and scalar multiplication defined in this manner we can check that the set of all such sequences, square summable sequences form a ((Refer Time: 08:49)) form states in a linear vector space.

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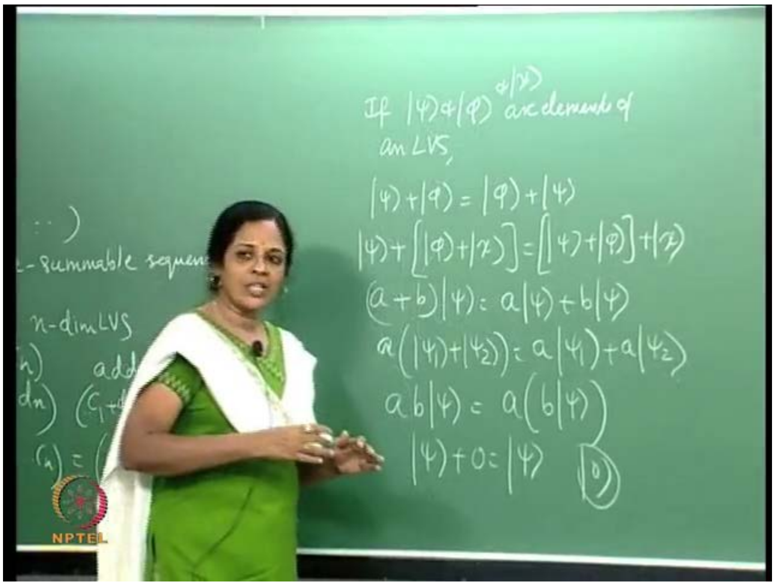

They belong to a linear vector space and to recall the properties of a linear vector space, if phi and phi are elements of the linear vector space. First of all, there is commutativity under addition which indeed is true here. (Refer Slide Time: 06:40) Because I could have written: addition, scalar addition, point wise addition, this sequence with that sequence giving me c 1 plus d 1, c 2 plus d 2 equivalently d 1 plus c 1, d 2 plus c 2, d n plus c n and so on. Then there is associativity and chi now is simply psi plus phi plus chi which is evident from the manner in which (Refer Slide Time: 06:40) I have defined addition for these sequences. Then a plus b psi, where a and b are scalars, is a psi plus b psi which follows from this. (Refer Slide Time: 06:40) And then, I could have a times psi 1 plus psi 2, that is a psi 1 plus a psi 2. Then of course, a b psi is a multiplying b psi.

I introduce the null vector. The null's a vector, the analogue of the null vector in this case (Refer Slide Time: 06:40) would be all elements here, all entries being 0. I emphasize that I do not want to call this 0, because this would usually represent the ground state of a system and that is not the null vector.

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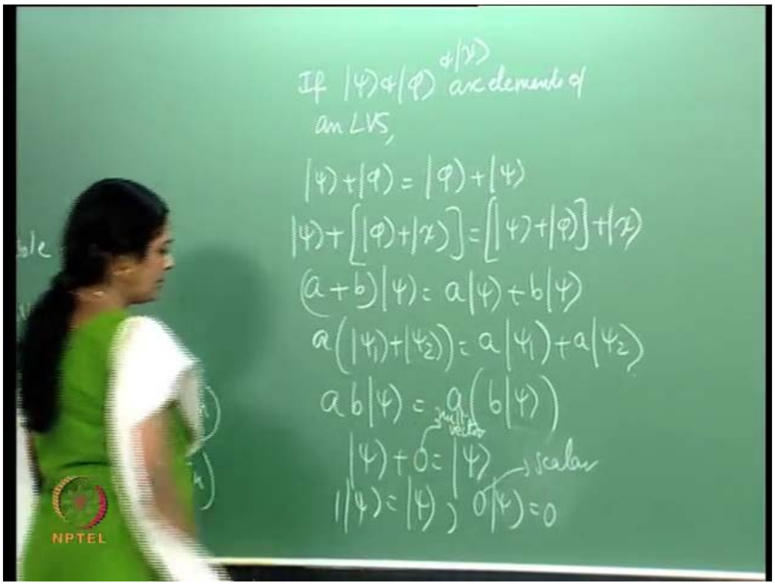

Then scalar multiplication 1 psi is psi and 0 psi. This is a scalar, is 0 and this is a null vector. So, you can see that all these properties are satisfied by (Refer Slide Time: 06:40) subsequences where addition and multiplication are defined, scalar multiplication are defined in the following manner.

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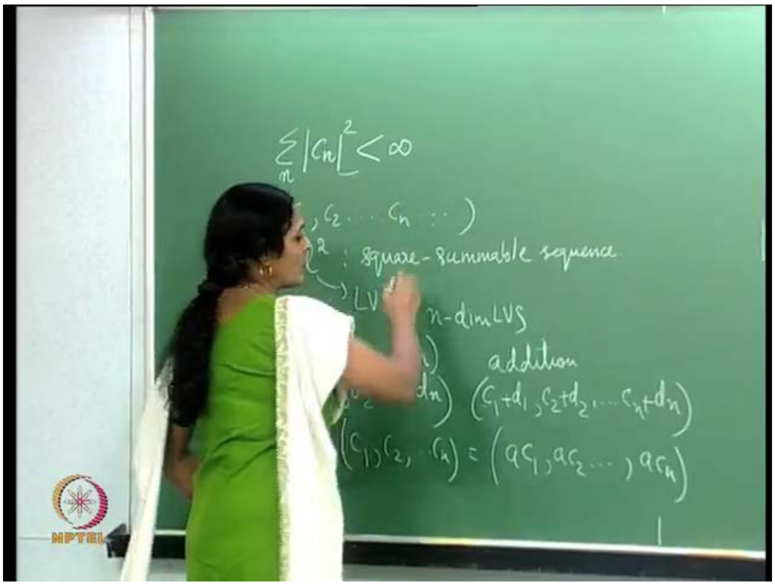

This is a space of square summable sequences. It is evident that if I need the probabilistic interpretation of quantum mechanics, that is I need to say that summation over n mod C n square equals 1 where C n's are the expansion coefficient of state of the system in terms of the basis states, a set of basis states. Then, when I go to infinite dimensional spaces, I need to impose this restriction, because there is no need to believe that an infinite sequence of numbers or vectors will converge to a point that there is a limit to such a sequence is not at all evident and we need to impose that as a condition.

I will talk more about this convergence property and what are known as Cauchy sequences a little later.

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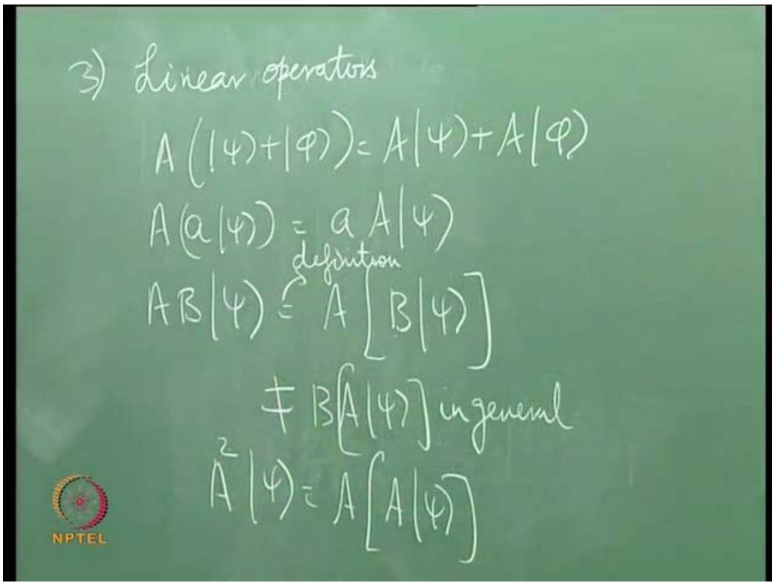

(Refer Slide Time: 01:00) But right now, apart from this probabilistic interpretation we had some more structure in the finite dimensional vector space, linear vector space. One of them being this, that operators were linear operators. So, if I have an operator A and it acts on the state psi plus phi, it acts linearly. Further, an operator acting on this state where a is a scalar is a times A psi and if I have two operators A and B acting on psi, that is the same as first acting with B on psi and then with A on psi. This is not equal to B A psi in general, because the operators A and B the examples that you have seen were represented by matrices and two matrices need not commute with each other.

So, these are the properties of a linear operator and this is a definition, I define it this way. All operators that we will deal with will be linear operators in quantum mechanics even in infinite dimensional linear vector spaces and while I can have functions of operators like this; A square on psi and so on that would be the same as A A psi, following this. This is what is meant by operators act linearly on states of a system. A crucial point is now already evident by a state of the system we need not necessarily mean a column vector.

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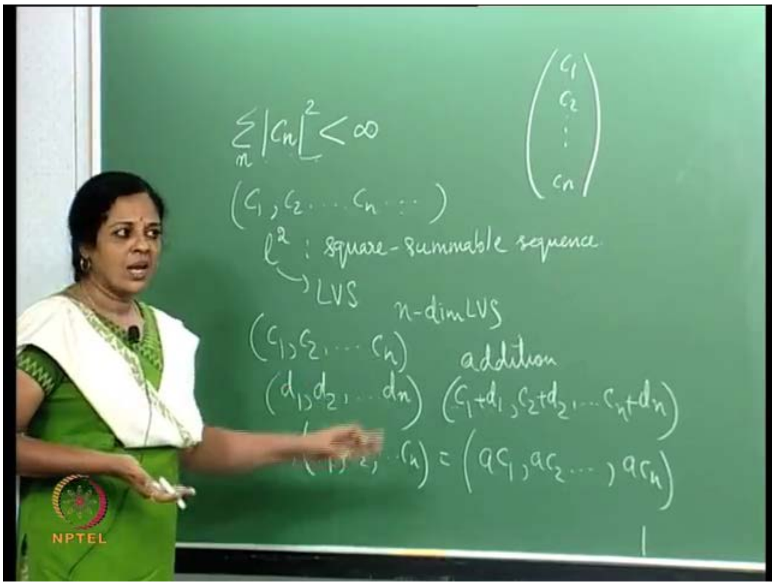

Here for instance, is a sequence. Of course, I could have written it as a column. I could have written it as a sequence as c 1 c 2 c n. And, this sequence if I know all the elements of this sequence. I know the state of the system because mod C n square gives me the probability of the component of the state psi along the basis state phi n, where psi is a state that I have expanded in terms of the basis set phi n.

So, ((Refer Time: 15:11)) a concept of linear operators again continues to be in infinite dimensional linear vector spaces.

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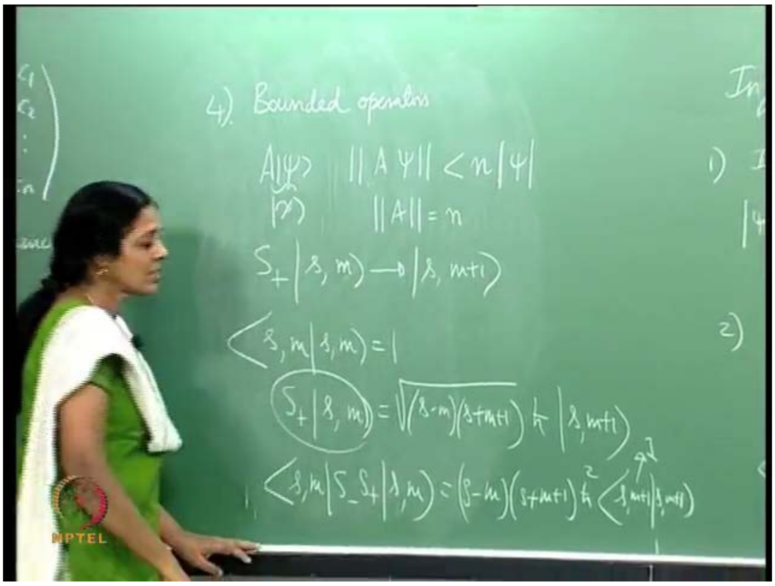

There was another crucial aspect in linear vector spaces which we saw, and that was bounded operators. Operators in a finite dimensional linear vector space were bounded in the following sense: if A is an operator and it acts on ket psi. Now, suppose I take the norm of that state, this is some state chi so I am taking the norm of chi. That was always less than some positive number times norm of psi and this was very important for normalization of the new vector. For instance, if we go back to the example of spins we have S plus which was the operator, the non Hermitian operator that took the state s, m to s, m plus 1.

The state s, m was orthogonal. The set of states were orthogonal to each other and each was normalized to 1. Now, what happen was this: the detailed calculation showed us that this was root of s minus m times s plus m plus 1 h cross s, m plus 1. So, this is the new state which is analogous to chi out there and surely this new state is not normalized to 1 where s minus is the dagger of s plus which is what I will have when I take the complex conjugate, a Hermitian conjugate of the state s plus s m. That gives me s minus m times s plus m plus 1 h cross squared and then of course, s m plus 1, s m plus 1 and this was 1. But then, you see I am left with this coefficient.

But the norm of this is some number times the norm of the original state. The norm of chi was less than or n times the norm of psi and that n is out here, this is in fact an equality. But, you see this helps me get you a new state which is normalized to 1. Now, I can suitably scale down chi such that the new state is normalized to 1. The smallest number n here which satisfies this is said to be the norm of A. So, one defines the norm of an operator as a smallest number that satisfies this. It turns out that all operators in a finite dimensional linear vector space are bounded operators and this is merely an example of what we saw. But, in an infinite dimensional vector space there is no need to imagine that all operators will be bounded operators. In fact, there are many unbounded operators and we will see some of them as we go along.

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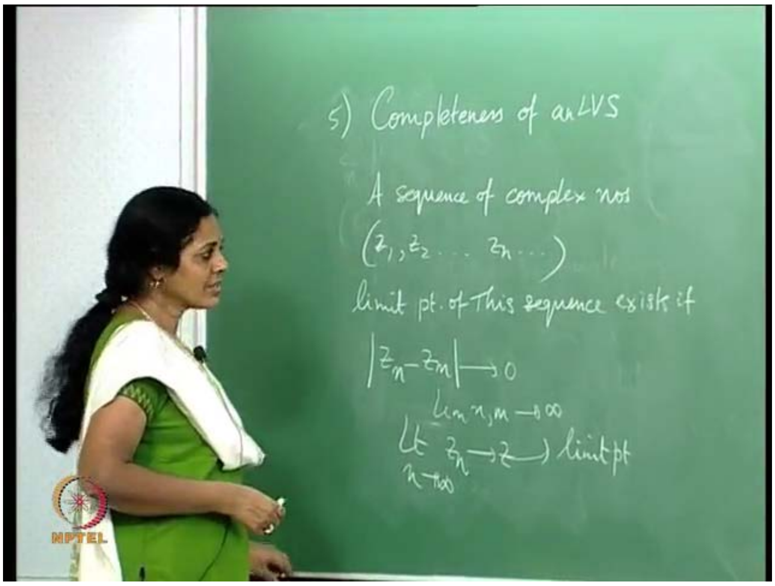

There is another important concept that we have come across and that is the concept of completeness: completeness of a linear vector space, completeness of an l v s. This concept is very important when it comes to infinite dimensional linear vector spaces and not at all obvious. To explain completeness, we do the following: You consider first a set of complex numbers, a sequence of complex numbers z 1, z 2, z n and so on. The limit point of this sequence exists. If modulus of z n minus z m goes to 0 in the limit n m going to infinity such a limit point need not exist for an infinite sequence. The series need not even converge.

Now, if this holds then you are guaranteed that limit n going to infinity of z n is some number z and that is the limit point. So, this guarantees this. Now, when it comes to vectors in the linear vector space we replace these numbers by vectors and we have the following.

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Suppose, you have a set of vectors: psi 1, psi 2, psi n and so on. You replace the modulus by the norm of psi n minus psi m and if that goes to 0 in the limit n m going to infinity. Then you are guaranteed that limit psi n goes to psi as n the limit n goes to infinity and psi is the limit vector.

So, the definition is analogous to what you have in the case of a sequence of complex numbers except that these could be vectors in a linear vector space and then limit vectors are defined in this fashion. Now, a Cauchy sequence is a sequence which has a limit point. The limit point exist for a Cauchy sequence.

So, you can have a Cauchy sequence of numbers, you can have a Cauchy sequence of vectors. It is not at all guaranteed but even if limit points of sequences exists in the sense that every sequence that you can think of in the linear vector space of concerned. Even if it is a Cauchy sequence there is no need imagine or assume that the limit point or the limit vector exists in that linear vector space. So, we have another statement.

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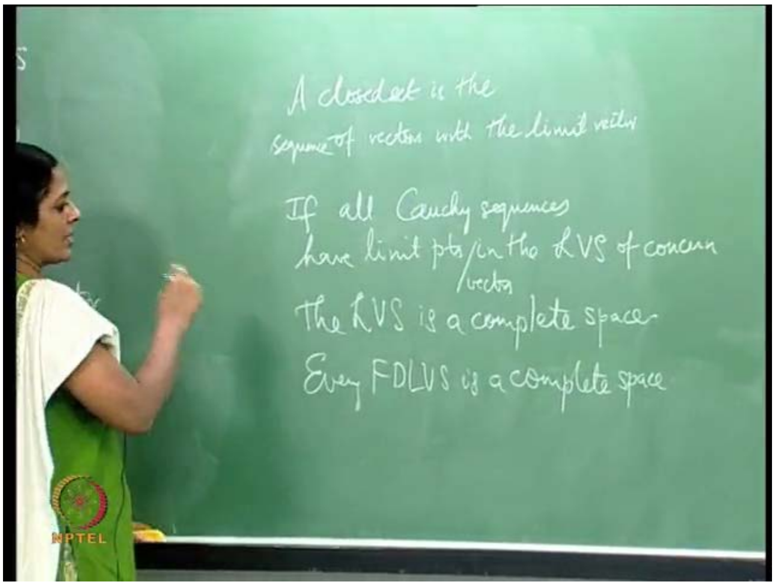

If, a closed set is the sequence of vectors with the limit point or with the limit vector in this case. So, you consider such a sequence and include psi, (Refer Slide Time: 21:13) the limit vector and that is the closed set. If all Cauchy sequences have limit points in the linear vector space or limit vectors in the linear vector space of concerned. Such a space is a complete space. We only look at spaces which are complete spaces. Simply because, we do not want the action of an operator to take a state, one of the states in the sequence psi n to take the state out to a limit point or a limit vector which does not even belong to that space. So, we will only look at complete spaces.

The action of operators on sequences should result in state sequences of vectors or any one of the vectors in the sequence should result in a state which belongs to that linear vector space and not take it out of that linear vector space. The state that I get after the action of the operator should be expandable in terms of the basis states of the linear vector space of consideration and therefore, this is a very crucial requirement that the linear vector space is a complete space.

Now, we come to the concept of a Hilbert space. (Refer Slide Time: 21:13) So as it turns out, if you have a finite dimensional vector space you are guaranteed that it is a complete space. The problem arises only for infinite dimensional vector spaces. So, let us just see why every finite dimensional linear vector space is a complete space. So, the way to prove this, I will indicate the proof to you right away.

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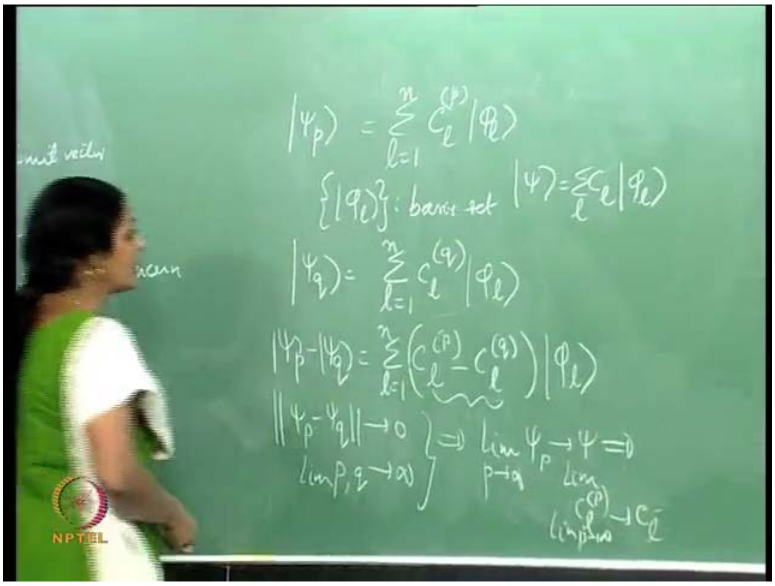
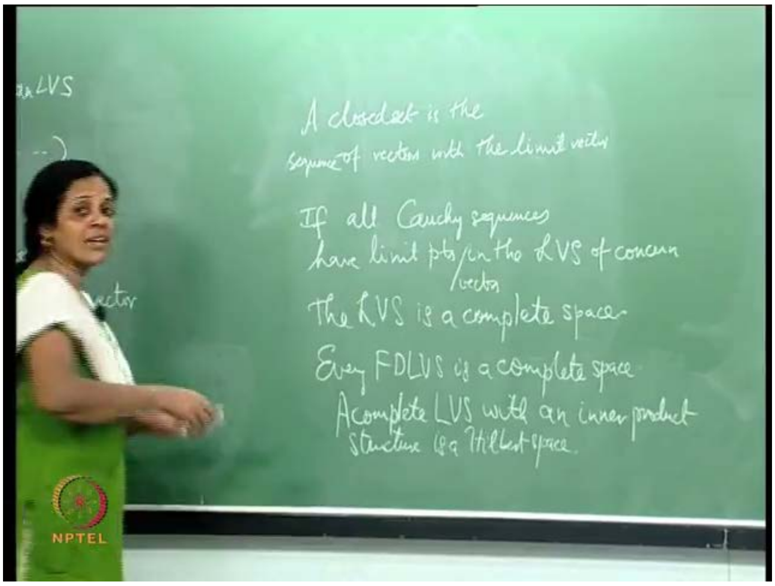


Consider a vector psi p in an n dimensional vector space and therefore, it is expanded in terms of the basis set phi l and there are n of them because it is an n dimensional linear vector space. Similarly, I have psi q and this state is also expanded in terms of the basis set except that the coefficients are different and therefore, I put a p there and a q here.

Now, I consider psi p minus psi q and this is clearly given by this object. Now, if this is a Cauchy sequence then in the limit p q going to infinity, this difference the norm of this goes to 0 limit p q going to infinity which automatically implies that this goes to 0 which means that limit p going to infinity of psi p goes to the limit vector psi. Correspondingly, this object C l of p minus C l of q, this implies that C l p minus C l q goes to some C or limit C l p in the limit p going to infinity goes to c, some value c l because there are l of them.

So, the fact that there is a limit vector translates to this statement for the coefficients that the limit of such a sequence goes to some value C l. But now you see, I can define a vector psi which is summation over l C l phi l and since, that is a linear superposition of the basis states that is a vector in that space. By the postulate of quantum mechanics, “any state which is a linear superposition of the basis states is a possible state, is an allowed state of the system”. And therefore, this belongs to that linear vector space. So, the limit vector also belongs to that linear vector space.

So, in a finite dimensional vector space, linear vector space I did not worry about whether the space is complete or not. The space is complete; it is guaranteed to be complete. (Refer Slide Time: 29:23)



But in an infinite dimensional linear vector space I have to impose that as an extra condition. Cauchy sequences are guaranteed to have limit points where all Cauchy sequences have limiting vectors or limit points in the linear vector space of concerned. And then, this is called a complete space. So, a complete linear vector space with an inner product structure defined on it and therefore, the norm of vectors defined on it is a Hilbert space.

Hilbert spaces are what we use in quantum mechanics. Since, every finite dimensional vector space is a complete space and since we have defined inner product structures. You have already looked at several examples of Hilbert spaces except that I have not used the word. The linear vector space corresponding to the two level atom, the linear vector space corresponding to the three level atom. The translated into the language of the spin system: the spin half system, the spin one system and so on. All of them are examples of complete spaces because they are finite dimensional linear vector spaces and since there was an inner product structure defined on them, they happen to be examples of Hilbert spaces.

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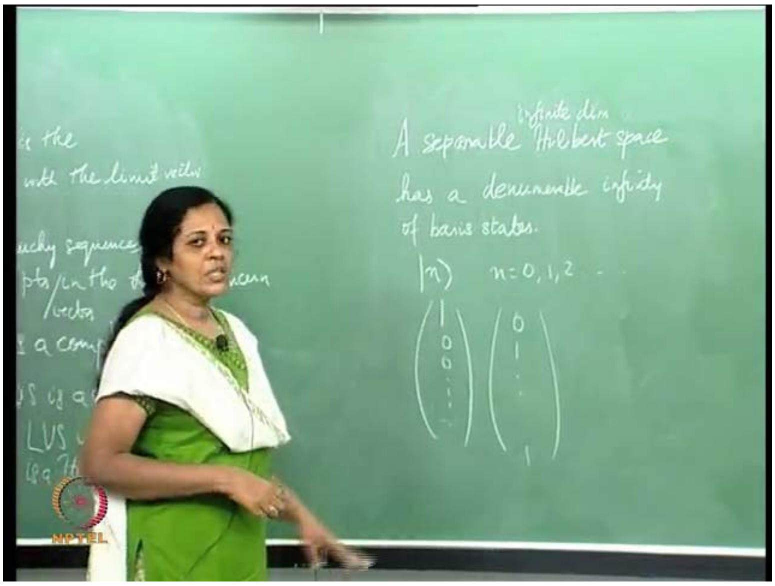

Now, the very crucial statement is the following: a separable Hilbert space is a complete space with an inner product structure defined on it. A separable Hilbert space has, and we are talking about an infinite dimensional linear vector space. Infinite dimensional has the denumerable, countable infinity of basis states. Again count the basis states, if you wish I can call them phi 1 all the way and therefore, I could call it just by a label n where n takes values: 0, 1, 2 all the way to infinity.

It is possible therefore, once I am given a denumerable infinity of basis states to write them in the column vector representation and I could refer to them as 1 0 0 0, 0 1 0 0 all the way to infinity. These are infinite columns. Now, it turns out that a bounded linear operator can be represented by an infinite dimensional matrix and here is a situation where I have a denumerable infinity of basis states. And therefore, I can choose these states to be of this type, analogous to what you had in the case of the two level and a three level atom. So, in this case, in systems which, where I can write the basis states in the following manner and the operators are represented by infinite dimensional matrices.

I can take over the machinery of matrix multiplication and so on which is what we have used in the case of finite dimensional linear vector spaces that we have seen. I can carry that over to such systems although they are infinite dimensional linear vector spaced systems. Such a situation would arise when we study the simple harmonic oscillator which we will be doing shortly in one of the few lectures that comes right after this.

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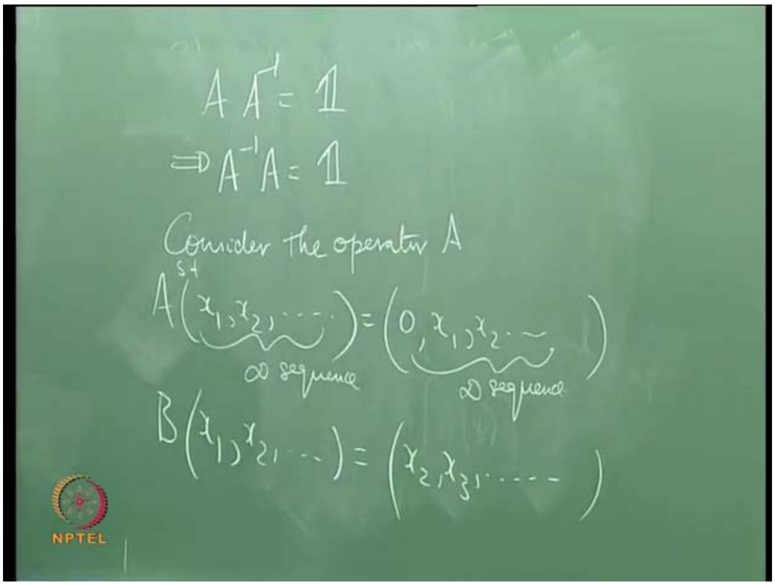

Now, having said this it is evident that there are going to be major differences between infinite dimensional vector spaces and finite dimensional vector spaces. The concepts are already getting to be somewhat more difficult. To give a simple example, suppose I were to talk of inverse of operators. In a finite dimensional vector space consider operator A and suppose we find an A inverse by using some standard procedure such that A A inverse is the identity operator. I do not have to check that A inverse A is identity. It follows that if A A inverse is identity, A inverse A is also identity. I cannot make that statement in infinite dimensions.

For instance, consider the linear operator A which acts on this infinite sequence such that A acts on sequence: x 1, x 2 to give me 0, x 1, x 2 and so on and this is an infinite sequence. So, that is another infinite sequence. Consider the operator B. Now, B acts on the sequence and it has the following effect. It simply removes x 1, the 1st entry is removed and it leaves behind another infinite sequence. So, what is B, A acting on this sequence?

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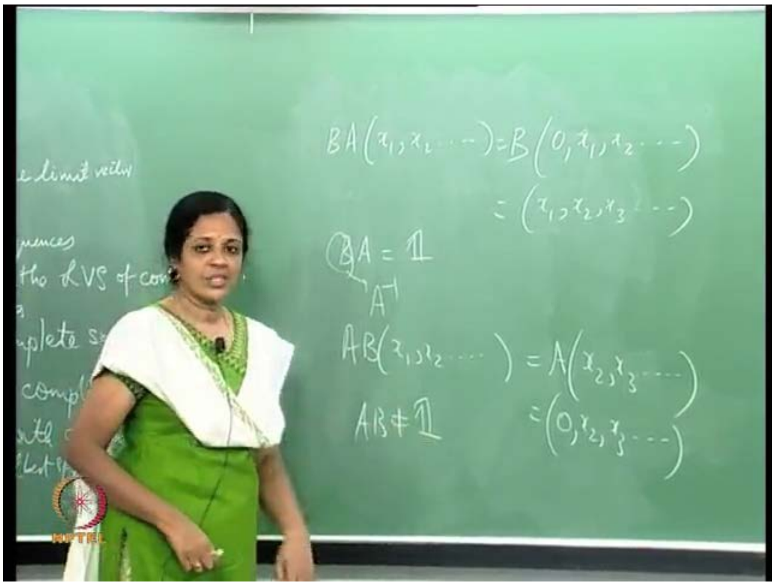

B A acting on the sequence: x 1, x 2 and so on. Since it acts linearly it is B acting on 0, x 1, x 2 and so on. But the action of B is to remove the 1st entry and therefore, it restores the original sequence and therefore, B A acts like the identity operator. So, it looks like B is A inverse because A inverse A is the identity operator. But now, let us find out A B acting on the sequence. Now this is A, B acts on the sequence to just give me: x 2, x 3 and so on and A simply puts a 0 in front and therefore, I do not get the original sequence back. That means that, A B is not equal to the identity operator. So, the fact that there is a left inverse does not guarantee that there is a right inverse. It does not guarantee that A A inverse is identity where I have identified B with A inverse in this problem.

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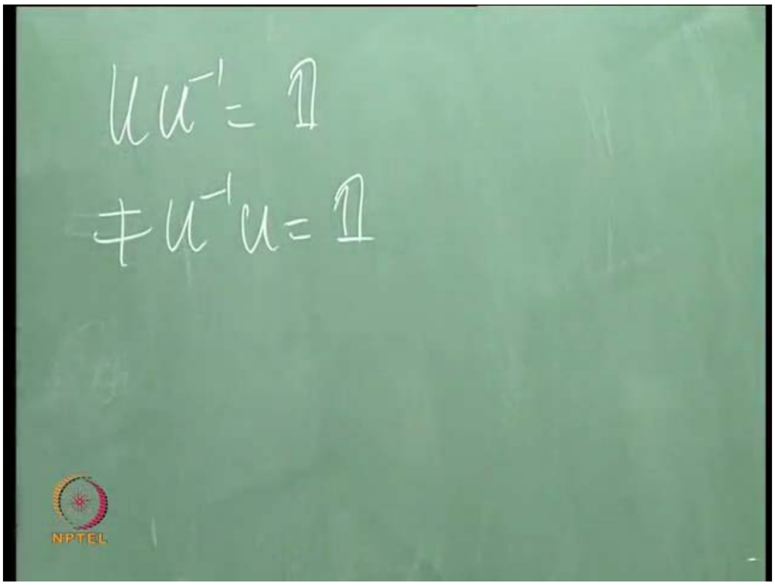

This is just one of the several aspects which are different between infinite dimensional linear vector spaces and finite dimensional linear vector spaces. The fact that there is a unitary operator u which satisfies u, u inverse equals identity does not imply that u, inverse u is identity so on for orthogonal matrices and so on. You have to check the fact that this condition satisfied does not mean this and you have to check for this independently, separately. So, these are some of the differences that you see between infinite dimensional linear vector spaces and finite dimensional linear vector spaces.

Now, one thing emerges that if we have a finite dimensional linear vector space, as we did in a in the examples that we have looked at till now, all finite dimensional linear vector spaces are isomorphic to the Euclidean space of the same dimensions.

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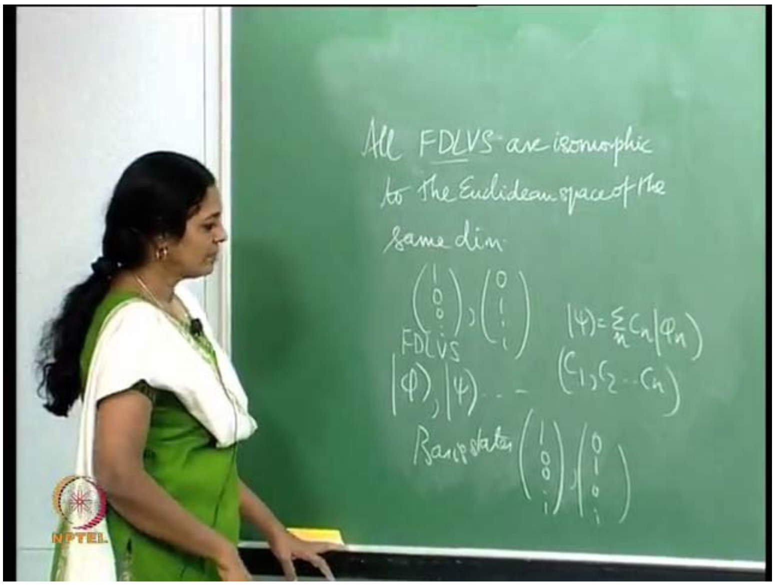

Finite dimensional linear vector spaces are isomorphic; that means that there is a one to one mapping to the Euclidean space of the same dimension. In other words, in a Euclidean space I would choose basis states: 1 0, 0 1 and so on depending upon, the number of entries here would depend upon the dimension of Euclidean space.

Since, all finite dimensional linear vector spaces are isomorphic to the Euclidean space. I could choose these column vectors as the basis states of a finite dimensional linear vector space and indeed that is what I have been doing till now without as much as saying. So, we found that we would always choose basis states: 1 0, 0 1 in the case of the two dimensional linear vector space, 1 0 0, 0 1 0, 0 0 1 for the three dimensional linear vector space and so on. So, a state in a linear vector space could be represented by ket phi, ket psi and so on.

This is an abstract notation. If it is a finite dimensional linear vector space you could choose basis states: 1 0 0, 0 1 0 and so on. This is finite dimensional linear vector space and any state can be expanded as a sequence C n where the state itself is written as summation over n C n phi n where phi n are the basis states of that linear vector space. Knowledge of: C 1, C 2 and so on till C n has all knowledge about the state psi and understanding the state psi would amount to measuring: mod c 1 square, mod c 2 square, mod c 3 square which are the probabilities.

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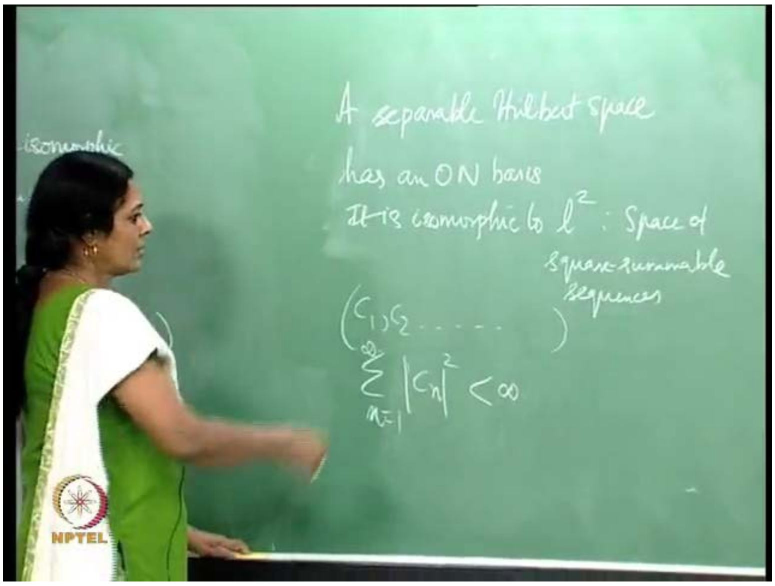

Now, an infinite dimensional linear vector space which is a separable space that means that there is a denumerable infinity of basis states or a Hilbert space, a separable Hilbert space has an Orthonormal basis. You are guaranteed that because there is a denumerable infinity of basis states and I could write them in the form 1 0 0 and so on. That is one way of writing it. I could also represent it in many other ways and I will talk about it shortly. A separable Hilbert space has an Orthonormal basis. It is isomorphic to L 2, the space of square summable sequences.

So, I just have this infinite sequence, infinite string of numbers, complex numbers in general: c 1, c 2, c 3 and so on. So that every state is represented by a unique set: c 1, c 2 satisfying summation over n equals 1 to infinity mod c n square, is less than infinity. That is what is square summable and that as we have already seen is required for the probabilistic interpretation of quantum mechanics. But, as I said basis states are a matter of convenience choice preference. Now, you may choose to work with one set of basis states in a given problem and I could choose another set of basis states. In fact there are n infinite set of basis states. So, instead of choosing this basis state I could have chosen something else, where set of states that belong to the linear vector space.

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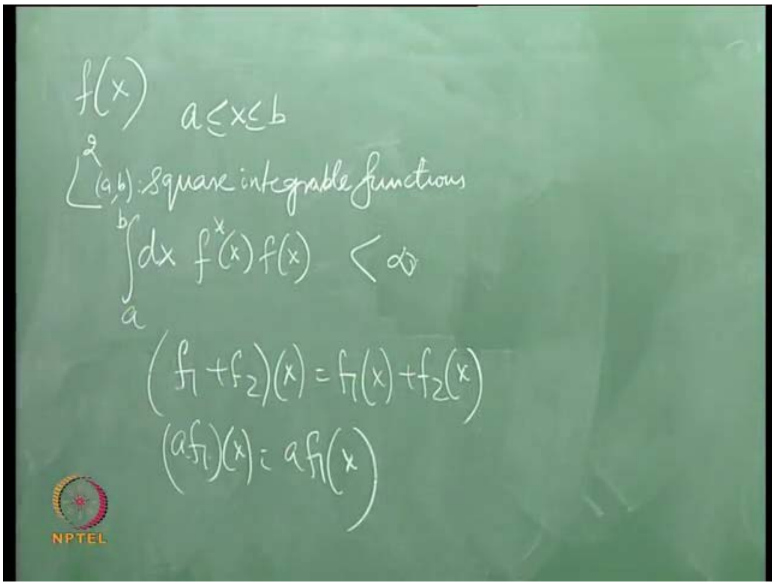

One of the choice is that I can have is a set of functions of some argument x satisfying the following property, square integrability. The states are simply square integrable functions where this L 2 is defined in the range a b that means x takes values, the argument takes values between a and b, f's could in general be complex. It is less than infinity.

So, this is simply direct extrapolation of the concept of square summability and here I have square integrability. It is clear that now we are talking about continuous functions of an argument x. Now, can these objects be basis states in a linear vector space? It is possible. Let us give an example. Suppose we are looking at L 2 of 0 1. 1st of all we have to check that these satisfy the requirements of states in a linear vector space. It follows directly. If you look at the properties of the linear vector space that we have gone over and you define functions in the following fashion: Function f 1 plus function f 2 of x is f 1 of x, plus f 2 of x, a f 1 of x where a is a scalar, is a times the function of x.

So, I defined addition and multiplication, scalar multiplication in the following manner. Then it is easy to check that such functions would also be states in a linear vector space. They are elements of a linear vector space and that brings us to a crucial point. A state in a linear vector space need not necessarily have to be a column vector. In abstract notation it is simply the ket. The state could well be a column vector. It could be a function, the space considered itself could be a function space and so on. And, with this definition we can check that this class of functions the square integrable functions are very respectable members of a linear vector space, states in a linear vector space. So, the basis states would also be functions.

So, let us look at square integrable functions, specific examples of square integrable functions.

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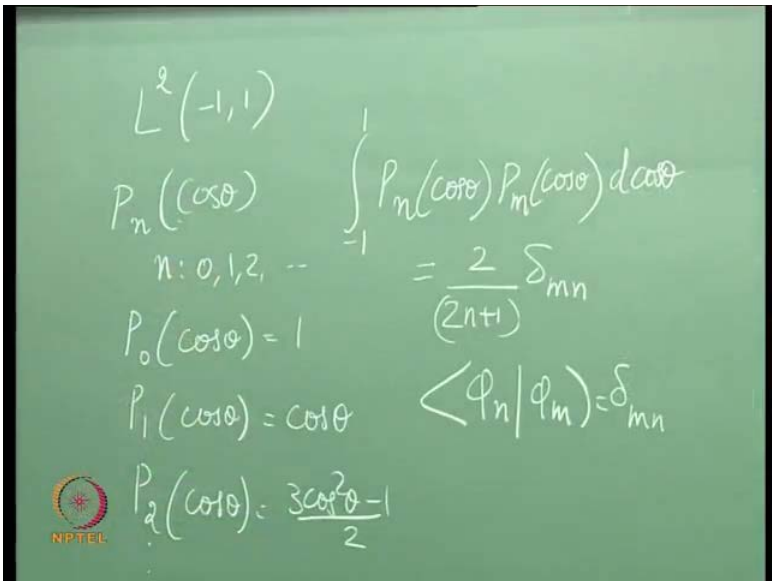

Let me consider L 2 of minus 1 1. What kind of functions, which are orthogonal to each other? Can I think of that could be a part of the basis set here? Suppose, I take the argument of the function to be cos theta, cos theta certainly goes over the range minus 1 1 and if I consider the P n of cos theta, these special functions would be good basis states in this space of square integrable functions. To begin with, with n taking values: 0, 1, 2 and so on P 0 of cos theta is 1, P 1 of cos theta is cos theta itself, P 2 of cos theta is 3 cos square theta minus 1 by 2 and so on.

There is an interesting property here. An odd function, an even function of cos theta and so on. So, these are specific parity properties which I will discuss later. They also satisfy Orthonormality conditions, Integral minus 1 to 1, P n of cos theta P m of cos theta. Now these are all real functions, so I do not put a star there d cos theta is 2 by 2 n plus 1 delta m n. Now, this is the Orthonormality property that the P n satisfy. The delta m n takes care of the fact that these are orthogonal to each other if n is not equal to m. Of course, you have a constant, a number out here 2 by 2 n plus 1. But, this is certainly the analogue of the statement phi n phi m is delta m n.

You can have different types of special functions which act as basis states in function spaces. For instance, if instead of minus 1 and 1 here, I have minus infinity infinity. The Hermite polynomials are an example of basis states in L 2 of minus infinity infinity.

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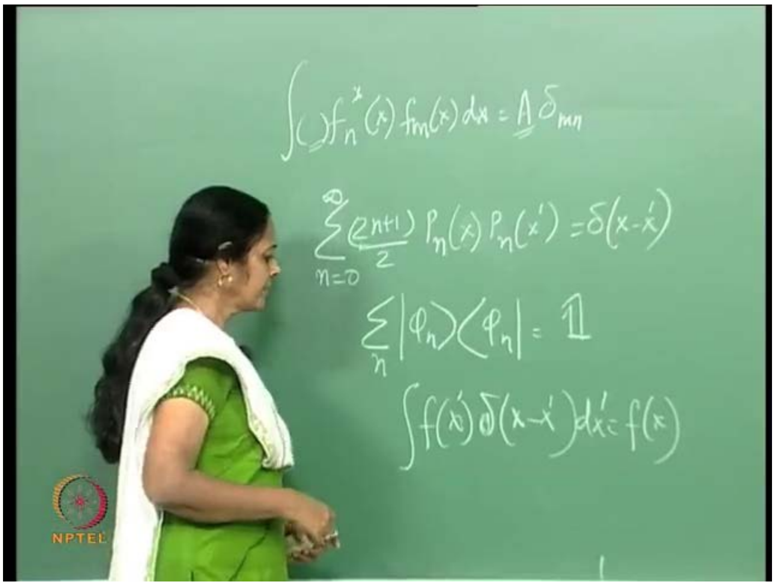

Now, in general you would expect the Orthonormality property to be of the form f n star of x, f m of x, d x and there could be a measure here, is equal to A delta m n. So, there is a measure here and there is A there, like (Refer Slide Time: 45:25) 2 by 2 n plus 1. And, in the case of the Hermite polynomials, this is a Gaussian. It is e to the minus x square, of course, in that case the range would be minus infinity to infinity.

Apart from this, I also have a completeness relation. But, before we put that down look at the following: (Refer Slide Time: 45:25) n is a discrete index. So, there is a denumerable infinity of basis states in this separable Hilbert space. But, each basis state is a function. In this case it is a function of cos theta. So, why the number of basis states is countably infinite? You have now replaced column vectors of the type: 1 0, 0 1 and so on with functions. (Refer Slide Time: 45:25) I can use functions as basis states in a linear vector space and for such functions there is a completeness relation. For instance, in the case of the Legendre polynomials summation n equals 0 to infinity 2 n plus 1 by 2 P n of x P n of x prime is delta x minus x prime, where this is the Dirac delta function.

Now, in terms of the Dirac notation, earlier the completeness relation was written as follows. We would have summation over n phi n phi n is the identity operator. So, this is clearly the analogue of that. Where you know that integral f of x, f of x prime delta of x minus x prime d x prime is f of x. So, it is in that sense that instead of the identity operator we have delta of x minus x prime here.

The important thing is the following: Most of you would have seen the Schrodinger equation where the wave function psi of x is involved. Psi of x is a function of x and psi of x can be expanded normally in terms of basis states which are also functions of x. That is how function spaces become very important. Because, the Schrodinger formulation of quantum mechanics which we normally refer to as wave mechanics relies on basis states in a function space. And, we will talk about square integrable functions, the space of square integrable functions. It is for that reason that one discusses special functions of this kind in the context of quantum mechanics.