Quantum Mechanics - I
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Lecture - 19
Addition of Angular Momenta III

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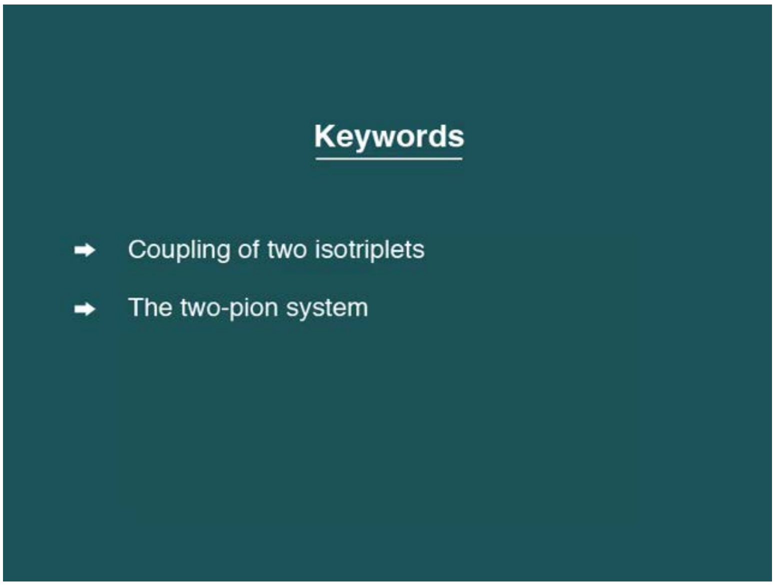

In the last class, we were discussing addition of angular momenta. And we had worked out certain examples, where J 1 is half and J 2 is half, also J 1 is 1 and J 2 is half. In this class I will continue working out a specific problem in addition of angular momenta and that is J 1 is 1 and J 2 is 1. This has some very interesting consequences and certainly helps one understand subtleties related to addition of angular momentum and the Clebsch-Gordan coefficients.

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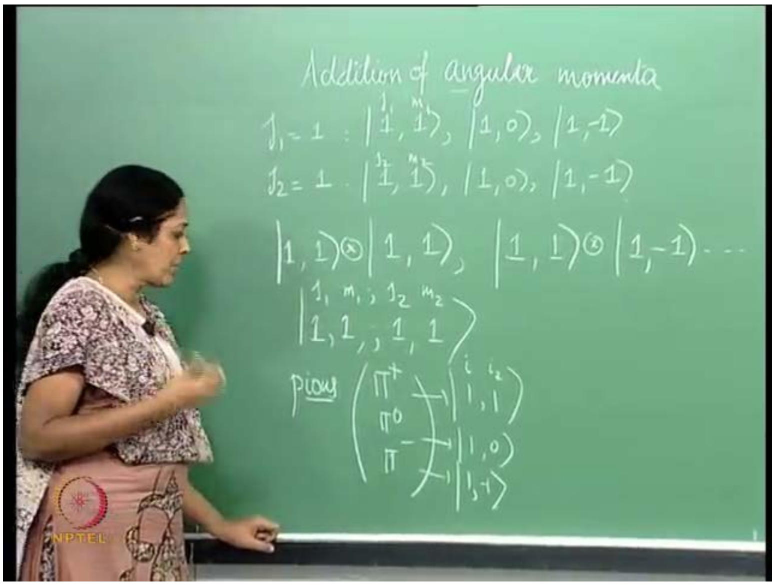

So, I would work with addition of angular momenta. So I have J 1 is 1 and J 2 is 1. So these are two spin one objects or isospin one objects. So, if J 1 is 1 the various uncoupled states corresponding to this are 1, 1, 1, 0 and 1 minus 1. Similarly, for J 2 equals 1 I have the states 1, 1, 1, 0 and 1 minus 1where the 1st index is J 1 and the 2nd is m 1 here the 1st index is J 2 and 2nd is m 2 there, m 2 takes values minus J 2 to plus J 2 in steps of 1 similarly, m 1. So, the uncoupled basis states would be 1, 1 with 1, 1, 1, 1 with 1, 0 by which, I mean direct product states. So the uncoupled basis states would be of the form and so on sub states.

Of course, using my notation I represent that as J 1 m 1; J 2 m 2 and therefore, this state for instance would just be 1, 1, 1, 1. I wish to take a very specific example. In this case I would like to talk about an isospin one object coupling with another iso spin one object. As I have said earlier, the pions from an iso triplet, there are 2 pions two of them with charges plus e and minus e and the other with charge 0 so it is a neutral pion. These are the charge pions and these have iso spin and i z values 1, 1, 1, 0 and 1 minus 1 respectively. So, this forms an iso triplet and as I have mentioned earlier, every particle is defined by a state which is labeled by a set of quantum numbers and certainly i and i z are two such quantum numbers, which label the state of the particle apart from charge, spin, rest mass and so on.

So this is an iso triplet, that means i is 1 and i z is analogous to m, i itself is analogous to J that I have written there and i z takes values plus i to minus i in steps of 1. And that is how I have these three distinct states of pions, these are bosons they have got spin 0 in units of h cross. Now, I would like to look at this problem as a coupling of two pions. So, 2 i equals one objects couple. So I write i 1 is 1, i 2 is 1 and 2 iso spin one objects or 2 iso triplets, in this case specifically 2 pions couple. And we want to look at the couple basis and write the couple states in terms of the uncoupled basis states as a super position of uncoupled basis states and therefore, having a lot of Clebsch Gordon coefficients arising in the process.

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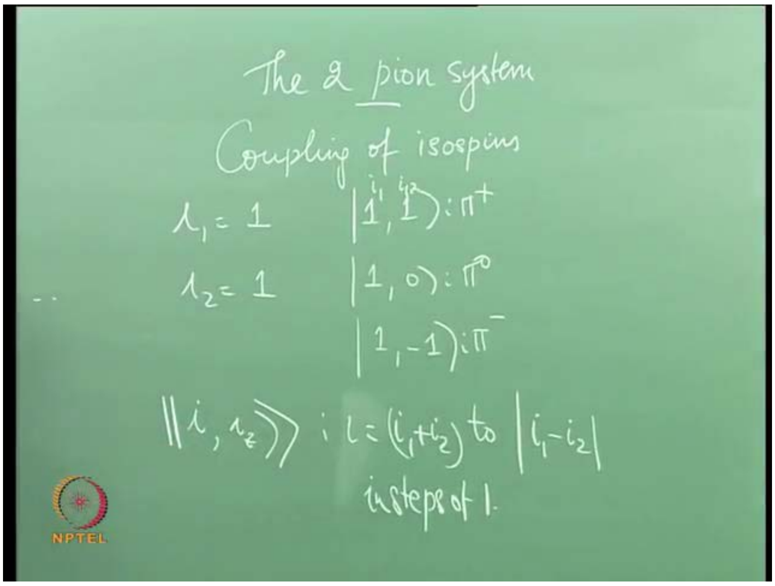

So, I have the 2 pion system, coupling of iso spins. So, I have equals i 1 equals 1, i 2 equals 1 which means I have the state 1, 1 which is the pi plus 1, 0 which is the pi naught and 1, minus 1 which is the pi minus. As I said earlier just I have to replace J by i and m by i z and the entire angular momentum machinery follows as such. So, the coupled state has i and i z. This is i 1 z in my notation, this is i 1 this is i 1 z perhaps i 2, i 2 z and so on. So, this could well be i 2, i 2 z as well. So, the coupled state has i and i z, i itself takes values i 1 plus i 2 to modulus of i 1 minus i 2 in steps of 1 and for a given value of i, i z takes values minus i 2 plus i in steps of 1 and that is 2 i plus 1 values.

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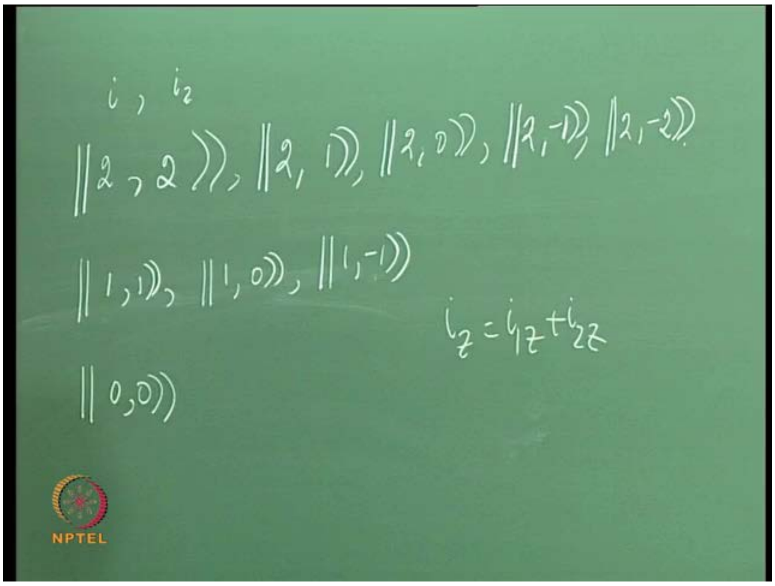

Now, let us look at the various coupled states in the two pion systems. So, that gives me i 1 plus i 2 to i 1 minus i 2 in steps of 1 and because, it is a coupled state I denote it by double braces as a matter of notation, I have a 2 lines out here. So, I have the following couple states: 2, 2, 2, 1, 2, 0, 2 minus 1 and 2 minus 2. Similarly, if i is 1 I have 1, 1, 1, 0, and 1 minus 1. The 2nd entry is i z so we have discussing the coupled states. This is the total isospin, the net isospin of the couple state system of 2 pions and this is the net i z value. And it is good to remember that i z is i 1 z plus i 2 z, that means take the i z values of the 1st pion, the i z value of the 2nd pion add them up like you would to scalars and you get i z. And then of course, I have 0 0. So, these are the complete list of coupled states that I can have when I combine 2 pions. My aim is to write the couple states in terms of the uncoupled basis.

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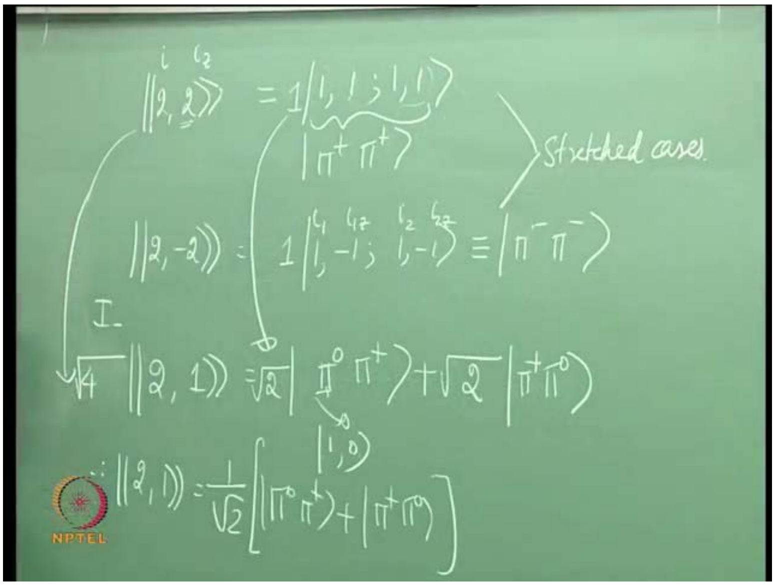

So, let me start with the coupled state 2, 2. Our aim as I have said earlier is to express this state in terms of the uncoupled basis. This is i and that is i z, the isospin of the coupled state which we have got by combining two iso triplets and this is the i z value. How could I have got this value for i z? This should have come from i 1 z plus i 2 z. So, clearly the contribution has come from the uncoupled basis 1, 1 and another 1, 1 I have combined an i is equal i 1 is 1, i 1 z is 1 with i 2 is 1 and i 2 z is 1 and my scalar addition these two i 1 z and i 2 z add up to give me to 2. There is no other possibility in this problem of getting a coupled state, which as isospin 2 and i z 2. Since, this is the only state the c g coefficient there is 1 that corresponds to the stretched case where the i z value is the same as the i value. So that is an example of a stretched case.

The c g coefficient being 1 now, in terms of pions what would this state be i 1 is 1, i 1 z is 1 so this must be the pi plus and that is another pi plus. So, basically we have combined two positively charged pions to get this particular coupled state 2, 2. There is another stretched case as well and that is 2, minus 2. On similar line I can argue and I can see that minus 2 can only arise from the uncoupled basis states 1, minus 1 and another 1, minus 1. This is i 1 that is i 1 z that is i 2 and that is i 2 z so minus 2 really came from minus 1 plus minus 1. Now, in terms of particle content, this state with isospin 1 and the 3rd component of isospin being minus 1 is the pi minus and therefore, I have this case 2, minus 2 c g coefficient 1 and that really came from the uncoupled basis states pi minus pi minus. So, this is another stretched case.

So in a stretched case the 3rd component has the value i or minus i in terms of our old notation, where we spoke of J and m. When m takes the value J or minus J, we call it as stretched case and the c g coefficient is 1. So, these are examples of stretched cases and there are other states as well. For instance, there is this state 2, 1 after all i z can take values from plus i to minus i in steps of 1. So, how do I get 2, 1? I can use an i minus on 2, 2. If I did that I use the fact that the coefficient in this case would be root of i plus m times i minus m plus 1 m being is z here.

So, i plus i z times i minus i z plus 1 so that gives me the state 2 , 2 and on this side I have to work with i 1 minus on this state. Keeping this state fixed, in terms of particle content the 2nd entry is not changed. The 1st entry the i 1 z value is brought down by 1 and therefore, I go to pi 0 because pi 0 you will recall is a 1, 0. This is i 1 and that is i 1 z now and what is the coefficient? It is a 1 plus 1 times a 1 minus 1 plus 1. Now, I work with an i 2 minus on this state. Again by the same argument leaving the 1st entry untouched, i get that. In other words, I have 2 , 1 is 1 by root 2 pi 0 pi plus plus pi plus pi 0. Look at the i 3 values or i z values. Here, i z is 1 pi naught has i z 0 and pi plus has i z 1 so they add up to give me 1 similarly, here. That is the matter that should be checked out at every stage that the i z values are all right.

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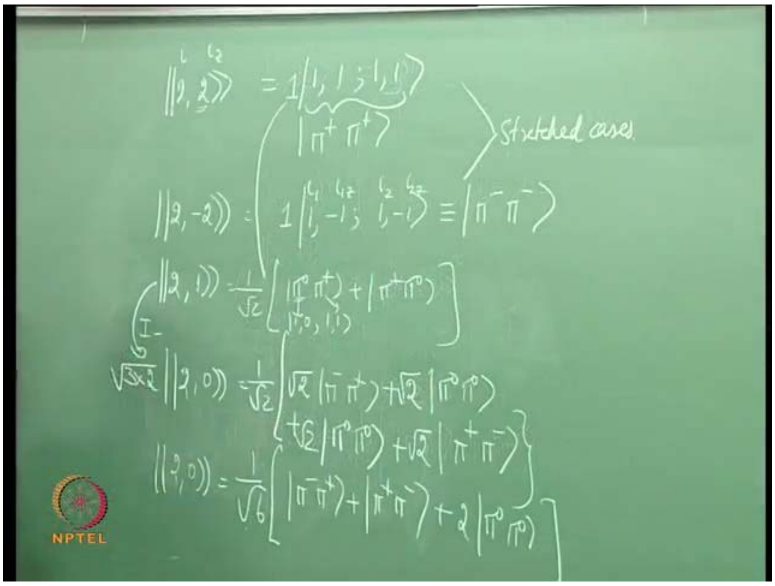

So, let me remove this part of the algebra and I can just write 2, 1 as 1 by root 2 pi 0 pi plus plus pi plus pi 0. I will comment on those symmetric properties a little later so that is what we have. The next thing is 2, 0 reduce the i z value by 1. How do I get that? By using i minus on this state, what is the coefficient? It is an i plus i z times i minus i z plus 1 so that gives me a root 6. And on this side there is an overall factor 1 by root 2. I repeat the same kind of argument that I had earlier on. So, this object has i equals 1, i z equals 0 and this object has i equals 1, i z equals 1. This is i 1, i 1 z, i 2, i 2 z. So, apart from the factor 1 by root 2 outside, pi 0 goes to pi minus with an overall coefficient 1 plus 0 times 1 minus 0 plus 1 so that is all I have here.

Once more, this time pi 0 is unchanged, pi plus goes to pi 0 and what is the overall coefficient? 1 plus 1 times 1 minus 1 plus 1 so that is root 2 again. The 3rd term comes from here with pi plus going to pi 0 again pulling out a root two as earlier. The 4th term is a pi plus pi minus because pi 0 goes to pi minus pulling out the coefficient root 2 so root 2 is canceled out. And what do I have? I have 2, 0 is 1 by root 6 pi minus pi plus plus pi plus pi minus plus twice pi 0 pi 0. It is a pi 0 pi 0 there and pi 0 pi 0 here. Let us mod square this. This gives me between these two I get a two sixth and that is a four sixth, that gives me a total probability of 1.

At every stage one has to check, that the mod square of the individual c g coefficients when summed over gives me 1. So, that is a half from here and a half from there that gives me 1 and so on.

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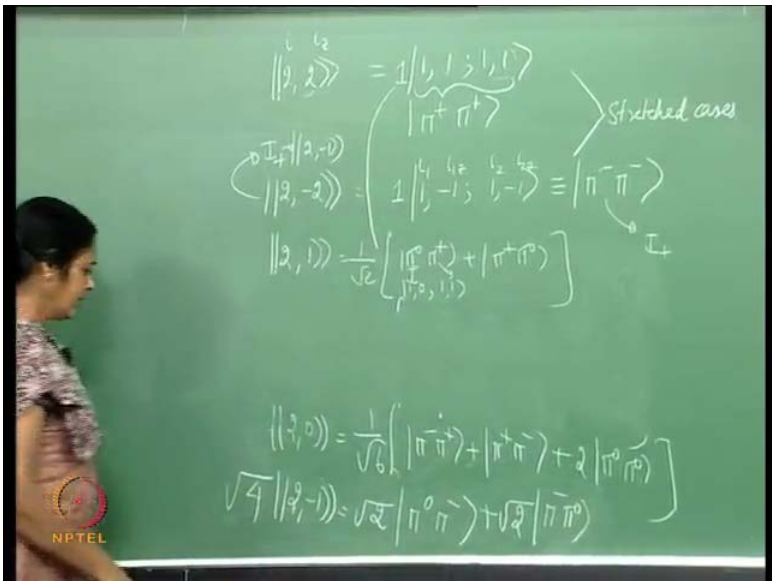

So, how do I get 2, minus 1? That is the only thing that is left behind. I could have done that by using an i minus on 2, 0. But this involves 3 states, the algebras a little bit more messy. So, instead let me start with 2, minus 2. Now, if I did that, I get an i plus acting on 2, minus 2 gives me 2, minus 1. What is the overall coefficient? It is root 4 because it is i minus m times i plus m plus 1 that is on the left hand side. On the right hand side I use an i plus on this.

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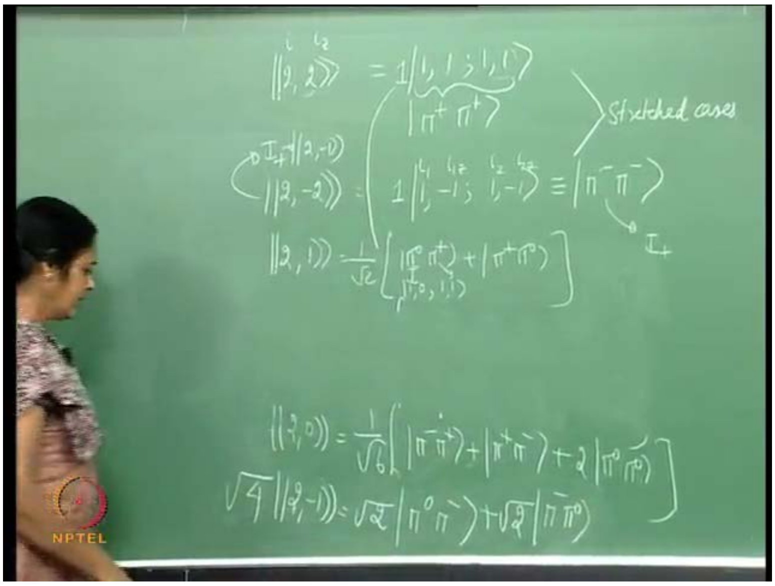

So, the 1st pi minus goes to pi 0 leaving the 2nd entry as such, the coefficient itself gives me a 1 plus 1 there and then of course, again I have a root 2 pi minus pi 0. In other words, 2, minus 1 is 1 by root 2 pi 0 pi minus plus pi minus pi 0. Look at the i 3 values; this is minus 1 that is a minus 1 plus 0 which is a minus 1. Here it is a 0, this has i 3 minus 1 that has i 3 plus 1. So, term by term you can find that the i 3 values match. So, indeed we have got all these states. Now, before we proceed it is worth understanding two things, first of all when we say pi 0 pi plus plus pi plus pi 0 as in the coupled state 2 , 1. How do you distinguish between the two states? The particle content is the same and basically, when we say pi 0 pi plus. We mean that there are two beams of pions and the pi 0 is from the 1st beam and the pi plus is from the 2nd.

Similarly, when we say pi plus pi 0 the pi plus is from the 1st beam and the pi 0 is from the 2nd. So, this is the way we distinguish between the state pi 0 pi plus where pi 0 is the 1st entry and pi plus as the 2nd entry, in contrast to a state with pi plus as the 1st entry and pi 0 is the 2nd entry. The same argument holds when we discussed the coupled state 2, 0 for instance where we have pi minus pi plus plus pi plus pi minus plus 2 pi 0 pi 0. Now, the 2nd remark is on the symmetry properties. In all these states take for instance, the stretch case 2, 2 if you interchange i 1 z with i 2 z, the state remains the same. In 2, 1 for instance that amounts to interchanging pi 0 with pi plus. Such an interchange simply picks up an overall plus sin and the state is symmetric under that interchange.

Similarly, in the coupled state 2, minus 1 you interchange pi 0 with pi minus, nothing changes there is just an overall plus sign that is picked up. So, all these states with isospin two are symmetric and the interchange of i 1 z and i 2 z. I will comment on this later. (Refer Slide Time: 06:52) Now, let us look at the triplet state 1, 1, 1, 0, 1, minus 1. Now 1, 1 is orthogonal to 2, 1 of course, 1, 1 is also orthogonal to all the other coupled states. But here we are trying to fix the c g coefficients in the expansion of the couple state 1, 1 and for this purpose we will only choose other coupled states with the same i z value that is i z equals 1.

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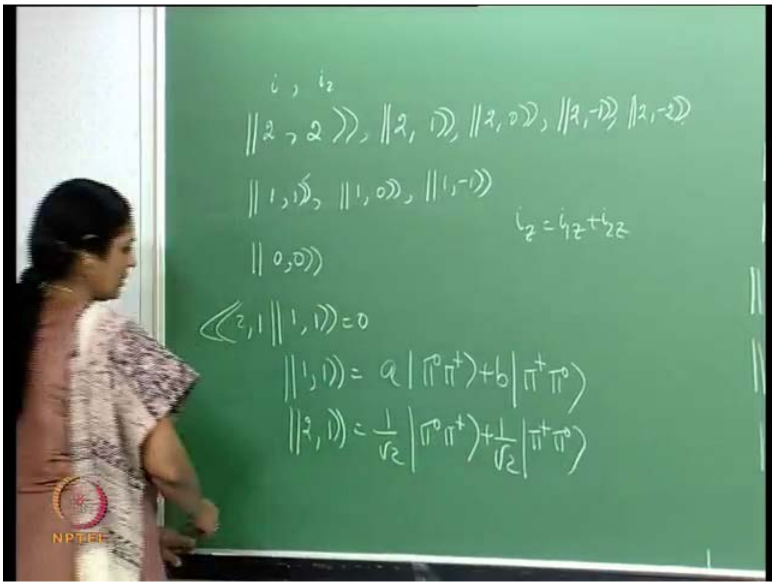

So, I know 2, 1 and I know that 1, 1 is expanded in the following manner similar to 2, 1. (Refer Slide Time: 14:42) It will get a contribution from the state pi naught pi plus also from the state pi plus pi naught. I wish to reemphasize the fact that by this we need pi naught from the 1st beam and pi plus from the 2nd beam, by this we mean pi plus from the 1st beam and pi naught from the 2nd beam and if in your mind you imagine that, these particles came with badges, white badges for this beam and black badges for that beam. Then you know the difference. This comes from the 1st beam, this from the 2nd, this comes from the 1st beam and that from the 2nd.

So there is a definite difference although, the particle content is the same. There is one neutral pion and one positively charged pion. So, returning to 1, 1. I know that it should be a superposition of pi naught pi plus and pi plus pi naught. These are the only states that can contribute to 1, 1, but since I know 2, 1 to be the symmetric state 1 by root 2 pi naught pi plus plus 1 by root 2 pi plus pi naught.

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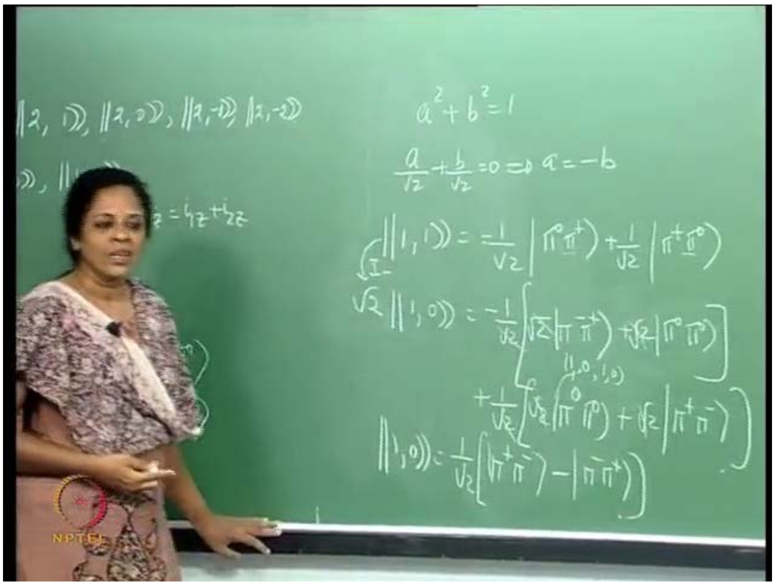

And I also know that a squared plus b squared is 1. So, I have a squared plus b squared is 1. We check the c g coefficients to be real without loss of generality. And they will come with phase, otherwise and we would write mod a square plus mod b square equals 1. Now, taking the c g coefficients to be real I have this (Refer Slide Time: 22:23) and therefore, from the fact that 2, 1 is orthogonal to 1, 1. I have a by root 2 plus b by root 2 0, which implies that a is minus b. I need to use my convention now. So, I find the 1 , 1 could be written as 1 by root 2 pi 0 pi plus minus 1 by root 2 pi plus pi 0 or minus 1 by root 2 pi 0 pi plus plus 1 by root 2 pi plus pi 0.

The both would be all right. What is important is the relative negative sign, but I choose a convention and I use that convention, the same convention throughout this problem. My convention is a following: Look at the 2nd entry, this is the pi plus here and a pi 0 there. Pi 0 has an m value or i z value which is smaller than the i z value of the pi plus. So, the 2nd entry has a smaller m value here and I would use a plus there and a minus there, same as this. That is simply a convention. Now, you could have gone ahead and put a plus here and minus there and you have to stick to that convention throughout the problem. So, this is 1, 1. Now, I need to get 1, 0. Notice that 1, 1 is an antisymmetric state, it is antisymmetric under interchange of the i 1 z and i 2 z values. Because this state has an overall negative sign and that has an overall positive sign here and therefore, when I interchange pi naught with pi plus, which is the same as interchanging the i z values of the individual particles I pickup an overall negative sign.

So, this is antisymmetric, and the interchange of the isospin labels. Now, if you look at 1, 0 there are two ways of getting this. I could have used an i minus on this, done it the way (Refer Slide Time: 14:42) I got 2, 1 from 2, 2. Another way of doing this is by realizing that (Refer Slide Time: 22:23) 1, 0 is orthogonal to 2, 0 unfortunately that cannot be used at this stage because it is also orthogonal to 0, 0. And since, this is also unknown in the sense I do not know how to expand this in terms of the uncoupled basis yet. I cannot use the fact that this state is orthogonal to those two states and find the c g coefficients. I will be forced to use i minus.

So, this is a point to note. I did not face this problem when I found out 1, 1 in terms of the uncoupled basis states, because there was exactly one state it was orthogonal to and that was 2, 1. I knew 2, 1 therefore, I could get 1, 1. But here I just have to use i minus so when i do that I pickup a coefficient root 2 1, 0 is minus 1 by root 2. The pi 0 goes to pi minus picking up root 2 leaving the pi plus alone. Then the pi plus goes to pi 0 with an overall coefficient root 2 because that is a 1 plus 1. This i value is 1 the i z value is 1 so it is root of 1 plus 1 times 1 minus 1 plus 1 so there we are.

But there is an overall minus 1 by root 2, then the 2nd term plus 1 by root 2. Similarly, pi plus goes to pi 0 with a root 2. The last term is pi 0 goes to pi minus with a root 2. Canceling out the root 2 allover I have 1, 0 is 1 by root 2. The neutral pions do not seem to contribute I just have pi plus pi minus minus pi minus pi plus. This is an important point for the following reason: that if you told me that I need to form an i equals 1 i z equals 0 state from two iso triplets, I would expect a contribution to come from the i equals 1, i z equals 0, i equals 1, i z equals 0 combination as well. Because i 1 z is 0 i 2 z is 0 and i is 0, but it turns out once I workout the algebra in detail, it turns out that two neutral pions cannot be in the i equals 1 i equal i 3 equal i z equals 0 coupled state.

This is not just an accident. The fact that pi naught pi naught cannot exist in the coupled state i equals 1, i z equals 0 is really statement of very important symmetry in particle physics called Bose symmetry. I will get to that later right now, it is nearly an observation. The details of the algebra show me that there is no contribution from two neutral pions to the 1, 0 states.

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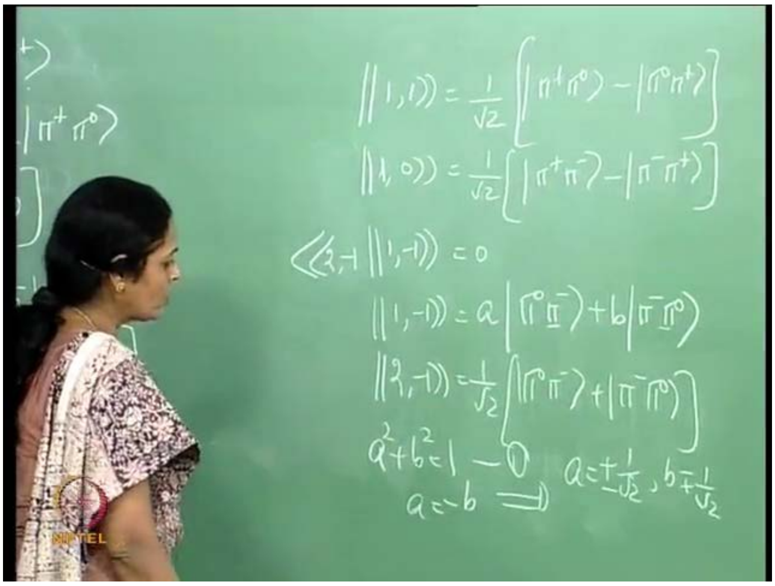

So, let me list those states here, in any case. So, I have 1, 1 is an antisymmetric state. It is 1 by root 2 pi plus pi naught minus pi naught pi plus 1, 0 is another antisymmetric state, that is here. It is 1 by root 2 pi plus pi minus minus pi minus pi plus. I need to get 1, minus 1 I can use the fact that this is orthogonal to 2, minus 1 and I know 2, minus 1. (Refer Slide Time: 14:42) So, 2, minus 1 well, 1, minus 1 is also expandable in terms of pi naught pi minus plus pi minus pi naught. And 2, minus 1 is the symmetric state 1 by root 2 pi naught pi minus plus pi minus pi naught. I have a squared plus b squared equals 1 as always and this gives me the fact that 2, minus 1 is orthogonal to 1, minus 1 gives me the 2nd relation, a equals minus b which implies that if a is plus 1 by root 2 b is minus 1 by root 2 and vice versa. I use the convention that I used earlier. Look at this state, look at the 2nd entry that is my convention. Pi minus has an i z value which is less than pi naught I put a plus sign here and a minus sign there. So, I choose a equals 1 by root 2 and b equals minus 1 by root 2.

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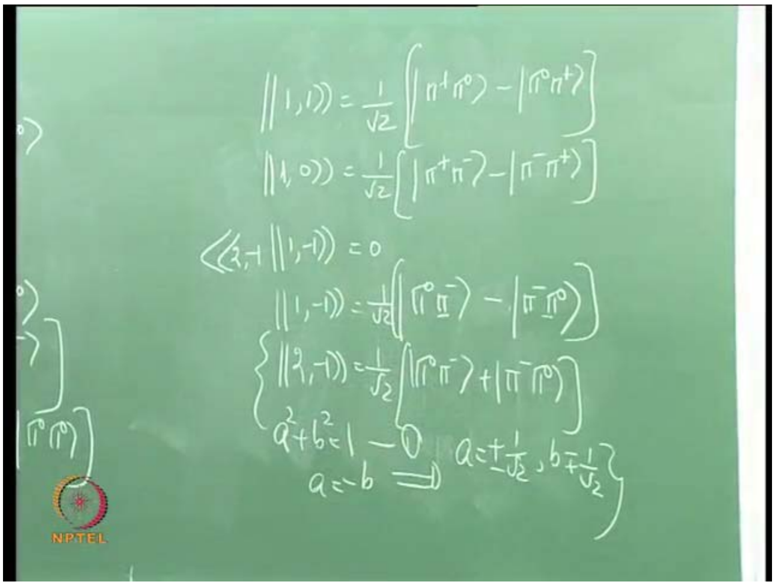

So, this is the working out of the thing a is 1 by root 2 and b is minus 1 by root 2. So, there we are that is 1, minus 1. So, we have the 3 i equals one state let me just put down all the states in 1 place. (Refer Slide Time: 22:23) We are only left with 0, 0 which we will find shortly.

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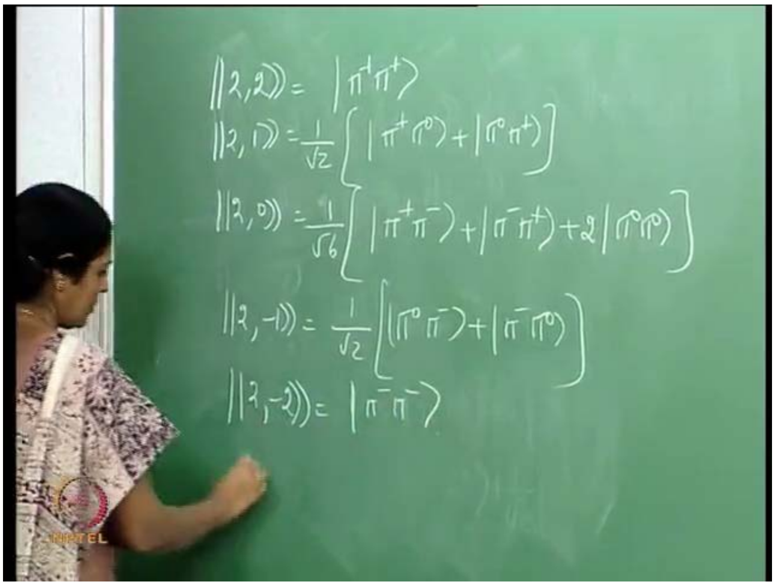

The i equals two states are totally symmetric states. That symmetric under interchange of the i z labels so that is a pi plus pi plus 2, 1 is 1 by root 2 pi plus pi 0 plus pi 0 pi plus. 2, 0 had more contributions. (Refer Slide Time: 14:42) This is 2, 1 out here we got 2, minus 1. We need to get 2, 0 as well and 2, 0 had many contributions to it. So, let me write that down. So, it had two neutral pions contributing to it, 2, minus 1 is (Refer Slide Time: 24:00) simply 1 by root 2 pi 0 pi minus plus pi minus pi 0, 2, minus 2 was a stretch case 1, 1 was the antisymmetric case.

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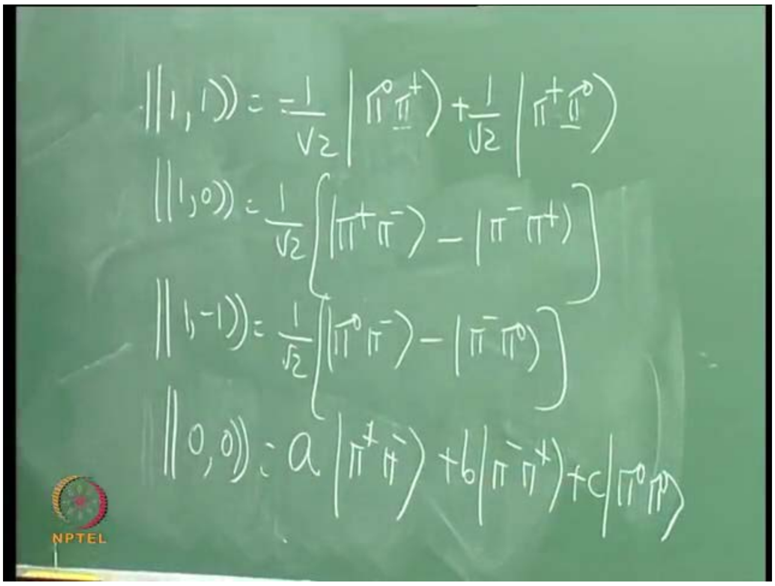

So, let me just write that down (Refer Slide Time: 24:00) 1, 1 is out here, it is minus 1 by root 2 pi 0 pi plus plus 1 by root 2 pi plus pi 0. 1, 0 is 1 by root 2 pi plus pi minus minus pi minus pi plus. These are the two antisymmetric states (Refer Slide Time: 14:42) and 1, minus 1 is out there; it is 1 by root 2 pi 0 pi minus minus pi minus pi 0. I need to now find the coupled state 0, 0 the iso singlet state. In terms of the uncoupled basis, I know that the contributions come from the same set of states that contributed to (Refer Slide Time: 33:17) 2, 0 or 1, 0 in general 2, 0. Recall that 1, 0 did not have pi naught pi naught in it. So, basically I can write this as a times pi plus pi minus plus b pi minus pi plus plus c pi naught pi naught.

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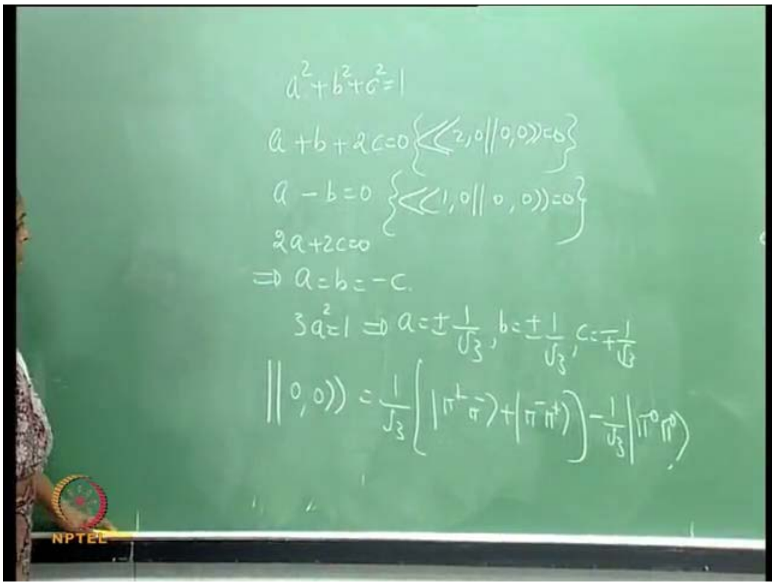

I have the following relations. I have three unknowns and I can determine them using the fact that a squared plus b squared plus c squared equals 1. That is the 1st thing and then 0 0 is orthogonal to 2 0 (Refer Slide Time: 33:17) so that gives me a by root 6 plus b by root 6 plus 2 c by root 6 equals 0. So, a plus b plus 2 c equals 0. This came from the fact that 2, 0 was orthogonal to 0, 0. But 0, 0 is also orthogonal to 1, 0 and therefore, I have a by root 2 minus b by root 2 equals 0. So, a equals b or a minus b equals 0, this came from the fact that 1, 0 is orthogonal to 0, 0. So, I have a equals b so here I have 2 a plus 2 c equals 0 which implies that a equals b equals minus c. So, I go back there and I have a squared plus b squared plus c squared equals 1 which implies that a is plus or minus 1 by root 3. Correspondingly, b is plus or minus 1 by root 3 and c is minus or plus 1 by root 3.

I could choose any of this, but once more I follow the convention and I write 0, 0 (Refer Slide Time: 35:15) the 3rd entry c is pi 0. Of course, pi 0 is has an isospin less than pi plus and more than pi minus so I could choose any of these conventions and I have 1 by root 3 pi plus pi minus plus pi minus pi plus minus 1 by root 3 pi naught pi naught. This is again a symmetric state. Symmetric under interchange of the 3rd component of isospin term by term so this is 0, 0.

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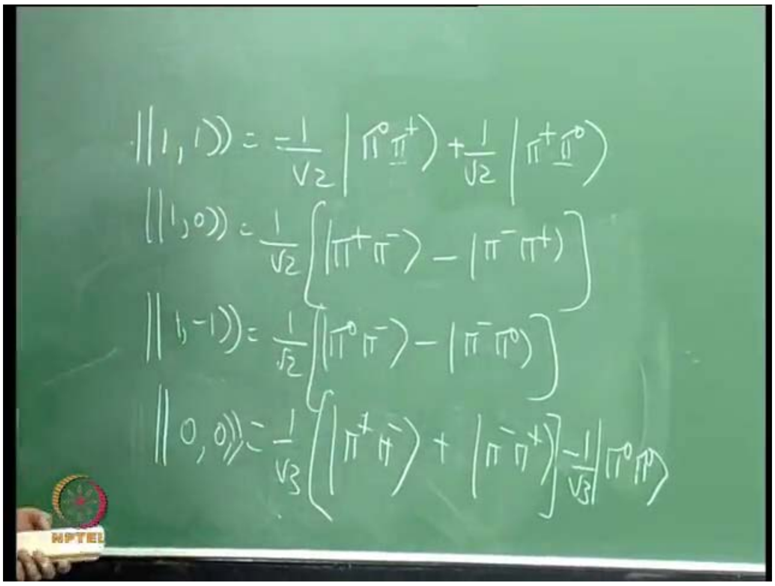

I have now listed out all the states, that is a minus 1 by root 3 here. I have now listed out all the states and now, we have the following comments following observations to make. The 1st observation is this. The stretched case that means 2, 2, 2, minus 2 and anything else in that multiplet, turn out to be symmetric states under interchange of the isospin labels. Whatever, I say for isospin also holds for any J 1 and J 2, but in this context we will be discussing a certain consequence I have already mentioned it both symmetry. (Refer Slide Time: 33:17)

But, now coming back to this, this set of state is symmetric under the interchange of m 1 and m 2, the 3rd component of isospin in this context. We would like to prove this in general, that if the J, J state in this case the i, i state is symmetric under interchange of the labels. All the other states in that multiplet also satisfy the same symmetric property. Now, if you look at this. Again if you look at this, this state is antisymmetric under interchange of the 3rd component labels and all the other states in that multiplet are again antisymmetric.

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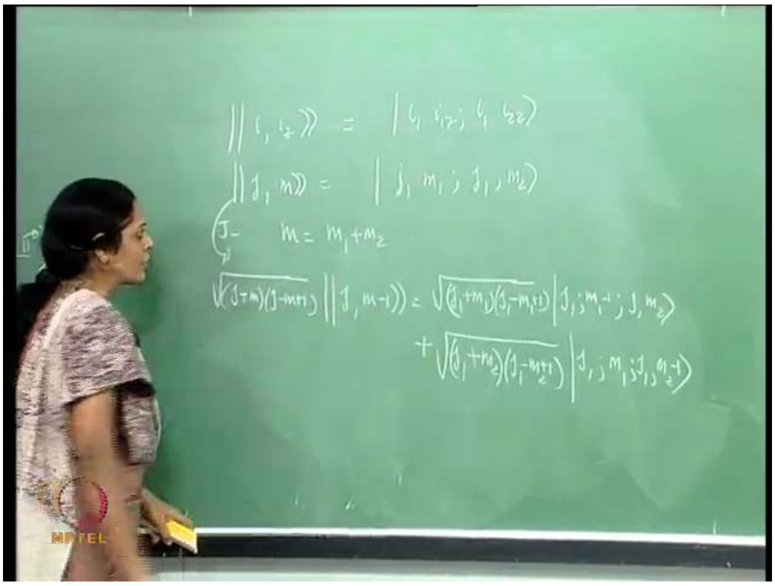

I need to show that in general, you have a coupled state J m. Let us start with a state which is symmetric which is J 1, m 1. My J 1 was equal to J 2 because I am considering 2 i equals one state so in this problem perhaps it would be more sensible to write i 1, i 1 z, i 1, i 2 z and call this i and i z. In general I would call this J m for the coupled states and this is like pi plus pi plus so for instance it is J 1 m 1, it is a same J 1 m 2 because J 1 equals J 2 in my problem it is 1 and remember that m is equal to m 1 plus m 2. So, this is symmetric under interchange of m 1 and m 2 it is clear.

So, now i use J minus on this. It gives me root of J plus m times J minus m plus 1 apart from the h crosses which I have said equal to 1 J , m minus 1; this is what I have on the left hand side. On the right hand side I have the 1st term gives me root of J 1 plus m 1 times J 1 minus m 1 plus 1 and the state itself becomes J 1 m 1 minus 1 J 1 m 2 that is a 1st term. The 2nd time the operator acts on this leaving this alone. So, I have root of J 1 plus m 2 times J 1 minus m 2 plus 1. The 1st states stays put the 2nd one the m 2 value changes by 1. So, this is what I would have got. The original state was symmetric under interchange of m 1 and m 2.

Now, if you look at this state again I see the same thing. If I interchange m 1 with m 2 this becomes J 1 plus m 2 times J 1 minus m 2 plus 1 J 1, m 2 minus 1, J 1, m 1. So, you see this state becomes that and that state becomes this. This state goes to that and that state goes to this is an overall plus sign and this is just an overall coefficient. So, I can see that using J minus on a state which is symmetric is not going to change the symmetric properties of a state. (Refer Slide Time: 33:17) So, that is why, if 2, 2 was symmetric all the other states in the i equals 2 multiplet they are also symmetric under interchange of the labels. Similarly, if I have a state which is antisymmetric under interchange of the labels I would pick up the same antisymmetric property when I do a J minus on that state. (Refer Slide Time: 39:12) and that is what we have in the 1, 1, 1, 0 1, minus 1 series.

So, symmetry and antisymmetric properties are preserved under the action of J minus and J plus. We now go back to the observation that we made earlier. The two neutral Pions do not contribute to the i equals 1, i z equals 0 state.

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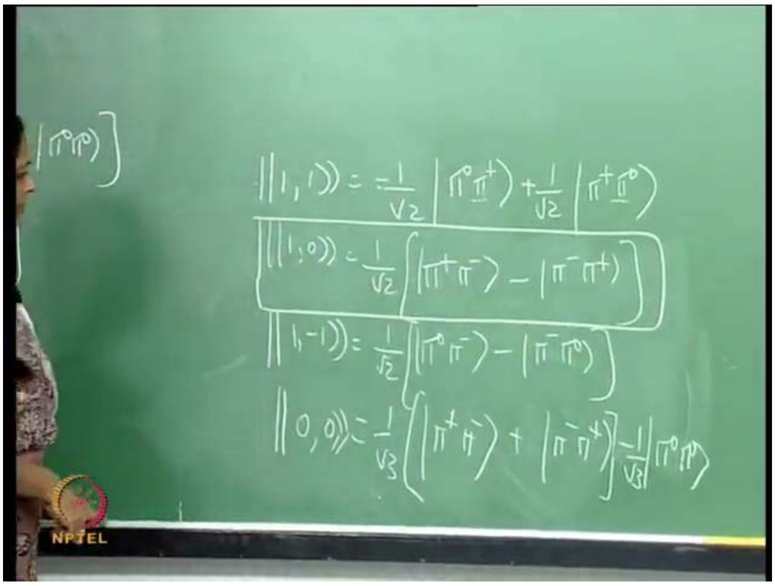

I am looking at this particular one here where I have expanded the coupled basis in terms of these uncoupled basis and I wonder if there is a deeper symmetry which prevents a contribution from pi naught pi naught to this state, because certainly if m is 0 or i z is 0, i 1 z can 0 and i 2 z can be 0 that is allowed. (Refer Slide Time: 33:17) So, at the face of it I would expect pi naught pi naught contribution here the same that there was the pi naught pi naught contribution to 2, 0 and also to 0, 0 out there.

So, why is it that this is happening? This is happening for a very deep reason it is called the generalized Bose symmetry. Pions or bosons the statement is the following, for fermion there is an analogous statement, but I am now looking at bosons. The two neutral pions are identical bosons. They also happen to be iso-partners in the sense if you take a pi 0 and take it is i z value to minus i z nothing changes. It is its own partner pi naught goes to pi naught under i z going to minus i z, because the neutral pi naught has i z equals 0.

Whereas, pi plus and pi minus are iso-partners when I take i z equals 1 to minus 1 I get pi minus vice versa. So, pi plus pi minus are so-partners the pi naught pi naught again it is they are iso-partners, pi naught is its own iso-partner if you know what I mean. So, you see the statement is this: if I have two identical bosons like pi naught pi naught the total state in any case for a set of particles is going to carry a list of quantum numbers. Some quantum numbers related to space could be the space coordinates then the spin quantum numbers and then the isospin quantum numbers and so on.

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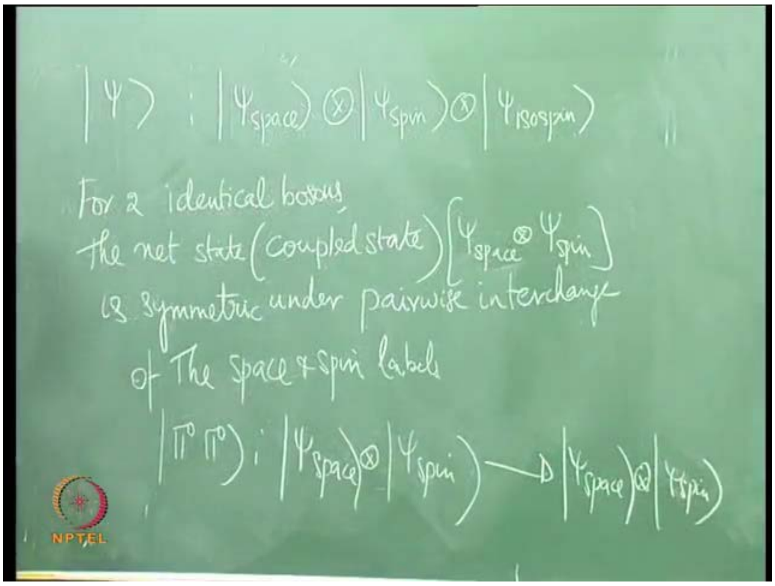

The total state of the particle the pi naught pi naught system or any two particles. We will look at this problem specifically, is made up of a space part which involves the space coordinates of the two particles. The spin labels s and s z of the two particles and the isospin labels of the two particles. Now, for two identical bosons this is like if you go back to the harmonic isolator problem, where I had two harmonic oscillators. The next state of the two dimensional oscillator was given in my notation by ket n a direct product with ket n b, where n a was the quantum number corresponding to the 1st oscillator and n b the quantum number, corresponding to the 2nd oscillator. So, these are different quantum numbers.

There are some labels here perhaps psi space in the case of the 2 pi naughts would be psi space would be a function of r 1 and r 2 where r 1 is the coordinate corresponding to the 1st pi naught and r 2 to the 2nd pi naught and so on. Similarly, if you look at psi spin, it will have s of the 1st pi naught s z of the 1st pi naught, s of the 2nd pi naught and s z value of the 2nd pi naught isospin i, i 1 z, i 1, i 1 z, i 2, i 2 z as we have said in this problem. So, for two identical bosons the net state or the coupled state is symmetric under pairwise interchange of the space and spin labels. So, if you look at pi naught pi naught. They are two identical bosons and if you interchange the space and spin labels, the psi space psi spin the net state which is psi space psi spin picks up an overall positive sign. So, it is symmetric under pairwise interchange, goes to itself picks up an overall positive sign.

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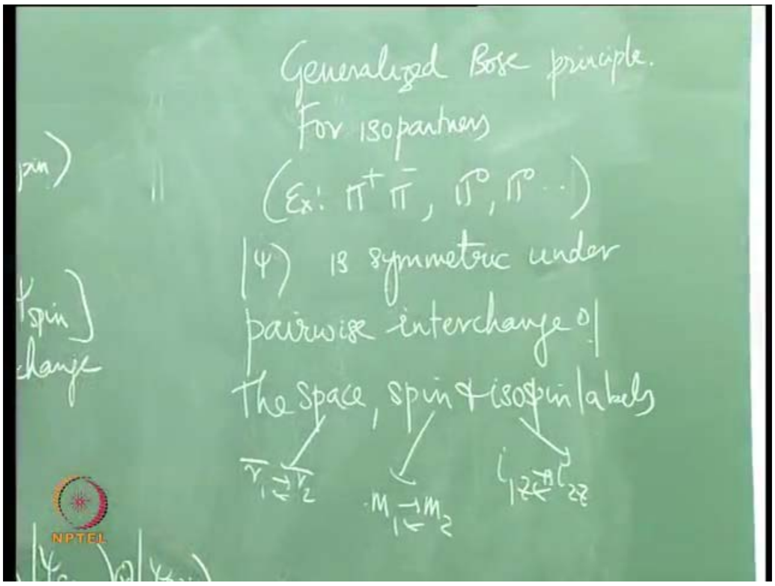

Now, for iso-partners, example pi plus pi minus, pi 0, pi 0 and so on. The net state is symmetric under pairwise interchange of the space spin and isospin labels that means i 1 z and i 2 z. By this we mean i 1 z going to i 2 z and i 2 z going to i 1 z by interchange of the spin labels we mean M 1 going to M 2 and M 2 going to M 1, whereby M 1 and M 2 I mean the 3rd component values of spin. And space would mean r 1 going to r 2 and r 2 going to r 1, where r 1 and r 2 are the coordinates of the two particles. The net wave function, the net state is symmetric under pairwise interchange of all these labels, not one at time, all of them, all these labels. (Refer Slide Time: 46:51) So, let us get back to pi 0 pi 0 the case of pi 0 0. This is the generalized Bose symmetry or Bose principle.

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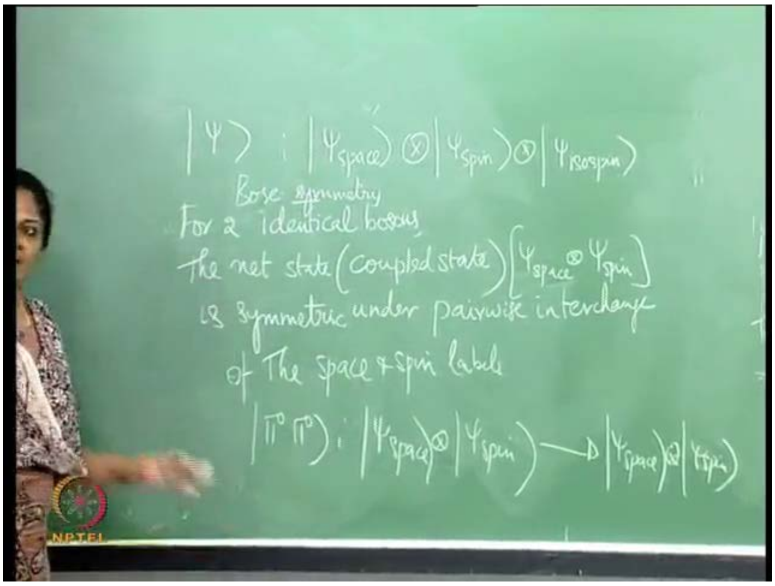

This is the Bose principle, Bose symmetry. We have already seen that since pi 0 pi 0 are identical bosons under interchange of the space and spin labels. The net space spin part of its state does not change, just picks up an overall positive sign. But, they also happen with the iso-partner pi naught is its own iso-partner. So, now if you look at pi naught pi naught and use this. (Refer Slide Time: 50:01) Since psi space psi spin is already symmetric under interchange of the space spin labels. And the net state which includes isospin as well should be symmetric under interchange of all the labels, just psi isospin alone should be symmetric under interchange of i 1 z and i 2 z. Which means that the isospin coupled state cannot be antisymmetric under interchange of the 2 pi naughts (Refer Slide Time: 33:17)

Look at 2, 0 here. This is symmetric under interchange of the 2 pi naughts, there is no problem anyway it would be symmetric under interchange of the 2 pi naughts, because they are identical particles in any case. (Refer Slide Time: 44:38) And therefore, they cannot be present here. This is antisymmetric under interchange of pi plus pi minus picks up an overall negative sign, but that is ok because these are not identical bosons. They are iso-partner bosons and between the isospin labels and the space spin labels, I would expect the total state of pi plus pi minus to be symmetric. (Refer Slide Time: 51:46) Whereas, in the case of pi naught pi naught already just the space spin part is symmetric and therefore, by the generalized Bose principle the isospin part alone should be symmetric under interchange of the two labels. (Refer Slide Time: 44:38) Therefore, it does not contribute to an antisymmetric isospin wave function. So, there is a deep principle behind what we saw by brute force algebra that 1, 0 surprisingly did not have a contribution from two neutral pions the reason is Bose symmetry.