Quantum Mechanics - I
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Lecture - 18
Action of Angular Momenta – II

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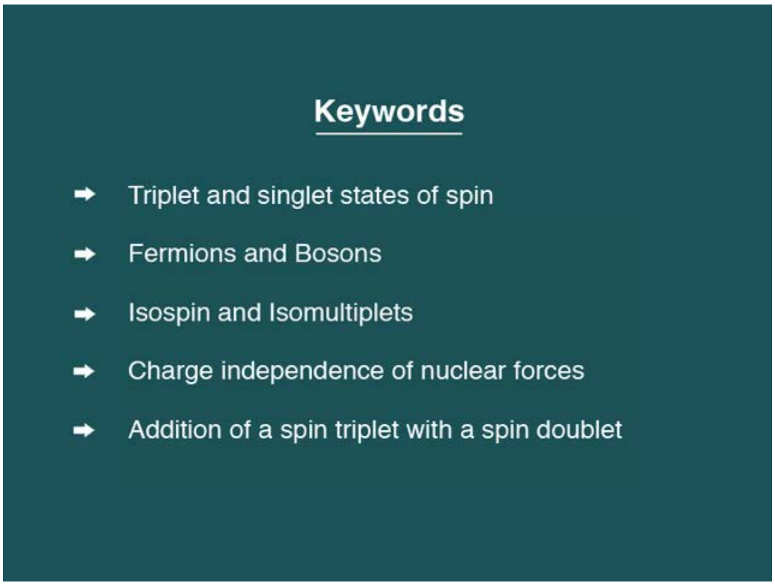

In the last lecture, I told you how to add angular momenta and so we demonstrated how to start with 2 spin half objects for instance and add them to get coupled states which are either spin triplets states or a spin singlet state.

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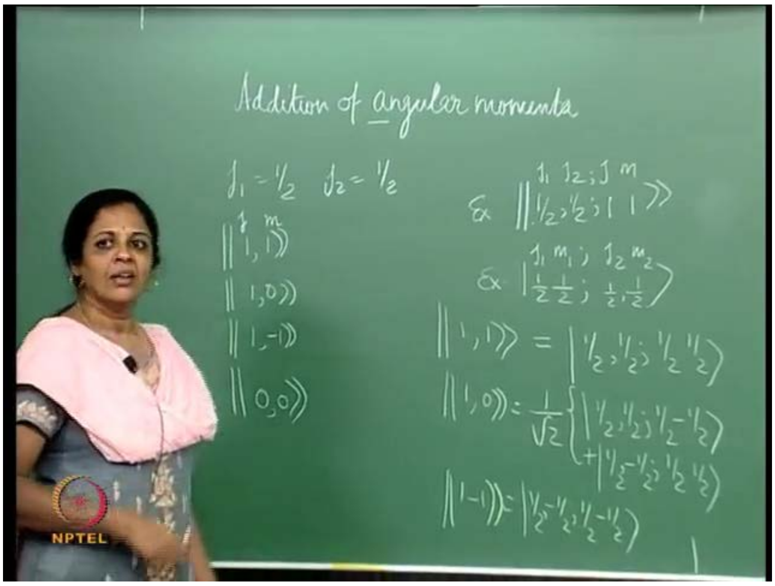

So, we will continue with addition of angular momenta. And yesterday we worked with j 1 equals half in units of h cross j 2 equals half and this gave us coupled states the spin triplet state. By spin triplet, I mean j is 1 the 3rd component m is 1, j is 1, m is 0 and j is 1, m is minus 1. This is the triplet state of angular momentum and if indeed by j 1 and j 2 we mean spin then it is the spin triplet state. And we can have a spin singlet state as well, that is just one state with j equals 0 and therefore, m equals 0.

We also learnt how to write this coupled basis states in terms of the uncoupled set and we had the following. Suppose my notation is J 1, J 2, J m for the coupled basis J 1 and J 2 are anyway fixed as half and half and j is 1 and m is 1. Like this state can be written in terms of the uncoupled basis state. The uncoupled base states are represented by j 1 m 1, j 2 m 2 and I would not use the double braces for that just to show you that this is the uncoupled basis set.

So this would be half if I want m equals 1, then this should have come from m 1 equals half and m 2 equals half. So, this is an example of the uncoupled basis set and indeed we would have gotten this coupled basis state in terms of the uncoupled bases state by using half, half for j 1 and m 1 and half, half for j 2 and m 2. We wrote it in compact notation and I simply said that 1, 1 the 1st entry being j, the 2nd entry being m was simply half half, half half 1, 0 had a combination.

I could have started with m 1 equals half and m 2 equals minus half or the other way round and we showed that this was 1 by root 2 half half, half minus half, plus half minus half, half half. So this was 1, 0 and 1, minus 1 was another stretched case which simply came from m 1 equals minus half and m 2 equals minus half.

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There was the other state which was the spin singlet state. This is j and that is m. (Refer Slide Time: 00:37) The states that contributed to the spin singlet state, the states that superposed in order to get the spin singlet state were precisely these states, except that the combination was different and we got to a point where we said that we could write this in the following fashion: minus of half minus half half half. This is what we had.

There is a relative negative sign and that is what is important. I could have started with this having a negative sign and that state coming with the positive sign in front of it. It would not have mattered I stick to our convention and keep that during a particular exercise or when I work out a single problem I stick to a certain convention. The convention will be best demonstrated when I do more examples. I want to comment a little bit about the importance of the results that we have got. The best way in my opinion to understand this is by looking at the world of elementary particles. So, elementary particles are labelled by quantum numbers. In other words, they are represented by states in an appropriate linear vector space.

The state labels are various quantum numbers corresponding to the elementary particles. Examples of elementary particles of course, could be protons, neutrons and the pions, the charged pions and the netural pions, the kaons, the charged kaons and the neutral kaons. By that what I mean the deltas some of them, one of them is neutral and the others carry electrical charge and so on.

In fact, there is a wide variety of particles and depending upon the spin quantum number of the particle, particles are classified as fermions and bosons. Fermions are particles which have spin half in units of h cross, half integer spin in units of h cross. So, examples are protons and neutrons, electrons these come with s equals half S z of course, could be of m could be plus half or minus half. That is ok and then you have objects like the deltas. Now, these have s equals 3 by 2 in units of h cross. So, these are examples of spin 3 by 2 particles while this pi plus the pions, pi 0 and pi minus. The suffix denotes the electric change these are examples of and there are many more particles.

But these are examples of spin 0 objects so they are bosons. Bosons have 0 or integer spin and fermions have half integer spins. So, spin is a quantum number which certainly is important in representing the particle. Now, apart from spin there is another quantum number called isospin and this is very relevant particularly in the context of particles which can interact strongly with each other or decay strongly. The nuclear force is an example of a strong force.

So, examples of objects that can interact strongly are protons, neutrons not the electrons, the deltas, that is delta plus plus, delta plus, delta 0, delta minus. Of course, pi plus pi 0 pi minus all these and many more are examples of strongly interacting particles and strongly interacting particles are called hadrons. I want to emphasise, that hadrons interact strongly and can also interact electromagnetically, weakly as the case may be. The definition is simply that these particles are capable of interacting strongly. They could have electromagnetic interactions or weak interactions, may or may not have it that is not the issue. The point is hadrons are particles which are capable of strong interactions. These are examples of such particles. If, you look at the proton and the neutron, so I am going to look at some of the hadrons.

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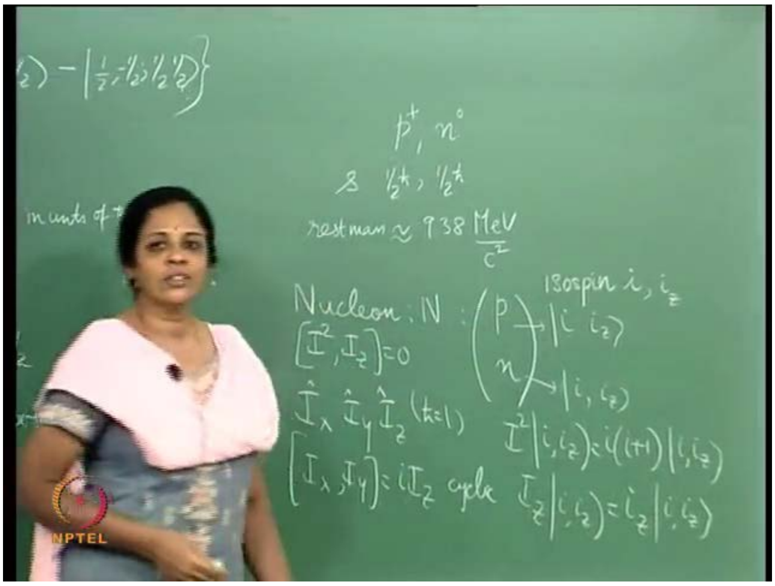

So, if you look at the proton and the neutron for instance. Of course, this carries a charge plus e and this has 0 electric charges, electrically neutral; that is certainly a difference between the proton and the neutron. But, if you look at the other quantum numbers both of them have spin half h cross. If you look at their rest mass, that is approximately 938 million electron Volts by C square, it is conventional in particle physics and they are useful to talk in terms of energy units and therefore, if I convert the rest mass into energy by multiplying the rest mass by C square I get energy 938 M e V approximately.

There are other quantum numbers which I need not introduce now, such as barion numbers, strangeness and so on. It turns out that the proton and the neutron have the same values for these quantum numbers. So, I would like to look at it in the following fashion. Of course, I can see the difference between the electrical charge of the proton and the neutron if I turn on the electromagnetic field, because the charge particle deflects in the presence of an electromagnetic field. In other words, it interacts with the field quanta, the photon and exchanges momentum and energy with it and deflects in its track.

A neutron does not do that because it is electrically neutral. But suppose I did not switch on the electromagnetic field at all. If I were only considering a world of strong interactions and not anything else, not the electromagnetic force or a relatively negligible forces like the weak force and the gravitational force. Suppose, I did not consider any of that, in a world of strong interactions there is no way of seeing the difference between a proton and a neutron. Because, I cannot see the difference in their electric charge, since I am not using electromagnetic field at all not switching it on.

In other words, I am not considering the fact that the proton can interact electromagnetically with itself for instance, 2 protons can interact electromagnetically and 2 neutrons cannot. A neutron cannot interact electromagnetically with the proton. Suppose I did not talk about electromagnetism. I cannot see the difference between the proton and the neutron, because all other quantum numbers are the same and the rest masses are approximately the same.

Now, you could always tell me that I can see the difference in the rest mass. I could, but now it is known that this difference is small difference in the rest mass of the 2 particles really arises from the fact that the protons can interact electromagnetically and the neutrons cannot. Therefore, if I switch off the electromagnetic field, I do not see the difference between the proton and the neutron. I could think of them as 2 states of a single object called the nucleon. I could think of the proton and the neutron in a world of strong interactions only as a nucleon doublet.

The proton being one of the states of the nucleon, and the neutron being the other state of the nucleon, and then it turns out that there is a new quantum number, which can be associated with these objects and that is called isospin. To distinguish it from the spin that I have been talking about or orbital angular momentum and the 3rd component of isospin you could call that i z or i 3 and that is the analogue of m s, m.

So I could label the proton and the neutron as a doublet with the certain value of i and i z. This is another value of i and i z. These are the isospin states of the proton and the neutron I would give entries i and i z for the proton and the neutron. Isospin is analogous to spin in the sense that there are 3 Hermitian generators of isospin. I could call them I X, I Y and I Z these are Hermitian operators and they satisfy the isospin algebra which is the same as spin.

Therefore, it follows that all that I said about spin or angular momentum can now be told in context of isospin simply replacing j by i, which means J X by I X, J Y by I Y and J Z by I Z little j by little i. Because, I square acting on a state i i z is i times i plus 1 i i z and I Z the operator i z acting on a state with labels i and i z give me this. It is evident that I square and I z commute with each other and that is why I am able to do this. Now, getting to this like a spin doublet I could treat the proton and the neutron since there are only 2 states of the nucleon. I could treat the proton and the neutron as an isospin doublet.

The proton being in the i is equal to half and i 3 equals half state. The neutron also being in the i is equal to half and i 3 equals minus half state. This is analogous to the spin doublet. You could think of the electron in the upstate as a spin half, m equals half state of the electron and the electron in the down state as an s is equal to half, n equals minus half state of the electron. Similarly, I talk of an i equals half, i z equals plus half state and an i equals half i z equals minus half state of the nucleon. The half, half state is the proton and the half, minus half state is a neutron. It is not merely the protons and the neutrons that form isomultiplets it turns out that all hadrons form isomultiplets.

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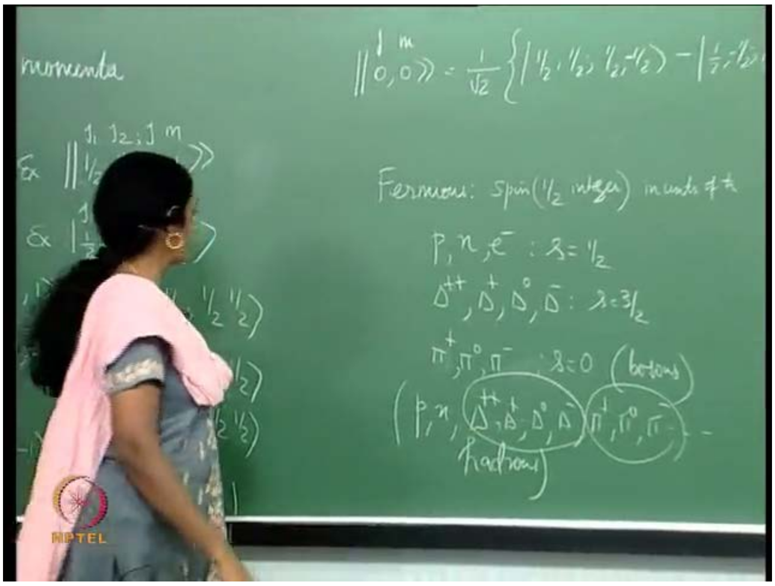

These objects delta plus plus, delta plus, delta 0, delta minus happen to be an isoquanted. In other words they are 4 state they are set of a 4 objects that transform as an isomultiplet.

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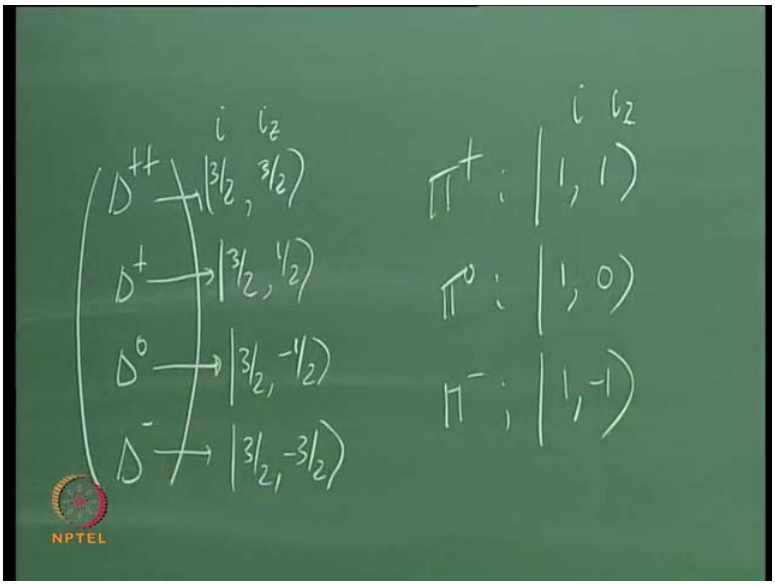

So they transform analogous to a spin 3 by 2 object. This will be an i equals 3 by 2, i z equals 3 by 2. This is an i equals 3 by 2, i z equals half state. This is an i equals 3 by 2 i z equals minus half state and this is an i equals 3 by 2 i z equals minus 3 by 2 state. So, I have an isoquanted and therefore, i is fixed it is a single multiplet where isospin is 3 by 2 and i z takes values minus i to plus i in steps of 1. Now, if you look at the pi plus pi 0 pi minus they transform as a triplet under a isospin. So, indeed I have pi plus which is the i equals 1, i z equals 1 state, pi 0 i equals 1 i z equals 0 state and pi minus which is i equals 1 i z equals minus 1 state. Isospin and i z are 2 labels like the charge or like the spin of a particle. They are quantum numbers corresponding to a particle and 2 particles will differ at least in 1 quantum number. These are just examples of quantum numbers which label particle states.

(Refer Slide Time: 00:37) In any case that was a digression, but in the context of what I have done here I could think of this whole exercise as being done in the context of combining 2 isospin equals half objects. In other words, I could think of this as i 1 the isospin of the 1st particle and that as i 2 the isospin of the 2nd particle. As I said this is possible because isospin follows the same algebra as the angular momentum, orbital angular momentum or spin. And therefore, I simply have to replace j by i, j z by i z and carry on for bookkeeping purposes. (Refer Slide Time: 16:20) So, returning to this problem. I have the following situation suppose I have 2 beans interacting strongly, one is 2 beams of nucleons. So, beam 1 has protons and neutrons beam 2 has protons and neutrons.

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Now, when I look at it as a couple system, as a composite system of 2 nucleons of course, if I think of them as protons and neutrons the proton state is half, half. As I have just now explained the neutron state is half, minus half. This is the uncoupled situation where I think of them as independent subsystems, but I could think of 2 protons, one from beam 1 and the other from beam 2.

Now, if I did that it would be represented by half half with half half. This is beam 1 and this is beam 2. If you wish you could call this beam a and that as beam b. Now I could think of 2 neutrons and that would be a half minus half with the half minus half. These are basis states in the uncouple basis. Of course, I could use my shorthand notation and just call this half, half semicolon half, half. So, that will be like i i z, i 1 i 1 z, i 2 i 2 z and so on. So, that is analogous to j 1 m, 1 j 2 m 2. But, I could think of them as a coupled state. So, for instance I can have protons from the 1st beam interacting with protons from the 2nd beam that is the initial state to give me a final state of 2 protons. So, the possibilities are the following: The question to be asked is, how do I couple to i equals half objects and what is the coupled state?

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As you know the couple states are 1 1. So, this is i i z in the couple basis that is my notation, because I am not able to put it in bold as a bold ket and therefore, I do this. So, the uncouple basis states would be half half, half minus half half half half half. But, that is like combining 2 protons and therefore, this would be just be identical to the statement that in the uncouple basis I have 2 protons. In the coupled basis that amounts to an i equals 1, i z equals 1 state. Then I have i equals 1 i z equals minus 1 state in the couple basis. This should have come by working with i 1 z equals minus half, i 2 z equals minus half because I could have got i z, the z component in the couple basis to be minus 1 only if I had a minus half here and a minus half there.

This is simply analogous to m equals m 1 plus m 2 and that is like saying that I have combined 2 neutrons. So, I could have written this. But, I can also have a situation where I can have i equals 1 with i z equals 0 and that could have come by taking a proton from the 1st beam combining it with the neutron from the 2nd beam or by taking neutron from the 1st beam and combining it with the proton from 2nd beam and this is my notation. This is for the 1st beam, 2nd beam, 1st beam, 2nd beam.

But these are different states because this is a times half half, half minus half plus b multiplying half minus half half half. But we had worked out these c g coefficients earlier and we know that 1 0 is simply 1 by root 2 n p plus p n. The singlet state 0 0 is orthogonal to the state and we could have written that and we showed that this is 1 by root 2 n p minus p n. In the language of protons and neutrons and isospin this is what it is.

(Refer Slide Time: 19:26) Earlier on I have written in terms of j and m in the following manner. So if I call this i and I call that i z, that is a proton neutron minus neutron proton. (Refer Slide Time: 21:05) Well, I have written neutron proton minus proton neutron here, but as I said it is a relative sign that is important and I have a pulled out of 1 by root 2 and put in a relative negative sign. So, this is the singlet state of isospin and these are the triplet states of isospin.

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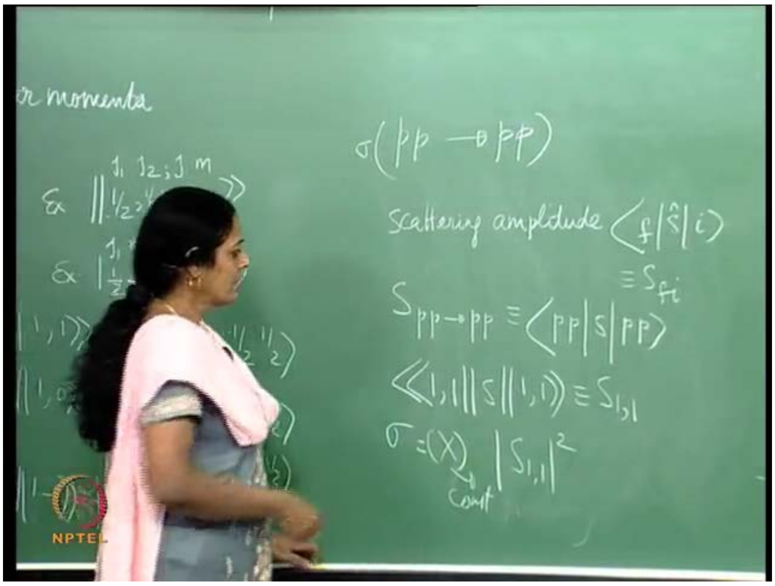

Now, given these I ask the following question: Suppose I look at the process proton proton interacting strongly to produce 2 protons. Experimentally, what we observe in the laboratory is the cross section for this process the scattering cross section for 2 protons interacting to produce 2 protons strongly.

Now, if that is what I was looking at. I compute the scattering cross section in 2 stages. 1st of all there is a scattering amplitude which I will denote by S initial state to final state. This is an operator it depends upon the interaction Hamiltonian. I could just call it S f i, so this is an operator and this is the matrix element it takes it from an initial state to a final state. So, in this context I have the scattering amplitude for p p going to p p and that is the same in my notation as this object.

In other words, I have to find out what is the probability that if I started with the combination 1, 1 I end up with the same combination after the interaction I get 1, 1. I want to denote this by S 1, 1 that is what I put down there. The scattering cross section is essentially apart from some constant factors which would not change for all these processes, because it is based on kinematics, it is based on the phase space available, it is based on the energy and the momentum available and that does not change. Apart from this overall constant, this is modulus of S 1, 1 the whole square.

And this is the matter of derivation, but we will take this as a definition. The scattering cross section is measured experimentally. This is like an amplitude which mod squares to give me the cross section. It is the probability that 2 protons interact with each other to produce 2 protons.

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Similarly, I can ask what is the probability that if I prepare an initial state very carefully with i equals 1, i z equals 0. What is the probability of starting with that initial state and ending with that same initial state? In other words, what is the scattering amplitude for a state which is prepared very carefully as n p plus p n by root 2 going to a final state which is also given as a n p plus p n by root 2. So, sigma for this process the scattering cross section for an initial state carefully prepared as n p plus p n by root 2, going to n p plus p n by root 2 is essentially X S 1, 0 mod square.

Similarly, sigma n n going to n n and now the notation is evident an initial state of 2 neutrons going to a final state of 2 neutrons is S 1, minus 1 the whole square. Now, I could have prepared it in the state ket n p by root 2 minus ket p n by root 2 which is what I have there. So, the probability S n p minus p n by root 2 going to itself. This mod square apart from a constant factor is sigma n p minus p n ket by root 2 going to n p minus p n by root 2. And, this is what I have. These are the various possibilities that I have, assuming that strong interactions conserve isospin and i z.

(Refer Slide Time: 27:22) So I will not for instance ask: What is the probability that 1, 1 goes to 1, 0? Because in strong interactions i z is conserved which means that the initial state's i z value is equal to the final state's i z value. Similarly, the initial state's isospin value is equal to the final state's isospin value and therefore, there is nothing like 1, 1 going to 1, 0. The labels have to be the same in both the cases. So, these are the only possibilities.

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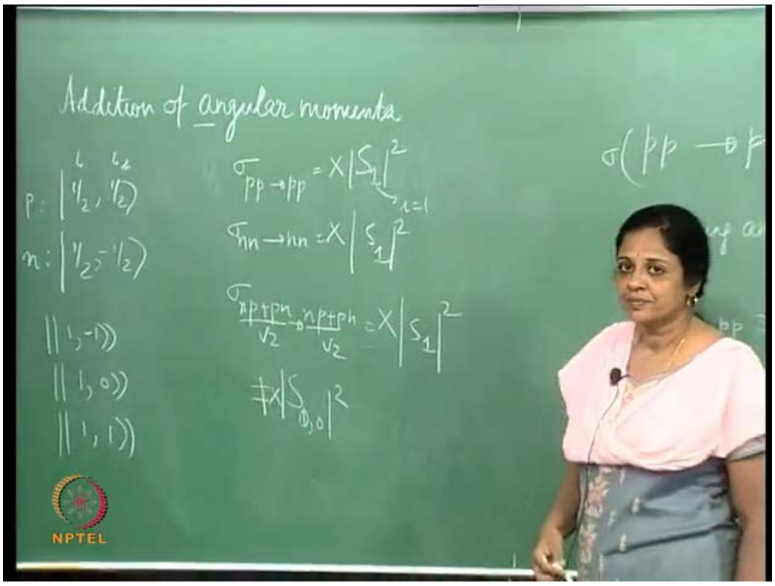

Now I find something interesting I know that I cannot see the difference between the proton and the neutron. This is an i equals half i z equals half, this is an i equals half i z equals minus half in a world of strong interactions. So, given an isomultiplet that means once you fix i, changing i z in steps of 1, takes you to various particles within that same multiplet and with strong interactions alone I cannot see the difference between the various particles that belong to the same isomultiplet. Obviously therefore, I cannot see the difference between 1, minus 1 1, 0 and 1 1, 1 because these are all states with the same isospin value, made of the same state of particles 2 nucleon system with i equals 1, but the i z value changing and therefore, I will use a shorthand notation.

(Refer Slide Time: 27:22) I will drop the 3rd component because it does not matter cannot see the difference between couple state 1, 0 1, minus 1 and 1, 1 and therefore, the cross section can be now written in the following manner: sigma p p going to p p is an overall constant S 1 mod square this 1 stands for i equals 1 sigma n n going to n n so same S 1 mod square the only difference being that this is the 1, minus 1 state, but as I said the 2nd index does not matter within an isomultiplet and sigma n p plus p n by root 2 which is the i equals 1 combination going to itself .

But clearly this is in general not equal to 0, 0. Because, this is a different isospin valued object. This is an isosinglet and that is the part of the same isotriplet. What is that mean? It means the following: the cross section of the probability of 2 protons in the initial state interacting strongly to produce 2 protons. This probability of the cross section for p p going to p p is the same, is equal to the cross section for 2 neutrons interacting to produce 2 neutrons. And indeed is the same for a neutron from the 1st beam and the proton from the 2nd beam, a proton from the 1st beam and a neutron from the 2nd beam taken and prepared very carefully in the isospin 1 state that is as part of the isotriplet going to itself.

So, these cross sections are the same and that is not the same in general as the cross section for the isosinglet going to itself. So, if you prepare the state combining a neutron and a proton as n p plus p n by root 2 going to itself. That cross section is the same as a cross section for p p going to p p or n n going to n n, but if you prepare this combination in the antisymmetric state n p minus p n by root 2. By antisymmetric I mean that if you interchange the neutron with the proton you pick up an overall negative sign. A symmetric one means that you pick up an overall positive sign as happens here.

So, if you prepared it in the antisymmetric state or in the isosinglet state the cross section is not the same as these. This is popularly called charge independence of nuclear forces, because nuclear forces I told you is an example of the strong force. So, charge independence of nuclear forces simply means the following, it is irrelevant whether you take neutrons or protons, the fact that the proton is electrically charged and the neutron is electrically neutral does not matter. Provided you prepare the 2 nucleon system in the same isospin state that means in i equals one state or the isotriplet state.

Then, the cross sections are the same. The only difference that can be seen in the laboratory is because of this, because when a neutron interacts with the proton it could be either in the i equals 1 state or in the i equals 0 state, and the small difference between this and that comes because of this extra piece which makes a contribution in neutron proton interactions. This has been experimentally verified and is popularly it goes under the name charge independence of the nuclear forces.

So, nuclear forces are charge independent, provided we are talking about the same isospin state i equals 1 or isotriplet. In the isotriplet state, I find that these cross sections are the same and that is not the same as the cross section for the isosinglet going to itself. So, this is a very important and useful experimentally verified example of addition of angular momenta except that I have also use this opportunity to introduce a new quantum number isospin. It is very important in strong interaction physics and i and i z are important quantum numbers which label particles that interact strongly hadrons.

Now, having demonstrated this I will go ahead and illustrate addition of angular momenta in more detail by conducting the following exercise. I will now combine a j equals 1 state with the j equals half state.

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In other words, the exercise is the following: j 1 is 1, j 2 is half. So, we are going to combine these 2 objects. So, j takes values 1 plus half to 1 minus half in steps of 1. So, j takes values 3 by 2 and half. Corresponding to j equals 3 by 2; m can take various values. So, let me write all the coupled states j equals 3 by 2 so m can be 3 by 2, but m can be half. It goes down in steps of 1, m can be minus half and m can be minus 3 by 2 and if j equals half; m can be half m can be minus half. So, these are the various couple states that are possible in this context. What about the uncoupled basis state? So this is the coupled basis state.

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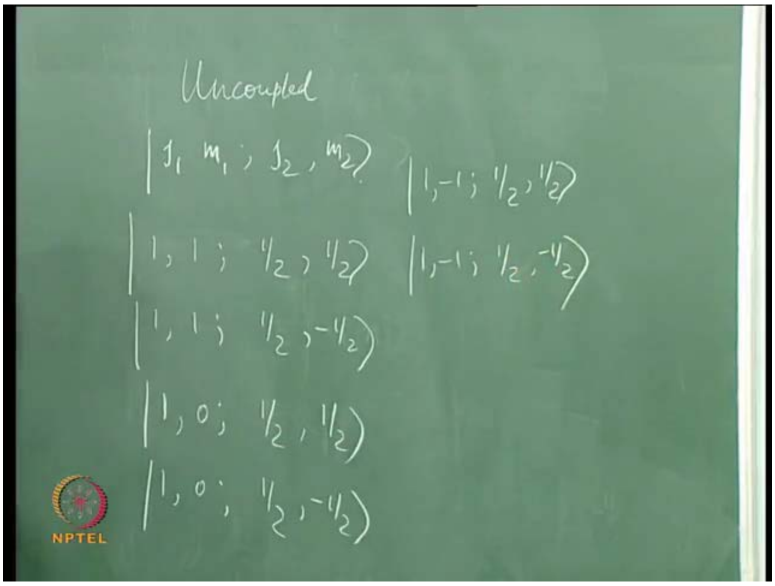

Now, the uncoupled basis that j 1, m 1, j 2, m 2 I have half half combining with 1 so perhaps I could write 1 1 combining with half half, 1 1 combining with half minus half then 1 0 combining with half half, 1 0 combining with half minus half, 1 minus 1 combining with half half and 1 minus 1 combining with half minus half. So, the labels are clear that is j 1, m 1, j 2 m 2 that is what I have. So, I have the following 6 states. You will recall that there is 2 j 1 plus 1 times 2 j 2 plus 1 states.

(Refer Slide Time: 36:34) And here too I have 6 states that is four of them are here. This is a j equals to 3 by 2 quanterd and this is a j equals half doublet and therefore, I have 6 states. So, the number of basis states match and this is a very important thing to verify when one does a calculation. So, I would like to find the c j coefficients in this context by expanding the coupled states in terms of uncoupled basis.

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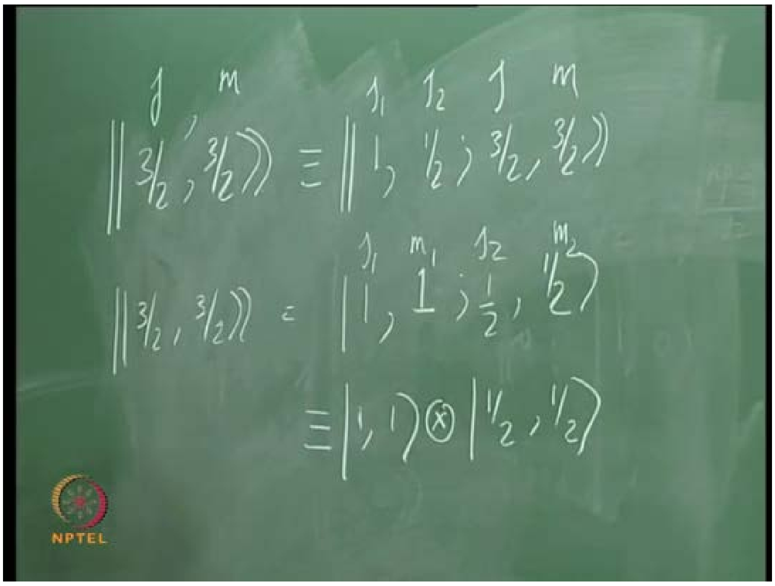

So, let us start with 3 by 2, 3 by 2. Remember that this really amounts to j 1 is 1, j 2 is half, j is 3 by 2 and m is 3 by 2, but I am dropping j 1 and j 2 and just using j and m as the labels for the coupled state. But 3 by 2, 3 by 2 could have come only in the following fashion. It is a stretched case, because if I want m equals 3 by 2 then m 1 should have been 1 and m 2 should have been half. The only possibility is for m 1, as you can see are 1 0 and minus 1 and for m 2 are half and minus half.

So, I just have 1, 1 with half, half and this is the uncoupled basis and therefore I do this. This is j 1, m 1, j 2, m 2 of course, by this I mean the following: j 1, m 1, j 2, m 2, but I am going to use this as a simple way of writing it. So, this is a stretched case there is no other way I could have got 3 by 2, 3 by 2 and therefore, the c g coefficient is 1 and that is a stretched case.

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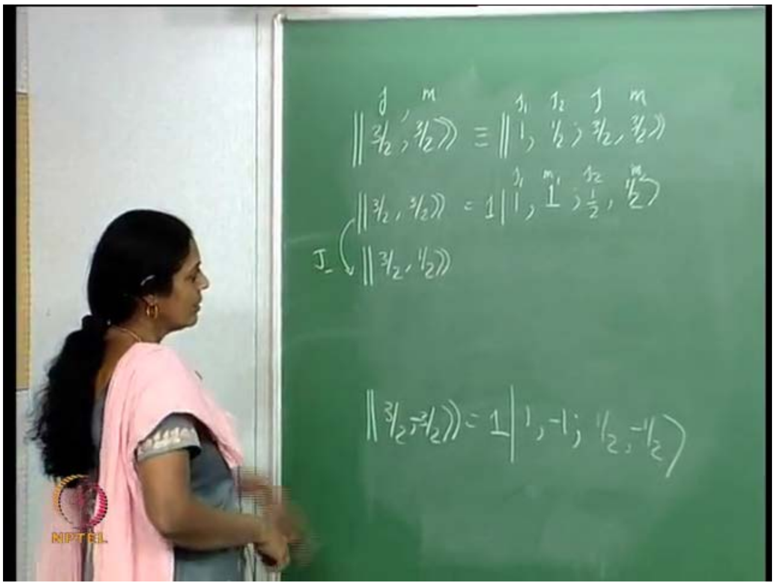

Similarly, 3 by 2, minus 3 by 2 should have come by combining m 1 equals minus 1 with m 2 equals minus half. So, while j 1 and j 2 are fixed, m 1 and m 2 are now minus 1 and minus half and that is only way I could have got 3 by 2 minus 3 by 2 therefore, that is a stretched case. I need to find 3 by 2 half and, as I pointed out yesterday I can do this by using j minus here correspondingly, j 1 minus and j 2 minus there.

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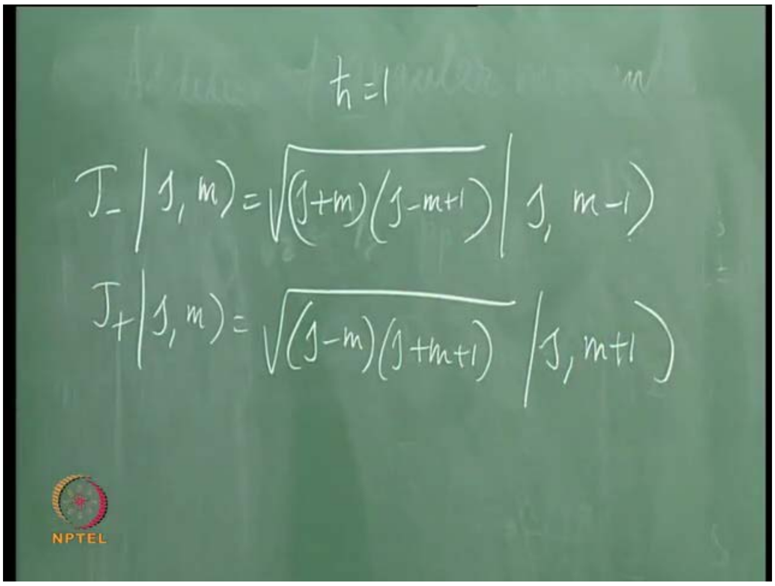

I need to remember the following: j minus acts on a state j m to give me root of j plus m times j minus m plus 1. I have set h cross equals 1, j, m minus 1. And, j plus acts on j m to give me root of j minus m times j plus m plus 1 j m plus 1. These are the lowering and raising operators respectively, so I use that.

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And therefore, when I use j minus here the coefficient is root 3 where I have used j is 3 by 2 and m is 3 by 2. Here j 1 minus acts on this leaving that alone, pulls out a coefficient j 1 plus m 1 times j 1 minus m 1 plus 1, reduces m 1 to 0, leaves j 2 and m 2 as such, j 2 minus acts on this and leaves this alone. So, j 1 and m 1 do not change and I pick up a coefficient root of j 2 plus m 2 times j 2 minus m 2 plus 1 and m 2 gets reduced by 1, so I have this. In other words, I can write 3 by 2 half as the state root of 2 thirds 1 0 half half plus root of 1 thirds 1 1 half minus half. So, this is what I have.

I need to get the state 3 by 2 minus half in terms of the uncoupled basis. It is worth finding out the mod square of the c j coefficient sum together gives me unity; that is the 2 third plus 1 thirds and this is the thing that needs to be checked out. Further, if I want a half here that is the 0 plus half and that is the 1 minus half. These are things that should be checked out at various stages. So, if I want to do 3 by 2 minus half, I can use j minus on this. Well, there are 2 terms and a little bit more work to be done.

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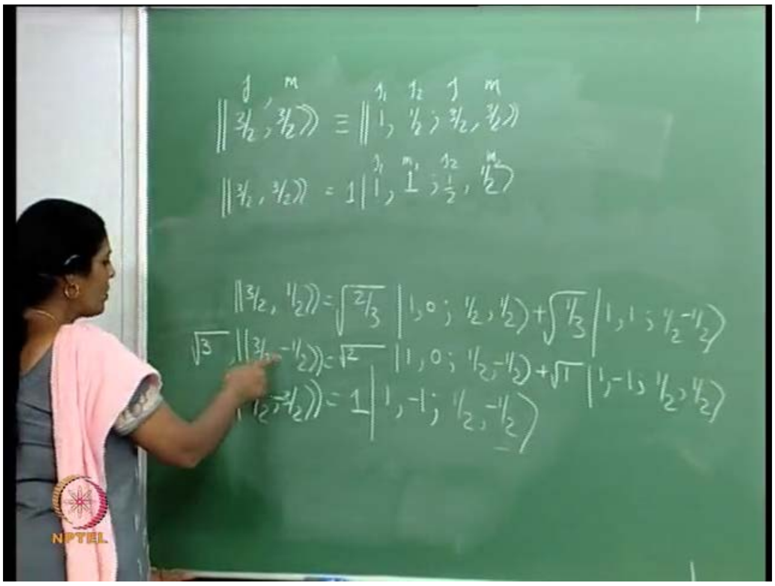

I could have done a j plus on this, that is just one term on which I need to work. So, if use a j plus I remember that the coefficient here is j minus m times j plus m plus 1 so I have this and here I use j 1 plus which is a j minus m times j plus m plus 1 takes 1, minus 1 to 1, 0 and leaves the other state as such .

Then I have j 2 plus acting on this. It leaves 1, minus 1 alone and pulls out half plus half times half minus half plus 1 and then takes minus half to plus half.

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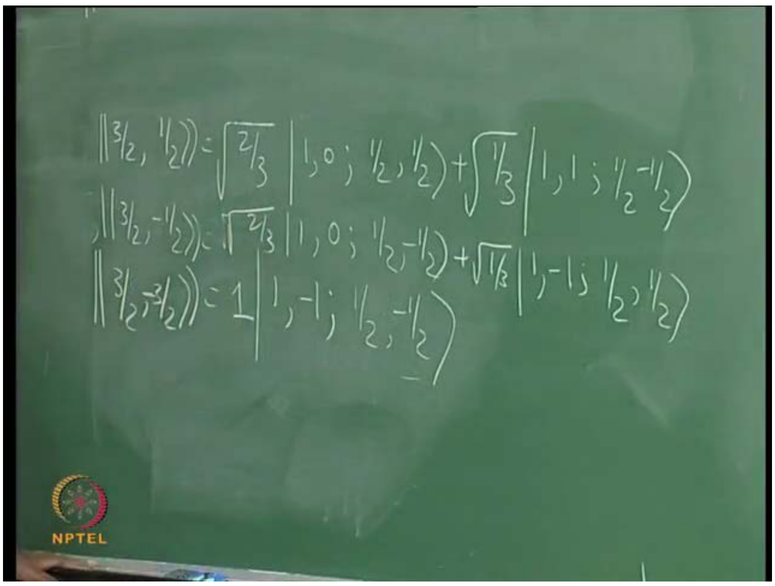

In other words, 3 by 2 minus half is root 2 third times this plus root 1 third times the other state so I can write this as root of 2 by 3 plus root of 1 by 3. Once more it is clear that things add up root 2 3rd square plus root 1 3rd square is 1.

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So, I have the various states here. These are the stretched cases and then I got these by using j minus and j plus so I have already written these four states in terms of the uncouple basis. I need to find half, half and half, minus half.

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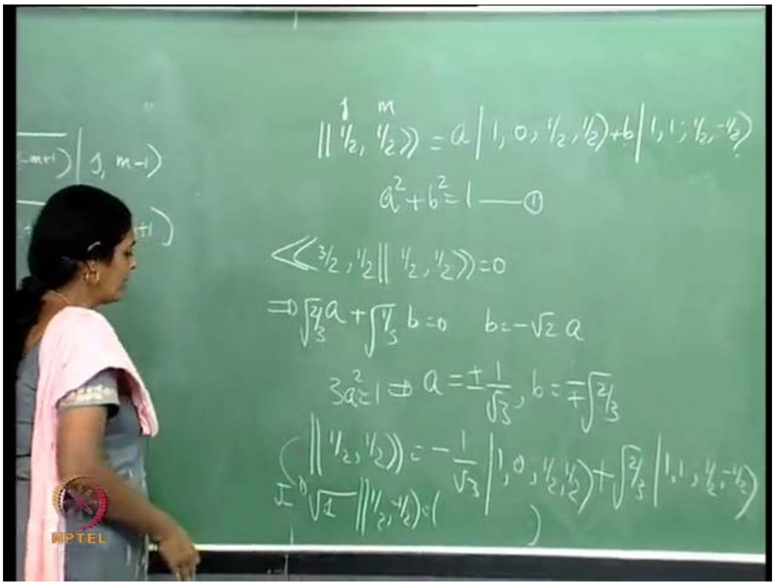

Now, I do the following. Consider the state half, half this is j and that is m. (Refer Slide Time: 45:25) Half, half is like 3 by 2, half in the sense, it has the same uncoupled states contributing to it 0 plus half and a 1 minus half, but the coefficients would be different. So I write half, half as a times 1, 0 half, plus b that is the coefficient and I have 1, 1 half, minus half.

Given the fact that a square plus b square is 1; I need to determine these c j coefficients. This is easily done because this is one equation the other equation is this 3 by 2 half is orthogonal to half half. Of course, all these states are orthogonal to half, half, but what is of relevance is the fact that 3 by 2 half is orthogonal to half half, because in the uncoupled basis they are superpositions of the same state, and this implies that root of 2 by 3 a plus root of 1 by 3 b equals 0 or b is minus root 2 a.

So, I can use that and so I have a square plus b square and that gives me a 2 a square. So, I have 3 a square equals 1 which gives me a is plus or minus 1 by root 3 which tells me that b is minus or plus root of 2 by 3. This is where I can use the convention. I am going to look at the 2nd state and look at the m values well, this as a higher value compare to that. So, my convention is to put positive sign here and a negative sign there.

I stick to this convention throughout the problem. So I have half, half is minus 1 by root 3 1 0 half half plus root of 2 by 3 1 1 half minus half. Again the mod square when summed over add to 1 that is the one third plus two third which gives me a 1 and that is what I have for half, half. I could have written plus 1 by root 3 here and a minus root of 2 by 3 there. As I said, I have used this convention we can get half, minus half in two ways. (Refer Slide Time: 45:25) We can use the facts that half, minus half is orthogonal to 3 by 2, minus half and repeat the procedure or can use a j minus on this and get half, minus half so that gives me a 1 half, minus half. Minus 1 by root 2 is fixed j 1 minus acts on this leaving this alone plus j 2 minus acts on that leaving this alone.

Similarly, plus root of 2 by 3 j 1 minus acting on leaving that alone and j 2 minus acting on that leaving this alone. And, once that is done we have the state half, minus half which can be simply found out. That is one way of doing it. As I said the other way is to use the fact that half, (Refer Slide Time: 45:25) minus half is orthogonal to 3 by 2, minus half and find out the coefficients which too can be done, but remember the convention I compare the 2nd entries and this m 2 there is smaller than m 2 here. That comes with the positive sign and this comes with a negative sign. I stick to that convention during the problem. I leave it to you complete this exercise.