Quantum Mechanics- I

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Lecture - 17
Addition of angular Moment – I

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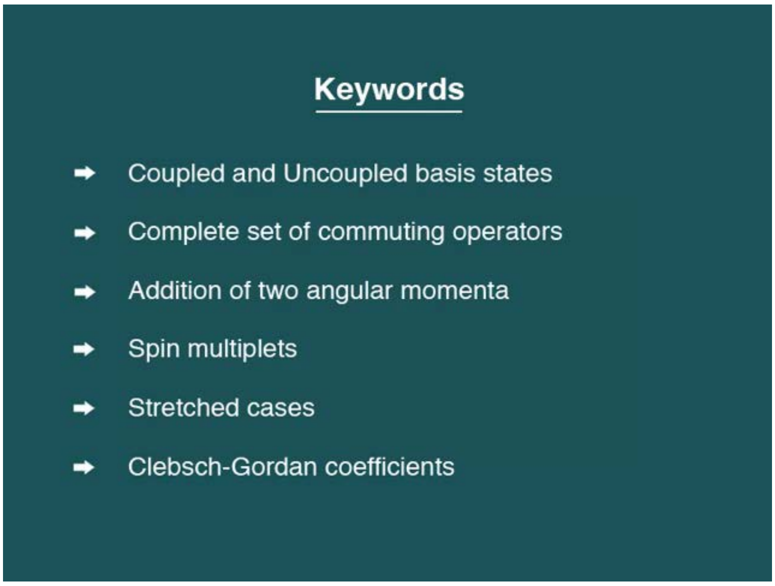

We have been looking at, two composite systems, by composite systems. I mean a system which is made up of two subsystems, for instances, the oscillator, which had a simple harmonic oscillator, along the x axis and another along the y axis if you wish. That total system was a composite system. Yesterday, we looked the beam splitter. Now, the beam splitter again has two input ports and I feed beams of photons, through both the input ports and look at the system has a whole. And what happens to the system after it passes through the beam splitter. Today, I wish to look at another composite system, which is two spins, spin 1 and spin 2.

But, in general these need not be spins. They could be any form of angular momentum and therefore, I would be looking at 2 particle perhaps, which we have got angular momenta, J 1 and J 2 respectively. 1 and 2 being labels for the particles, particle 1 and particle 2.

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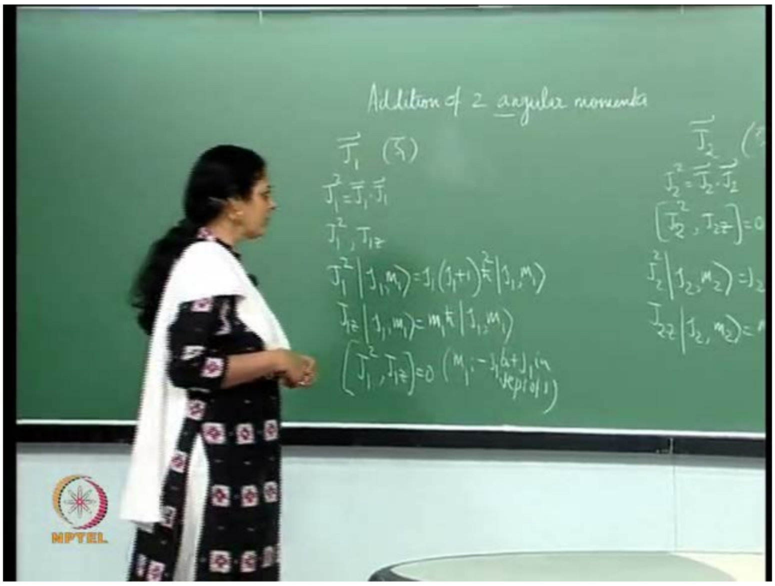

So, now today I will be talking about, addition of 2 angular momenta. So, there are two systems, given by vectors J 1 and J 2. J 1 squared is J 1 dot J 1 and I can find simultaneous Eigen states of J 1 squared and J 1 z. The 1 here, as I have mentioned earlier, refers to the fact that we are talking about the 1st system, one of the two systems. So, J 1 squared acting on a state, label by J 1 m 1 gives me, J 1 into J 1 plus 1 h cross squared, J 1 m 1 and J 1 z acting on the state, J 1 m 1 gives me, m 1 h cross J 1 m 1. So, the commuting operators, for the 1st system are J 1 square and J 1 z. And we can have a complete set of Eigen states, label by J 1 comma m 1, with m 1 taking values, minus J 1 to plus J 1 in steps of 1.

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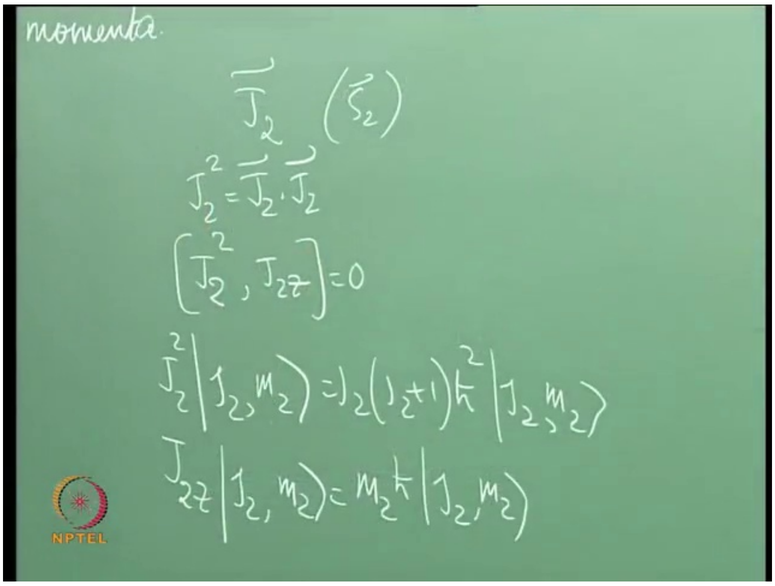

Similarly, I have the 2nd system. And here my notation is going to be following. I have J 2, the 2 stands for the 2nd system and J 2 squared, is J 2 dot J 2. We have already seen that, J 2 as also J 1 are vectors under rotations. Once more I have J 2 squared and J 2 z as the operators with simultaneous commute with each other, for the system 2 and I can write the following. There are Eigen states common Eigen states, a complete set of common Eigen states, of J 2 square and J 2 z and the Eigen value equation is this.

So, here we are, now if these 2 systems, these 2 spin state, now, if I were talking about spins. If you recall I would use the notation, s 1 and s 2, to denote the spin of the 1st system and the spin of the 2nd system. So, I could talk of 2 spin multiples for instance. In general I would like to refer to them as J 1 and J 2, to show that they could be any kind of angular momentum that is any kind of generators with satisfies the angular momentum algebra.

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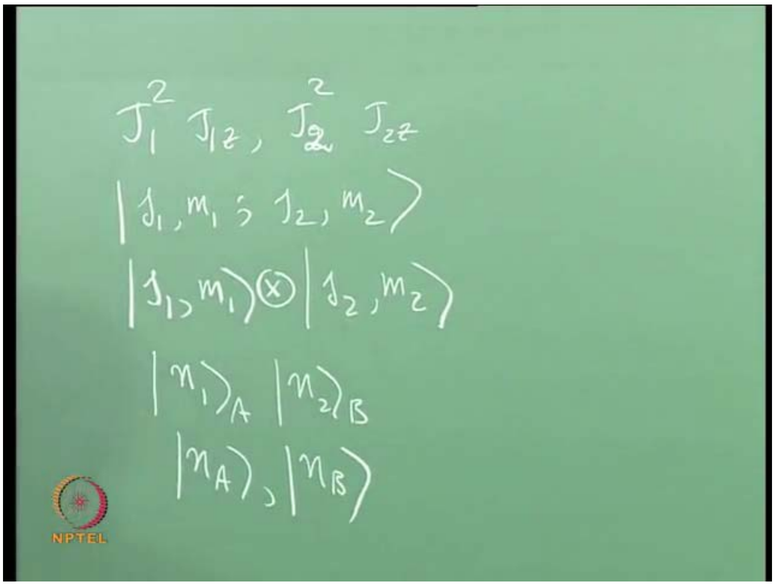

Now, if I were talking about these 2 systems, 1 and 2 as non interacting systems. Clearly it is obvious that, the following operators commute with each other. J 1 squared J 1 z J 2 squared and J 2 z commute with each other because clearly operators corresponding to the 1st system, commute with operators corresponding to the 2nd system. And therefore, if I have to describe the 2 systems as a whole, I would use the following labels. J 1 m 1 J 2 m 2 by this short hand notation I mean the following. We have come across this kind of situation earlier when we discussed composite systems.

You will recall that in the case of the harmonic oscillator. We had state labels, n 1 corresponding to system A and n 2 corresponding to system B, perhaps you would like to call it n A and n B. In the same sense here, I have the following basis sates J 1 m 1, for the 1st system and J 2 m 2 for the 2nd system.

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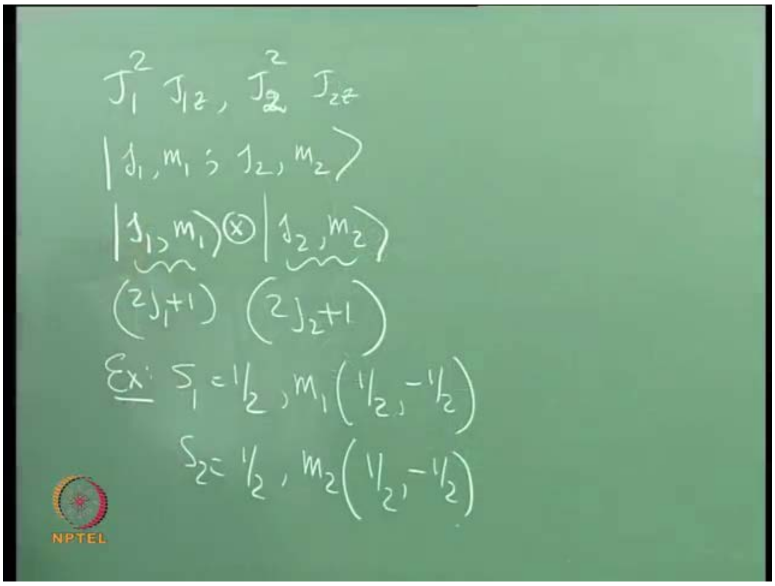

There are 2 J 1 plus 1 states here, because, m 1 can take values, minus J 1 to plus J 1 in steps of 1. There are 2 J 2 plus 1 states here, corresponding to this system. And therefore, the total number of states, the total number of basis states, of the 2 systems taken together. That is as a composite system, the total number of basis states are 2 J 1 plus 1 times 2 J 2 plus 1. Let me given an example, suppose, we were talking about 2 spin half particles. So, let me call them, S 1 is half m 1 can take values half and minus half. Similarly, S 2 is half and m 2 can take values half and minus half.

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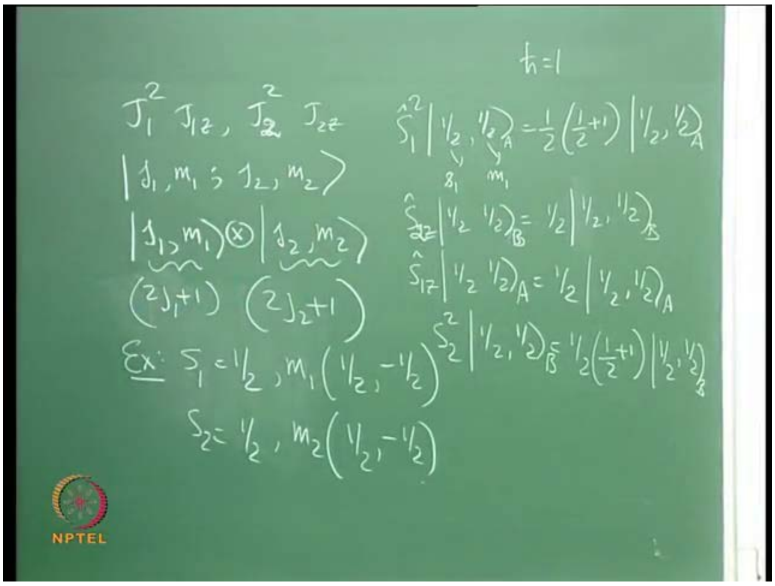

The understanding being this, that S 1 squared acting on the state half half. This is s 1 this is an operator. This is m 1, gives me half times, half plus 1, in units of h cross equals 1 half into half plus 1 half half. And S 1 z acting on half half, is half in units of h cross half, half. Similarly, for s 2, I have s 2 square acting on the system 2 state half half, gives me half times half plus 1 half half. I could do the following thing, (Refer Slide Time: 06:50) instead of J 1 and J 2, I have used S 1 and s 2. And therefore, I can make it convenient by denoting this, by A and this by B. As I did in the case of the harmonic oscillator, to show that system 1 has kets denoted by a subscript A and system 2 has kets denoted by subscript B, S 2 z acting on half half, gives me half half half.

So, that is the way it is, similarly, S 1 z acting on half half A, gives me half, ket half half A. S 2 squared acting on half half B, is half times half plus 1 half, half B and so on. The notation must now be evident. System 1 has kets with a subscript A here and system 2 has kets, with subscript B. The point is, what are basis states of the couple system, or the composite system?

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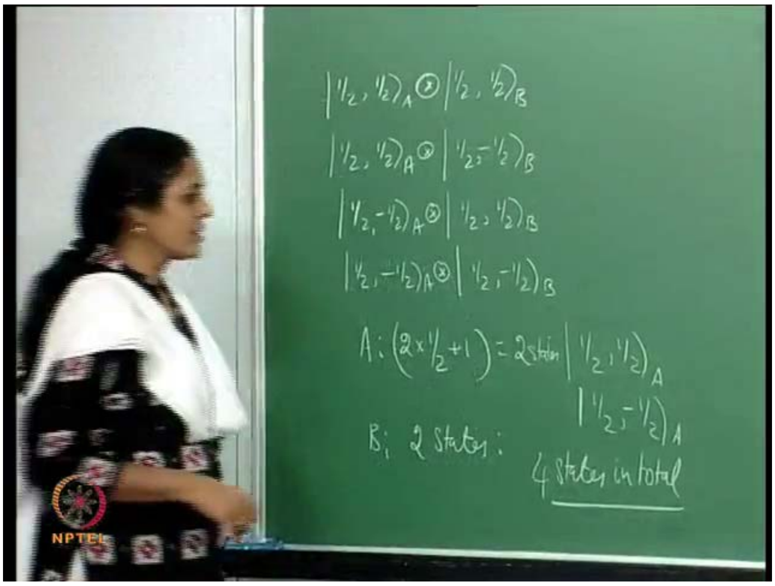

Well the basis states of the composite system, would be the following. They would be half half of A, half half of B, half half corresponding to A. With half minus half corresponding to B, half minus half corresponding to, with half, half corresponding to B and half minus half corresponding to A, with half minus half corresponding to B. So, I have 4 states. System A has 2 J 1, or 2 x 1 plus 1 states, which is 2 states. They are half

half A and half minus half A. Similarly, system B has 2 states and the total is 2 times 2, which is 4 states in total. So, these are the basis states of the 2 system taken as a whole and I have given you the 4 states here.

This is nearly as example in general therefore, (Refer Slide Time: 06:50) we have 2 J 1 plus 1 states, time 2 J 2 plus 1 states. And this is going to be the complete set of basis states, for the composite systems. I could well work with this set of basis states, provided there is no interaction between, J 1 and J 2. On the other hand if there is interaction between J 1 and J 2. Then, I could land the problem. It might be wiser to talk about the total angular momentum of the system as a whole and the corresponding 3rd component of angular momentum.

Because, the Eigen values, the Eigen states corresponding to the subsystems, may not be the relevant basis states here. The Eigen values corresponding to J 1 and J 2, may not be good labels to use and in fact they need not be conserved quantities because of the interaction. And therefore, it would be more sensible to look at the composite system in terms of the total angular momentum of the composite system and it is 3rd component and anything else it is commutes with.

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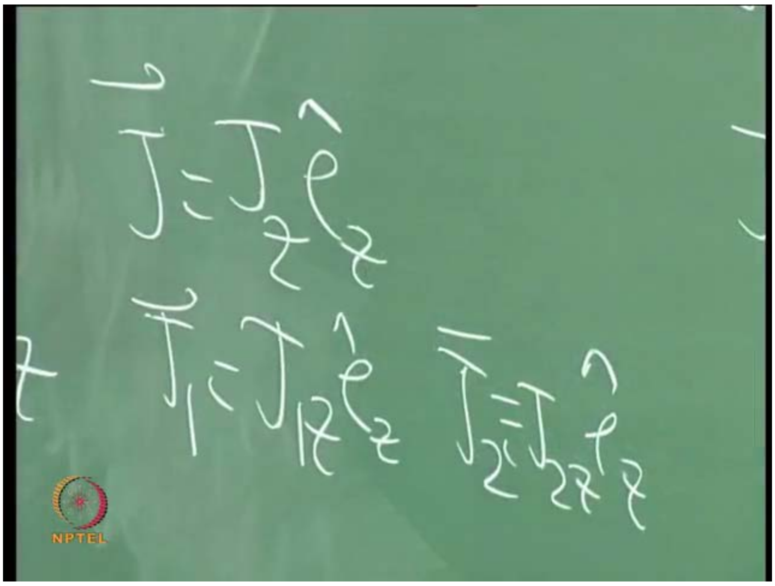

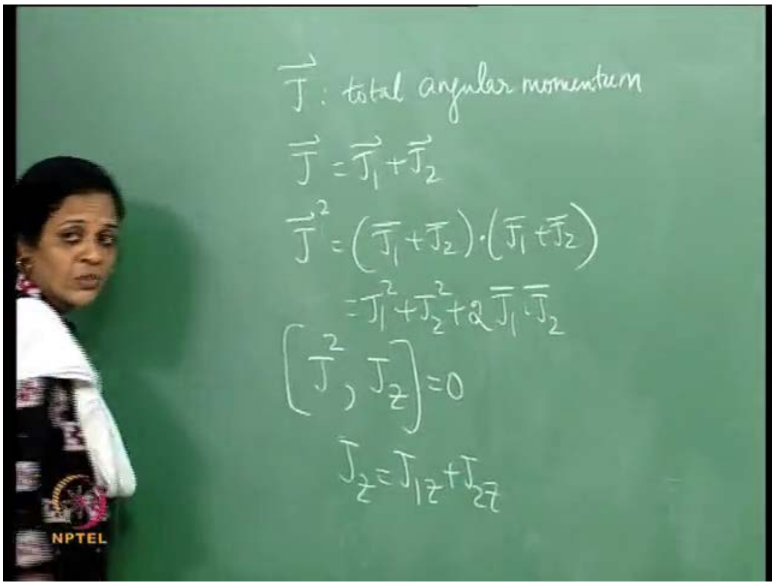

Now, if I way to talk of the total angular momentum, of the composite system. It is clear, that you should behave like any angular momentum. It should behave like a vector under rotations. And therefore, like all vectors the total angular momentum, must follow vector addition and I have J equals J 1 plus J 2, and therefore, J squared is J 1 plus J 2, dotted with J 1 plus J 2. But, that gives me cross term because, I have 2 J 1 dot J 2. Specifically, I should write J 1 dot J 2 plus J 2 dot J 1 and that is what I have written here, as 2 J 1 dot J 2, remember J 1 and J 2 commute with each other they are two different systems.

Now suppose, I find out the set of operators, that commute with J squared. It is clear that for corresponding to J squared, I can have J z, which will commute with J squared. But, J z is the 3rd component of the vector J, is a z component of the vector J. And like 3rd components, J z added simply adds up, J z is J 1 z plus J 2 z. You could think of J as a specific example, J z e z and J 1 as J 1 z e z. Similarly, J 2 as J 2 z e z and it is clear that J z is J 1 z plus J 2 z. That is in a very specific case. But, in general like components of a vector J z simply adds up, not vectorially. But, like scalars would add up and therefore, the Eigen value for J z, if I call that m, m would be equal to m 1 plus m 2, where m 1 and m 2 are given there.

So certainly, I can talk about J, where J times J plus 1 h cross square, is the Eigen value corresponding to the J square. And m the Eigen value corresponding to J z, J squared and J z having a complete set of common Eigen states and m being equal to m 1 plus m 2. But, these are not the only 2 operators that commute with each other.

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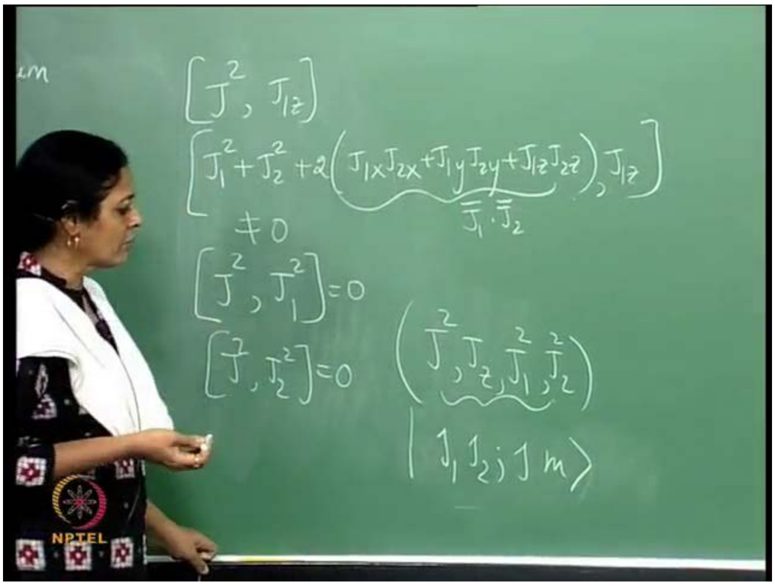

Look at the following, J squared commutator with J 1 z. So, this is the same, as J 1 squared plus J 2 squared, plus twice J 1 x J 2 x plus J 1 y J 2 y, plus J 1 z J 2 z. This is J squared and we wish to find the commutator with J 1 z. J 1 squared certainly commutes with J 1 z. They have a complete set of common Eigen states; label by J 1 m 1, J 2 squared commutes with J 1 z because they are two different systems. But, here J 1 x, J 1 z do not commute with each other. In fact they give me J 1 y. Similarly, J 1 y commutator with J 1 z, is proportional to J 1 x, the 3rd term of course commutes.

So, in general this is not 0. It is not possible for me to find a complete set of common Eigen states. of J squared and J 1 z. But, what about J squared and J 1 squared this is all right. Because, when I expand J squared in this manner, J 1 squared commutes with itself. J 2 square and J 1 squared commute, because, they are two different systems and this is. J 1 dot J 2, J 1 commutes with J 1 squared and J 2 commutes with J 1 squared.

Therefore, this is 0. Similarly, J squared, with J 2 squared is 0. So, this is what I have. I have a set of operators, J squared J z, J 1 squared J 2 squared, all of them commuting with each other. And therefore, if I were talking about the composite system as a whole and I would like to talk in terms of the coupled basis. By coupled basis I mean, when you coupled the individual systems perhaps through an interaction. And then, I look at the basis states relevant to describe the composite system as a whole. In this case, since these 4 operators commute with each other, I can label my state as j m and of course, j 1 and j 2.

This would be, the manner in which, I would label the basis states, when I look at the couple basis. If I were looking at the uncoupled basis (Refer Slide Time: 07:51) that is where the 2 spins were not interacting with each other. Then, I could well used a J 1 m 1 J 2 m 2, into get this notation nicely, I would like to call this, J 1 J 2 J m and so, that we may be clear in our minds, that this is the coupled basis and that is the uncoupled basis. I would like to draw this thick compare to that which is just (Refer Slide Time: 07:51) single lines, thin lines like that.

But, since that would be difficult, I am Just indicating it, by this notation. This notation is simply a notation, to show that I am now working with the coupled basis, whereas this was the uncoupled basis. I could work with the coupled basis, or the uncoupled basis, to describe the composite system, which is this sum of 2 angular momenta, sum within quotations. How we sum up? It is different matter. So let us look at the values that J and m can take.

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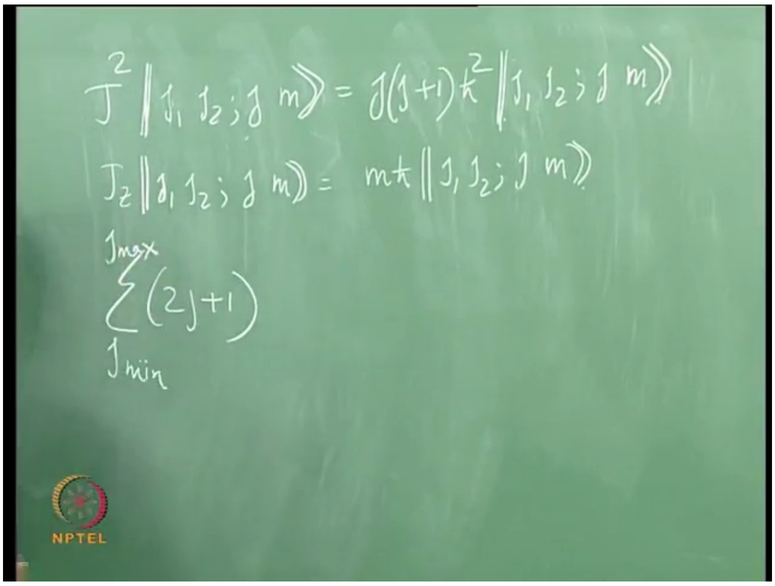

I now, have the following established. Because, we are talking about the couple states angular momentum, and therefore, I pull out J times J plus 1 h cross squared, J 1 J 2 J m. And J z acting on this state, gives me m h cross. So, this is what I have expect that because, these are angular momentum operators, m takes values minus J to plus J, in steps of 1. So, there are 2 J plus 1 values that m takes.

And therefore, the total number of basis states, for the couple basis, would be 2 J plus 1 expect that I have to sum over the minimum value of J min all the way to J max, in quantized steps, which I have to figured out. So, I need to find out the range of the values that J can take, corresponding to each value of J, m takes 2 J plus 1 values in steps of 1. J max itself can be found in the following manner.

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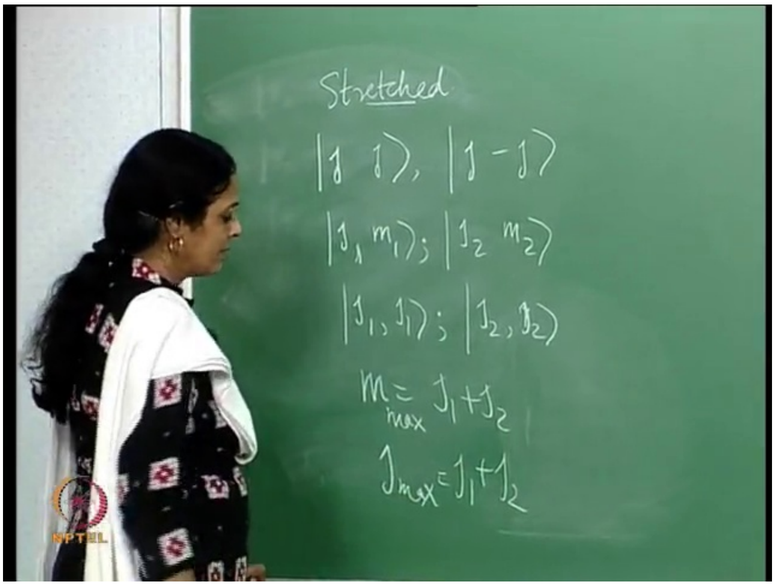

I define something called stretched case, when m takes the value, j or minus j I call them the stretched case. So ,this is the maximum value that m can take, for a given j and this is the minimum value that m can take, for the same j. Remember that a j multiplied, for a given value of j, has 2 j plus 1 states, with m ranging from minus j to plus j, in steps of 1. And therefore, in my example, where, I have J 1 m 1 and J 2 m 2, as the uncoupled basis states.

The maximum value that m 1 can take, is J 1 and the maximum value that m 2 can take, is J 2 and since, (Refer Slide Time: 12:35) J z is J 1 z plus J 2 z. The maximum value that m can take, is J 1 plus J 2. If m takes this maximum value, clearly J should at least be equal to m and therefore, the maximum value the J takes is J 1 plus J 2.

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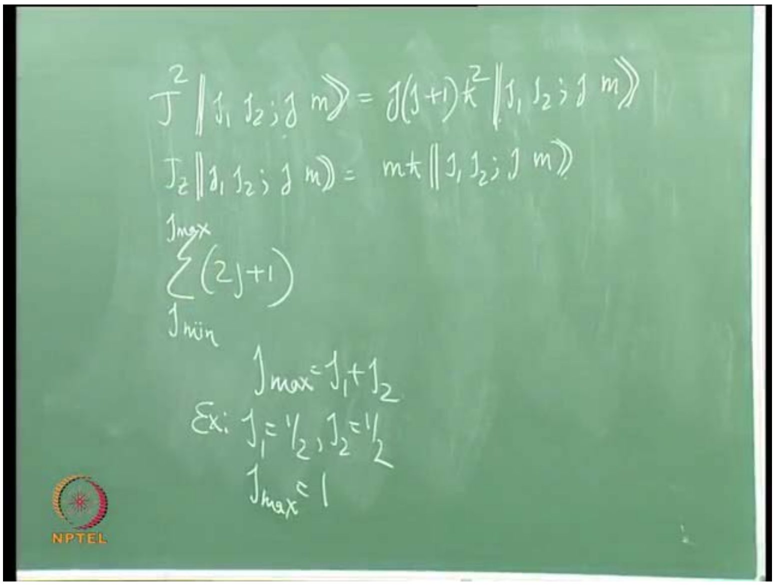

So, in my summation here, J max is J 1 plus J 2. As an example suppose, I where combining J 1 equals half, with J 2 equals half, J max is 1 and so on. I now have to find, J min but, before that I would like to see. What is the nature of quantization? What is the next value that j can take in this summation and that can be simply done as follows.

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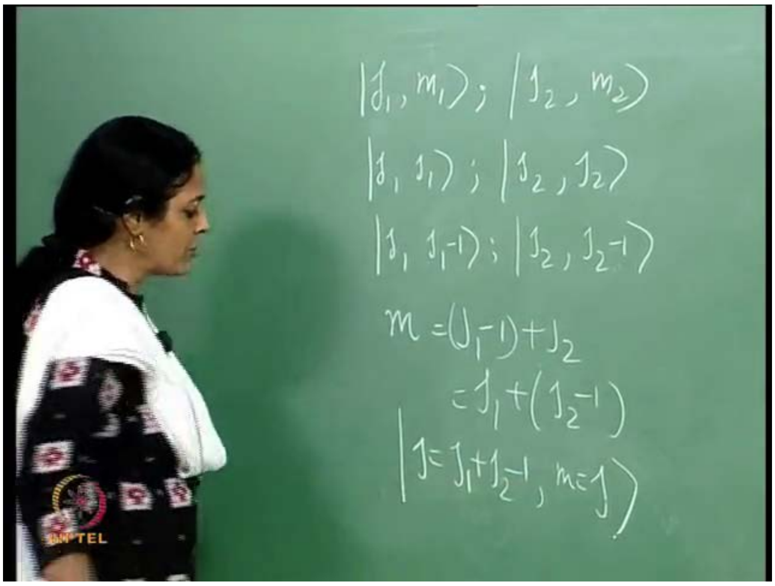

Given J 1 m 1 and J 2 m 2, the highest values that m 1 and m 2 can take, are J 1 and J 2. The next value could be this, because, m reduces in steps of 1 and therefore, the value that m can take, m is m 1 plus m 2 could be J 1 minus 1 from here, with J 2 from there. I could have got it the other way. This value could also be J 1 plus J 2 minus 1 from here combining with J 1 from there. Surely if m takes this value, J 1 plus J 2 minus 1, I could think of the stretched case, where j is equal to j 1 plus j 2 minus 1 and m equals j.

That is one manner that is one way in which m takes values j 1 plus j 2 minus 1. But, I have 2 states; I have 2 ways of combining m 1 and m 2. I have 2 different choices, which will give me the same value of m and clearly these are 2 different states. So, while one of them can be accounted for by saying, that j is j 1 plus j 2 minus 1 and m as a stretched possibility is also, j 1 plus j 2 minus 1. The other one came from here (Refer Slide Time: 21:33) itself.

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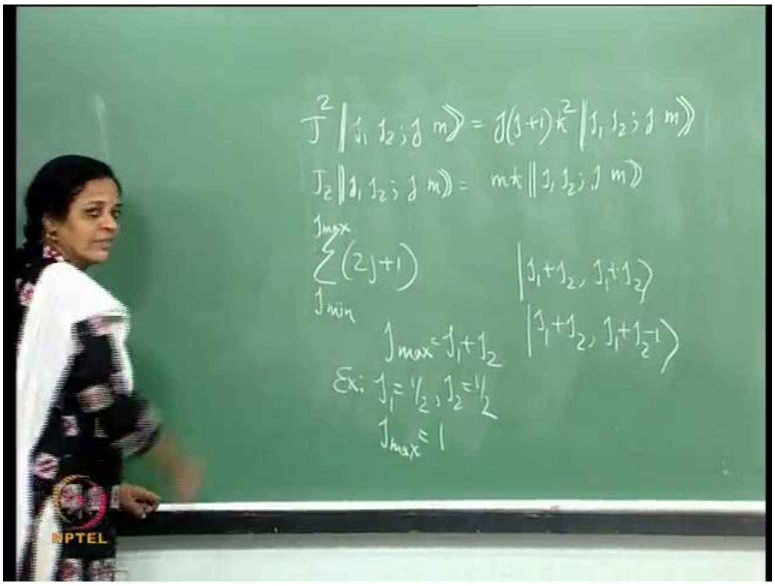

I started with j max equals j 1 plus j 2. So, what are the corresponding states? j 1 plus j 2 with j 1 plus j 2. The next state would be j 1 plus j 2, with j 1 plus j 2 minus 1. So, that is one possibility. That, I start with j 1 plus j 2 being the value of j and m reduces in steps of 1, I have 2 j plus 1 states, in this multiplate, where j is j 1 plus j 2. The other possibility for (Refer Slide Time: 23:49) m to be j 1 plus j 2 minus 1 is through this. So, the next value of j, is j 1 plus j 2 minus 1, I can argue further.

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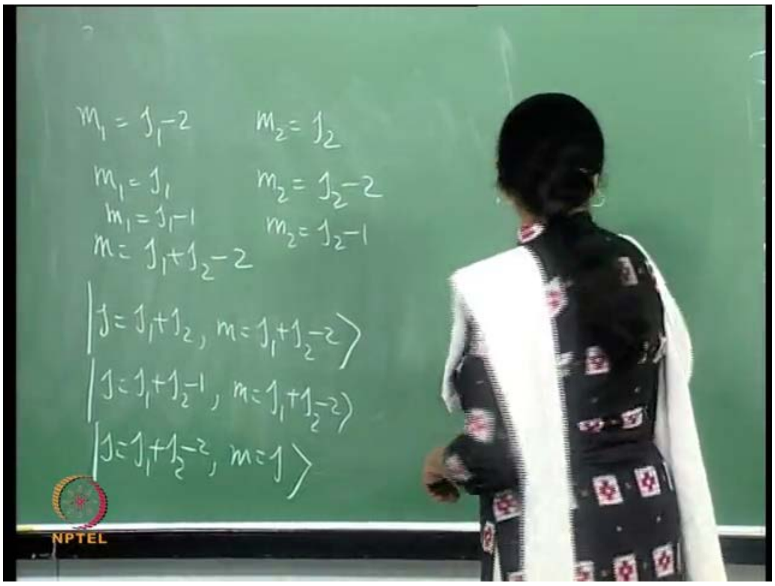

In the next stage, I would like at the following, m 1 is j 1 minus 2, m 2 is j 2, m 1 is j 1 m 2 is j 2 minus 2. Surely these are possibilities. Then m is j 1 plus j 2 minus 2. I can also have m 1 equal's j 1 minus 1, m 2 equals j 2 minus 1. So, there are three ways in which, m takes values j 1 plus j 2 minus 2. Surely this can come from j equals j 1 plus j 2, m equals j 1 plus j 2 minus 2. It can come from j equals j 1 plus j 2 minus 1, m equals j 1 plus h 2 minus 2 and the 3rd possibility, would have arisen from, j is j 1 plus j 2 minus 2 and m equals j. That is the same thing, j 1 plus j 2 minus 2.

So, I have accounted for all the three possibilities and these are three distinct states. I go on this manner and therefore, I realized that (Refer Slide Time: 25:38) J reduces insteps of 1. So, j starts with j 1 plus j 2, comes down insteps of 1 and those are the possible values of j. Expect that, I should now know what is j minimum.

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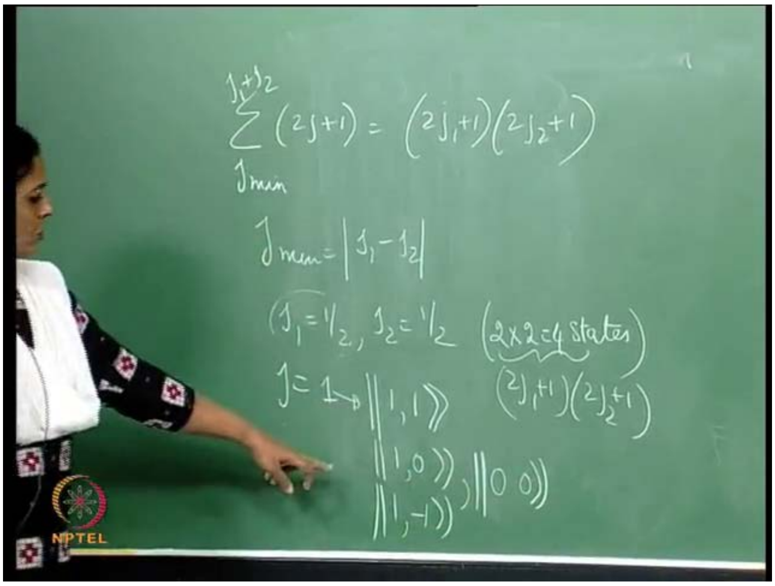

And that can be easily found out, by arguing in the following manner. In the couple basis, I have summation j minimum to j 1 plus j 2. Remember, this summation happens in steps of 1, of 2 j plus 1. Should be equal to 2 j 1 plus 1 times 2 j 2 plus 1 and then, I can work out. I can do the summation and work out what is j minimum and j minimum is modulus of j 1 minus j 2. Set of doing that I would illustrate this, again let us start with j 1 is half, j 2 is half. So, j the number of states would be 4, because here, I have 2 j plus 1 that is 2 states. Similarly, for j 2 I have 2 states. So, 2 j 1 plus 1, times 2 j 2 plus 1 in my example is 4 states.

Now, j takes values j 1 plus j 2, to j min. Well this certainly gives me 3 states, j is 1 m is 1 and these are coupled states, j is 1 m is 0 and j is 1 m is minus 1. That is 3 states, the minimum value is modulus of j 1 minus j 2, which is 0. That gives me one more state j is 0 and therefore, m is 0. So, these are 4 states in the couple basis. Can I have 4 states in the uncoupled basis? So, I find that the number of states match and that is the way you thought to be, quite independent of the kind of basis states that we select, they should form a complete set, not only that. The number of basis states, must be the same in various choices that we make for the basis states.

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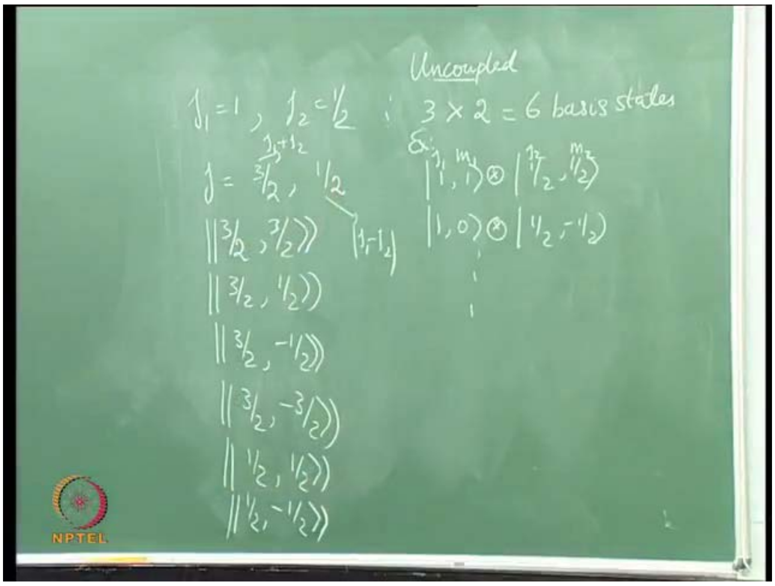

So, let me illustrate this with some more examples, j 1 is 1 j 2 is half. Recall from whatever, was done earlier on spin systems, that the value that j can take. In that case you would have call it as, the values would be either integer, or half integer positive values, in units of h cross. So, I have j 1 is 1 and j 2 is half. So, j takes values 1 plus half 2 1 minus half, modulus of 1 minus half, in steps of 1. What are the number of states in the uncoupled basis? I have states 2 j plus 1 states here, which is 3 states here and that is 2 states there.

So, I have 6 basis states, example: 1 1 with half half, or 1 0 with half minus half and so on. This is j 1, this is m 1, this is j 2 and that is m 2. I should account for 6 states here and that is true they are 2 j plus 1 states. So, that is for each j they are 2 j plus 1 states and for each j value, these are the m values. So, that is 4 states, corresponding to j equals 3 by 2, 3 half's and m takes values 3 half half minus half and minus 3 half's. So, this is the case j, minus j. That is the case j, j and m comes down in steps of 1. Then, I have j equals half.

So, that gives me other 2 states, half half and half minus half. That is a counting for 6 states. So, j reduces from j 1 plus j 2, to modulus of j 1 minus j 2 in steps of 1. Which is what we have got here and we have counted for the basis states. Now, we are in a position to look at addition of angular momenta, the problem simply reduces to this. How do you express a couple state, or state expressed in the coupled basis, in terms of the uncoupled basis. In other words how do you express 3 half's 3 half's for instance, in terms of combinations like this, or how, do you express 3 half’s minus half, in terms of appropriate basis states there. And that is all that we look at in the contest of addition of angular momenta.

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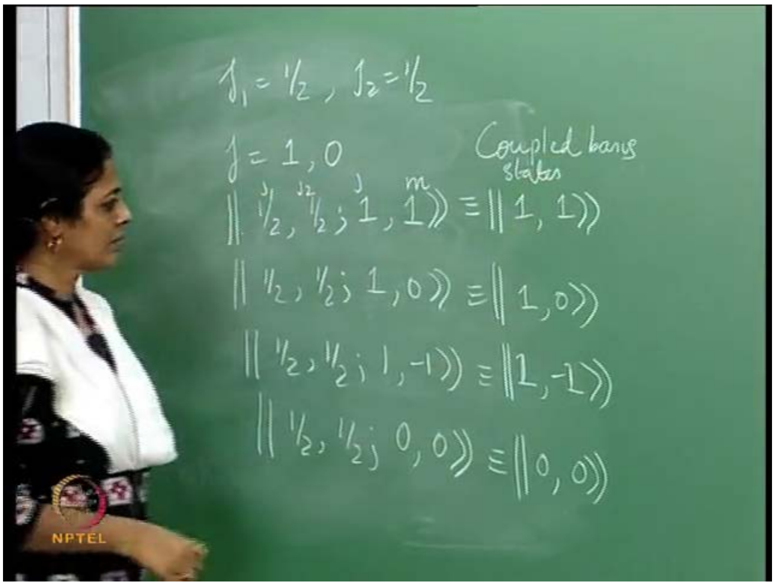

So, let me look at simple example: combining 2 spin doublets, j 1 is half j 2 is half. So, j is 1, or 0 and therefore, the coupled basis states, are 1,1 this is j this is m, of course, I should write j 1 j 2 j m. So, let us do that, half for j 1, half for j 2, j is 1 m is 1. So, this is j 1 j 2 j m. Then of course, I have half half 1 0, half half 1 minus 1, half half 0 0. To begin, with there are 4 states. Because, it is 2 j 1 plus 1 times 2 j 2 plus j 1. Now, I have 2 j plus 1 states corresponding to j is 1 and 2 j plus 1 states corresponding j is 2.

So, this is what I have. I would like to shorten the notation further and not write down j 1 and j 2 at all. So, this is simply identical to 1 1, this is identical to 1 0, this is identical to 1 minus 1 and this is identical to 0 0. This is a notation, it is a convenient notation. I would like to suppress the indices j 1 and the label j 1 and j 2 and just keep j and m to represent the coupled basis states.

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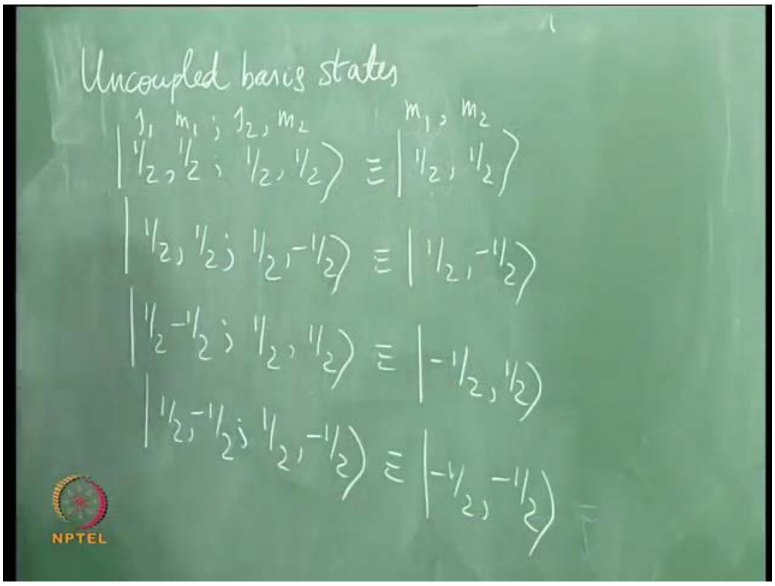

For the uncoupled basis states in this problem, I have j 1 is half; j 2 is half that is j 1, or J 1 m 1, J 2 m 2. So, that is half half with half half. Then of course, I can have half half, with half minus half. And then, I have half minus half with half half and I have half minus half, with half minus half. Again, I would like to shorten my notation and simply use m 1 and m 2, remembering j 1 and j 2, at the back of my mind. Just keeping them in my mind. So, I would like to write this state, as half half. This as half minus half, this as minus half half and this is minus half minus half and these are the uncoupled basis states.

So, by way of short hand notation, I represent the uncoupled basis set, using the labels m 1 and m 2. I should have written j 1 m 1 j 2 m 2. But, I am just trying to say sometime and write a compact notation for this. So, I write m 1 and m 2. Similarly, for the coupled base states, I should have written (Refer Slide Time: 33:48) j 1 j 2 J m. But, once more I just remember j 1 and j 2 at the back of my mind and I keep the labels as j and m. The question that is to be addressed is, how do you write the coupled basis states?

In terms of the uncoupled basis states, if I were talking about spin, I would talk of the spin triplet, when I talk about these. Because, s is equal to 1 S z is 1, or m equals 1 s is 1 m is 0 and s is 1 m is minus 1 and I talk of this, as the singlet state of spin because, this exactly 1 basis states there.

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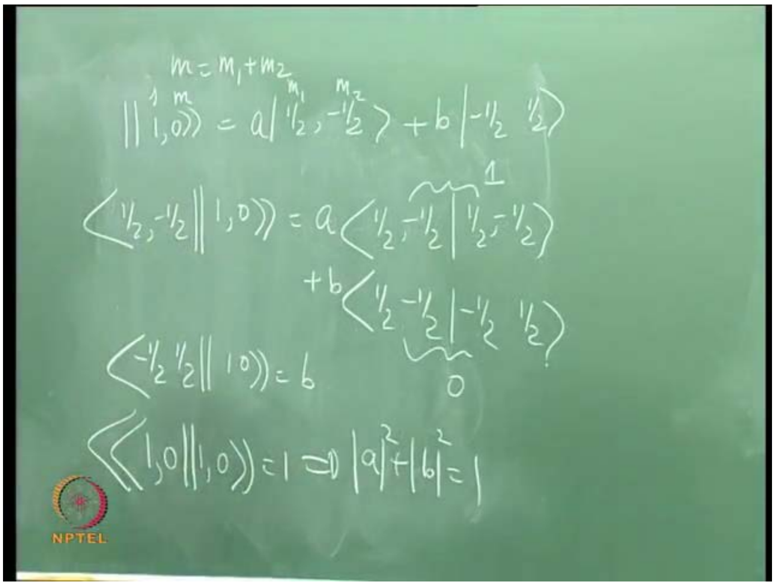

So, now, let me look at the state 1 0. Well instead of writing this as 1 0, I could have written this in terms of the uncoupled basis. I would have got an m equals 0, this is j this is m. I would have got an m equals 0, only by using (Refer Slide Time: 35:45) m 1 is half and m 2 is minus half. So, this would have come from m 1 is half, m 2 is minus half. But, it could also have come from (Refer Slide Time: 35:45) m 1 is minus half and m 2 is half. So, it should be a superposition. There is no other way, I could have used, m 1 and m 2, given the values of m 1 and m 2. There is no other way, I could have produced, m equals m 1 plus m 2. Expect to combine half with minus half and minus half with half, these are distinct states and therefore, there is a superposition of these 2 states, to give me 1 0.

Suppose, I want to find out this coefficient, It is clear, that if I did this. That means take the inner product of half minus half with 1 0, that should give me the following. But, these are orthogonal states. They have different quantum numbers, 2 states are orthogonal are distinct states, if at least 1 label is different, between the 2 states. And here the m 1 labels are different, whereas this is 1, because, I have normalized all states to 1, that is the assumption and therefore, a is simply this object. Similarly, b is minus half half 1 0 this inner product is b.

Well this state is a superposition of 2 of the uncoupled basis states, and we have a way finding out, as always. We uses a same method, to find out the coefficients a and b and since, these are normalized states and this state 2 is normalized to 1. This implies where in general a and b can be complex numbers and therefore, mod a square plus mod b square equals 1. The interpretation is very clear. If you expand the coupled basis state 1,0 in terms of the uncoupled basis states, half, minus half and minus half, half. The coefficients appearing here a and b when mod square is added should give me 1.

Because, after all this state has to be expanded completely in terms of these. And therefore, the total probability of it being expanded in terms of these states, should be 1, a and b are refer as Clebsch Gordan coefficients, or the c g coefficients in short and they satisfies the following relation. Now suppose, we look at the state 1 1, that is the state out here, 1, 1. (Refer Slide Time: 33:48)

Let me repeat this. So, 1,1 is to be expanded in terms of the uncoupled basis states. The only way by which I would have got m equals 1, is by using m 1 equals 1 and m 2 equals m 1 equals half and m 2 equals half. There is no other possibility, I should have had (Refer Slide Time: 35:45) a half plus half to give me 1, because, this gives me 0 that gives me 0 and minus half minus half this minus 1.

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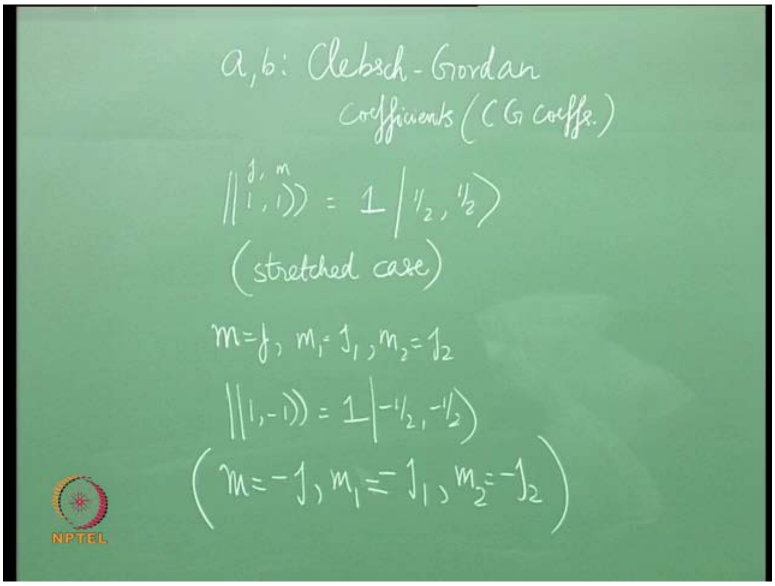

So, when I write 1,1 in terms half, half. There is exactly 1 basis state, appearing here and the coefficient is 1. This is called stretched case. Stretched case because m equals j, m 1 equals j 1, m 2 equals j 2. I would say the same thing for 1, minus 1. Because, that would have come only from m 1 equals minus half and m 2 equals minus half and that is also a stretched case. So, stretched case is this, or m equals minus j m 1 equals minus j 1, m 2 equals minus j 2. When we discuss a stretched case, there is a exactly one c g coefficient and that is 1, that is simply unity.

So, looked from the coupled basis framework, the basis state is 1, minus 1 in the uncoupled basis it is simply translates to minus half, minus half. Again, I emphasize, that by this object by 1, 1 I mean (Refer Slide Time: 33:48) j 1 is half j 2 is half j is 1 and m is 1. There by half, half, I mean (Refer Slide Time: 35:45) j 1 is half m 1 is half j 2 is half m 2 is half. So, whenever m is equals to j, or minus j corresponding to which, m 1 is plus j 1, or minus j 1 and m 2 is plus j 2, or minus j 2 is called as stretched case.

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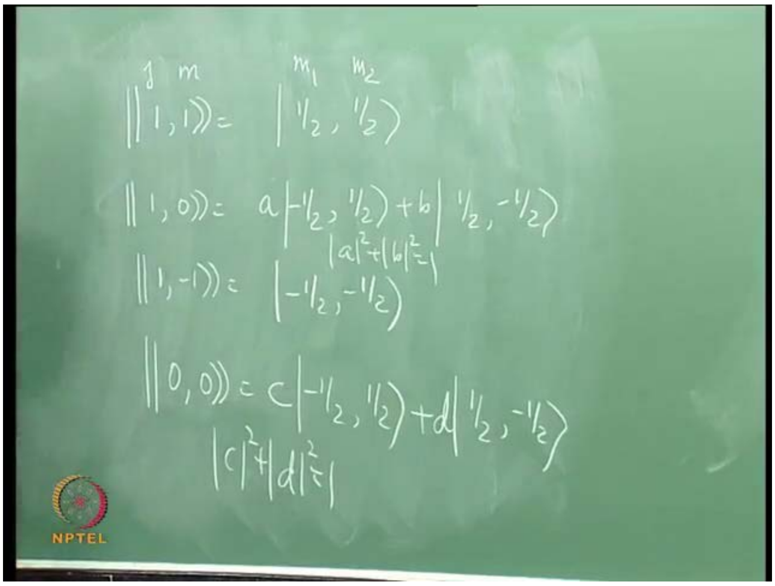

So, in this problem: I can now write the following. I have 1,1 is half, half here. This is m 1 m 2 and this is j m. 1,0 is a times minus half, half plus b times half, minus half and 1, minus 1 is half minus half, minus half. These are the stretched cases. We have to find out a and b. Then there is a singlet state, which is 0, 0. Again, if n is equal to 0, clearly it could have come from m 1 is minus half and m 2 is half. It could also have come from m 1 is plus half, m 2 is minus half. Expect that, this state is distinctly different from this; they are orthogonal to each other and therefore, they are 2 different coefficients there c and d.

Expect that, mod c square plus mod d squared is 1. The same way as mod a squared plus mod b squared equals 1. So, the problem now reduces to finding out to estimating the values of to determine values of a b c and d, given these conditions and given the fact that these states are normalized to 1 and they are orthogonal to each other. The manner in which this is done, is straight forward. We know the action of j minus and j plus, the raising and lowering operators on the angular momentum states.

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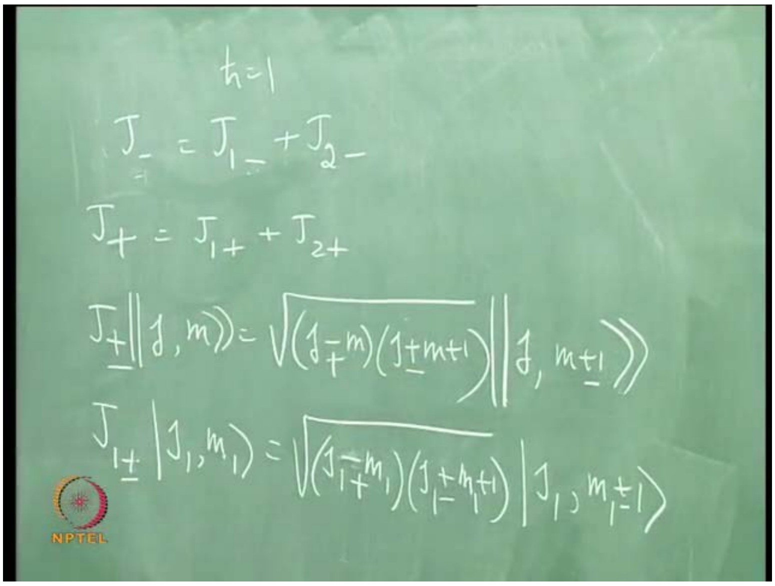

J minus is J 1 minus plus J 2 minus. J minus is the operator acting on the coupled basis. J 1 minus on system 1 and J 2 minus on system 2, is clear. And similarly, J plus is J 1 plus, plus J 2 plus, so let us let us write this better, J plus is J 1 plus, plus J 2 plus. The effect is as follows, J plus acting on the state j m, is root of j minus m, into j plus m plus 1. I have set h cross is equals 1, in all these problems and I have j m plus 1. I am using double braces because, these are coupled states and j minus acting on j m, gives me j plus m j minus m plus 1 j, m minus 1.

J 1 minus acting on the state j 1 m 1 does the same thing. It is j 1, oh let us starts with J 1 plus it is j 1, minus m 1, times j 1 plus m 1 plus 1, j 1 m 1 plus 1. J 1 minus acting on the state j 1 m 1, is j 1 plus m 1, times j 1 minus m 1 plus 1, with the minus sign. So, this is the way they act. Similarly J 2, J 2 acts on the state the operator J 2 plus acts on the states j 2, m 2 and so on.

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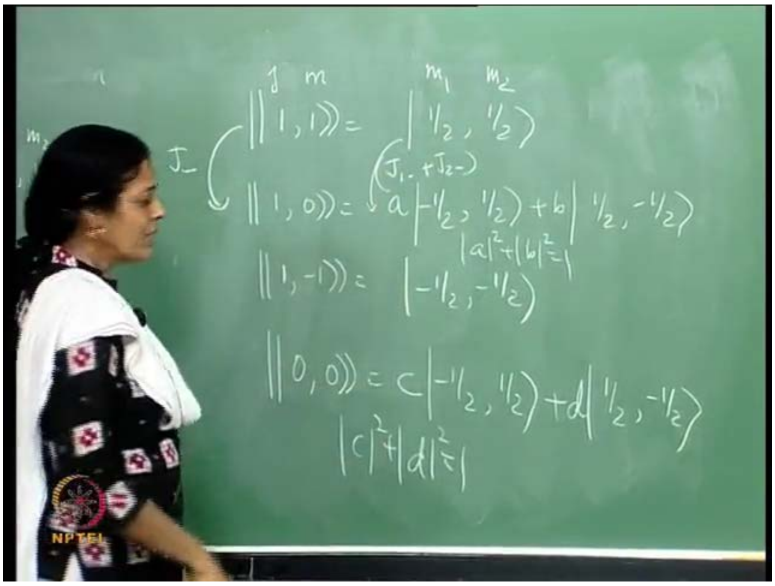

So, given this I can now do the following, if I used J minus on the state 1 1. Apart from the coefficients put down there, I should get the state 1 0. That amounts to doing J 1 minus plus J 2 minus, on the state half half. J 1 minus of course, acts only on J 1, m 1 and J 2 minus acts on J 2, m 2 and so on. And this is how I would be able to estimate, get the values of a and b.

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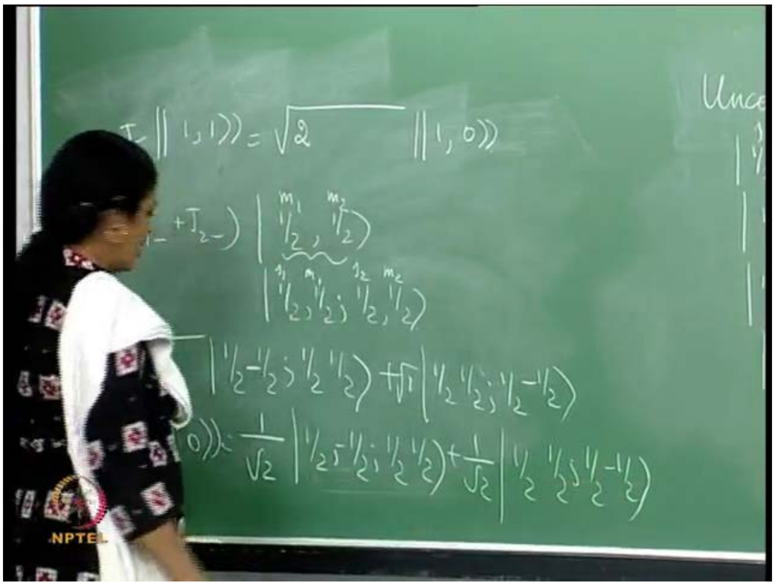

So, let me do that now. So, let us start with 1, 1. J minus acting on 1,1 is root of J plus m, times J minus m plus 1, which is just root 2 1,0 and J 1 minus plus J 2 minus acting on half, half. This state is to be understood, as j 1 is half, m 1 is half, j 2 is half, m 2 is half. This object is simply going to be, J 1 acts on this giving me root of j plus m 2 j minus m plus 1 and does nothing to the other state, plus J 2 minus acts on j 2 m 2. This is j 2 that is m 2, this is j 1, that is m 1. Leaving this state alone untouched and pulls out a j 2 plus m 2 times j 2 minus m 2 plus 1. Expect that it lowers the m 2 value.

So, in the 1st case, the m 2 became minus half, the m 1 became minus half and the m 2 continue to be half. In the 2nd case the m 1 continuous to be half and the m 2 becomes minus half. And therefore, I have, 1 0 is 1 by root 2 times half minus half, half half, plus 1 by root 2 half half, half minus half. In other words, I can now fill up this column.

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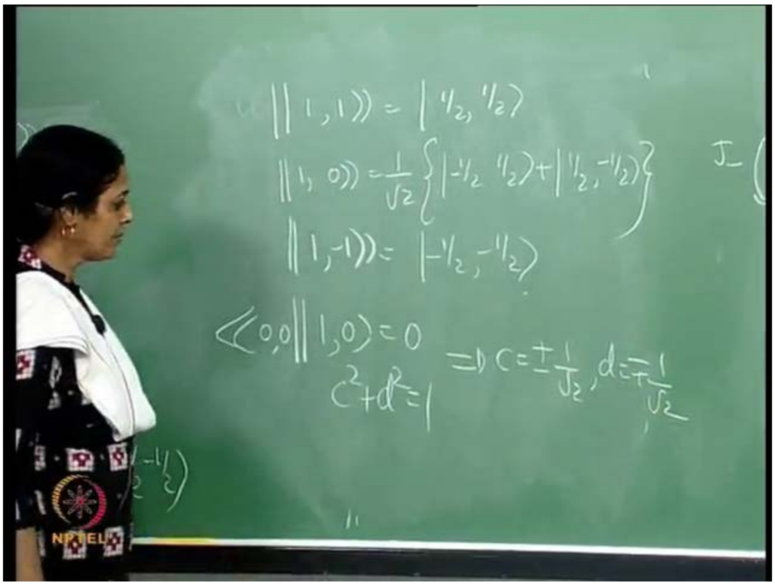

I will have the following, 1 1 is half half. 1 0 is 1 by root 2 times half half, half minus half half plus half minus half and 1 minus 1 is the stretched case any way, that is just minus half minus half. So, I have got a and b and it clear that a squared plus b squared, each of them being 1 by root 2 a square plus n square is 1. 0 0 is orthogonal to 1 0 and this together with a fact that is c squared plus d squared equals 1. Without loss of generality, I am choosing c n d to be real values. Otherwise, I would have done mod c square plus mod d square, I can chose them to be real, because, there is a phase and I am only concern with the squares, there addition to be 1 and this together with c squared plus d squared equals 1.

Since, I know n b would give me that c is plus or minus 1 by root 2 and correspondingly d is minus, or plus 1 by root 2, either of them is correct. I have to chose a convention, but, as it happens, I can write 0 0 as 1 by root 2 times minus half half, minus 1 by root 2, 2 times half minus half, 1 by root 2 half minus half, or the other way around, either is right. A convention has to be used and a certain convention once decided up on, must be used throughout the problem. In this case, we do not have any issues on this matter, we can use, c to be plus or minus 1 by root 2 and the correspondingly to be minus or plus 1 by root 2. Therefore, I have got the c g coefficients, for both the triplet states and the singlet state.