Quantum Mechanics - I
Prof. Dr. S. Lakshmi Bala
Department of Physics
Indian Institute of Technology, Madras

Module - 1
Lecture - 1
Quantum Mechanics - An Introduction

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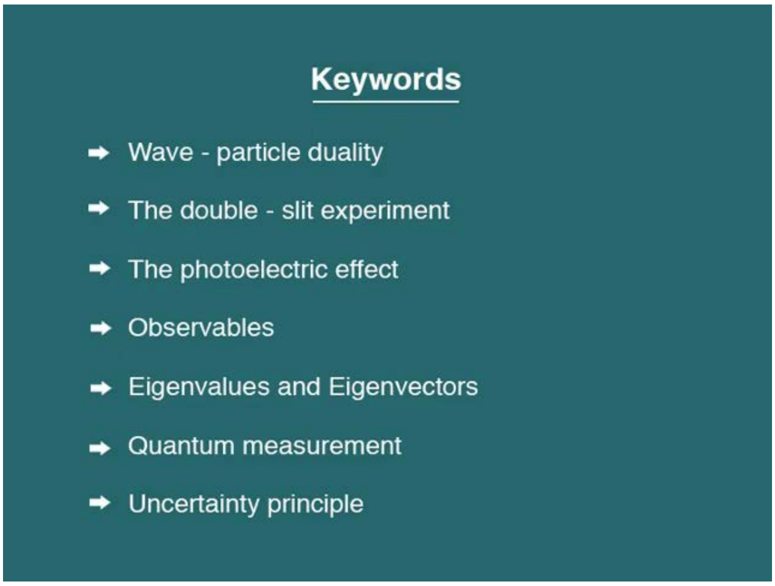

I will be giving a set of lectures, pertaining to an introductory course in quantum mechanics. So this is going to be quantum mechanics.

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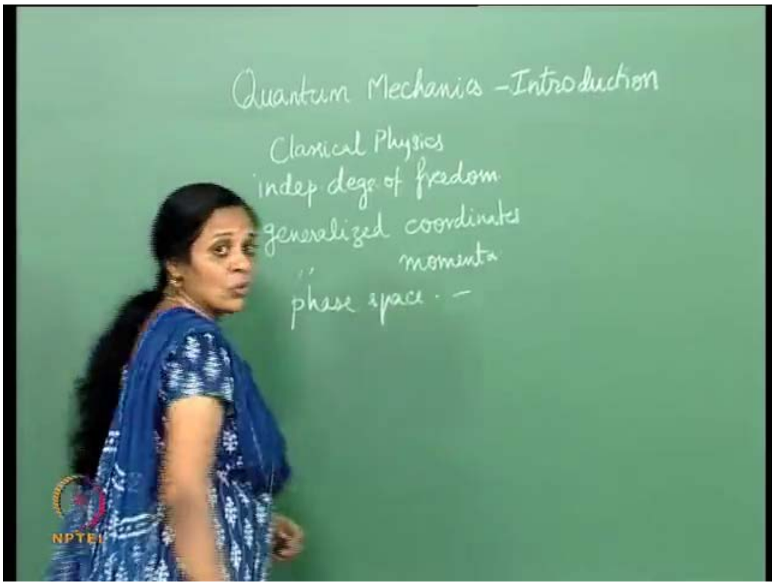

Many of you would be familiar with concepts from classical physics, presumably you know what is meant by independent degrees of freedom, this is all from classical physics; generalized coordinates of course, generalized momenta, phase space and so on. Some of you might even be aware of the richness of the structure of phase space. Now, when it comes to quantum mechanics these concepts could undergo some alterations. The alterations were not obvious under history of quantum mechanics is as revealing as it was dramatic. The reason is that classical mechanics could really not give any pointers to understanding the quantum world; it made mockery of intuition of human beings and it became a challenge to the best and brightest of minds to find out how exactly the quantum world works.

The features relating to the quantum world were really obvious only through the atomic and sub microscopic phenomena that is not to say that quantum mechanics does not hold for macroscopic objects. In fact, several macroscopic phenomena, I can think of ferromagnetism right away can be explained only on the basis of quantum physics. Having said that we normally can understand most of things that happen around us in daily life, most of the macroscopic world just based on our physical intuition, our prejudices and our common sense which has really been in us due to our experiences in daily life. So as long as, we are dealing with objects of length scales, mass scales and time scales that we are used to in daily life. Most of the time we can get by with our physical intuition and with classical laws which seem to confirm predictions based on our intuition.

Unfortunately, and interestingly we cannot do that with quantum laws. So, it turns out that in the history of quantum mechanics; it is in the context of atomic phenomena and sub microscopic particles that these laws really came out into the open. Einstein is supposed to have remarked that our physical intuition is a collection of prejudices that we have developed till age eighteen, and prejudices and intuition do not really explain things in the quantum world. Many concepts in quantum physics are very dramatic, it is very difficult for the human mind to comprehend it atleast, it was at that stage in history and I am talking about the end of the 19th century where most classical laws were pretty well understood.

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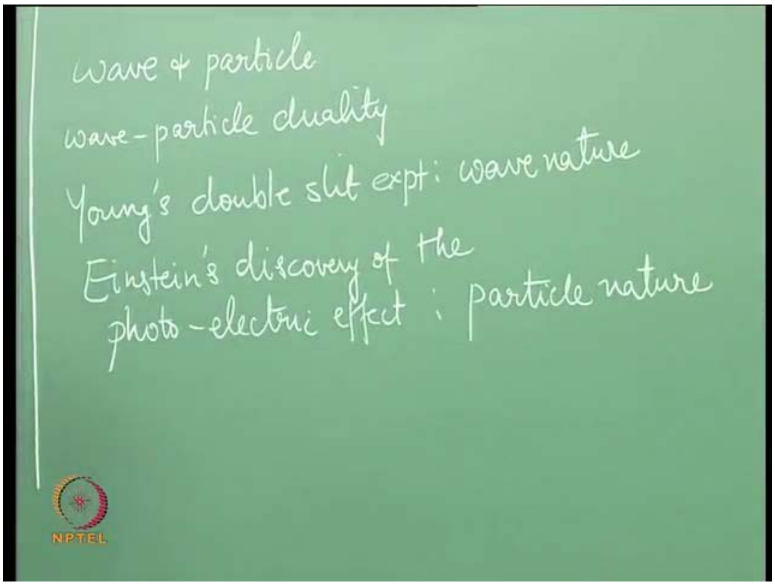

The first thing, that was very different when it comes to quantum physics is the fact that; an object can be thought of as a wave and as a particle. This is normally called the wave particle duality in most books and the series of experiment showed that matter behaves like waves and also like particles. I am normally fond of two of this class of experiments; Young's double slit experiment which demonstrates the wave nature of objects; and Einstein's discovery of the photo electric effect, and this demonstrates the particle nature of objects. So, I will just give a rather a brief description of the Young's double slit experiment first and we will take it up from there.

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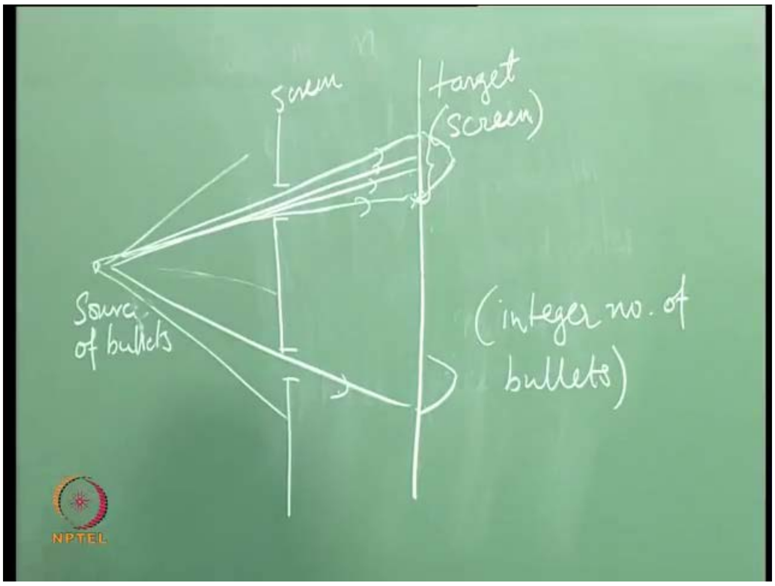

Let us first look at it from a classical point of view. Start with the source of bullets. The bullets are shot out in various directions and they hit a target; the target is a screen. Now, as the bullet strike the target you could perhaps use detectors and count the number of bullets that have hit the target. Clearly, there will be an integer number of bullets. You could modify this experiment by putting a screen out here with two slits. And now see what is the pattern that emerges on the target screen by pattern I mean that if you plot the distance along this versus the number of particles that strike the screen, you would like to see how exactly this plot looks.

So from the source, there are bullets that go and you would therefore, expect the plot of distance versus number of bullets to have a peak out there and similarly, out here because the bullets that come here. The pattern would be that of two peaks one there and one here and nothing else in between. Now, this is what you would expect from classical physics. Let us replace this source of bullets with an electron gun so that we do a quantum experiment, now we start off with a source of electrons.

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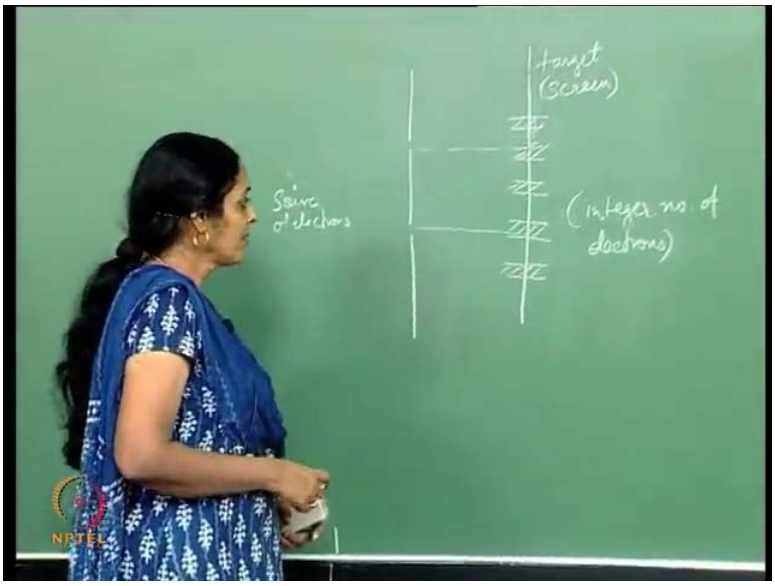

Once more, we have a screen on which these electrons fall and if we have detectors and counts the number of electrons that reach the screen, we would count an integer number of electrons. Once more, we can put a screen out here with 2 slits through which the electrons have to pass in order to hit the target. You would normally expect once more a pattern like that, if you plot a distance versus number of electrons that strike if the electrons behave like bullets or like particles.

On the other hand, something very dramatic happens on this screen instead of a pattern like this what one sees is a alternate bright and dark fringes. This is the kind of property that you would expect, if the electrons behave like waves and if they were waves that emerged out of these slits they could interfere and depending on whether there is constructive interference or destructive interference you would expect an interference pattern like this. This is intensity versus distance from the screen: a more intense area than a less intense area than a more intense area and so on. This is very striking, because this region for instance is a region where no electrons can reach; if the electrons are thought of as particles.

On the other hand, the fact that there is an overlap between waves that interfere has created this pattern the Young's double slit experiment therefore, demonstrates very clearly the wave nature of the electron. Consider on the other hand, the photo electric effect.

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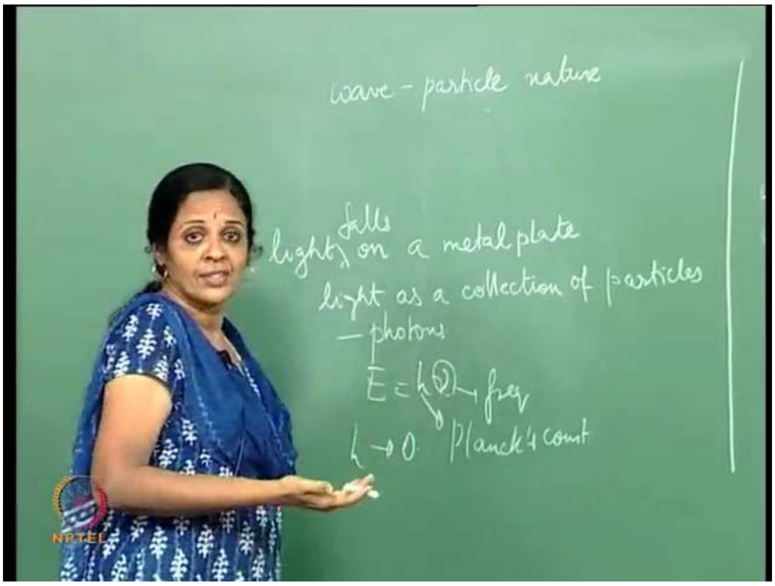

Now, in the photo electric effect light is shown on matter perhaps a metal and electrons are emitted. Now, if the light were thought of as electromagnetic waves and electrons come out of this metal plate you would normally expect the energy of the electron that comes out to be related to the intensity of the incident light, In other words, as the intensity of the incident light changes, you would expect the energy of the electron to change, For light of very low intensity; you would expect low energy electrons, for light of high intensity; you would expect high energy electrons, but that is not what happens. And what happens, can be explained only on the basis of the Corpuscular theory in other words, you treat light as a collection of particles namely; photons, each one with energy given by h nu where nu is the frequency of the light and h is Planck's constant, has the same dimensions as action rule second, those could be the units.

So, it turns out that photons interact with the electrons in the metal plate and transfer energy to them and pull them out, So that, the energy of the electron does not really change with the intensity of the incident light. The more intense the incident light; the more the number of photons, the less intense the incident light; the less the number of photons and therefore, the energy of the emitted electrons do not depend upon the intensity of the light. On the other hand, they would depend upon the frequency of the light the higher the frequency the more the energy.

Now, these two statements have been experimentally verified; clearly demonstrating that particles and waves are both attributes of the same object, the electron, it is not nearly the electron if you look at photons, protons, neutrons or any form of matter you can associate waves with all forms of matter .And therefore, even for large systems there are matter waves associated with them it is simply that as I said before you understand many things about macroscopic systems based on classical laws..

Classical mechanics is really an approximation it is not the exact theory and finally, when everything was in place it was clear that when the Planck's constant, in the limit planck's constant going to zero and if the limits are taken properly, you can retrieve the classical laws from the quantum laws. So, given these 2 we understand that in the quantum world there is this wave particle nature, attributed to all objects and we need a theory to explain this we have to learn to accept waves and particles being properties of the same entity and not waves or particles which is the kind of lesson that the classical world has taught us. To explain this, was difficult.

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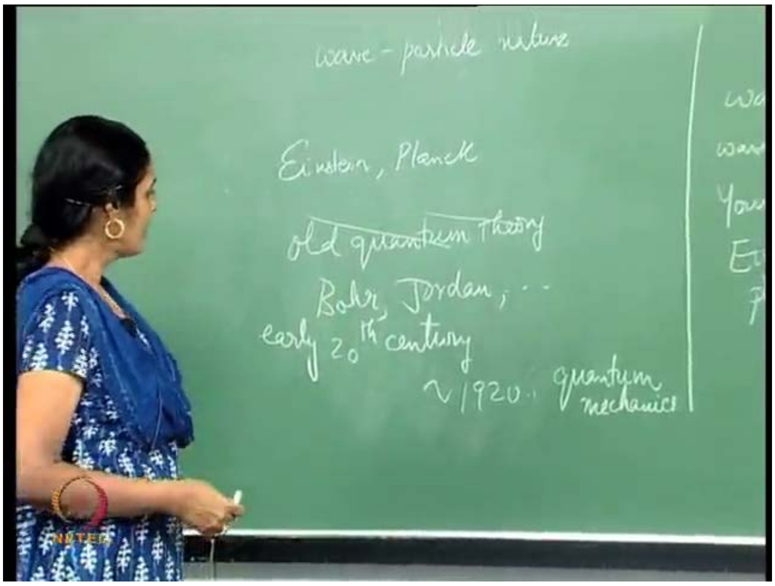

Einstein and Planck, and many other physicists were involved in this explanation and something called the old quantum theory was proposed primarily by Bohr Jordan. However, it was also realized soon enough that the old quantum theory was rather adhoc in nature and cannot be used as a strong edifice or a good mathematical structure for explaining, what happens in the quantum world, a new theory had to be brought in.

Think of it this way, physical intuition is not helpful and in a manner of speaking classical physics is merely shrugged her shoulders. There is a certain helplessness, there are no pointers as to what should be the central objects around which the theory should be built, what should be the central objects which should be used in explaining things like the wave particle nature of matter, and then physicists brilliant minds working in different parts largely in Europe Copenhagen Berlin; got into aggressive debates about what should be the right theory of the quantum world.

This was a period in the early twentieth century, which was very dramatic educator and revealing because by the 1920s it culminated, in what we now know as quantum mechanics. So, I will not make much ado about the old quantum theory was not very satisfactory and what we now work with is quantum mechanics as was given to us largely by 2 master minds; Heisenberg and Schrodinger.

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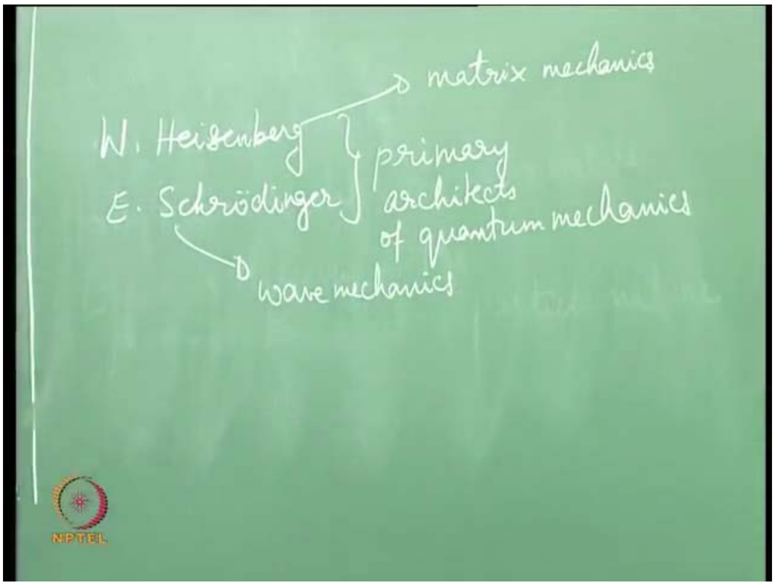

The primary architects of quantum mechanics as we see in the text books today, there are alternative formalisms of quantum mechanics but during the course of my lectures I will be only talking about Heisenberg’s; matrix mechanics and Schrodinger’s; wave mechanics both being equivalent formalisms of quantum mechanics. Having got no pointers from classical physics, as to what should be the central issues central objects around which the quantum theory should be built. Heisenberg fought that observables, that are objects that you measure in experiments should be the focus of attention and the theory should be built around these objects.

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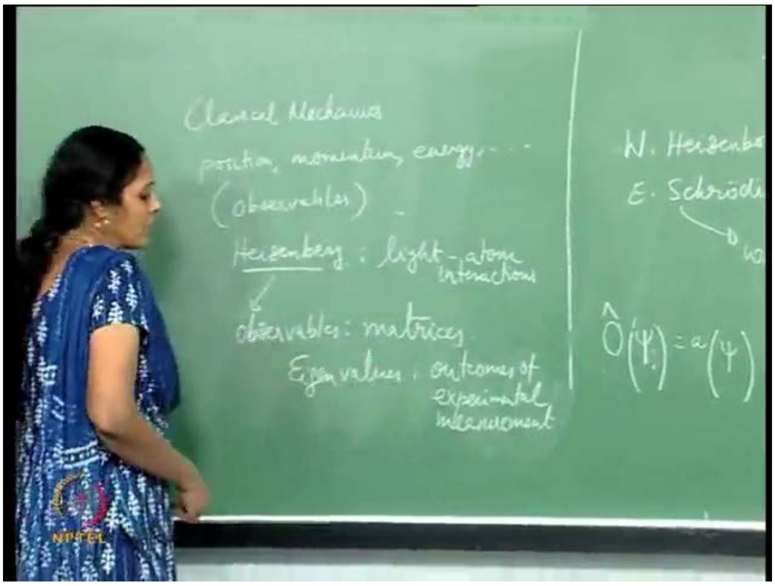

In classical mechanics, we are familiar with objects that can be experimentally measured like position, momentum, energy, and so on. Now, these objects are the observables in an experiment. In principle, a clever experiment on any physical system should be able to tell us what is a value of observables and in quantum mechanics Heisenberg said; that observables are represented by matrices.

Matrix mechanics, itself came about because of the inspiration that Heisenberg drew from light atom interactions because there are atomic transitions that result due to interaction of the atom with light and parameters pertaining to these atomic transitions, which can be measured. It was realized by Heisenberg and others that, these observables are best represented through matrices because the Eigen values of the matrices, are the outcomes of experiments.

So, suppose one way measuring energy in an experiment: the value of the energy which one would get on measurement would be one of the set of Eigen values of the matrix that represents energy. Energy is re-observable, the state of the system itself would be the Eigen vector corresponding to that Eigen value. Of course, there are cases where for a given Eigen value there is more than one Eigen vector but such degeneracies will be dealt with by me later in subsequent lectures. And therefore, you have Eigen value equations there is an observable.

I will put a hat on that to show that it is an observable or an operator, it acts on an Eigen vector. For the moment I, call that psi it is a column vector because this is a matrix that gives me a number may be a number a; which is the Eigen value or a set of Eigen values. This is a typical Eigen value equation, where a matrix acts on a column vector or Eigen vector and you have a number with the same Eigen vector on that side. If, I diagonalize the matrix the diagonal elements would be the Eigen values and the outcome of the experimental measurement would be one of this set of Eigen values.

There is a crucial difference between classical mechanics and quantum mechanics. It is a very interesting difference and is being studied even now in great detail in measurement theory. In a classical system, if I make a measurement of the energy and I get a certain value I can insist, that before I made the measurement the energy of the system was the same ,the same value that I got on measurement .Whereas, in quantum systems I cannot do that, the measurement outcome nearly tells me that on measurement, this is the energy of the system and this is the state of the system. I cannot extrapolate and say that before measurement, the energy of the system was the same nor can I insist that the state of the system after measurement, is the same as the state of the system there was before the measurement.

Now, this is another crucial difference between classical and quantum physics. So while, Heisenberg developed matrix mechanics by which observables were given importance as central objects, and outcomes of measurements are the Eigen values of these observables. There was an alternative formalism given by Schrodinger.

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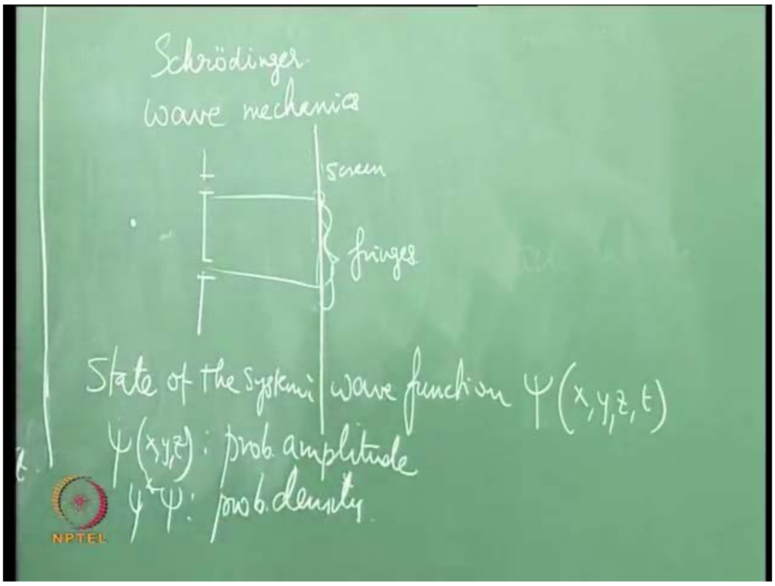

Wave mechanics: Let us go back to the Young's double slit experiment. In the Young's double slit experiment there was a non zero probability of the electron being in the region between the slits on the screen recall the experiment. Since, we had fringes here also clearly, the electrons were there the waves were interfering even in this region. So, there is a non zero probability of finding the electron here, here, here, here and of course, all over there. This makes the whole subject probabilistic in nature, you can only talk of the probability of the electron being there or the electron being here or the electron being there or the electron being there, you cannot say with certainty, that the electron was there or was here and not here, because if it were not here, one would not see fringes out here.

It is this wave like nature of the electron and any other physical system which perhaps inspired Schrodinger, to develop wave mechanics, because Schrodinger starts with the state of the system, being a wave function. And in fact, we will see in subsequent lectures that psi could in general be complex we could, work with psi as a function of space time and even if we were working in a situation where we were not worried about the time evolution of the system, means the probability amplitude of the system being; in the region x two x plus d x, y two y plus d y, and z two z plus d z, psi star psi is therefore, the probability density and the total probability of seeing the system anywhere in space will be one.

So, observables are represented by matrices and the Eigen values of observables are the outcomes of experiments but when you do a measurement the answer must be a real number and therefore, observables are represented by Hermitian matrices because Eigen values of Hermitian matrices are real.

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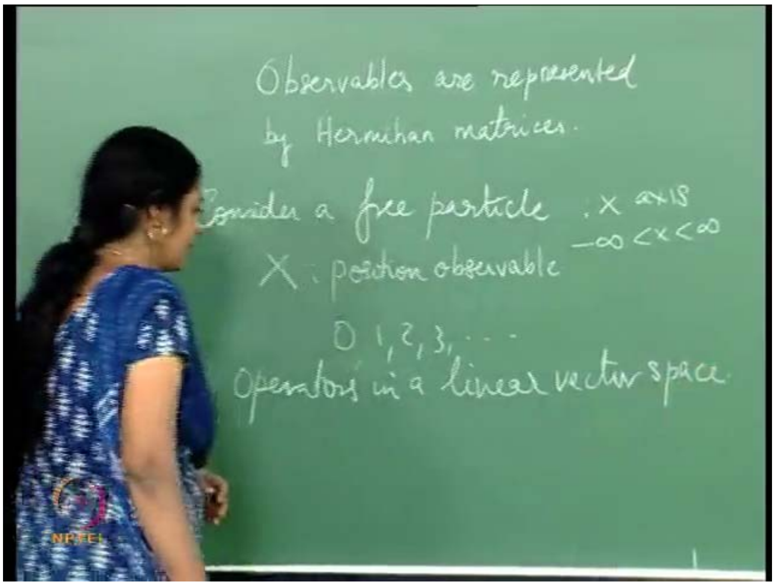

It is possible, to make measurements of two or more observables. Each observable, represented by a Hermitian matrix and as I said earlier the system collapses to an Eigen state, the Eigen state is going to be a common Eigen state of all these observables. It was soon realized that matrices are really representations of operators, in an appropriate linear vector space. The state of the system, the quantum state lives in this linear vector space.

Now, this needs some explanation: the linear vector space should not be confused with the usual physical space in which we live where we have the usual Cartesian coordinates: x y and z and time t .I will give an example, to illustrate this point. Consider a free particle, moving along the: x axis. So in principle, the particle can be anywhere it continuously moves from minus infinity to infinity. It is pretty clear therefore, that if I measure the position of the particle, the observable is positioned; there will be an infinity of Eigen values available in this problem, which means that the Hermitian matrix representing the position observable, should be an infinite dimensional matrix and therefore, the linear vector space in consideration is an infinite dimensional linear vector space.

So, here we have a problem of a particle moving in one dimensions but the linear vector space in question is an infinite dimensional linear vector space. There is another wrinkle, to this problem the Eigen values themselves form a continuous infinity it is not as if the Eigen values are discrete, like this. It is a continuous infinity of Eigen values because the particle is gliding along the x axis and therefore, we have to modify our opinions of matrices and extend it and we have to allow for objects, which have a continuous infinity of rows and a continuous infinity of columns. In other words, we have to accept the concept of operators in linear vector spaces.

The state of the system in this problem is of course,, an infinite column vector. In a physical situation, we would like to track the state of the system as it moves in space time and therefore, there is a prescription this prescription tells us, how to go from this abstract Eigen vector in some linear vector space to a function psi of space time coordinates.

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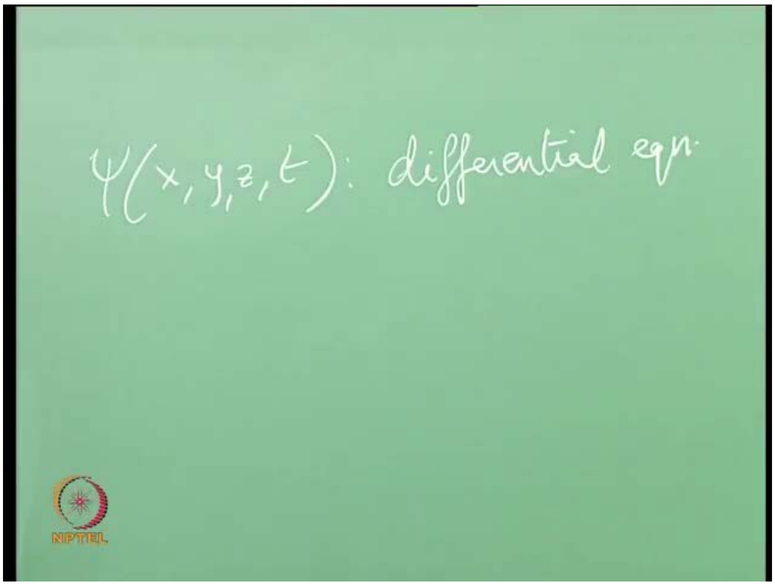

Now, given this prescription it is possible to talk about the dynamics of the system as time evolves and the dynamics is normally, given in terms of differential equations. Functional spaces, have in a very natural fashion got into the picture because this is a function of x y z and t. Thus, linear vector spaces, function spaces, operators, matrices differential equations, all these have become a very natural ingredient of the understanding of quantum mechanics. So, we go into the next question how do we make measurements? How do we find out what is the value of the observable in question? Normally, when we do an experiment we conduct several trials on a system, repeated measurements, many trials and then take the arithmetic mean of all the values that we have got as outcomes of the measurement, and that is going to be the value of the observable. Now, in quantum mechanics as I have already stated if you make one measurement on a system, the state is no longer the original state which was there pre measurement. Post measurement the state collapses, to one of the possible Eigen states of the system.

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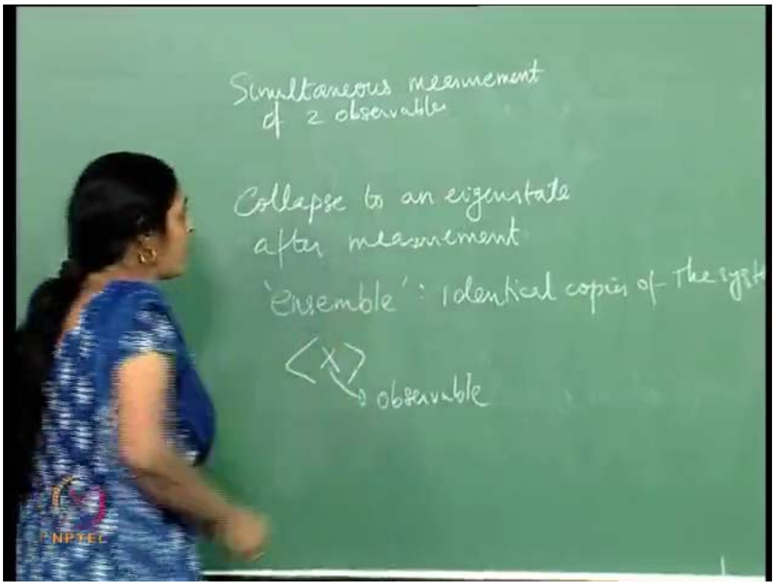

Immediately, after measurement the system collapses to an Eigen state and therefore, there is no point in repeating the measurement on this Eigen state or the new state of the system because this was not the system initially. The only way to handle this problem, is to make an ensemble: identical copies of the system, on which we conduct the measurement and then make one measurement on each copy. Take the arithmetic mean and that is going to be the value, the average value of the observable in question. The average is represented like this where x is the observable, this is merely a notation and the average value is always with reference to a particular state of the system. So, this is the way you represent quantum averages.

Now, we have an interesting question. Suppose, we make the measurement and suppose we make simultaneous measurement of 2 observables or more observables. Is it possible in a simultaneous measurement of these observables, the word simultaneous is very important. Is it possible, to get accurate values for both these observables. The answer in quantum mechanics is generally no, it is not possible! And you can trace it back to the fact that the matrices corresponding to the 2 observables need not commute with each other. In general, they do not and when 2 matrices do not commute with each other, we can show that it is not possible to get accurate values for both the observables in question.

Of course, we are allowing for and if we make simultaneous measurements, that is a very important thing. You allow for human errors, you allow for instrumentation errors and after that also it is not possible in a simultaneous measurement of 2 observables in general, to get accurate values for both of them. This is an inherent feature of quantum mechanics, this is quite unlike classical physics. It is an inherent feature of quantum mechanics and it has lead to a very many interesting relations one of them being the: Heisenberg uncertainty relation.

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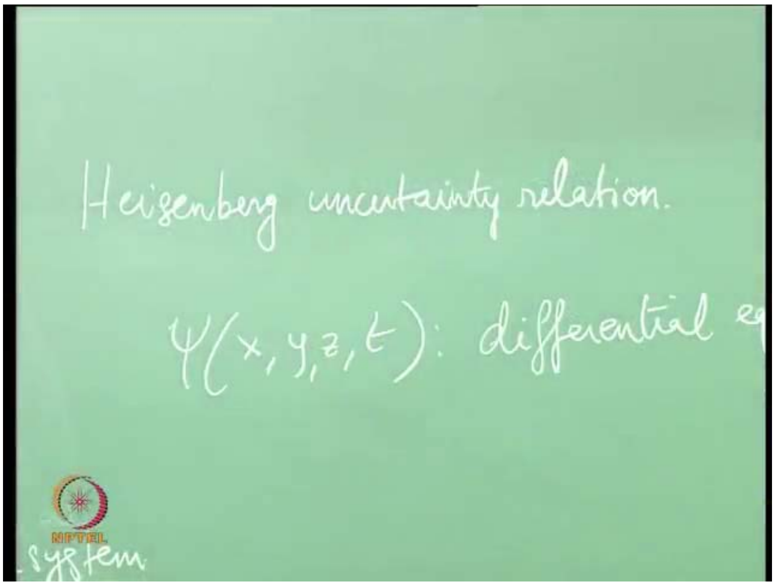

So, normally referred to as the uncertainty principle, so what is the Heisenberg uncertainty relation?

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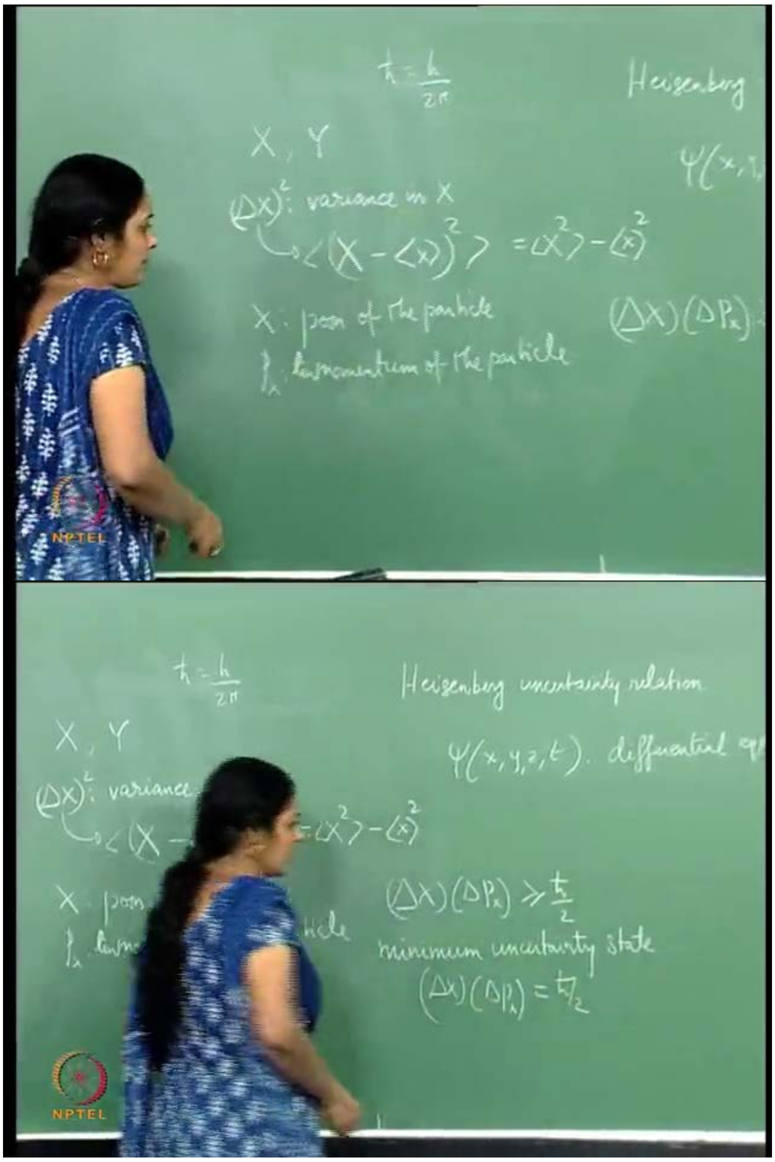

Suppose, we consider 2 observables: x and y and delta x: is the variance in x I make measurements in a certain state of the system. So delta x, is simply equal to: x minus expectation value of x the whole squared average. This as most of you know is, expectation x squared minus expectation x the whole squared that is the variance in x. Similarly, I define delta y in the particular case of a particle moving along the x axis let x denote the position of the particle and p sub x the momentum linear momentum, the Heisenberg uncertainty principle in this context says, that delta x, delta p sub x should always be greater than or equal to h cross by two, where h is the Planck's constant and h cross is h by two pi. In a minimum uncertainty state, the equality is satisfied.

In general, it is greater than this bound there is no way by reducing instrumentation errors improving human effort and bringing this down to zero whereas, in classical physics it is possible that the measurement gives you delta x delta p x equal to zero such a thing is not possible in quantum physics.

It is not just true for position and momentum but for any 2 observables that do not commute there are corresponding uncertainty relations. If you look for instance at the ground state of a quantized linear harmonic oscillator that turns out to be a minimum uncertainty state in the position and momentum variables there are very interesting states in optics, states of light which obey the minimum uncertainty relation. You look at ideal, laser light, it is a coherent state of light and there in terms of another pair of variables which I will call the quadrature variables, such a relation is satisfied, such an equality relation is satisfied and we should remember, that there x does not mean position and certainly the momentum is not involved but they are 2 pairs of a pair of observables for that context. Then there is squeeze state of light: squeeze state of light is one where either delta x or delta p x is less than root of h cross by two.

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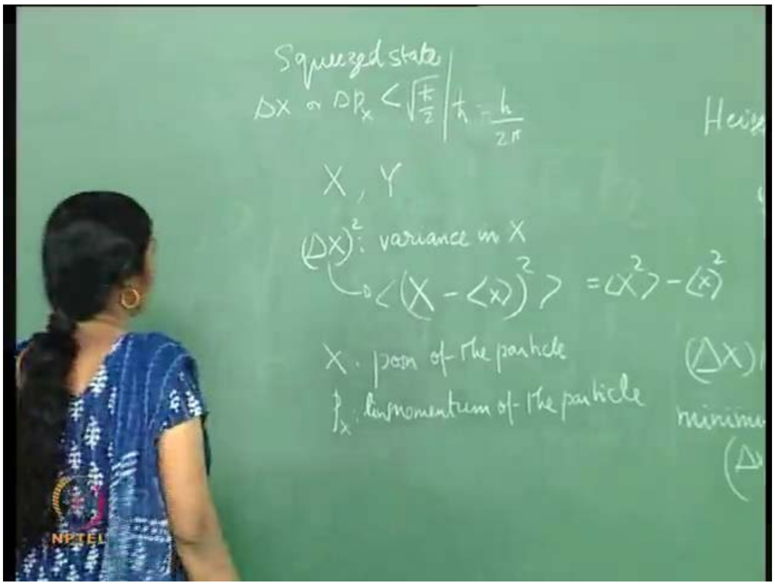

So squeeze light, squeezed state of light. Once again, I should emphasize that x and piece of x would not stand for position and linear momentum but for 2 appropriate observables. So, in quantum physics it is possible to have different types of states satisfying uncertainty relation. In a subsequent lecture, I will talk about the Heisenberg uncertainty relation and also about variations and extensions of the uncertainty principle.

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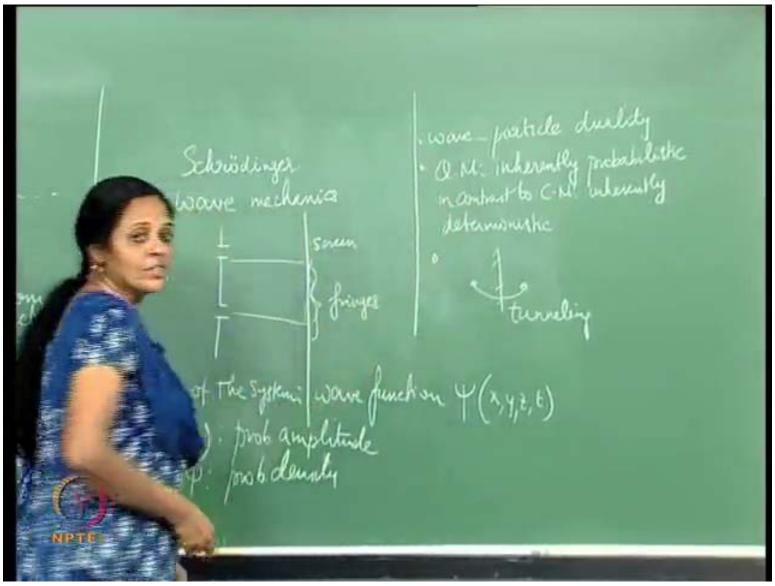

So already, we have seen some unfamiliar things we have seen wave particle duality. Quantum mechanics, is inherently probabilistic in contrast to classical mechanics which is, inherently deterministic. And in this context, I have to mention something else in classical physics, suppose there were a particle which is kept on this side of a barrier, an impenetrable barrier the probability of seeing the particle on that side of the barrier is, strictly zero whereas, in quantum physics because of the wave nature there is a non zero probability of seeing the particle on this side of the barrier as well if it is a quantum particle.

This phenomenon, that there is a non zero probability of seeing the particle there here there and there is due to something called tunneling this again is a third very important difference which manifests itself in the quantum world which you do not see in the classical world.

It is very important that, states of a system can be tampered with particularly in the context of quantum optics you will see very many interesting states of light. In atom light interactions, it is possible to tune the state of the atom, it is possible to make atomic transitions, various types of light sources could be used for this purpose.

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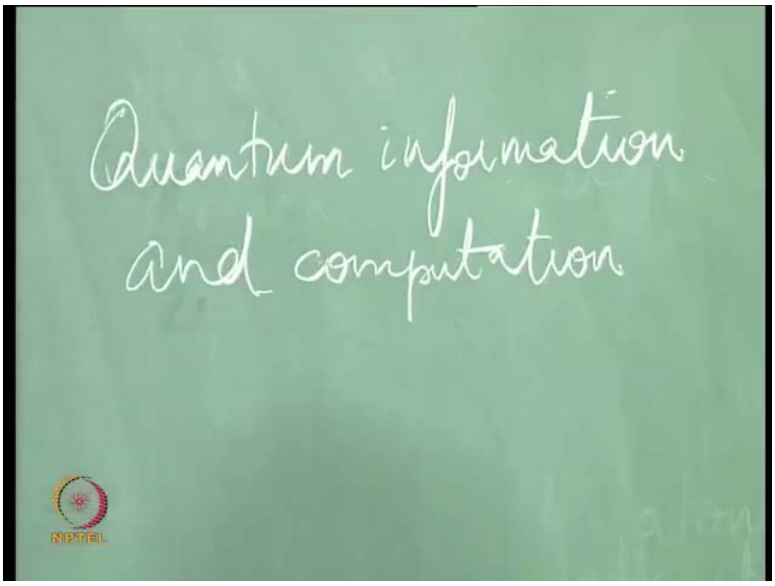

And even in active areas like quantum information and computation, we work with quantum states, specific quantum states, tinker with them, use them for logic gate operations like you would do logic gate operations with classical systems with bits. You could use quantum states, to store quantum information, use it for teleporting information from one place to another and these are areas of extreme research activity even now. All this, leads to a certain picture of the state of the system.

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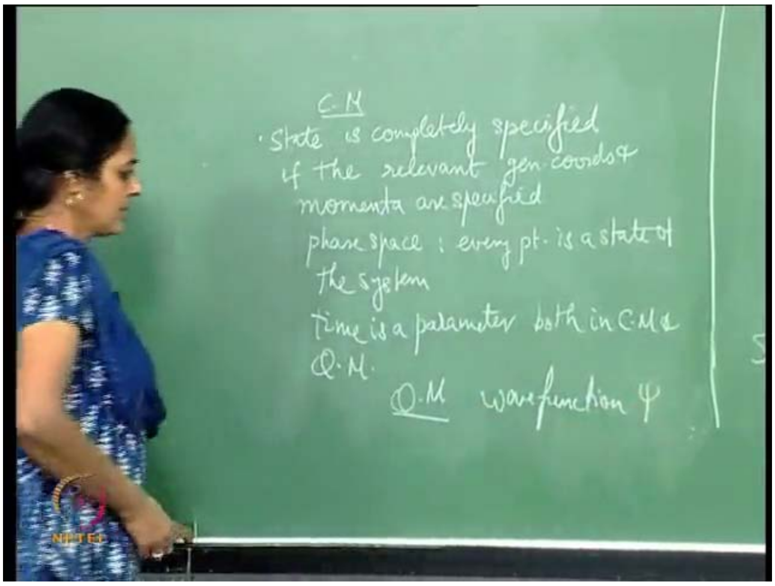

In classical physics, the state of the system is completely specified. Classical mechanics, the state is completely specified if, the relevant generalized coordinates and momenta are specified and if you have a phase space, of these coordinates and momenta every point is a state of the system. Time is a parameter, and these coordinates and momenta; evolve in time. These become the observables, in quantum physics. Time is a parameter both in classical and quantum physics.

Observables are represented by matrices and the state of the system in quantum mechanics is prescribed by the wave function, psi. It is pretty clear, that this is dramatically different from classical physics. When all intuition fails, and you want to create a theory you have to take recourse to something very exact, something that will speak the truth.

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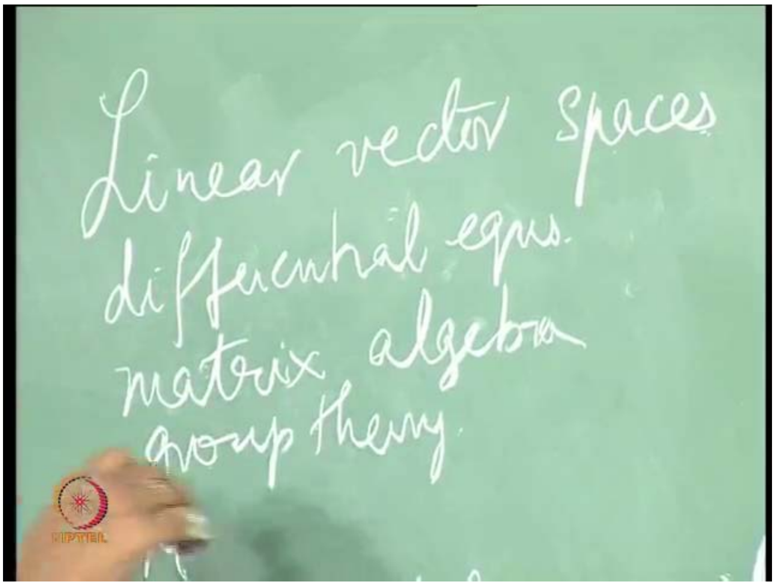

Help comes, only in the form of mathematics and the various branches of mathematics involved in the understanding of quantum mechanics are linear vector spaces, differential equations, matrix algebra, group theory. So, an understanding of the quantum world really amounts to seeing how exactly, mathematical structures are useful in describing the physics in the quantum world and in explaining quantum phenomena. And therefore, in subsequent lectures I will be first visiting some of these branches of mathematics.

Starting from, something we know and explaining some of the essential features of linear vector spaces in the next lecture and taking recourse to all these wherever necessary in order to explain the quantum phenomenon.

In this lecture, I have attempted to give you a first glimpse of the quantum world. I have tried to emphasize the contrast between a world of classical laws and a world of quantum laws, the surprises and the unexpected. I proposed to do the following in subsequent lectures: As I have already indicated, the state of the system lives in an abstract linear vector space and therefore, for something like the first half of my series of lectures I would be only looking, at the abstract state of the system. In order to, illustrate various examples I would work out exercises as we go along, rather explicitly taking into account the fact that perhaps you are heterogeneous audience of listeners and therefore, I will try to work out a lot of the mathematics explicitly.

Now, after about 20 lectures I will move on to the concept of the function space and talk about the wave function, psi: as a function of space. We will study that for a while and then proceed to do dynamics where psi is also an explicit function of time. Now and then I will intersperse my lectures with exercises, in order to illustrate the salient features. This is a good place to stop by way of introduction. In the next lecture I would begin with finite dimensional linear vector spaces calling upon ideas that we already have from vectors in 2 dimensions, vectors in 3 dimensions and so on and proceed from there.