Quantum Field Theory
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Module - 05
Radiative Corrections
Lecture - 37
Photon Self energy I

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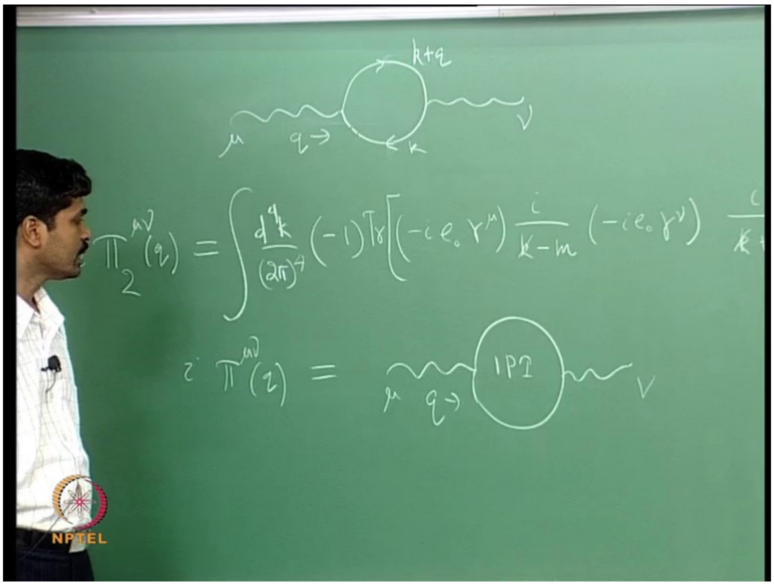

So, in the last lecture we saw the electrons self energy diagram, and we have seen how such a diagram gives contribution to the mass of the electrons. In this lecture, we will consider a diagram of this kind where there is a fermions in the loop. So, at one loop level this is last diagram that we need to consider. You have a virtual photon of momentum let us say q, and there is a pair created here which is subsequently annihilated. And finally, you again have a virtual photon; if I denote this momentum here to be k, then this will simply be k plus q. And we need to consider the contribution of this diagram, a diagram of this type to the photon propagator; this is what we will discuss in today’s lecture.

So, we can use the Feynman diagram, we know what. So, if we forget about these two photon propagators, then the contribution from this loop will be given by first you have a vertex here for which I will put a minus i e 0; in this lecture I will denote e 0 to be the charge of the electron instead of e. At the end of this lecture you will see why a 0 instead of e. So, you will have i e 0 gamma mu for this vertex. Then you have a fermions propagator which goes like i divided by k slash minus m and you have for this vertex again you have a factor of minus i e 0 gamma nu. And for this fermions propagator it is i divided by k slash plus q slash minus m, and there is a there is a closed fermions loop for which you have to put a factor of minus 1. And you need to trace over this and because there is loop here you have to also integrate over the loop variable case.

So, you have d 4 k divided by 2 pi to the power fourth and this. So, this is what you will get from a closed fermions loop like this. I will denote this to be i pi 2 mu nu of q if you integrate over k, the only thing that will be left is q. So, pi 2 will be a function of q, and as usual 2, 4 because it is second order in e 0 square, this and this; the contribution is of order e 0 square that is why I denote it as pi 2, and this is what you will get. We will evaluate this diagram explicitly little later. First we will discuss what the physics of such a contribution here. As usual we will denote all the one particle irreducible diagrams to be like this; I will write here as 1 PI.

This will be all diagrams which are usually irreducible; in the sense if you remove any of these internal lines, then the diagram is not separated into two parts. So, all such diagrams I will denote them like this and sum of all such diagrams I will denote by this symbol here. And I will define pi mu nu such that I pi mu nu of a q and this is q it is given by the contribution from such a diagram. So, this is what I will denote to be I pi mu nu of q.

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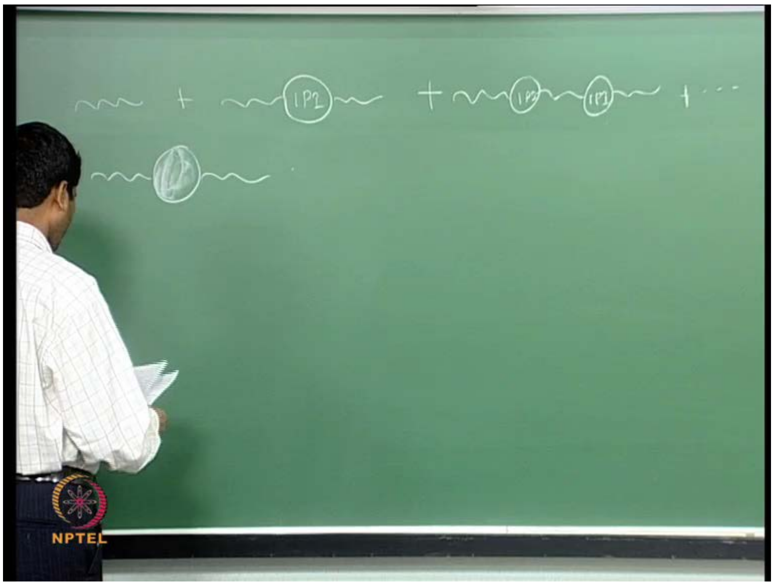

And then the full propagator will actually be given by the full proton propagator to all orders will be determined by taking sum from all such diagram. So, at tree level you simply have this; after that you have only one such term here and all such one particles irreducible diagrams included like this. So, this is what is going to give us the full proton propagator, and I will denote this by the symbol.

This full propagator is denoted by this symbol; this is what I will get. So, we will like to know what this quantity here is; even before we try to compute this quantity, we can see what for this object can take place. So, using symmetry arguments and ward identity we can get the most general for that that this guy can take place. First of all you know that this is the symmetric function and also it can only be functions of q.

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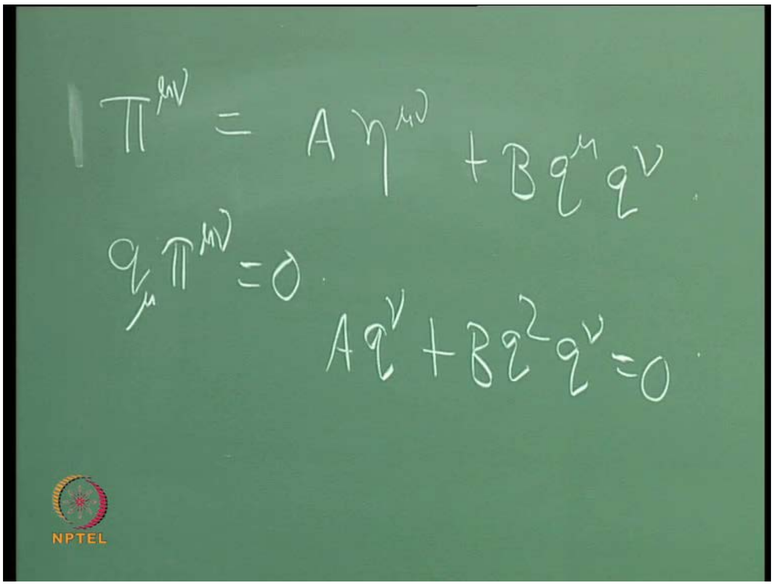

So, the only thing that it can be a function of its eta mu nu which is a symmetric second ring tensor, and in addition you can have q mu times q nu. So, the most general form for phi mu nu will be pi mu nu, for simplicity pi mu nu simply will be A times eta mu nu plus B times q mu q nu. But now if we use ward identity which basically says that q mu pi mu nu equal to 0, then this immediately give me. So, the first term A times q nu and from the second term I get plus B q square q mu equal to 0; therefore, A equal to minus B times q square.

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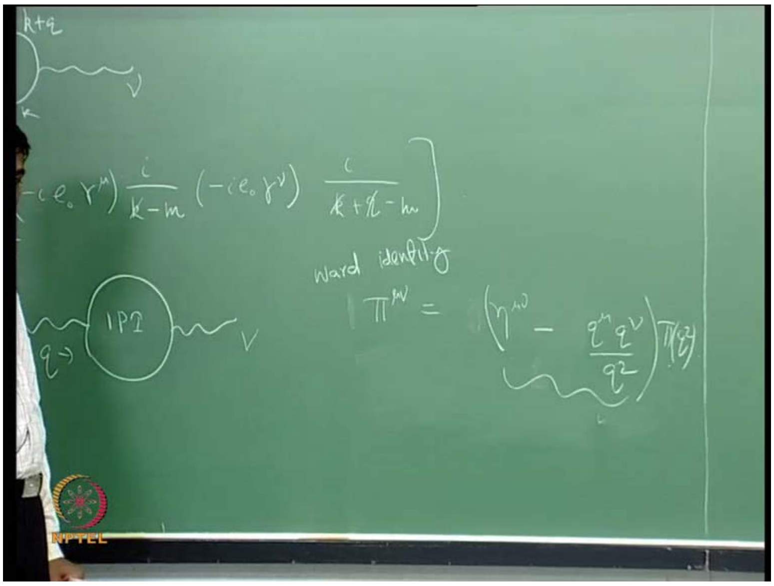

So, therefore, the ward identity implies this to be minus 1 over q square; this is what you will get, where this quantity can actually be a scalar function of q. So, therefore, this A I will denote this to be sum pi of q square where pi is the scalar function and then the tensor reel structure is given by this structure. So, this is the most general for that pi mu nu can take; therefore, all we need to do is we need to compute pi here.

So, once we know that one particle irreducible diagrams will give a contribution which is of this form, we can now come back and then we can compute the full propagator; the exact propagator to all orders in perturbation theory can be computed by summing over all such diagrams and this is given by.

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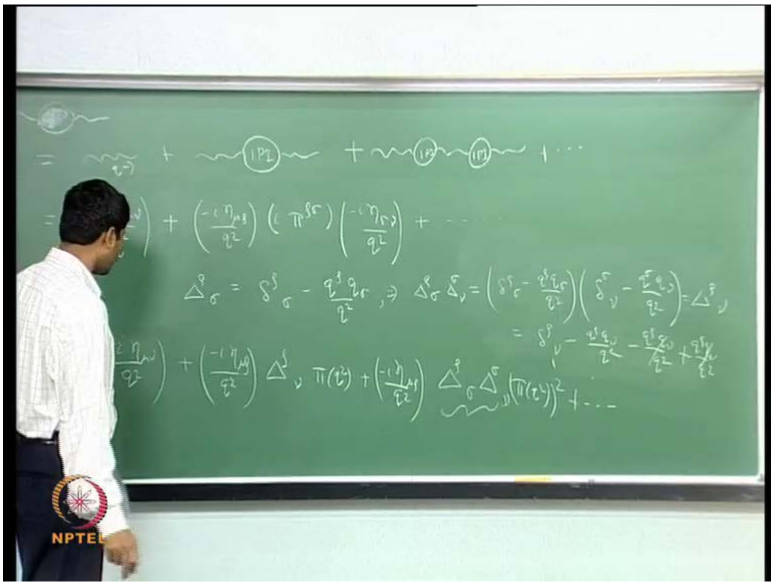

So, the tree level term will give me. So, I will write this equal to this. This at tree level I will have minus I eta mu nu; f q is the momentum then this will simply be q square. This is from this term; here you will have one proton propagator which will simply be minus i eta mu rho over q square. Then you will have this which I denote to be i pi of rho sigma and you have finally, the proton propagator here which is again given by minus i eta sigma nu divided by q square. Then you have a diagram here which will have two such terms and so on.

So, this is what you are going to get here. Now suppose I will introduce this function this tensor here delta of rho sigma to be delta rho sigma minus q rho q sigma divided by q square. Then you can see this quantity here if I multiply this with this, then i times minus i gives me plus 1, and this here will simply be.

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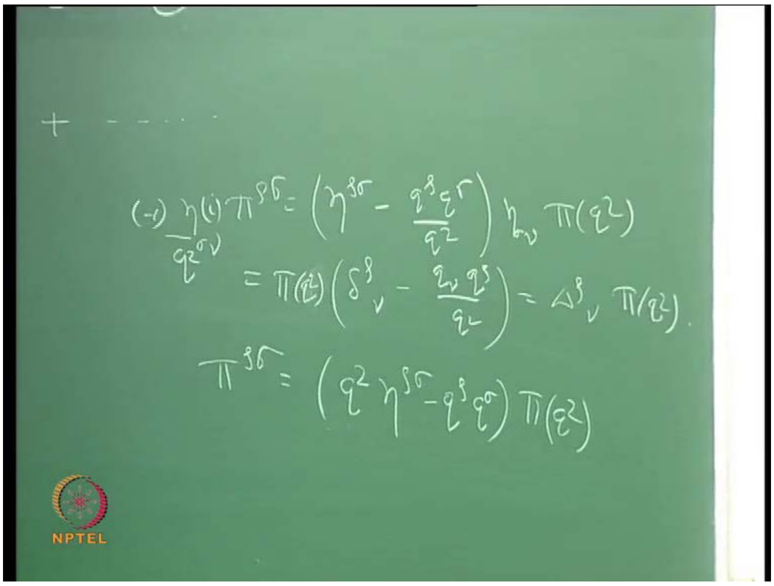

So, pi of rho sigma is simply eta rho sigma minus q rho q sigma divided by q square; to that I will multiply by eta sigma nu. So, if I have eta sigma nu then you have eta sigma nu which is basically delta of rho nu minus q nu q rho divided by q square. So, this is this delta of rho nu. So, this is what I will get if I multiply i eta sigma nu with i times pi rho sigma. For pi of q square I will use this notation here. Remember you have this q square and this term. So, I can as well write it as q square times this times pi of q square. So, my definition of pi of q square will involve a q square here. So, in terms of that this is just for convenience so that I will not have a 1 over q square here.

So, eat mu nu times eta rho sigma divided by q square minus i times i pi is given by this times pi of q square here. So, you will have a factor of pi of q square here, and then finally, you have pi of q square. So, this is just to say that if I leave the first term here, then the second and third term, the product of second and third term is nothing but delta rho nu times pi of q square. So, if this is my delta rho sigma, then what I get here is this term here is given by first term is going to remain as it is. So, it is i eta mu nu divided by q square, and the second term to emphasize what I said I denote pi of rho sigma. I know pi of rho sigma can only take this form; it can be given by q square eta rho sigma minus q rho q sigma.

Ward identity uniquely determines this form times any some scalar function of pi of q square which we need to determine by doing explicit computation. So, if this is what is pi of rho sigma, then this term is given by minus i eta mu rho divided by q square times the product of these two terms is simply delta, sigma is contracted. So, delta rho nu times pi of q square. The next term will again be written n power one minus i eta mu rho divided by q square. Then you will have delta of rho sigma, delta of sigma nu pi of q square whole square, and it will continue. So, this is what we are going to get, alright.

So, this first term is given by this; the second one is this one, and the third diagram will give a term like this and then so on. Now you notice if this is our delta rho sigma, then delta rho sigma delta sigma nu is nearly given by delta rho sigma minus q rho q sigma over q square times delta sigma nu minus q sigma q nu divided by q square. And the first term when it multiplies here gives me a delta rho nu, the second term minus q rho q nu divided by q square.

This term here minus q rho q nu divided by q square and the last term here plus you can see q rho q nu; again there will be 1 over q forth here but q sigma q sigma is again q square. So, this will be plus q rho q nu divided by q square. This will cancel, and as a result this is merely given by delta of rho nu. So, this quantity here is simply delta of rho nu, and that will keep continuing. So, what we can do is that we have a common factor i eta mu nu over q square or.

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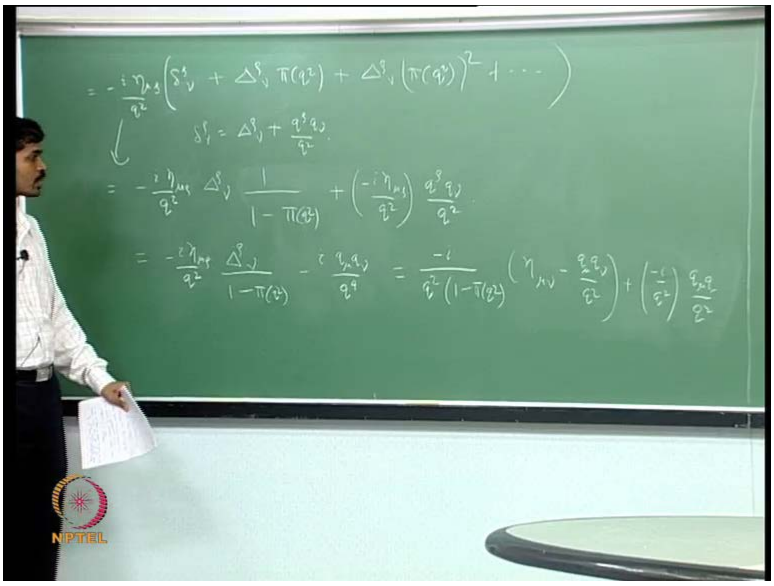

So, I can write the first term as minus i eta mi rho delta of rho nu over q square, then i eta nu rho is there in all terms. So, I can write it as i eta mu rho divided by q square times this chronicle delta rho mu, and then the second term will give me delta rho nu pi q square. The third term delta rho nu pi q square whole square and so on; this is what I am going to get. The first term also I can write it as delta rho nu is delta rho nu plus q rho q nu divided by q square. If I substitute it here, then this is going to give me minus i eta rho nu over q square; there will be delta rho nu in all terms. So, I will get delta rho nu and then whatever left is a geometrics series; it is 1 plus pi plus pi square and so on which I will simply denote as 1 over 1 minus pi of q square.

And finally, because I have a chronicle delta here instead of this, I will get an additional term here which is given by minus i eta nu rho divided by q square times q rho q nu divided by q square; this is what I am going to get at the end of the day, alright. So, what I have here it is a simply minus i eta rho nu over q square delta rho nu divided by 1 minus pi of q square and here minus I, this can contract and it will give me q mu q nu divided by q to the power fourth; this is what I am going to get. I can simplify this eta rho nu if I multiply it with; sorry, it is eta mu rho, it is not eta rho nu, it is eta mu rho. So, I have a eta mu rho in this term.

If I multiply eta mu rho by delta mu nu, what I will get is simply I take minus i q square times 1 minus pi of q square. So, minus i divided by q square 1 minus pi of q square times. This time this is merely given by eta mu nu minus q mu q nu divided by q square; that is the first term. And the second term is simply given by plus minus i over q square q mu q nu divided by q square. So, this is what I will get for the full propagator. Now this quantity has to be considered; for example, suppose we consider any physical process, then this will contribute to the s-matrix. When I evaluate this the matrix elements of the s-matrix, we already know this there will be three terms here; firs term eta mu nu times this, the second and third term are of this form.

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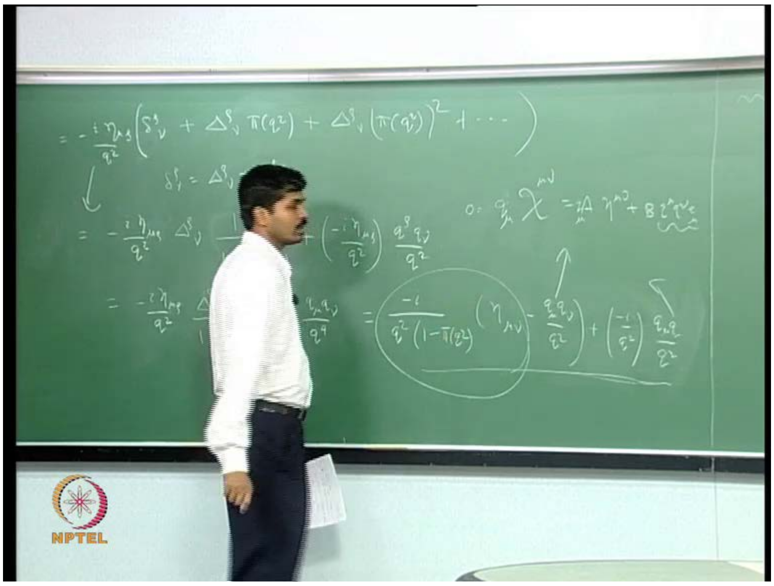

So, for example, finally, we know that whatever quantity we will get some chi mu nu kind of thing, it will have this form A eta mu nu plus B q mu q nu and so on. This quantity the s-matrix will have a term like that, and then it need to satisfy the ward identity. So, the ward identity basically says that q mu times this is equal to 0. So, therefore, you have a q mu here, you have a q mu here. So, if q square is nonzero then this B will have to be equal to 0; therefore, the ward identity tells that although the full propagator for the photon has all this term here. If you evaluate the s-matrix elements, then this term here in the second term in this expression as well as this term, they will not give any contribution.

The contribution coming from these two terms they will simply vanish; they will cancel and finally, it will become 0. So, for the purpose of computing the s-matrix element the full propagator is given by this term here; therefore, the full propagators we will simply write the full propagator to be. So, we can see that the ward identity plays a very important role here; without doing explicitly computation we can tell the form of the exact propagator by using the ward identity.

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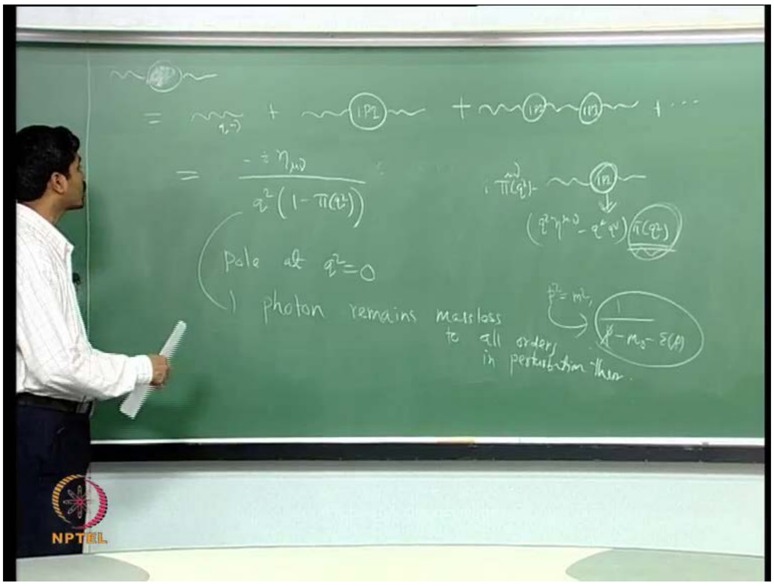

And for the proton propagator this is simply given by minus i eta mu nu divided by q square times 1 minus pi of q square; this is what is this propagator. Now what we will do is that we will see what this propagator can do for us. First of all this pi of q square, if you remember this is given by this one particle irreducible diagrams; this is our pi of q square. And so, this is pi mu nu q square i pi mu nu q square where pi mu nu is merely given by q square eta mu nu minus q mu q nu times pi of q square. And this diagram does not contain any internal proton line which is massless; therefore, because of this fact this pi of q square is going to remain finite.

If I just consider this term here this diagram contribution from this diagram, it does not have any massless propagators here. So, therefore, q square does not vanish; therefore, this quantity is finite when q square equal to 0, because q square the full proton propagator has this form. So, this result basically tells that the full propagator actually has a simple pole at q square equal to 0. So, it has a pole at q square equal to 0. Remember in contrast when we evaluated the electron self energy diagram, we did not have a simple pole; we had a simple pole at p square equal to m square, but in addition we had double pole triple pole and so on at p square equal to m square. And when we summed up all contributions, at the end of the day we got a simple pole which is given by p slash minus m 0 minus sigma of p slash or something like that.

When we summed all diagram we got once we got a term like this; therefore, this pole here is the simple pole actually got shifted because of the higher order contributions. Unlike the electrons self energy case, here the pole is not shifted; we still have because pi of q square does not have any pole. It is actually regular at q square equal to 0; because of that the exact propagator has a simple pole at q square equal to 0. And therefore, the pole is not shifted, and because of that the photon remains massless to all orders in perturbation theory; it is massless to all perturbative orders. So, therefore the mass of the photon is not shifted because of these corrections.

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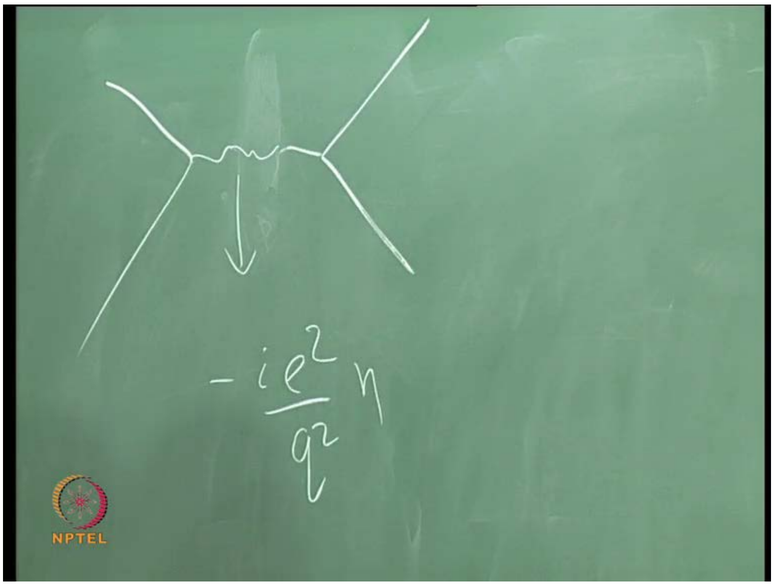

So, what do they do? What do these propagators do actually? To understand that, we can consider the s-matrix elements for some physical process; let us say for example, you have electron-electron scattering or something like that, and let us assume this scattering is actually by a low energy photon. Then the full diagram here will be given by this, the full proton propagator is given by this. So, at the tree level if you had nothing what you would have is for this propagator you would have got a contributions minus i e square divided by q square eta. So, you would have got a factor of minus e from here, minus e from here.

At the end of the day you would got, if e 0 is the charge of electron then you would have got e 0 divided by q square for such a diagram. Instead when you consider when you sum up all such diagram you do not get this if you include this, this quantity is simply replaced by this times eta mu nu; this is what you would have got, or this is what you get from this propagator at tree level. Now if we include all the quantum corrections, then instead of this we have just now showed that.

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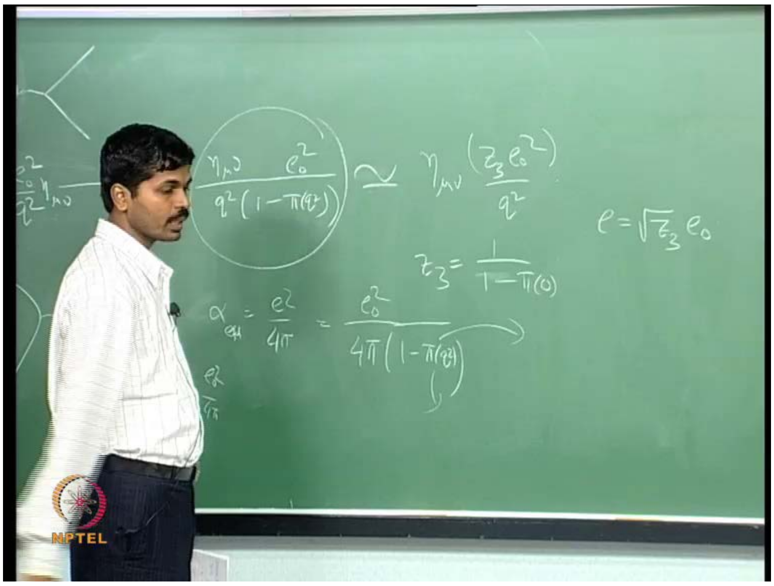

What we get instead of this is simply eta mu nu divided by q square times 1 minus pi of mu square times e 0 square. And suppose this is a low energy photon the virtual photon is low energy, then you can say that this is merely given by eta mu nu times sum z 3 e 0 square divided by q square where z 3 is given by 1 by 1 minus pi of 0. So, suppose you are evaluating this near q square equal to 0, then you can simply replace here instead of pi of q square, you can replace it by pi of 0. And I will define z 3 to be 1 over 1 minus pi of 0.

So, therefore, this quantity is nearly equal to this when q square is small. So, the role of these quantum corrections as we have seen instead of simply e 0 square eta mu nu which you get at tree level. When you include quantum corrections you get z 3 times e 0 square eta mu nu divided by q square. So, the role of this term here actually basically modifies the charge of the electrons. It renormalizes the charge of the electron, and e 0 is replaced by e which is given by square root of z 3 times e 0.

E 0 is known as the bear charge of electron; e is the physical charge of the electron and the bear charge times square root of z 3 gives the physical charge of the electron. Any physical process sees only the physical charge of the electron, and all these quantum corrections are encoded in this term pi of q square here or in the term z 3 here. So, this is what you will get for one q square is close to 0, but any finite q square you consider this is what is the contribution that you get. So, effectively you have an effective charge of the electrons which depend on q square; it is also the charge

So, the charge or if I just denote alpha equal to e square divided by 4 pi, then the effective alpha effective ion structure constant basically is given by e 0 square divided by 4 pi times 1 minus pi of q square, and the effective fine structure constant is dependent on q square and the full form of the fine structure constant is given by this expression. And this pi of q square contains the information about all the quantum corrections. At a tree level to zero th order if you consider merely a process like this, then you get alpha is simply e 0 square divided by 4 prime; however, the quantum corrections modify this, and then this is what you get.

So, if we know what is pi of q square then we know what is the effective coupling here, and this pi of q square we can explicitly determine order by order by explicitly computing all the diagrams. So, let us now compute this pi of q square at least to the second order; at one loop this pi of q square is simply given by pi 2 of q square and we know how to do loop integration. So, this is what we will discuss in the remaining part of our lecture. So, let us do a computation, and let us get the expression for pi of q square at one loop.

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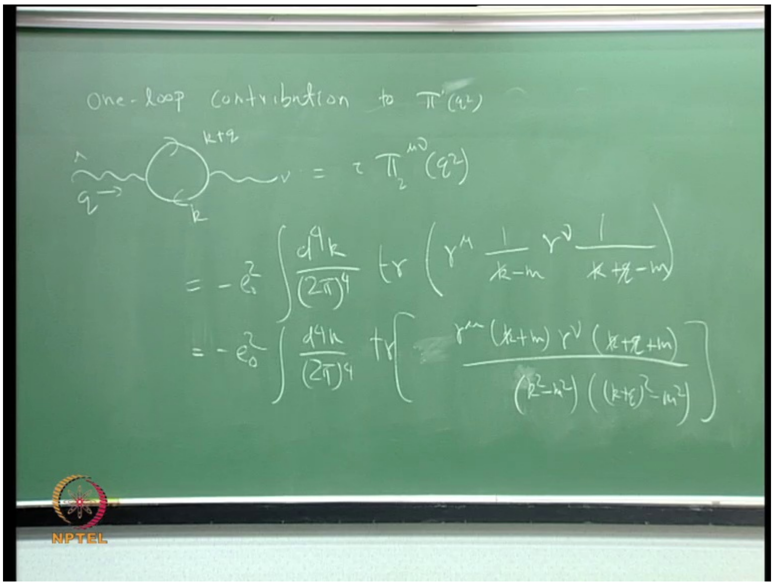

So, what we will like to do is one loop we will like to compute the one loop contribution to pi mu nu of q square or simply pi of q square, equivalently pi of q square. So, we need to consider this diagram here. This is q, this is k, and this is k plus q; sorry, this is mu nu. We know what is the amplitude for such a process, this is basically given by. So, this is i pi 2 mu nu of q square, and this is simply minus e 0 square integration d 4 k divided by 2 pi to the power fourth, trace of gamma mu 1 over k slash minus m gamma nu 1 over k slash plus q slash in a sum; this is what we have, and we would like to evaluate this integration here.

So, I will rewrite this term to be minus e 0 square d 4 k over 2 pi to the power fourth, trace of gamma mu. Then this will be k slash plus m here, gamma nu k slash plus q slash plus m divided by k slash k square minus m square times k plus q square minus m square. So, this denominator again as usual we will use the Feynman parameterization to rewrite this denominator as an integral of x, and the numerator it is just a trace involving four gamma matrices. So, it is very straightforward; we can work it out in explicitly.

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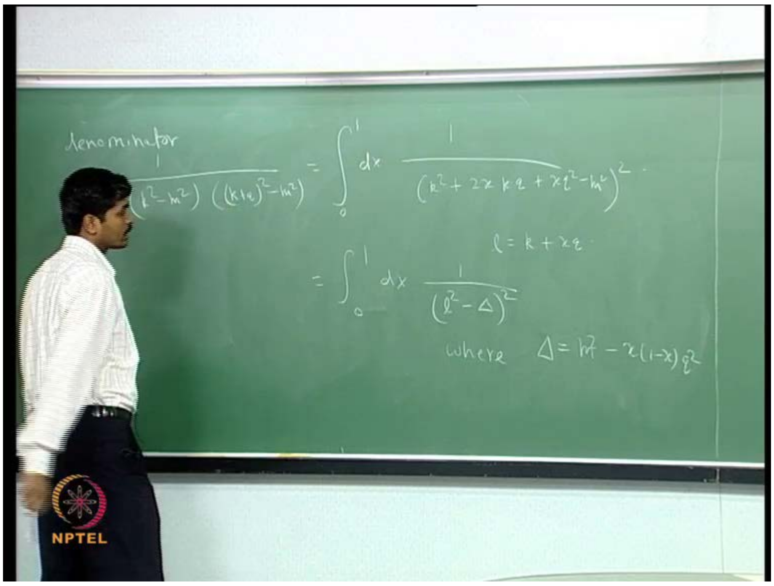

So, let us do that. The denominator is given by 1 over k square minus m square times k plus q whole square minus m square, and this quantity I will write it as integration 0 to 1 d x times 1 divided by k square plus 2 x k dot q plus x q square minus m square whole square; this is what I will get when I use the Feynman parameterization. And now again you see there is a linear term in k, and there is a quadratic term in k.

I want to complete this square. So, I will introduce the variable l which is basically k plus x cube, and I will express it in terms of l square. When I do that what I will get is it is simply 0 to 1 d x 1 over l square minus delta whole square, where delta basically is given by m square minus x into 1 minus x q square. So, this is what delta is. So, this is what I will get in the denominators; numerator again I can simplify this.

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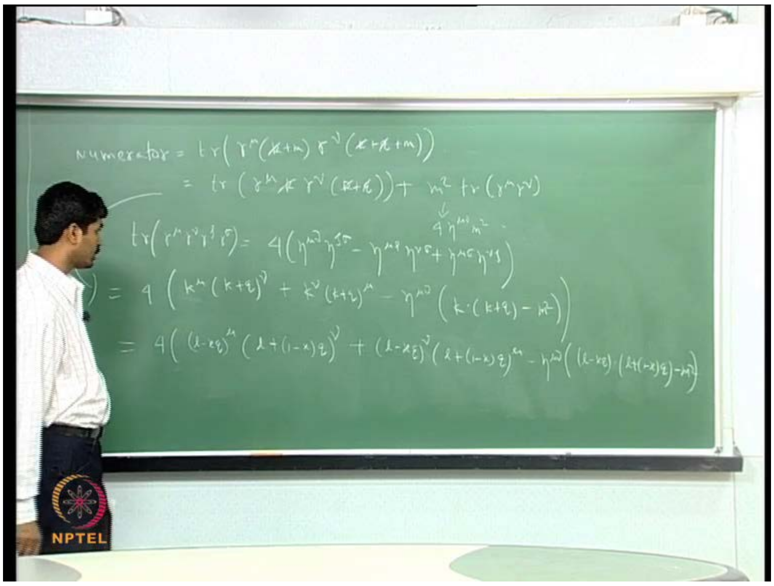

So, the numerator is given by trace gamma mu k slash plus m gamma nu k slash plus q slash plus m. And this is nothing but I can rewrite this as trace of gamma mu k slash gamma nu k slash plus q slash plus m square trace of gamma mu gamma nu; all other terms involve three gamma matrices whose trace vanish. So, this is what you will get, and this term here is merely given by 4 eta mu nu m square. And this term involves four gamma matrices; we know what is the trace of four gamma matrices. So, you have trace of gamma mu gamma nu gamma rho gamma sigma which is basically given by 4 eta mu nu eta rho sigma minus eta mu rho eta nu sigma plus eta mu sigma eta nu rho.

So, this is what we already know. Using this you can simplify that, and when we use that finally, what we will get is four times k mu k plus q mu plus k mu k plus q mu minus eta mu nu k dot k plus q minus m square. This is what is my numerator, but now I have introduced this variable l here which is k plus x q. And I would like to express this numerator in terms of l, and I know in the denominator, the denominator is a function of l square. So, I will rewrite it; I will write it in the variable l using the variable l, and I will keep only terms which contains even powers of l.

For example, if I have a l mu l nu I will keep it; if I have a term which is liner in element then that will give a zero contribution when I integrate it over all values of l. So, keeping that in mind what I have is for k equal to l minus x q. So, this is l minus x q mu l plus 1 minus x q mu plus l minus x q nu l plus 1 minus x q mu. And finally, here minus eta mu nu times l minus x cube dot l plus 1 minus x q minus m square; this is what I have.

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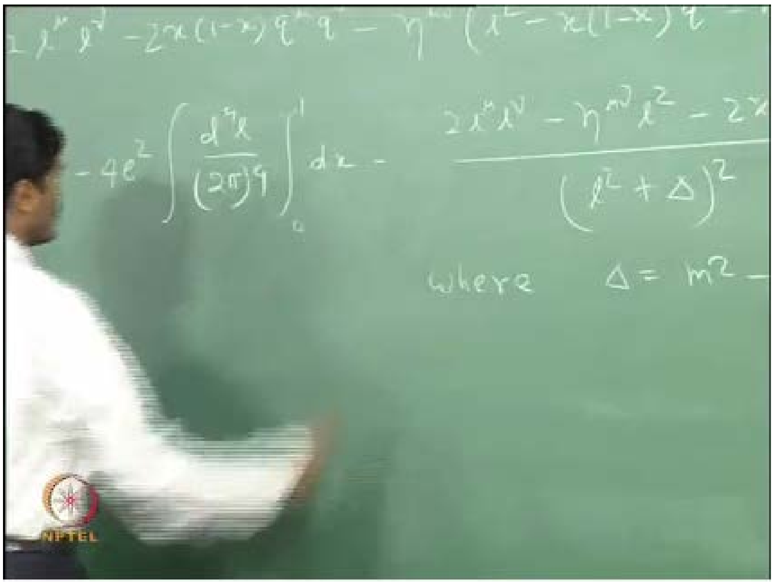

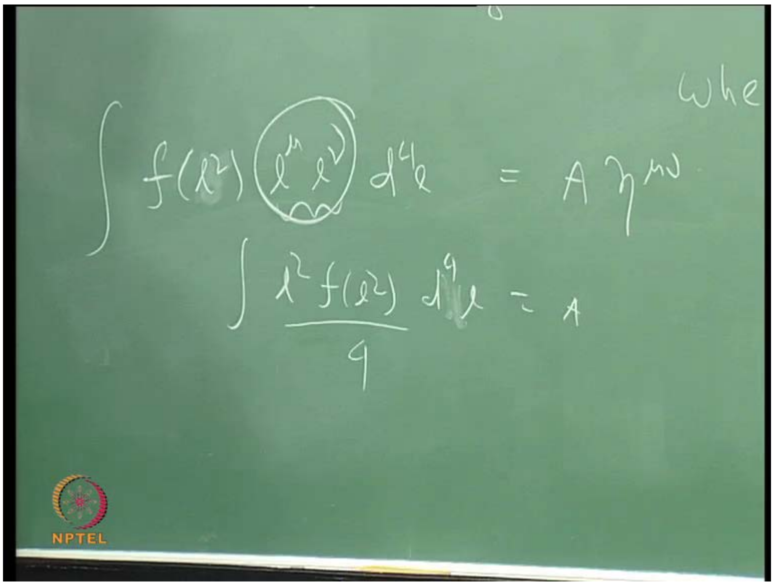

So, what I will get here is I will just get 4; this will give me a l mu l nu. This will again give me a l nu l mu. So, I have twice l mu l nu; l mu times this will give a term which is linear in l mu. So, I will not write it; however, this term multiplied by this term gives me minus x times 1 minus x q mu q nu. I will get one such term from here. So, this is minus 2 x 1 minus x q mu q nu. Finally, I have. So, this is the contribution from these two terms, and this term here will give me minus eta mu nu times, here it is l square, then minus x into 1 minus x q square; the other two terms are liner in l. So, I will not write them and finally, minus m square. So, this is what I have for the numerator.

So, when I put everything together what I will get now is i pi mu nu i pi 2 mu nu of q is going to be there is a minus e square and this is 4. So, minus 4 e square integration; now I have I am using the variable l. So, d 4 l divvied by 2 pi to the power fourth and then x is integrated from 0 to 1 d x; this is what you have in the numerator except the factor of 4. So, I have a 2 l mu l nu minus eta mu nu l square minus 2 x 1 minus x q mu q nu plus eta mu nu m square plus x into 1 minus x q square. This is in the numerator, and in the denominator I have l square plus delta whole square. We need to evaluate this integration here. We can see where delta is given by m square minus x into 1 minus x q square; this is what we will evaluate. We can further simplify this term here.

Remember, suppose we have some f of q square or f of l square some function of l square l mu l nu; when you integrate it over l, when to you integrate d 4 l at the end, because it is symmetric in l. The only thing that you can get is some quantity times eta mu nu. If you contract this with eta mu nu, what you get here is l square f of l square d x d 4 l is simply A times eta mu nu eta mu nu which is 4 A. On the other hand, if you consider l square. So, therefore, in this integration your A is merely given by this divided by 4; this equal to A is this divided by 4. So, in this integration here you can merely replace l mu l nu by l square divided by 4. So, this is what you can do.

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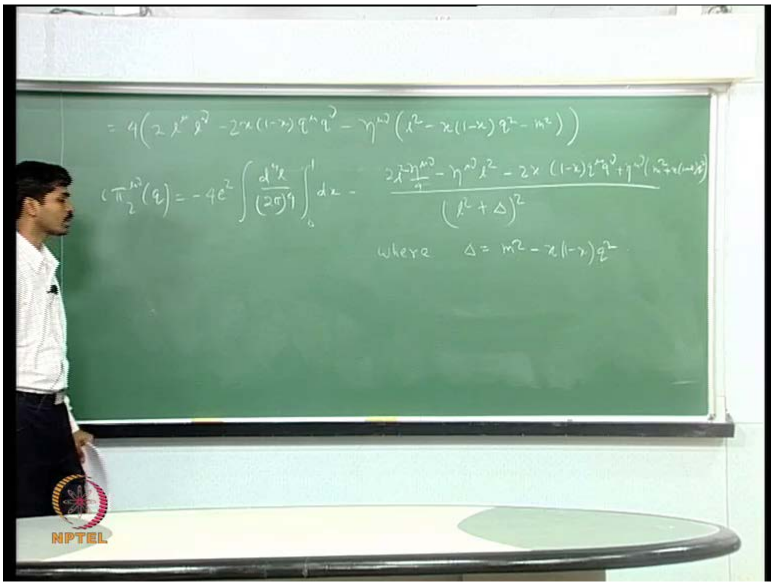

If you have l mu l nu inside the integration then this is merely given by eta mu nu l square divided by 4; this is what I can use here. So, when I do that I will simply get eta mu nu divided by 4 times l square. So, this we need to evaluate. This is again going to be divergent integral; first we need to do a week rotation Euclidian continuation and write it in terms of the variable of l e square. And then finally, we need to evaluate this integral here. It will again we divergent integral. In fact, it is going to be quadratically divergent. So, what we will do is in the next lecture we will first introduce a UV cutoff and then we will evaluate this integration explicitly. We will see that this is quadratically divergent.

In fact, if we use a UV cutoff, then pi mu nu does not preserve ward identity. It violates ward identity. So, in the next lecture we will introduce another regularization which is known as the dimensional regularization, and then we will evaluate this integration by using dimensional regularization. And then we will see that the dimensional regularization in fact, preserve ward identity. Finally, using the dimensional regularization we will evaluate the one loop contribution to the photon propagator. This is all we are going to do in the next lecture.

Thank you.