Quantum Field Theory
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Module - 5
Radiative Corrections
Lecture - 36
Electron Selfenergy

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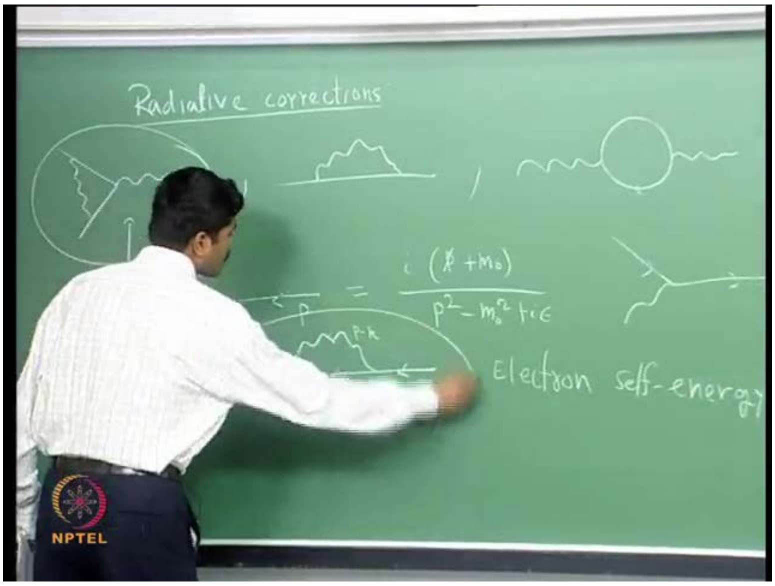

So, we have been discussing the Radiative Corrections. What we have seen is, at the one look level, these three diagrams are relevant this is one and then we have this diagram finally, we have a diagram like this, where there is fermion loop in between. We have discussed this the contribution of this diagram in great length and then we saw that this actually in fact gives corrections to the vertex function. The vertex function using symmetry argument and word identity, you can express it in terms of the form factors. And what this diagram does is that, it actually gives one loops contribution to the firm factors, and in the last few lectures, we have a computed the one loop contribution to the form factors, by evaluating the contribution from the this diagram explicitly.

In the next few lectures, we will discuss these two diagrams in great detail, this diagram especially gives the self-energy, ((Refer Time: 01:51)) this is called as the electron self energy diagram, and this is called as the photon self energy or the vacuum polarization diagram. This diagram contributes to the electron self energy, we will first discuss the contribution of this diagram in detail and then we will come back to this diagram. So, the two point function, we know already at any level it is basically given by this diagram here, and the contribution here is basically it is just free propagator, if I denote this to be p.

Then the propagator here is given by i p slash plus m 0 divided by p square minus m 0 square plus i epsilon, basically you consider this in between external lines. For example, if you have a process like this an something like that, the propagator part is given by this, this is at three level and at the one look level this is what is the diagram. So, this is the electron self energy at one loop as a given by this diagram here, I will call this to be p and this is again p, if I denote this to be k this will simply be p minus k.

And this will be next order contribution to the a propagator and so this is just again when I write this, I do not care about the external lines just like here. In this lecture we will focus basically on this diagram and then we will evaluate it an expressly in great detail, we can use the Feynman rules. And the amplitude the contribution from this diagram is a given by this, I will first write down the formula and then I will explain at in detail it is so basically this is, there are two propagators hear, and then there is loop here which contains one fermion propagator, one photon propagator.

So, we have to write all these propagators and then these vertex functions here and then because there is a loop here we have to integrate, the loop variable k that is what it is basically given here. And also I must say that, I have now used m 0 to be the mass of electron, you will see at the end of these lecture, why I have denoted it as a m 0instead of m, because in all over previous lecture. So, we used the notation m for electron mass basically that is the reason it is called self energy, it will give this diagram will give the correction to the mass of the electron, it will shift the mass of the electron. And that is what we will see when we do the explicit computation.

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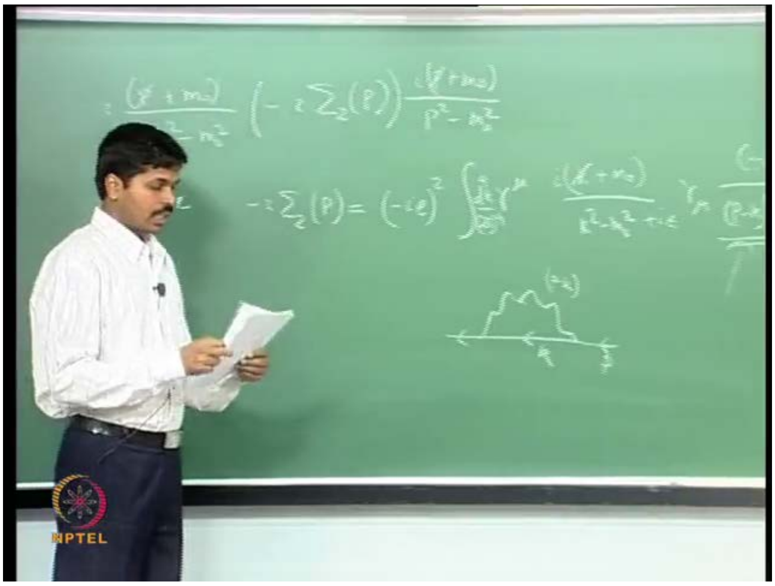

So, this diagram gives this factor here i p slash plus m 0 divided by p square minus m 0 square, then minus i sigma 2 of p i p slash plus m 0 divided by p square m 0 square. Where minus i sigma 2 of p is given by minus i square integration gamma mu i k slash plus m 0 divided by k square minus m 0 square plus i epsilon gamma mu minus i divided by p minus k square minus mu square plus i epsilon. So, to explain it in detail, this is the electron self energy diagram here, this is p and this fermion propagator here, the contribution from this propagator is given by this term here, i p slash plus m 0 divided by p square minus m square.

And here this is the this loop here, which I denote to be minus i sigma 2 of p, 2 here because it is as you can see it is a e square, because of that to for it is, because it is one loop contribution that why this sigma 2 here is. And then there is this fermion propagator here which is again given by i p slash plus m 0 divided by p square minus m square, this i sigma 2 is the contribution from this loop, as you can see for these two vertex. You have minus i e gamma mu for this and another minus i e gamma mu for this, then the fermion propagator here is given by this is k .

So, therefore, this is the contribution here is given by i k slash plus m 0 divided by k square minus m 0 square and then this vertex hears this gamma mu minus i e here. And this photon propagator, which is p minus k is the momentum carried by this photon virtual photon here. And this the contribution here is given by minus i over p minus k square, and because there is a loop we have to integrate over the loop where ever d 4 k over 2 pi to the power 4. Note that this diagram as also an infrared divergences, at this point we are not interested to study the infrared divergences.

What I have done is to cure infrared divergences I have put a cut of to the photon mass here that is given by the mu square, so there is a small photon mass I have given, so that there is no infrared divergences. If we have time we will discuss the infrared divergences in these processes in more detail, but at this moment we will just put a regulator, and we will evaluate this diagram here. So, again we know we have evaluated they diagrams like this, integration like this when we discuss the vertex correction, what we need to do is we need to use the Feynman parameterization.

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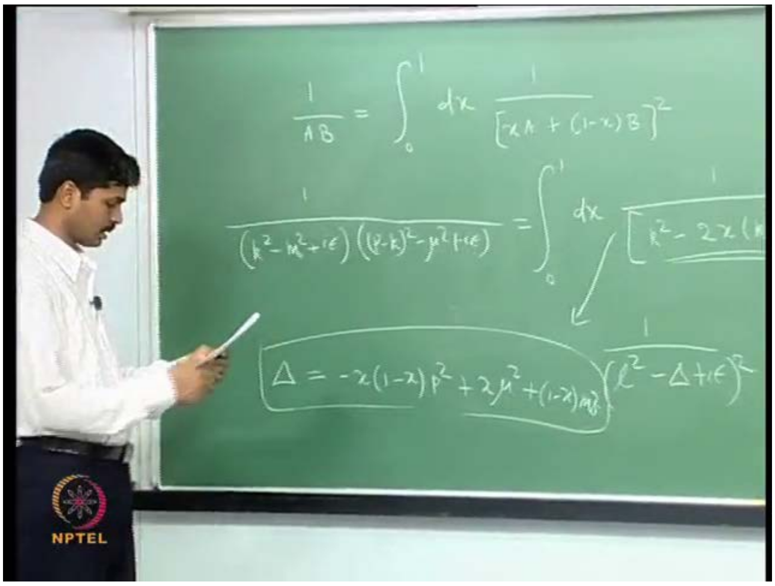

So, you have already seen this formula earlier 1 over A B, can be written as a integration 0 to 1 d x 1 divided by x A plus 1 minus x B whole square here, these two factors A and B. So, what I will do is that I am interested in writing this quantity 1 over k square minus m 0 square plus i epsilon times p minus k whole square minus mu square plus i epsilon in this for here. So, this is given by integration 0 to 1 d x 1 divided by k square minus 2 x k dot p plus x p square minus x mu square 1 minus x m 0 square plus i epsilon, this whole square, because that is there is a square here.

If I substitute this for A and B, then it is just straight for that, to see that this is what you will getting in the denominator, so what I want is I would like to again complete the square here, there is a quadratic term in k and there is a term which is linear in k. So, what I will do is that, I will observe this term here and I will write it as a some a complete square. So, that we can do the d 4 k integration remember, this k is the loop variable and then we have to integrate over all values of k and to do the integration, I need to accomplish this square here.

So, what I will do is I just consider this term here for example, k square minus 2 x k plus 2 x k dot p, so to observed this linear p is, I introduce this variable l which is a k minus x p then I can write the denominator in terms of l square and so on. So, in terms of instead of writing this in terms of variable k, if I write it in terms of the variable l, if I use the variable l, then what I will get in the denominator is the fowling. This denominator here can be written as 1 over l square, whatever is left I will denote that to be delta, so 1 over l square minus delta plus i epsilon which is there, this whole square.

So, you can see that if you use l equal to k minus x p, then this delta will be given by delta is equal to minus x times 1 minus x p square plus x mu square plus 1 minus x m 0 square, this is what is the expression for delta. So, this is what is the denominator here, and so if you come back to this diagram, what you have done is that we have used Feynman parameterization to write this. And the denominator here is basically given by integration 1 over or d x, where x goes to 0 to 1 l square minus delta plus i epsilon whole square, what about the numerator the numerator is given by this.

So, there is a minus i e square and there is a i here ((Refer Time: 14:53)) and what we are left with is gamma mu k slash plus m 0 gamma mu.

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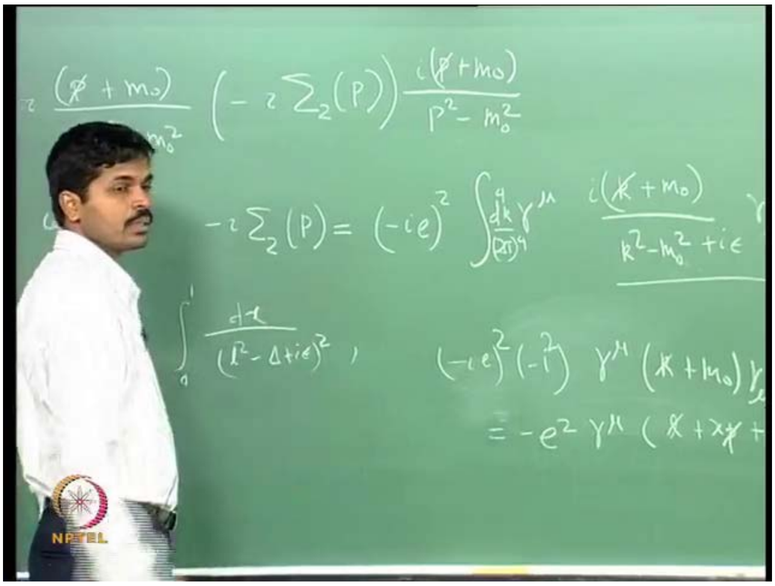

So, this is what is the denominator part, and the numerators is given by minus i e square minus i here, gamma mu k slash plus m 0 gamma mu, we can simplified this it is alertly straight forward. And you see that you will get, there is another minus i here, so what I have is minus i e square, so totally you get minus e square and gamma mu, we have used the definition l equal to k minus x p. Therefore, k equal to l plus x p, since over using instead of k were using the variable l as the integration variable.

So, I have to express this in terms of the variable l, and so this is simply given by l slash plus x p slash plus m 0 gamma mu, but now the denominator as only l square, and l takes all values from minus infinity to plus infinity. Therefore, the term which is linear in l in the numerator will be actually in all function of l, and when it is integrated for all values l it will give you 0 contribution. So, the linear terms in l will simply go away, when we perform the l integration here.

So, therefore, the numerator what we are left with is, it is simply minus e square gamma mu x plus x p slash plus m 0 gamma mu, this is what we are left with and this you can see if you take the gamma mu inside, gamma mu gamma mu will give you 4 here. So, you have 4 m 0 and gamma mu p slash gamma mu, we have evaluate it any number of time it is simply given by minus 2 p slash. So, at the end of the day this numerator here is simply given by minus e square minus 2 x p slash plus 4 m.

So, we will substitute this for the in numerator, so here l should be the integration variable and this is what for the denominator.

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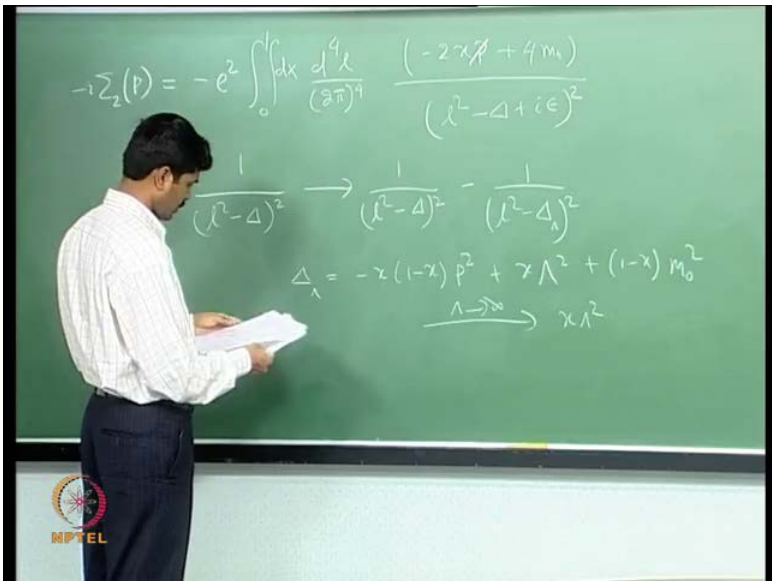

And at the end of day when I substitute all this things, what we will get for i sigma 2 is, it is simply given by minus i sigma 2 of p is equal to minus e square 0 to 1 d x and then also integration over d 4 l divided by 2 pi to the 4. Then in the numerator we got, minus 2 x p slash plus 4 m 0 and in the denominator we have l square minus delta plus i epsilon square, this is what we have from the loop. Now, you can just do the power counting here and then you can see that this l integration is actually divergent in fact, it is logarithmically divergent.

So, what you need to do is, you need to introduce cut of we have already done it when I discuss the vertex function, so what we need to do is here, we will just use the pauli-villars regularization, so we will put an upper cut off to the momentum k. So, the cut of the ultraviolet cut off UV cut off is denoted by lambda, so we will integrate instead of taking the all values of k, we will take the integration variable k from only up to the value lambda. So, this amounts to replacing this photon propagator here, simply by in the pauli-villars regularization you introduce the UV cut off to the momentum

To the photon momentum and that simply amounts to replacing this photon propagator, minus i divided by p minus k whole square minus mu square plus i epsilon by minus i divided by p minus k whole square minus mu square plus i epsilon minus i divided by p minus k whole square minus lambda square plus i epsilon, where this lambda is the UV cut of ultimately we will like to take the limit lambda goes to infinity. So, this is what we do instead of this we will substitute this.

So, then in i sigma 2, you will basically have two terms, one term is given by this ((Refer Time: 21:32)), the other term look will look exactly like this, except that instead of mu you will have this capital lambda. So, what you will have here is again everything will be exactly as it is, but instead of 1 over l square minus delta whole square, what you will have is 1 over l square minus delta whole square minus 1 over l square minus delta lambda, I will denote whatever the quantity to be delta lambda.

So, this is what you will get, instead of these the effect of UV cut of will be to introduce another term here 1 over l square minus delta lambda, we already know the expression for delta here, delta is given by minus x into 1 minus x p square plus x mu square plus 1 minus x m 0 square, this is what is our delta. Therefore, delta lambda the effect of the UV cut of is just to introduce another term, where this mu is replaced by lambda. So, delta lambda is merely given by this, were this mu square is substituted by lambda square.

And this ((Refer Time: 22:59)) quantity ultimately you will be interested in the limit, lambda goes to infinity, therefore in the lambda goes to infinity limit this is merely given by x lambda square. So, in place of delta lambda we will simply use x lambda square, so this is what we will be interested in evaluating. And so we have evaluated similar integrations, when I discussed the vertex correction.

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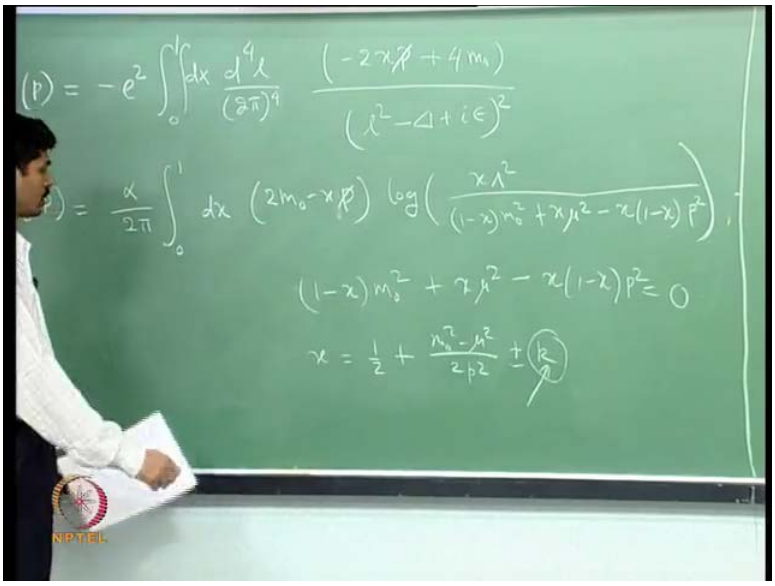

So, I will briefly summarize the result, what you have it is this integration d 4 l divided by 2 pi to the power 4 1 over l square minus delta whole square, I will recruited this, so what I will do is that instead of l 0, I will just write it as l E 0 which I will be denote as minus i l 0. And then I will do the integration in the Euclidian phase, so this and the regularization, the introduction of UV cut of together will amount to replacing this integration here by i divided by 4 pi whole square. Remember, I will introduce this and also I will integrate over the angular variables, the angular variables integration will give me a 2 pi square.

So, this 2 pi square will simply cancel this 2 pi to the power 4th, and what I will be left with is 4 pi square here ((Refer Time: 25:41)), this i because of the including the continuation. And finally, what I will be left with is an integration over d l E square early square is the integration variable divided by l E square divided by l E squared plus delta whole square minus l E square delta divided by l E square plus delta lambda whole square, this is what I have at the end of the day. And you can see that when you perform this integration here, you will get a finite peace, I will not be interested in the finite peace plus you will get a term which is logarithmically divergent.

For example, you can write it as l E square plus delta lambda minus delta lambda, the first term will give you 1 over l E square plus delta lambda, when you integrate it over, so this integration goes from 0 to infinity here, l E square values from 0 to infinity. And this here will have a finite peace term plus 1 over l E square plus delta lambda, and when you integrate it you will get log l E square plus delta lambda. And similarly here you will get a log l E square plus delta finally, when you evaluate it from 0 to infinity you will simply get log delta lambda divided by delta, with effect is i over 4 pi whole square.

So, finally, when you evaluate this integration this is what you will get here ((Refer Time: 27:45)), and this is only l integration and then you have the x integration here. So, what I will do is that, if I substitute whatever we have evaluated earlier, if I substitute for that, then I will get for minus i sigma 2 of p this minus e square and 4 pi will give me alpha divided by 2 pi. Finally, this i alpha I will observe hear, so this is simply given by 0 to 1 d x and in the numerator, if I first cancel this two here, 2 m 0 minus x p x times p slash.

And finally, log delta lambda divided by delta, delta lambda we already know is given by x capital Lambda square divided by 1 minus x m 0 square plus x mu square minus x 1 minus x p square, this is what you get for sigma 2. So, we will discussed what does it mean physically little later, but you can see already that this, because of this log here it as actually a brand skirt. And for any value of brand skirt, there is a brand skirt, whenever the denominator is negative for any value of x and it starts when this quantity becomes 0.

So, for any x the brand skirt starts when this quantity 1 minus x, when the denominator simply verses m 0 square equal to 0 plus x mu square minus x into 1 minus x p square this equal to 0. So, you can solve for it and then you can see that this quantity is negative for sufficiently large speed, so for any p if you have a real solution for x, if this equation admits a real solution for x, which lives between 0 to 1, then you have a brand skirt. And you can solve this equation and of course, this is a quadratic equation x and the solution is alertly trivial, it will be given by x equal to half plus m 0 square minus mu square divided by 2 p square plus or minus k I will not define this k here, but you can get it by solving this equation.

And this k is precisely the momentum, if you evaluate the threshold momentum for creation of two particles, then this will be given in the centre of mass frame, this is given by this k here. So, these are basically the analytical behavior of this expression here and then about the brand skirt what we will do now is, we will see what can we what can we tell about the exact propagator.

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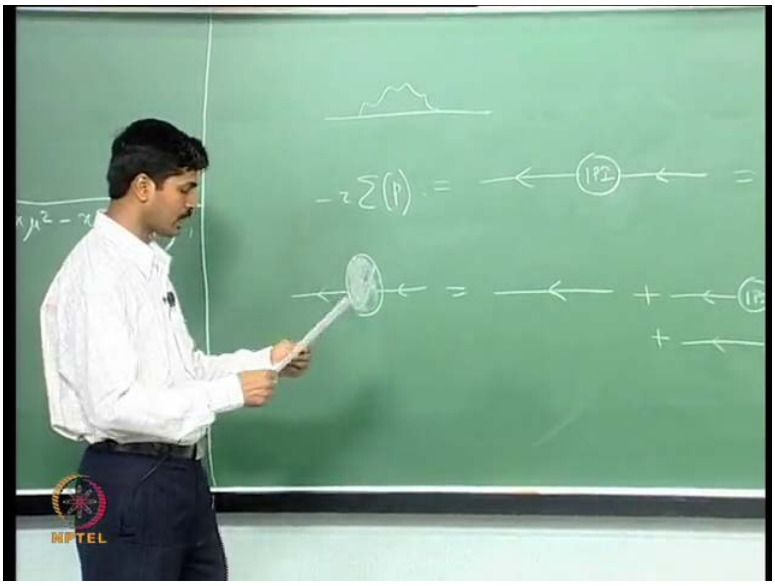

So, what we will do is that, we will introduce kind of course, that this is at one look level, but at two look level what can be except, you can have a diagram like this or you can have a diagram like this, at three look level you will have diagrams like this and so on. However, you can see if you look at this diagram or this diagram, if you cut here then this diagram is divided into two separate piece to one loop diagram. So, whereas, here if you remove any internal line you cannot separate it into two separate diagrams. So, for example, if you cut here ((Refer Time: 33:20)) or here or here wherever you want to cut you cannot simply separating in to two diagrams.

Therefore, this diagrams like this are known as one particle irreducible diagrams whereas, these are not irreducible diagrams these are reducible. So, these ones we will denote to be 1 p i, one particle irreducible diagram and at one loop level which is order e square, it is simply given by minus i sigma 2 to all orders I will denoted this one particle irreducible diagrams to by this simple well I will write 1 p i, for one particle irreducible. This is the one particle irreducible diagram to all orders, and these will be given by summing over all one particle irreducible diagrams.

So, you have diagram like this ((Refer Time: 34:51)), this is at two loop plus you have another diagram at two loop, which is given by this, then you will have three loop and so on, all such diagrams you sum over. And this is what you have, if this is what you denote as the one particle irreducible diagram, then the exact propagator basically will be given by some which involves all such one particle irreducible diagrams. So, the two point function to all orders I will basically denoted it like this, and this will be at three level of course, this is just the free propagator.

At then you have contribution from these then you have and so on, some over all such terms, will basically give you the exact propagator. This I will denote to be minus i sigma p and we would like to evaluate the exact propagator here, so the exact propagator basically will have the following for.

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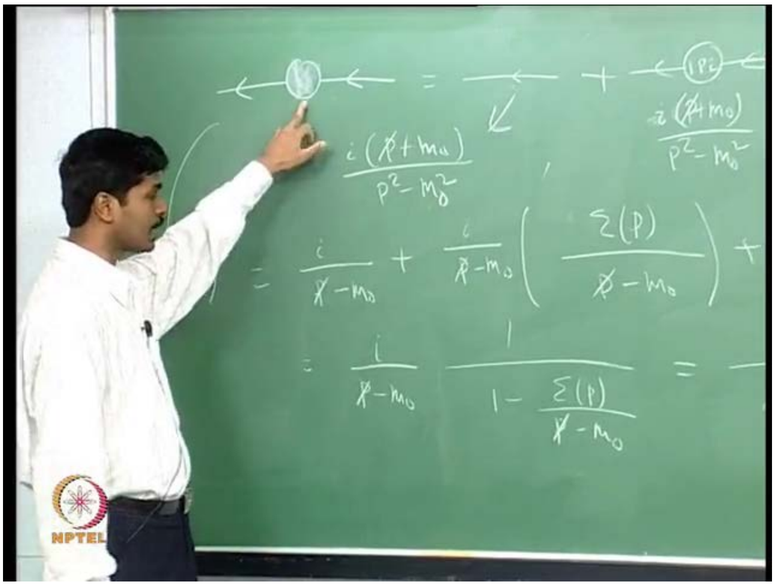

So, the three part of course, we all know to write it again, you have this is given by this plus and so on, this quantity here is merely given by i over p slash plus m 0 divided by p square minus m 0 square. Whereas, this quantity here you remember, this I am denoting this quantity to be minus i sigma p without these two propagator here, so if I include these two propagator also, what I have is for this propagator here, minus i p slash plus m 0 divided by p square minus m 0 square. And then this one I am denoting it to be minus i sigma and again here, i p slash plus m 0 divided by p square minus m 0 square so on.

So, this is the contribution from this term, similarly you can write down the contribution from this term which will involves, one fermion propagator here ((Refer Time: 38:43)), then minus i sigma for this, another fermion propagator minus i sigma another fermion propagator and so on. So, this you can write it like this, you can see that this the three level term as actually, it has a single pole at a p square equal to m 0 square, this term has a double pole at p square equal to m 0 square.

Because, it is 1 over this term contains 1 over p square minus m 0 square whole square, similarly this will have a triple pole and so on, so you will have all this diagram, the full propagator will involve higher order poles of all powers. So, this looks verse, but it is not as it looks, because as you can see it has a geometric matrix series, which you can sum up exactly. Because, you can sum up exactly, it basically what at the end of the day, what you will get is basically a simple poll, which is sifted instead of getting the pole at p square equals to m 0 square; we will see that we will have a simple pole which is shifted.

So, let us sum it up and see what we get it is of course, it is very straight forward to sum it up, I will re write this term here, I will simply write it as i, the first term is i p slash minus m 0, and the second term here is basically i divided by p slash minus m 0 times i sigma. So, this ((Refer Time: 40:39)) minus I will give you plus 1, so sigma of p divided by p slash minus m 0, and the next term will see again will be exactly like this, you will i p slash minus m 0 and this quantity will appear twice. So, you will sigma of p divided by p slash minus m 0 whole square and so on.

So, this you can of course, write it as i divided by p slash minus m 0 times 1 plus sigma p over p slash minus m 0 plus sigma p over p slash minus m 0 whole square and so on. You can this is a geometric series, you can sum it up a exactly and when you do at the end of the day what you get is, this times 1 over 1 minus sigma of p divided by p slash minus m 0. So, you can tack it inside, in the denominator and you can multiplied and what you have is simply 1 over p slash minus m 0 minus sigma of p slash, so everything will involve p slash only, this is what you will get.

So, you can see at the lowest order it looks like, there is a pole at p slash, the propagator as a pole at p slash equal to m 0, but when you considered the exact propagator this is what is the expression for the exact propagator. And the exact propagator as a pole which is not located at p slash equal to m 0, but the location of the pole is shifted, it is simply a pole and it is actually shifted, it is shifted to the point when denominator is 0.

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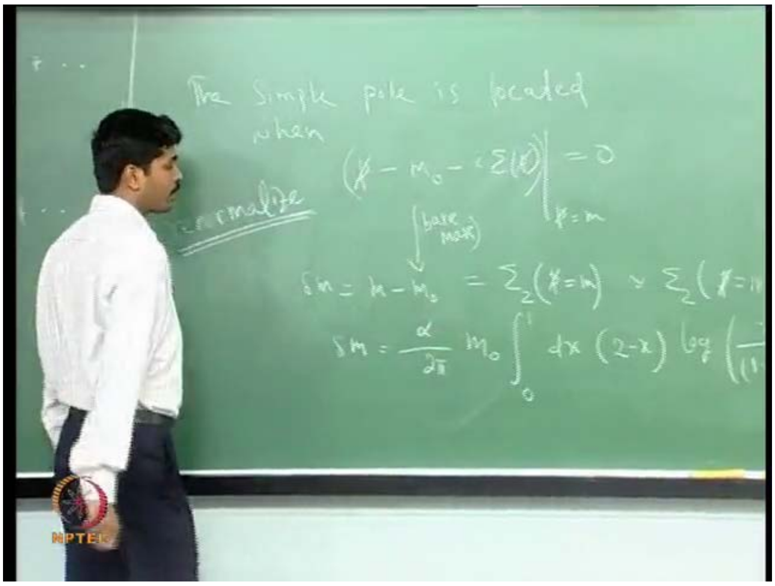

So, basically the simple pole is, when p slash minus m 0 minus i 0 sigma p slash is equal 0, we will denote the location to be m, so p slash equal to m the solution of this equation we will denote to be m. So, therefore, this equal to 0, when p slash is equal to m and then we see that this m which is actually the physical mass is not exactly equal to m 0, m 0 is known as the bare mass, and m is the physical mass. And the physical mass is basically determined by the simple pole of the exact propagator.

And we can compute this m, if we know what sigma p is and we know sigma p order by order we have just computed sigma p to the second order in e, so therefore we have an expression for m in terms of m 0 and so on, so that is what we will denote. So, they simply p slash equals to m, if I say delta m to be the difference between m and m 0, so this is basically m minus m 0. And delta m is what we have computed and this is given by sigma 2 at p slash equal to m, to the lowest order we can write it as sigma 2 evaluated at p slash equal to m 0.

So, we have already evaluated sigma 2 and if we simply substitute p slash equal to m 0 or p square equal to m 0 square, then what we will get is basically given by delta m equal to alpha divided by 2 pi m 0 integration m 0 1 d x 2 minus x log x lambda square divided by 1 minus x hole square m 0 square plus x mu square. So, therefore, this diagram here actually, so what we got sigma 2 comes from this diagram here ((Refer Time: 45:59)) and it basically gives mass shift for the electron, this diagram contributes to the mass of the electron.

And it is basically the shift in the mass of the electron is given by this diagram here, and you can see we have already discuss this diagram is actually divergent, you can see it because of the presence of lambda square. We had introduced a UV cut of here and ultimately we would like to tack this cut off to infinity, and when you tack last lambda goes to infinity this diagram, this term here is actually divergent. So, you have logarithmic divergent here, therefore it looks like the shift in mass is actually divergent, and you can say that the contribution diagram does not make sense.

Because, you are doing a expansion and then the next order term is basically divergent, at this point although it looks like a doest make any sense, it is because we have started with m 0, in the Lagrangian, when we did the cruelty Lagrangian with started with, at m 0 as the mass of the electron. And this ((Refer Time: 47:27)) m 0 it is self divergent that is why, this term here looks like divergent, instead of this m 0 known as the bare mass, and this computation tells that bare mass is not a finite quantity, it is a divergent quantity.

What is finite is the fiscal mass, the fiscal mass of the electron is known as divergent, it is finite quantity, so what you need to do is you need to renormalized the mass of the electron. So, what you need to do is that, you need to modify the Lagrangian, cruelty Lagrangian you started with appropriately, so that it contains finite quantities, if this Lagrangian contains finite quintiles, then theory of course, make perfect sense. And therefore, so by renormalizing what you do is that, you absorb the divergence in the bare mass.

And ultimately what you get is a finite quantity and if you do a contribution theory in terms of this finite quantity, then everything makes perfect sense. So, what we did here we have a discussed the electron self energy term in great detail, what we will do in the next lecture is, we will compute the photons self energy term, which is given by this diagram ((Refer Time: 49:12)) in a similar manner. And then we will see what this term basically gives.