Quantum Field Theory
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Module - 5
Radiative Correction
Lecture - 35
Vertex Correction IV

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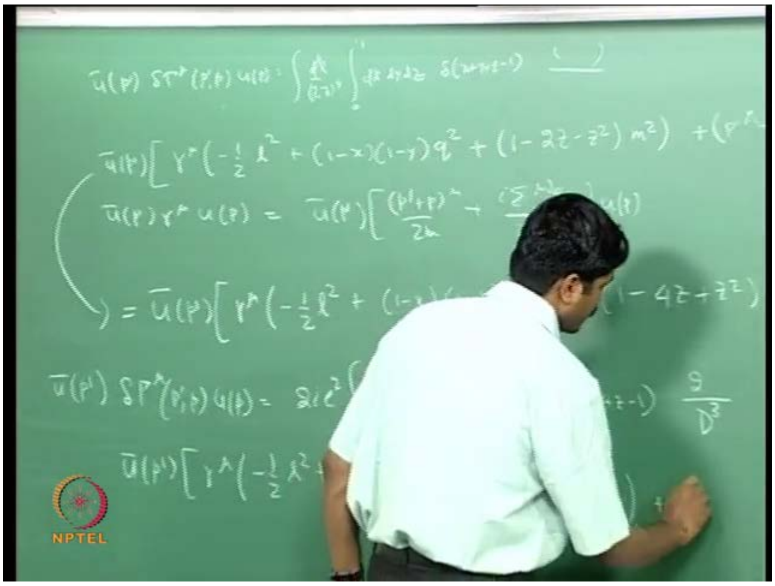

We are computing the one look contribution to the vertex operator u bar p prime delta gamma mu p prime p mu of p. What you have seen earlier is that, this can be expressed as a integration over d 4 l over 2 pi the power 4, and then you have introduced fine man parameters. So, there are integration over d x d y d z with the restriction that x plus y plus z is equal to 1, and then you have some numerator, these are the bracket numerator. So, we took the numerator, and then you have simplified it to certain extent, and at end of the day. What we so is that the numerator can be expressed is u bar p prime times gamma mu minus half l square plus 1 minus x into 1 minus y q square plus 1 minus 2 z minus z square m square.

Then u prime mu plus p mu m z into z minus 1, we want to use the golden identity and re write this sentence of gamma mu times some for factor of f 1 q square, and then sigma mu nu q nu times, so more there 4 factor f 2 f 2 of q square. So, we will use the golden identity, which is given by u bar p prime gamma mu u of p is equal to u bar p prime p prime plus p mu divided by 2 m plus i sigma mu nu q nu divided by 2 m u of p.

So, using that we will remove this term, and then we will rewrite the numerator, after we use the golden identity terms have to be u bar p prime gamma mu minus half l square plus 1 minus x 1 minus y q square plus 1 minus 4 z plus z square times m square. And then i sigma mu nu q nu divided by twice m that is 2 m z into 1 minus z u of p, this is the numerator, so what we have to do is that we have to take this thing here we have two put it and then carry out the integration, carry out the l integration.

So, when I put it here, what I get is u bar p prime delta gamma mu of p prime p u of p is 2 i e square, if I keep track of all the factors then d 4 l divided by 2 pi to the power forth. Integration 0 to 1 d x d y d z delta x plus y plus z minus 1 2 over d cube times, u bar p prime the whole thing here, gamma mu times minus half l square plus 1 minus x 1 minus y q square plus 1 minus 4 z plus z square m square plus i sigma mu nu q nu over 2 m twice m square here z into 1 minus z u of p.

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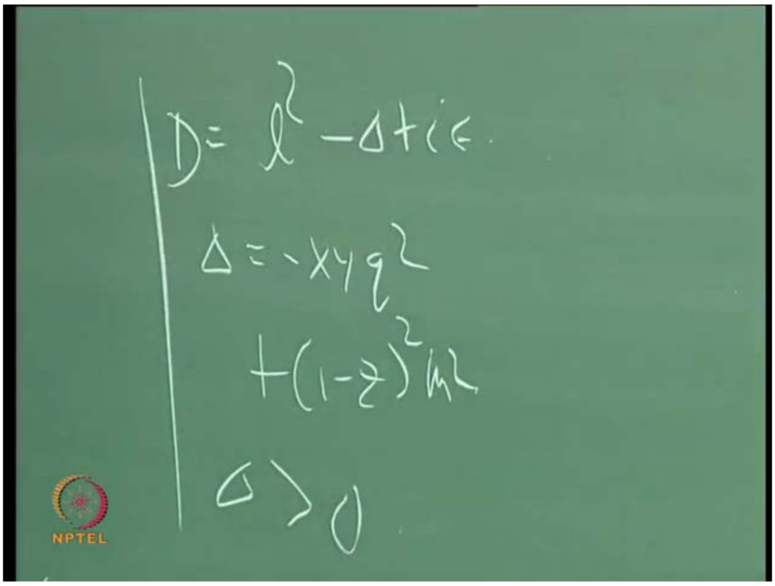

Remember, this d here, is a l square minus delta plus i epsilon, where delta is minus x y q square plus 1 minus z whole square m square, and we have argued earlier that the q square here is less than 0 therefore, this delta is greater than 0, this implies that delta is greater than 0. So, now what I will know, what we will do is that will have to carry out this d 4 l integration, you have to remember that this is actually mucus kern matrix, and then in this d square, you have l 0 square minus l square minus delta that is what comes in the denominator.

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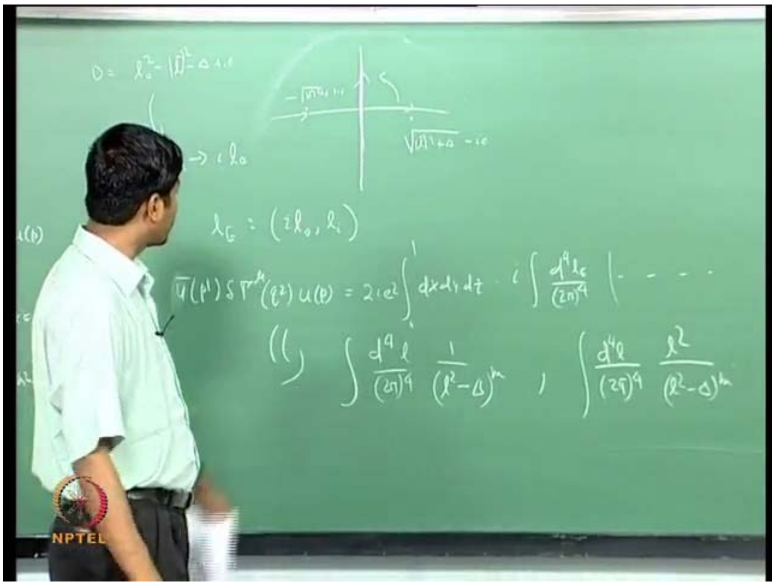

What we can do is we can perform a weak rotation, you remember that if you look at this quantity this d cube was like l to the power 6 at large l this. So, there are two things first of all there are if you look at complex l 0 plane, then there are to poles here and here, because here denominator contains d cube, where d is this plus i epsilon. So, at l 0 equal to square root of mod l square plus delta minus i epsilon, there is a pole, and then there is one at minus mod l square plus delta plus i epsilon.

You can considered the counter integration this integration, and then what you can do is that, because it if you close this you can do it in either in any way you want, you can close it. And then you can use residue theorem or what you can do is because the integrant actually when this is for large l 0, you can also perform a weak rotation, and then you can carry out this integration along this line, so this safely amongst to setting l 0 equal to i times l 0.

So, when I do a weak rotation, what I will do is that I will just denote the new variable after new integration variable after weak rotation to be l E, so the 4 vector l E is as that the 0 th component is i l 0, and the i th component is l i. So, then the integration here, in u bar p prime delta gamma mu of q square u of p, what you get is twice i e square integration d x d y d z 0 to 1 times i times d 4 th le divided by 2 pi to the power forth, and the whole lot of things.

In other words, so the integration, in both the terms here, if you notice here you have two different kinds of terms, one of them in bents integration over bunch of quantity times, l 0 l square divided by d cube, and then there are other things which is actually involves an integration which is of this for 1 over d cube. So, what we need to do, if you look at this is there are two different kind of, this gives this tills as that you need to evaluate integrations of this kind d 4 l divided by 2 pi to the power 4th 1 over l square minus delta to the power m.

For our purpose m is actually 3, and the other l integration that we need to evaluate is of this type, d 4 l divided by 2 pi to the power 4th l square divided by l square minus delta to the power m. These are the two different kind of integration, that we need to evaluate, if you perform a weak rotation that this integration d 4 l over 2 pi to the power four 1 over l square minus delta to the power m.

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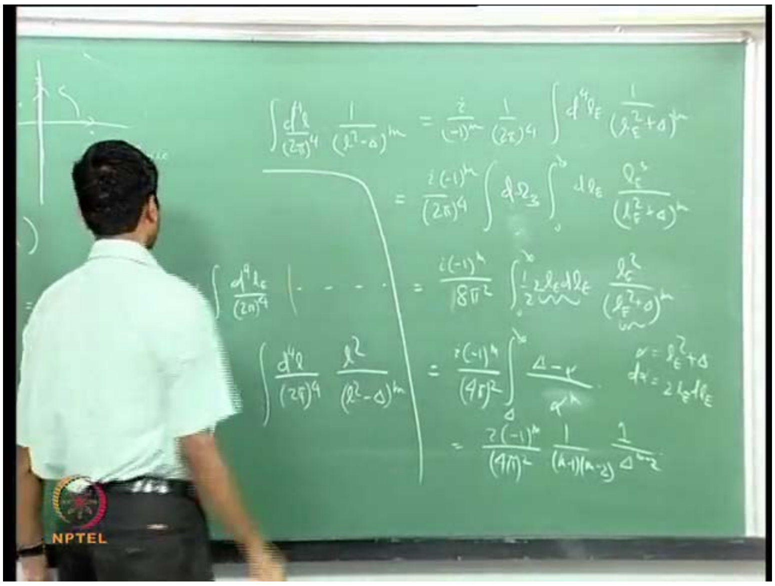

This goes to i because of the weak rotation divided by minus 1 to the power m, because the whole thing this is a minus sign, so minus 1 to the power m and 1 over 2 pi to the power 4 th d 4 l E 1 over l E square plus delta to the power m. Now, you work in the utility and space, and then the integrant depends on l E square the length of the vector l

E, so you can go to clerical polar coordinate in 4 diamonds null utility and space and then carry out this integration.

So, this integration here is why is there a minus sign, d l 0 is l 0 goes to, so let us call l E this is a subtract minus i, l 0 this is what you say l E minus I will define l E it to be i times l 0, d l 0, so l 0 is i times d l 0. So, you just take this to be there, is a minus i here and then is everything is fine, you just if the variable along this is l 0, then it this sign actually will shows the direction and along which you carry out the integration.

Now, it is i divided by i times minus 1 to the power m over 2 pi to the power 4th and d omega 3 0 to infinity d l E, in a 4 diamonds none utility and space d l E 4 is simply d omega e, where this d omega e gives you the volume element on unit 3 sphere, and then l E cube d l E. So, you substitute that you get l E cube divided by l E square plus delta to the power m, the integration over a unit 3 sphere will simply give you 2 pi square, just like if you consider an integration over a s 2.

It just gives the surface area of a unit to sphere which is just 4 pi, similarly the area of unit s 3 is simply 2 pi square nothing depends on a angular variable. So, this integration will nearly give you 2 pi square and so i minus 1 to the power m divided by eight pi square because of 2 pi square here 0 to infinity l E d l E square over. So, you just take l E square over delta to be some variable, let us say alpha, alpha is l E square plus delta this is very straight forward to carry out this integration, we will do it d alpha is just 2 l E d l E, so that we will substitute here.

So, there is a half times 2 l E d l E, which is just d alpha l E square is alpha minus delta and here, alpha to the power m. So, there is this 2 will multiply this 8, and then it will give me i minus 1 to the power m divided by 4 pi whole square and then here integration will range from delta to infinity, this is delta minus alpha divided by alpha to the power m. So, now, you know alpha to the power m, I have to carry out this integration, I will just write the answer it is a at the end of the day, it is just i minus 1 to the power m divided by 4 pi square 1 over m minus 1, m minus 2 1 over delta to the power m minus 2.

That is all you get when you carry out this integration, in a similar way you can carry out this integration with an l square in to numerator, so all you will get is there will be l fourth here. So, you have delta minus alpha whole square everything else will be exactly as it is l E square is alpha minus delta, so I will write down this answer to this integration here.

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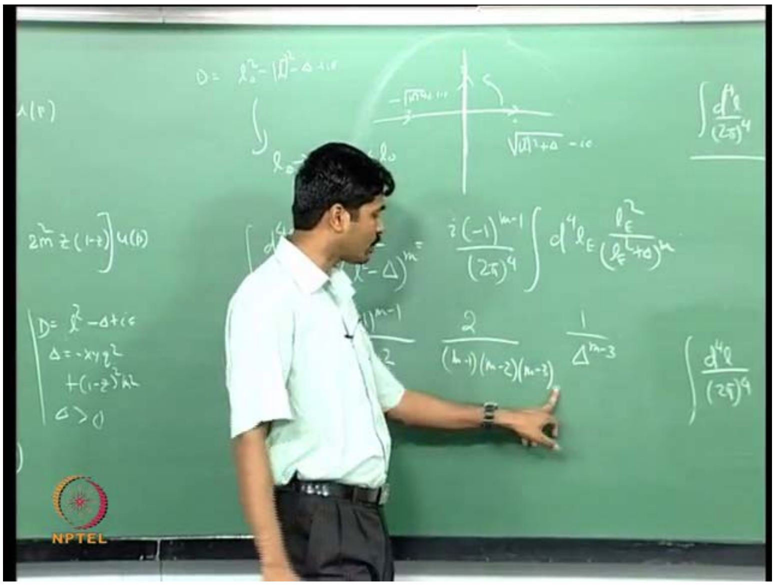

L square divided by l square minus delta to the power m, because here there is an additional l square here, there will be an additional minus sign. So, this is i minus 1 to the power m minus 1 over 2 pi to the power fourth the fourth l E l E square over l E square plus delta to the power m. And again you use the same redefinition of variables, then you will get i minus 1 to the power n minus 1 4 pi square 2 over m minus 1 m minus 2 m minus 3 1 over delta to the power m minus 3.

So, this gives you finite answer for m greater than 3, but as you can see m equal to 3 gives you infinity, if m equal to 3, if you simplicity m equal to three you get you get this integration divorcing. In fact, you can carry out this integration and then if by putting m equal to three here itself, you will see that you are getting logarithmic divergence. We will see where exactly this divergence appears here in this integration, or I have 2 for factors which is one of them f 1 q square. And then the other one f 2 q square it does not give any contribution to f 2 q square, because f 2 q square does not enable an integration of this type.

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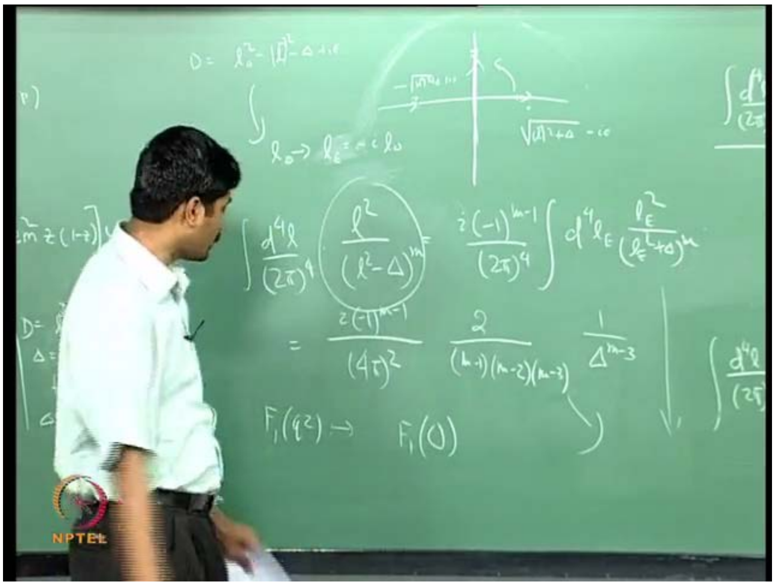

So, it is only f 1 q square, which is actually divergent, so we will not discuss a systematic procedure to cube this problem in today's lecture. What we will merely do is that we have already argued in one of the earlier lecture that f 1 of 0 actually should not receive any a one loop correction, here what you see is f 1 0, in fact actually is divorcing, because of these.

So, what we will do is that we will just a introduced a quote of in the integration, so that this quantity actually gives you finite answer or another words if you see the origin of this divergence here. The divergence appears, where because of the fact that the momentum variable l which takes all the value from 0 to infinity. If you introduce a quote of in the momentum, or another words you say that you have actually photons.

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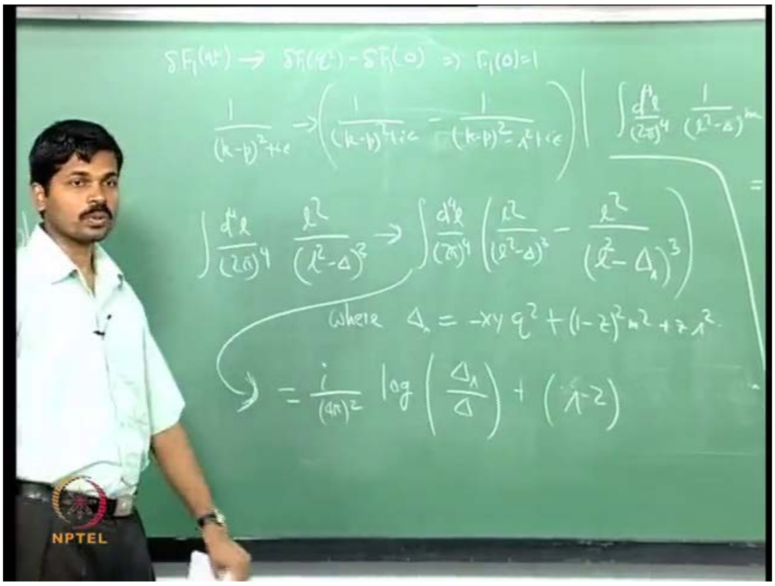

So, if you look closely this appears, because of this variable l, which takes all the way from 0 to infinity, so you just put a quote of in the momentum of the photon propagator. So, you have this intermediate photon, which for which the propagator is this and then the this here the k actually takes all the way from 0 to infinity. You say that you will only keep the photons up to some momentum k prime, and then you will ignore photons of all are momentum.

That amounts to replacing this photon propagator by 1 over a minus p whole square plus i epsilon minus 1 over k minus p whole square minus lambda square plus i epsilon. In the lambda goes to infinity, of course this term actually vanishes, so you recover this thing, so let us merely replace this photon propagator by this you will get two terms one of them will. So, basically here integration of d 4 l divided by 2 pi to the power 4 l square divided by l square minus delta whole cube, this will merely, because of this substitution.

It will simply become d 4 l over 2 pi to the power 4 l square divided by l square minus delta whole cube minus l square dived by l square minus. I will define something, which I will call as delta lambda whole cube, where delta lambda is nothing but minus x y q square plus 1 minus z, whole square m square plus z times lambda square or another words if this term as not there this would simply be delta, because of this lambda here. You get z lambda square, you remember this fine man parameter involve x times on factor, y times on factor and z times k plus or a k minus the whole square.

So, when you make this replacement here, you will get an additional term, z lambda k minus p square is finally, will give you a term z lambda square here. It is very easy to carry out this integration, you can x again you can make a substitution like these, and then you can explicitly carry out this integration. I will not do that what I claim is that when you carry out this integration, you will get i over 4 pi square look delta lambda over delta, and then there are other term which are finite.

They will go like lambda to the power minus 2 terms of order minus 2, so the divorcing term comes like these, so we will as I said this divorcing term actually gives a contribution to f 1 of q square. So, we will merely say that we will add out this prescription where delta f 1 the loop correction to the for factor q square we will merely substitute, it we will merely replace this by delta f 1 q square minus delta f 1 0. So, here substituting this infinite term by making this replacement, so that our f 1 0 becomes one at one loop.

Is this the only kind of diversion is appears here, it turns out that this is not the only diversion associated with this f 1 term, it also diverges at low momentum. So, these are all ultraviolet divergences, because this divergence comes, when the photon momentum goes to infinity, when the intermediate photon momentum goes to 0, then also there are other kind of divergences. So, these are known as the infrared divergences. So, we can verify that this f 1 of q square term also as an infrared divergence, this term here the contribution due to this term here, gives you infrared divergences, so we will see how these divergences appear in a moment.

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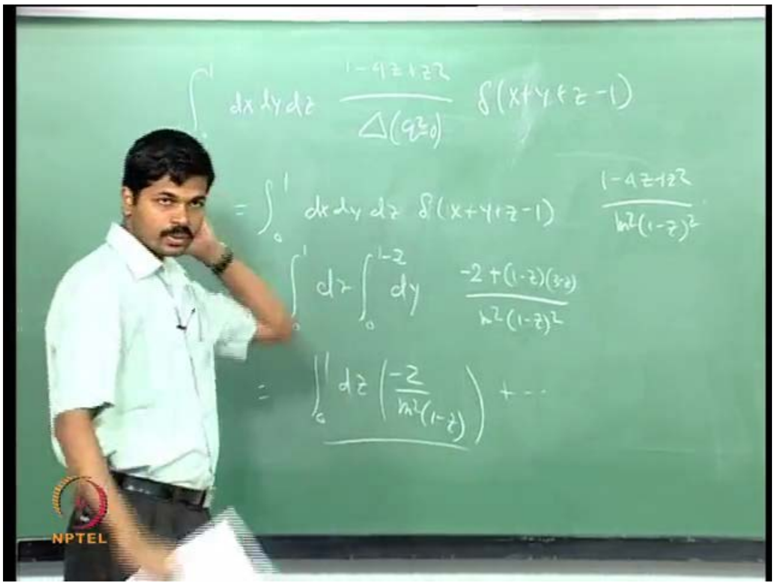

So, 0 to 1 d x d y d z, so the l integration gives you some finite answer we have already seen, if there is no l square in the numerator, then it is perfectly finite there is no problem with that. So, what we have from this term is 1 minus 4 z plus z square divided by delta q square, we are interested in q square equal to 0 behavior, so will just said q square equal to 0 you want to look at the low momentum behavior here. So, if q square you said to 0, then you remember delta is given by this, and when q square is 0, it is simply 1 minus z whole square times n square.

So, this integration here, and there is a delta x plus y plus z minus 1, so 0 to 1 d x d y d z delta x plus y plus z minus 1 and 1 minus 4 z plus z square divided by m square into 1 minus z whole square. You can see that this, you can just use this delta function to remove the x integration nothing difference on x, therefore this integration becomes 0 to 1 d z, 0 to 1 minus y d y 1 minus z minus y d x delta. So, this integration gives you one, because nothing in the integrant depends on x, so it simply 1 minus 4 z plus z square over m square 1 minus z x square.

I will just erase this part from here and I will rewrite this here, you notice that this is nothing but minus 2 the numerator plus 1 minus z 3 minus z m square 1 minus z whole square that is what this gives. So, now, you can see that, when you these all nothing depends on y, so the y integration will simply give you 1 minus z, so at the end of the day you will get 0 to 1 d z. And the first term will give you minus 2 over m square 1 minus z, and then you will get other term which does not have 1 minus z in the denominator, nothing in the denominator.

Those will be finite, when you do this integration, but you see because of this 1 minus z here, there is a divergence in this integration. So, this is a diversion term, we will say that this divergence actually can be removed, if you put a quote of a from the below, we will see how actually, I mean a systematically you conclude this diversion also, but not in today's lecture. The only thing that you need to remember is that all these things, the ultraviolet diversion as well as this infrared diversion, it appears only in the f 1 of q square term.

None of these terms appear in the f 2 of q square term, f 2 of q square is what to you are interested. In if you remember we wanted to compute the correction to the enamels magnetic moment of electron, which is given by the f 2 q square term the first order correction to f 2 q square. So, let us compute the correction to f 2 q square, you can read out the form vector from this integration here, and if you go back and check the definition of the form vector.

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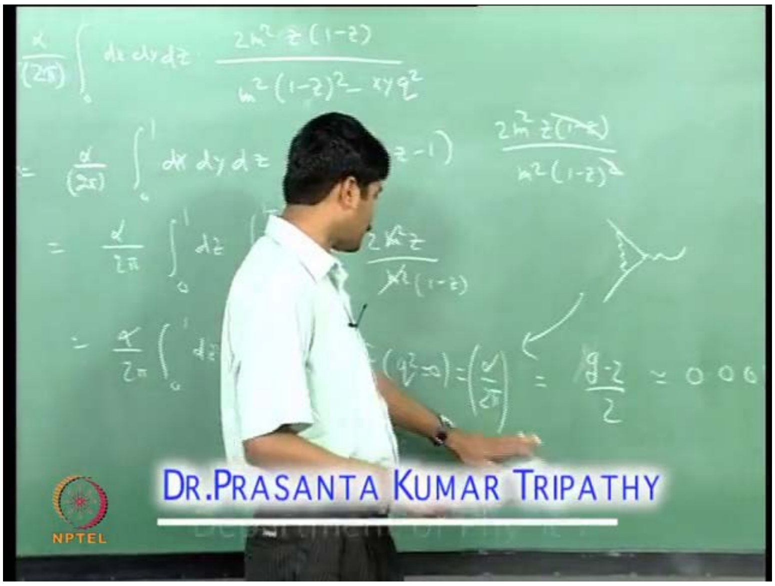

Then if you will find that f 2 q square is nothing but alpha over 2 pi times 0 to 1 d x d y d z 2 m square z into 1 minus z divided by m square 1 minus z, whole square minus x y q square, this is your delta. So, this term you understand the numerator 1, 2 m square z into z minus 1, denominator if you remember you have carried out.

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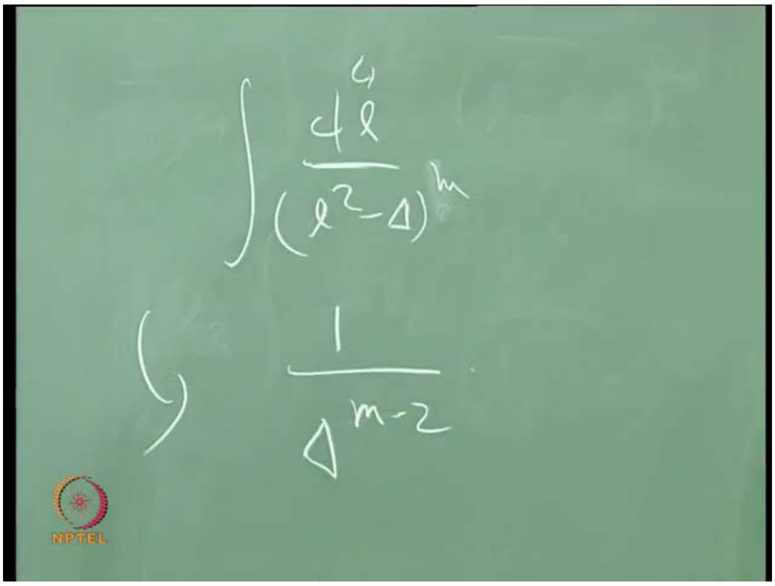

This integration 1 over d 4 l over l square minus delta to the power 3 or in general delta to the power m, and then this goes up to some factor like 1 over delta to the power m minus 2, and when m 3 you get only delta, which is what is given here.

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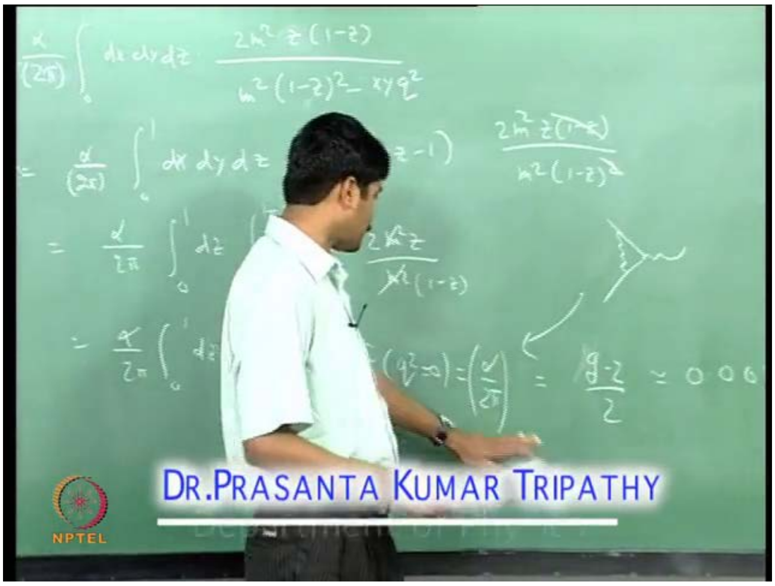

F 2 q square equal to 0 is what is the correction term, and when q square is equal to 0, what I get is alpha over 2 pi times 0 to 1 d x d y d z delta of x plus y plus z minus 1 2 m z into 1 minus z divided by m square 1 minus z whole square. So, now, you can see this 1 minus z cancels this one, again this x integration turns out to be trivial. And what you get is alpha over 2 pi 0 to 1 digit 0 to 1 minus z d y times, twice m z divided by m square 1 minus z twice m square. So, m square also cancels, this give you 1 minus z factor, which again cancels, so you at the end alpha over 2 pi 0 to 1 d z times twice z.

This is just one, this implies f 2 of q square equal to 0 is simply alpha over 2 pi, so there is no f 2 at 0th order term, f 2 equal to 0 for this diagram, what we have done is we have considered the one loop correction. We have consider at one loop this is the diagram, and then f 2 0 actually gets a contribution at finite contribution at one loop, and then this contribution, we can we have explicitly computed. And then we have shown that this contribution is actual, in fact equal to alpha over 2 pi, so this is what is the correction to the enamels magnetic moment, which is just g minus 2 divided by 2.

It is numerically this quantity, in fact terms out to be 0.0011614, you can, in fact a compute the higher loop corrections, also the correction of order alpha square and so on. That will change these numbers slightly experimentally, you can measure this enamels magnetic moment, and then you can compare these with the prediction given by quantum field theory. And then it turns out that this, in fact a remarkably agrees the experimentally measure result, agrees with the one that you compute from quantum field theory to a remarkable accuracy.