Quantum Field Theory

Prof. Dr. Prasanta Kumar Tripathy
Department of Physics
Indian Institute of Technology, Madras

Module - 5
Radiative Corrections
Lecture - 33
Vertex Correction II

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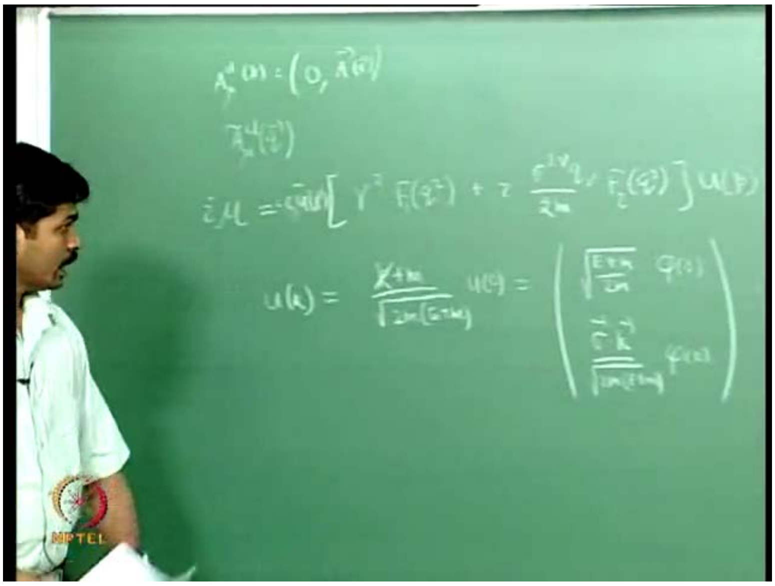

We were discussing interaction of electrons within external electromagnetic field. We started with a static magnetic field hence the vector potential A mu c l of x is given by 0 A of x, and if I denote A mu tilde classical of q to be before your transform of the vector potential. Then the fine man amplitude for the process is given by i times m is equal to i e gamma i F 1 of q square plus i sigma i mu q mu divided by 2 m F 2 of q square, with u bar of p prime i e.

And you have times p A tilde i classical of q this is what is Feynman amplitude, what we will do is we will evaluate this amplitude for a non relativistic electron. When the virtual photon is also weak in the sense that is a low energy, where the photon, so we will do that. So, what I will do that is I will use the exact expression for the direct spinners that we have derived when we quantized the derived field.

The exact expression is given by k slash plus m divided by 2 m s E plus m u of 0 in terms of the two component spinners this is given by E plus m divided by 2 m phi of 0 sigma dot k divided by 2 m E plus m i of 0. Where phi of 0 is a 2 component spinner, so for spin plus half phi of 0 is 1 0, and it is a 0 1 for spin minus half, this is what is the notation that we had used earlier. So, we will stick with this notation and we will work in the representation, where gamma 0 is equal to identity 0 0 minus identity gamma r is 0 sigma i minus sigma i 0.

And in addition we will assume that, then moment are all small compared to the mass of the electron. So, we will plug this and this formula here, let us first compute the first term which involves u bar p prime gamma i u of p, so what is this term we will write it as u dagger of p prime gamma 0 gamma i u of p. So, u dagger p prime 0 sigma i sigma i 0 u of p, now we will express this quantity we will put for u dagger p prime this expression. And also for u of p prime this, and then I will evaluate this, so what will you get.

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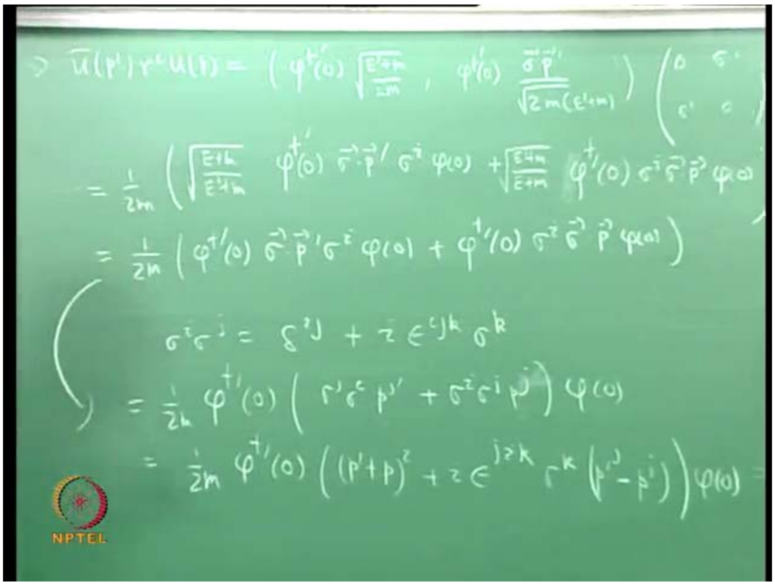

So, when I substitute that what I will get is this implies u bar p prime gamma i u of p is equal to the hermitian conjugate of this quantity here, which is nothing but phi dagger 0 times E prime plus m I will denote E prime to be e of prime. So, E prime plus m over 2 m, and the second component is phi dagger 0 sigma is hermitian, so sigma dot p prime divided by 2 m into E prime plus m. And then 0 sigma i sigma i 0 times E plus m over 2 m phi 0 sigma dot p over 2 E plus m square root phi of 0.

This is what we have for this quantity, now I can straight away multiply this and what I get is a result 1 over 2 m E plus m divided by E prime plus m phi dagger prime I will use for the first one. Under the moment of dependencies is out, you have to specify the spin and then the spin for both the things need not be the same, so just to distinguish is that I will just put a rime over here, phi dagger prime 0 sigma dot p prime sigma i phi of 0, and then square root of E prime plus m over E plus m phi dagger prime 0 sigma i sigma dot p phi of 0.

So, far we have not used any approximation, but now we will use the non relativistic limit, so in this limit E will be equal to E prime. And then this quantity will simply be one to order p square it will be one, so to the linear order in p this is just given by 1 over 2 m phi dagger phi 0 sigma dot p prime sigma i phi of 0 phi dagger i of 0 sigma i sigma dot p. Now, I will use this identity here, which is sigma i sigma j is equal to delta i j plus i epsilon i j k sigma k, you see here that here we have sigma i sigma j, here also we have sigma i sigma j.

So, you just use this formula and simplify that, when you do that what you get is this quantity simply 1 over to m phi dagger prime of 0 here it will be sigma j sigma i p j prime plus sigma i sigma j p i prime p i p j sigma dot p sigma i p j phi of 0, use this formula. Then I will simply get 1 over 2 m phi dagger prime of 0, this will give me 1 delta i j. So, this will be p prime i, this will also give me p i, so this p prime plus p i'th component of p prime plus p.

And then i epsilon i j k here j i is there, so you see epsilon j i k sigma k and here p prime j whereas here, epsilon i j k sigma k. So, if I take it comma I will get a minus sign, so p prime j minus p j phi of 0, so if I just want to keep the term which is linear in q j, then this will simply be phi dagger prime 0 and minus i over 2 m epsilon i j k q j sigma k phi of 0 plus all other term. So, this is what we get, when I look for the non relativistic limit of the first term.

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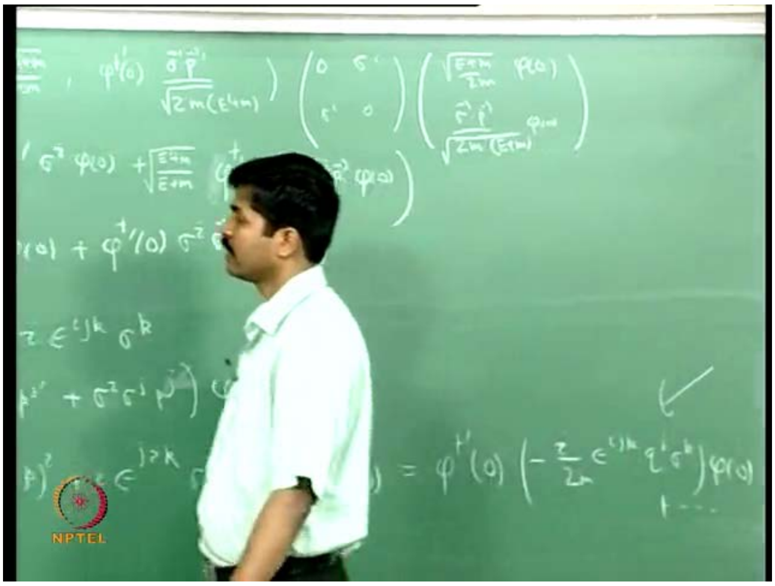

Now, we will similarly evaluate the second term and then we will look at the one which is linear in the momentum absorbed by the electron from the virtual photon.

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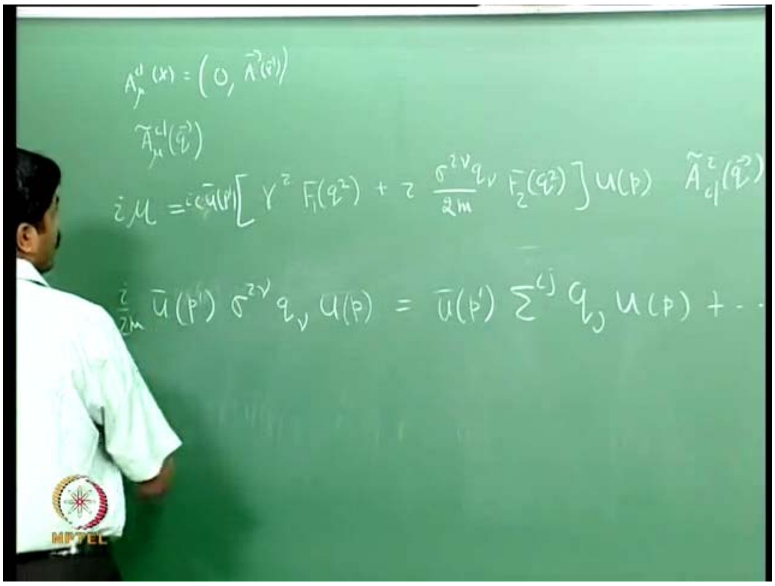

So, what is the second term, it just involves u bar p prime sigma i mu q mu u of p, this is what we need to evaluate. So, again I will look at only terms which are linear in q j, so that will tell me that this is u bar of p prime sigma i j q j u of p plus all other term.

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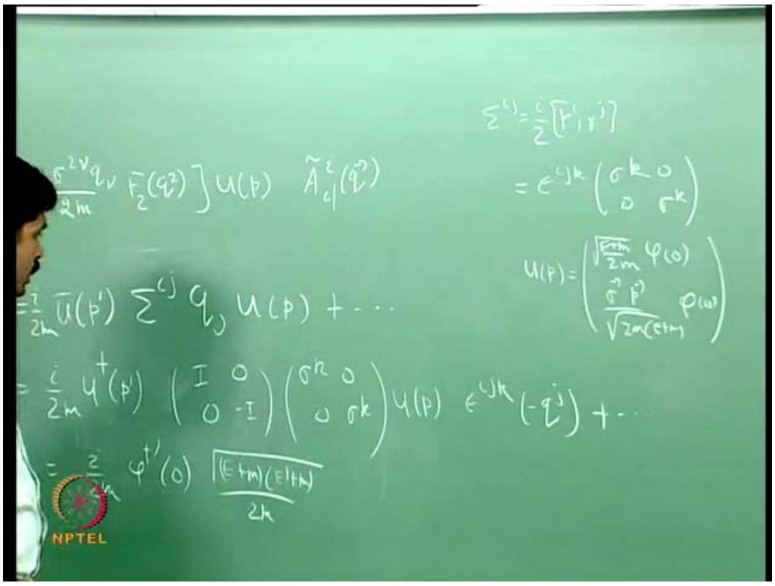

And sigma i j is i over 2 commutator of gamma i gamma j, in the representation that we have chosen, this is simply epsilon i j k sigma k 0 0 sigma k. So let us consider i over 2 m times this which is i over 2 m times this, so this is given by i over 2 m u bar p prime is u dagger p prime gamma 0 sigma k is this gamma 0 is 1 identity 0 0minus identity, sigma k 0 0 sigma k times u of p epsilon i j k q j right. So, this is the term which is linear in q, again you put this for u of p, you just put this formula E plus m over 2 m phi 0 sigma dot p over 2 m E plus m phi 0.

Since this is already linear in q I will set this to 0 because this if I keep this term it will give me a term which is cubic. So, when I do that what I will get is i over 2 m phi dagger prime of 0, and this will give m E plus m plus E prime plus m square root over 2 m and then sigma k phi of 0 epsilon i j k minus q j. This term I will set as 1 I will just put E and E prime to be m, so this is just one. So, what I will get at the end of the day is minus i over 2 m phi dagger prim of 0 this thing. So, we have evaluated both the terms here, so I will plug these expressions for this as well as this, here in the Feynman amplitude.

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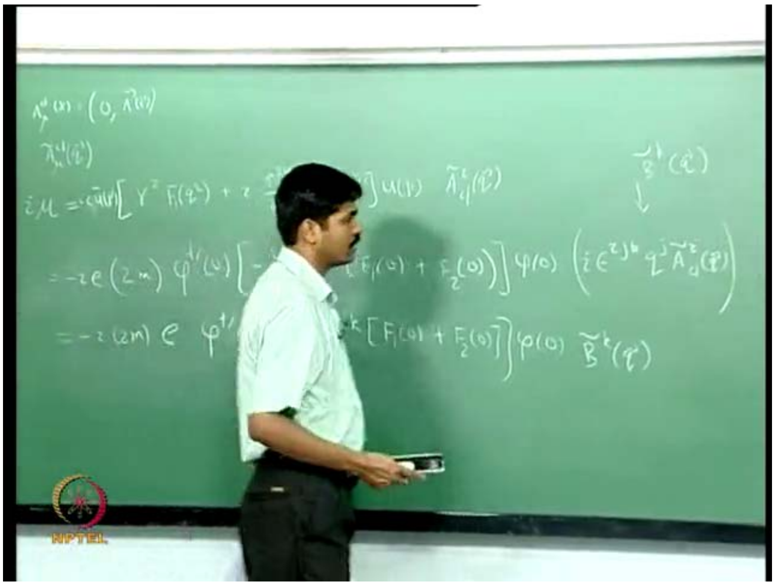

And then what I find when I do that is 2 m times this is what you are going to get, when you substitute it in this expression alright. Now, what is this quantity here, you consider B of x it is juts del cross A, so if A tilde q is the four a transform of A of x then what will be the Fourier transform of B of x it will simply this right. So, this quantity here is just b tilde k of q. So, I will substitute it here then the Feynman amplitude is minus i 2 m e times phi dagger prime of 0 minus 1 over 2 m sigma k F 1 0 plus F 2 0 phi of 0 B tilde k of k of q.

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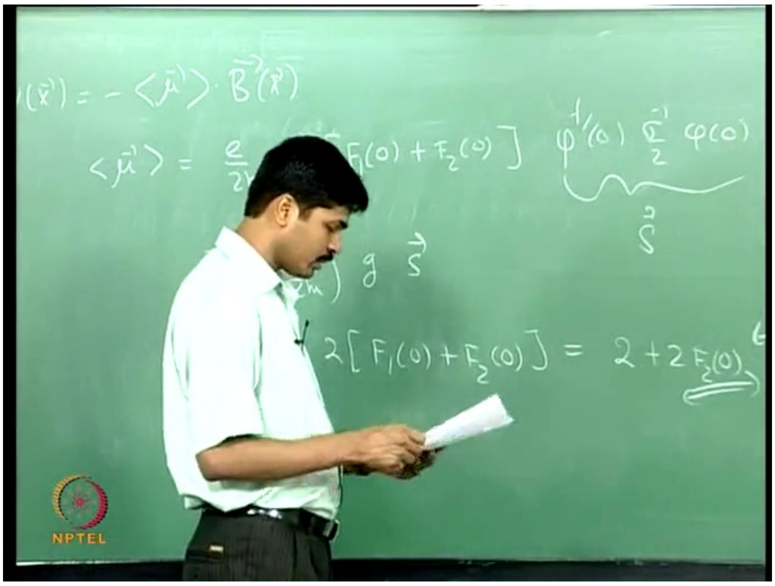

So, this quantity if you take a potential which is minus mu dot B of x then so you just consider the scattering of electron from this potential, use born approximation to evaluate the amplitude, this is what you are going to get. Here, mu is just e over 2 m twice F 1 of 0 plus F 2 of 0 phi dagger prime of 0 sigma over 2 phi of 0 this quantity is the spin operator which I will denote as S. And as this is of this form e over 2 m times g times s, where g is the lands g factor which is given by twice F 1 of 0 F 2 of 0.

We have already argued in the last lecture that this F 1 of 0, will not receive any correction, so F 1 of 0 I will just set as 1 2 F 2 of 0. So, to leading order g equal to 2 which is what is predicted by the Dirac's theory, and F 2 of 0 is something which is quantum field theory will give us. So, quantum field theory gives correction to the anomalous magnetic movement of electron, and we can compute this from this loop corrections to the vertex factor. And the loop for corrections to the vertex will give us correction to the anomalous magnetic movements alright.

So, what we will do now is that we will compute the correction to the vertex factor at one loop. So, let us now consider the Feynman diagram at one loop, we will just use the Feynman rules and then we will see what do we get, when we try to evaluate the Feynman amplitude by brute force.

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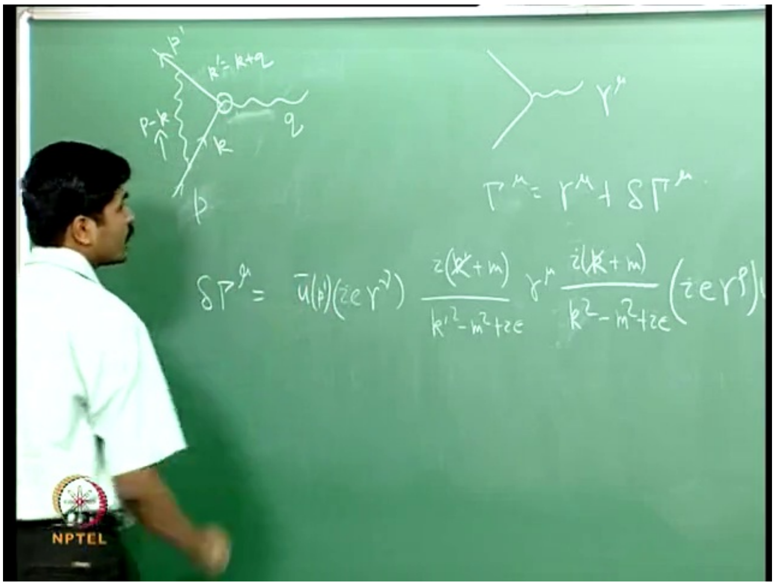

So, what is the diagram that will contribute at one loop, the diagram which contributed one loop is just given by e of momentum p from incoming electron, it emits the actual photon of momentum. Let us say p minus k this is k, a virtual photon of momentum q is absorbed at this vertex and finally, you have k prime which is k plus q, p prime is the momentum of the outgoing electron. So, leading order term is just this and the vertex factor for this is just gamma mu.

The vertex factor of gamma mu which I will set is, gamma mu plus delta gamma mu at this order. So, delta gamma mu I will read from this diagram, so what will be delta gamma mu for me I can use Feynman's rule to evaluate the amplitude here, and here instead of considering the photon propagator and i e gamma mu I will just put gamma mu here because that is what I am considering. So, let us use Feynman rule I will write u of p for the incoming electron here, then I have a vertex here.

So, what will I write for the vertex i e gamma mu, so I will use the notation gamma rho I will use, then I have a fermion propagator what is the factor for fermion propagator. It is i k slash plus m divided by k square minus m square plus i epsilon; this is for this vertex here. I will write gamma mu for this, and then I have another fermion propagator here, which I will write as i k prime slash plus m divided by k prime square minus m square plus i epsilon. Then I have a vertex here, which I will write as i e gamma sigma let us say gamma mu, and u bar p prime, and then I have this propagator here.

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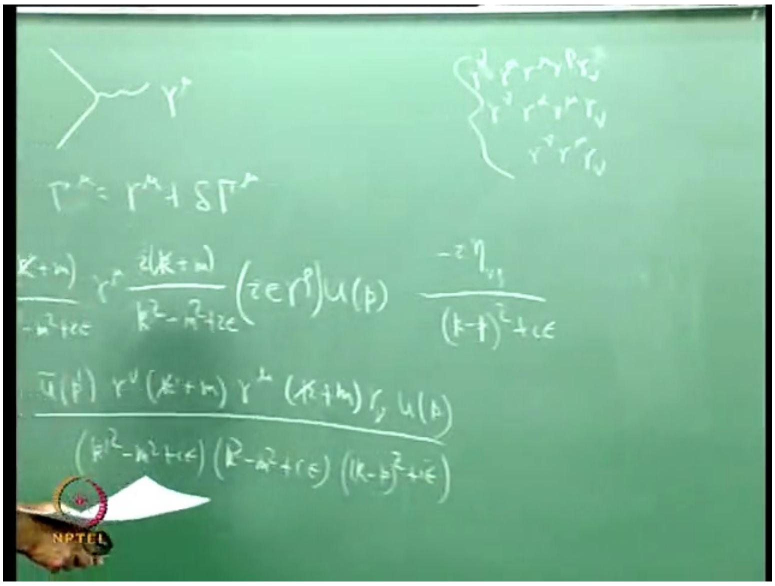

The photon propagator which is just given by minus i eta mu rho divided by k minus p whole square plus i epsilon, this is what I have for delta gamma. So, what I will do is that

I will simplify it a bit, you can see this is just given by i square i e square minus i and then u bar p prime gamma mu k prime slash plus m gamma mu k slash plus m gamma mu u of p divided by k prime square minus m square plus i epsilon k square minus m square plus i epsilon k minus p square plus i epsilon. This is what we have, you just notice this the numerator involves terms like this, gamma mu gamma alpha gamma mu gamma beta gamma mu gamma mu gamma alpha gamma mu gamma mu gamma mu gamma mu gamma mu.

It involves terms like these, and you know how to evaluate these gamma matrices even when you do not have trace you just know you have done it when you consider electron electron scattering. So, you can just use the identities for these ones we have derived earlier, and plug it in here it is a very straight forward thing. And finally, you can show that this quantity when you use these relations for.

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This is what it is thank you, so I will use these identities for these ones.

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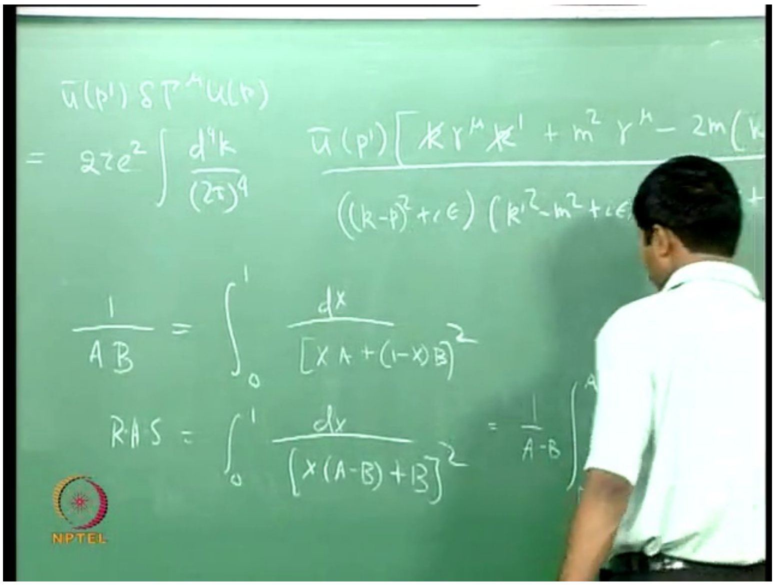

Then what I get is u bar p prime delta gamma mu u of p is given by and finally, I must remember that because you should have reminded me of this. Because, k is not fixed by the delta functions at all these vertices, it can be an arbitrary quantity here because you have k and b minus k here, k is not fixed at all. So, you need to integrate over k, so d 4 k over 2 pi to the power of 4'th. So, here also it is integrated d 4 k over 2 pi to the power of 4'th.

Whenever, you have a loop you always get an integration like that, so 2 i e square d 4 k over 2 pi to the power of 4'th u bar p prime k slash gamma mu k prime slash plus m square gamma mu minus 2 m k plus k prime mu u of p this divided by k minus p square plus i epsilon k prime square minus m square plus i epsilon, this is what you have now you have to evaluate this integration. So, how do you evaluate this integration it is horribly complicated expression.

What you will use is that you will use what is known as the Feynman parameters, we will introduce the Feynman parameters you will see how. And then we will carry out the d 4 k integration by that, so you notice that if you have some quantity 1 over A B then you can express it as, integration 0 to 1 d x divided by X A plus 1 minus X B it is very straight forward. Because, here you can write this the R.H.S is nothing but 0 to 1 d x X into A minus B plus B square that is a whole square here. You define this quantity here in the denominator to be t square, then you just lo this integration is simply 1 over A minus B d t over t square where the limit goes from B to A. So, when you evaluate this integration it is just 1 over A B it is very straight forward as this.

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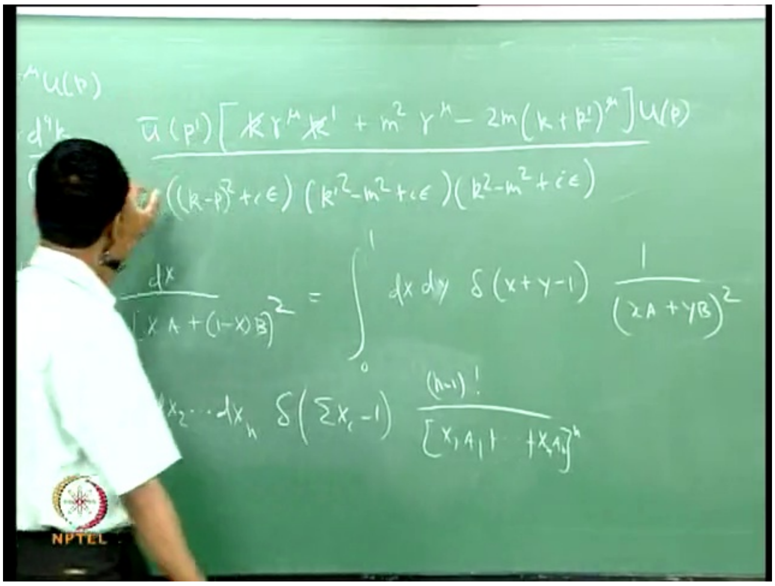

So, we will take that and then I will introduce a variable y this is integration over d x d y from 0 to 1 delta of x plus y minus 1. So, instead of 1 minus x I will just write it as y, and then I will introduce this delta function. So, 1 over x A plus y B whole square, this is what you have when you consider product of 2 terms, you can generalize this formula when you have n factors here in the denominator, if you have something like A 1 A 2 up to a n.

Then the generalization of this formula will simply be 0 to 1 d x 1 d x 2 up to d x n delta of sum over x i minus 1 n minus 1 factorial divided by x 1 A 1 plus up to x n a n whole to the power a n, you can prove this formula by induction I will not prove it for you. But, what we will use is that, we will use this formula to express this denominator here in terms of integration over x y z. So, let us do that, so the idea here is that we will simplify this denominator, we will use this formula and simplify this denominator and then we will simplify the numerator, and then carry out the d 4 k integration.

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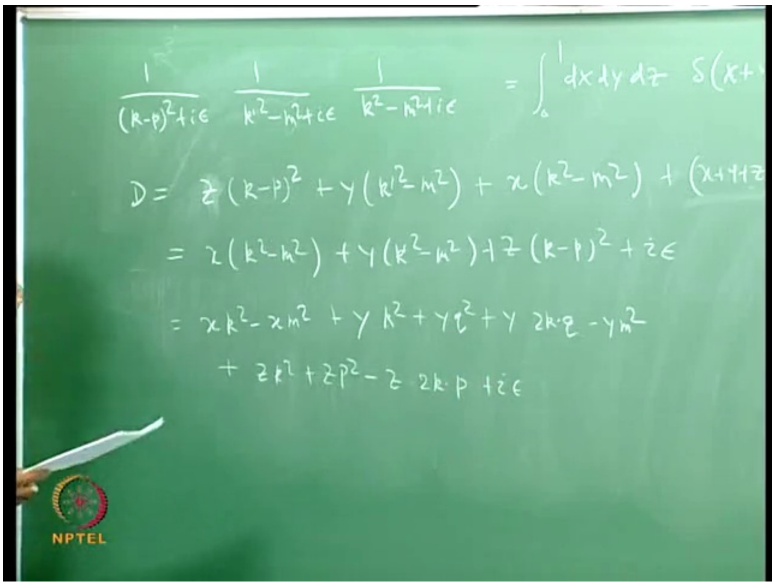

So, the denominator is 1 over k minus p whole square plus i epsilon 1 over k prime square minus m square plus i epsilon 1 over k square minus m square plus i epsilon, this is equal to 0 to 1 d x d y d z delta x plus y plus z minus 1 2 over D q. Where D is just x times k minus p whole square plus y times k prime minus m square plus let me parameterize in such a way that this is just z this is x times k square minus m square plus x plus y plus z times i epsilon right, this is very straight forward.

Now, I will simplify this denominator keeping in mind that x plus y plus z is 1, so whenever I write it is equal it is now on, these are the equalities which will hold inside this integration here. So, therefore, this quantity I can write it as x k square minus m square plus y k prime square minus m square plus z k minus p whole square plus i epsilon.

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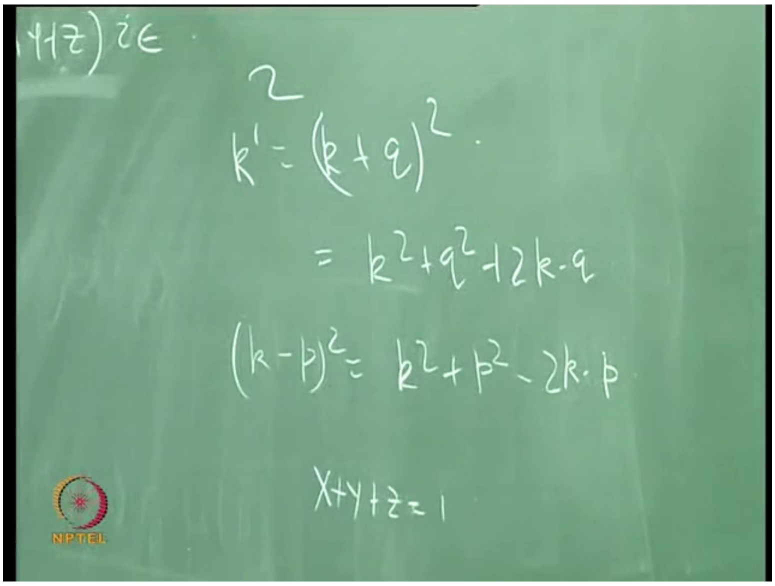

So, now, what I will use is I will use the fact that k prime is equal to k plus q, so k prime square is just k plus q whole square, which is k square plus q square plus 2 k dot q. Similarly, k minus p square is k square plus p square minus 2 k dot p, and substitute these here and what I get is x k square minus x m square plus y y times k prime square is y k square plus y q square plus y 2 k dot q minus y m square. And then z k square plus z p square minus z times 2 k dot p plus i epsilon is that right.

So, just z p square if you simplify this you can just see that you have an x k square, you have a y k square, you have a z k square. So, when I add them I will simply get k square and then you have these 2 quantities this and this, which will give me plus 2 k dot y q minus z p. And then you have y q square z p square, and then minus x minus y m square plus i epsilon, because I have again used the relation that x plus y plus z is equal to 1.

Now, you see that we are integrating variable k, now this comes in this form k square and the term as is linear in k. So, I can introduce a variable which I will call as l, so that this will be complete square, so to complete this square I will introduce l equal to k plus y q minus z p. And then this will be l square and some quantity which are independent of l right, so the integration variable I can change from k to l.

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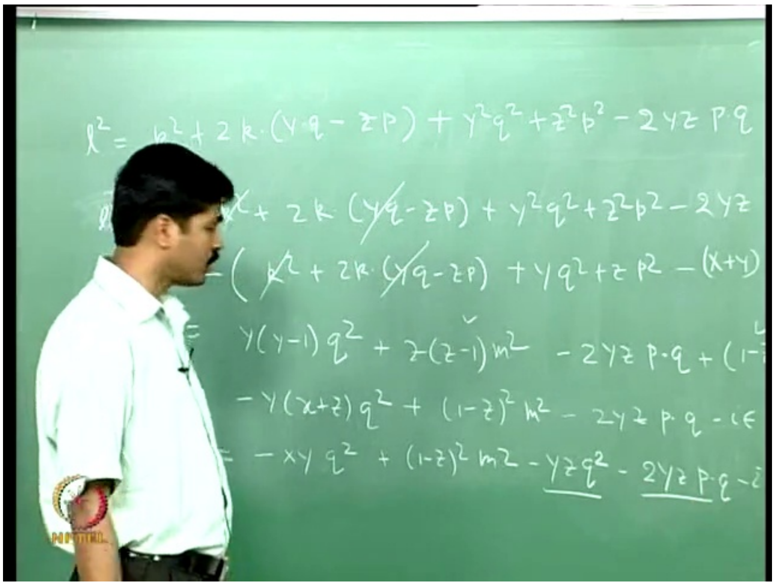

So, let us do that, so when you introduce l to be this is l square is k square plus twice k dot y q minus z p plus the square of this quantity, which is y square q square plus z square p square minus twice y z p dot q. So, this quantity already appears here therefore, my D will be or what I can do is l square minus D is just this quantity minus the quantity that I have there minus k square plus 2 k dot y q minus z p plus y q square plus z p square minus x plus y m square plus i epsilon p square is of course, m square.

So, this term cancels this term cancels remember they do not cancel here, so it is just y into y minus 1 q square q square is not 0, remember z into z minus 1 p square which is m square. And then x minus y minus 2 y z p dot q and this will give me plus x plus y which is 1 minus z x plus y is 1 minus z m square minus i epsilon, so this is y minus 1 I will write again I will write this relation minus y into x plus z q square.

And here z into z minus m square plus 1 minus z, so this if I combine this two terms here what I get, I just get this and this will give me 1 minus z whole square m square right and then minus 2 y z p dot q minus i epsilon. So, let us write it as minus x y q square plus 1 minus z square m square, and then I will take this minus y z q square minus 2 y z p dot q minus i epsilon. Now, if you combine these 2 term here, you just see you get y let us use this relation p prime equal to p plus q.

So, p prime square is p square plus q square plus 2 p dot q, but p square is p rime square which is equal to m square. So, q square plus 2 p dot q is equal to 0, I will use that relation here, then this plus this is simply 0, so what you are left with is if at the end this is your l square.

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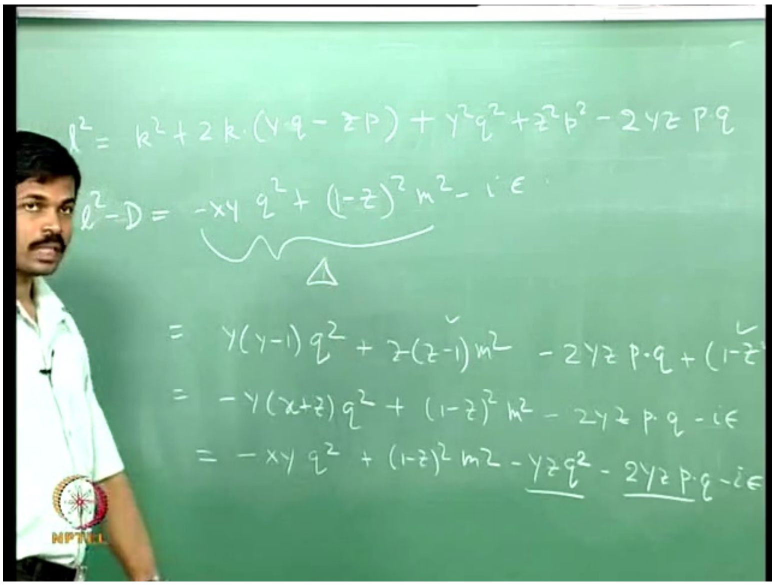

Then l square minus D is simply minus x y q square plus 1 minus plus z whole square m square minus i epsilon, I will introduce something which is I will call as delta this quantity I will denote as delta.

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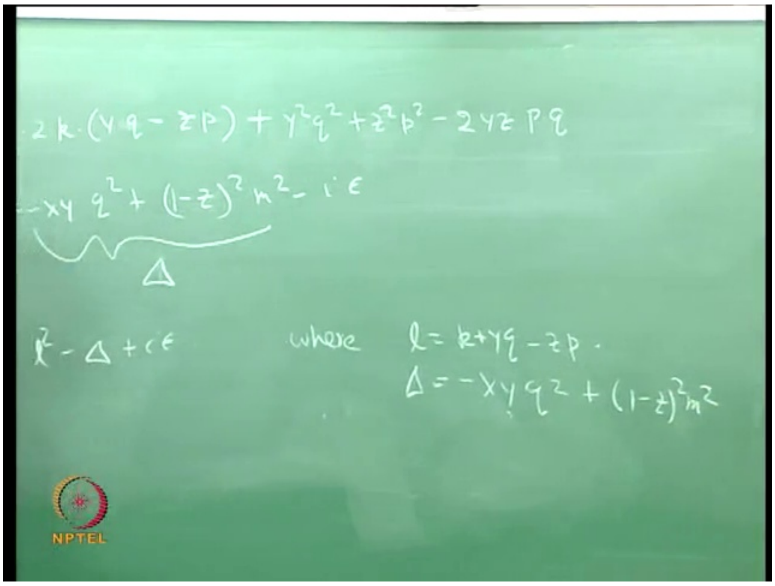

So, if this is the notation I will use then my D is simply l square minus delta plus i epsilon, where l equal to k plus y q minus z p and delta is equal to minus x y q square plus 1 minus z square m square. So, what we did is we have simplified the denominator, now in the next lecture we will look at the numerator and then we will simplify it, and then we will carry out the integration alright.