Quantum Field Theory
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Module - 5
Radiative Corrections
Lecture - 32
Vertex Correction I

So far, we have been discussing elementary processes at the tree level. So, if you look at the Feynman diagrams in the process, they do not contain any of the loops. They are the contributions which come from the lowest order term in the S matrix. What we will do in the next several lectures is to consider higher order terms in the S matrix. In other words, we will consider processes where the Feynman diagram also contains loops. In other words, we will consider the radiative corrections to the various processes.

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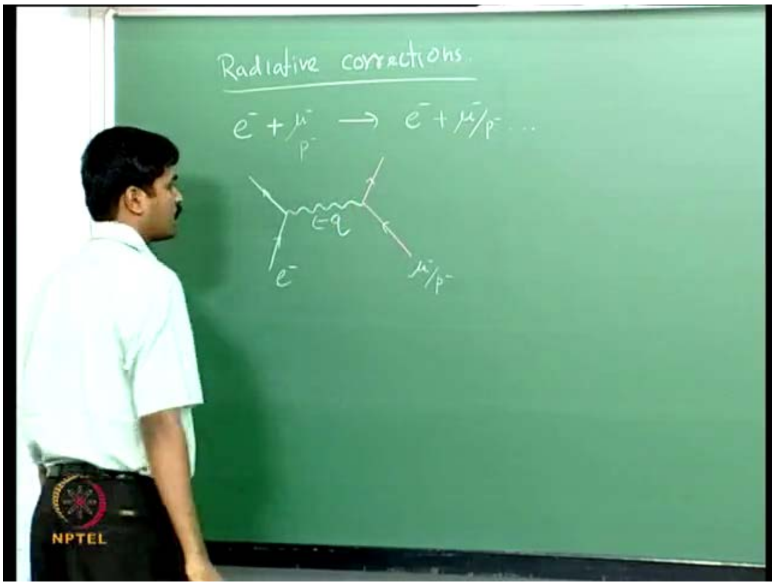

So, let us consider for this scattering of an electron from a very heavy particle, which can either be, let us say muon or a photon or whatever the mass of the particle should be much heavier than the mass of electron. It should interact with electron via electromagnetic interaction. That is the only requirement that we have. You will see in a moment why we want to consider heavy particle. So, we will consider this electron scattering from a heavy particle. Then we will see what happens when we consider the loops, so e minus plus mu minus or p minus or whatever, some heavy particle.

So, of course, the tree level process is that you have an electron. I will draw the heavy particle with a coloured line. So, you have an incoming electron and you have an incoming heavy particle, which could be mu minus or photon or whatever. This should exchange a virtual photon of momentum q. Then they scatter each other. This is the lowest order diagram. You know how to compute the cross section for this process.

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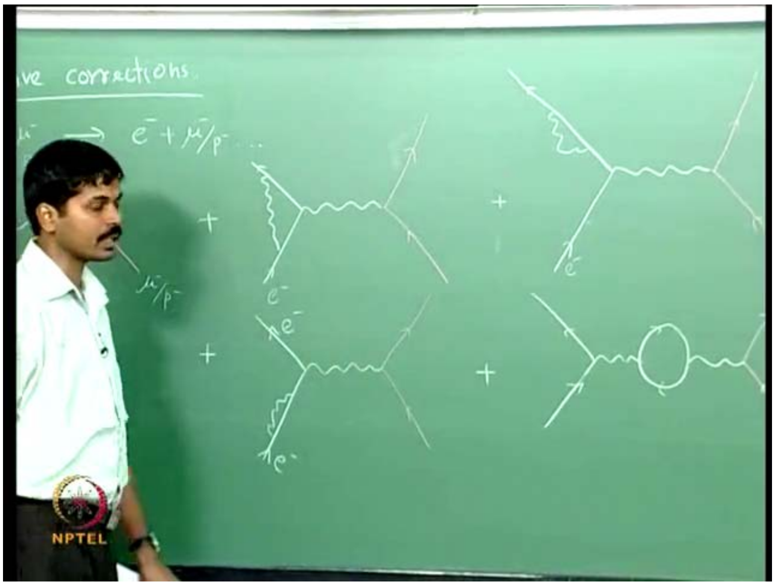

What we will do now is we will consider the next order contribution to this process. We will see what are the diagrams involved here. So, these are the diagrams which will contribute at the next order. So, here what happens is the incoming electron emits a virtual photon which is subsequently absorbed by the outgoing electron. The incoming electron exchanges a virtual photon with the heavy particle. Here, what happens is the outgoing electron emits and subsequently absorbs one virtual photon in addition to this. Here, the incoming electron emits and absorbs a virtual photon.

In this diagram, you have a pair created. The virtual photon here creates an electron positron pair which subsequently annihilate and the virtual photon is again created. Finally, it is absorbed by the electron. You might ask what happens? This heavy particle also can emit a virtual photon and subsequently absorb it and then this electron and the other particle, they can, for example, sense two virtual photons and all those things, all this will also contain one loop.

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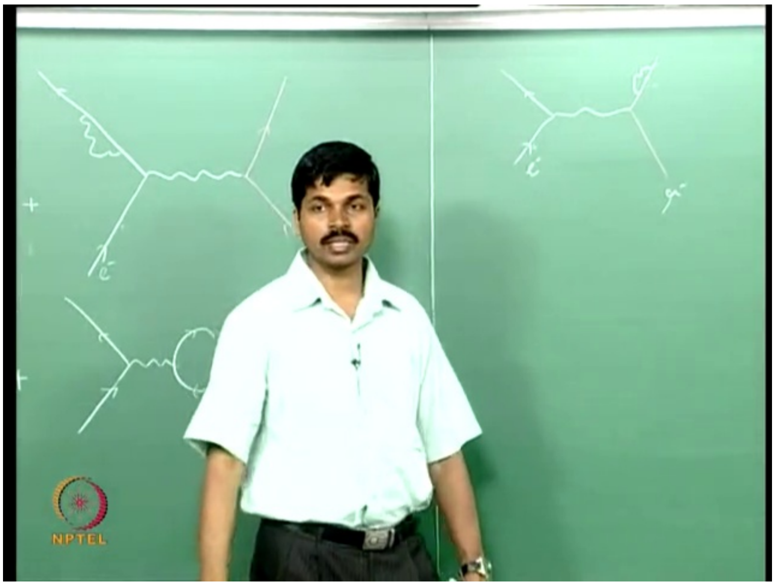

Just like here, all of these diagrams contain a loop, you can consider the loop where the heavy particle, which involves the heavy propagator of the heavy particle, but you might notice whenever that is the case for example, suppose you consider a process which goes like this. Let us say you have a process like this where the heavy particle is in the loop. You have an electron. You have a muon or so. Then this diagram also will be suppressed compared to this diagram by one over the mass of this heavy particle because it will contain an internal line of the heavy particle. Then you know the propagator contains inverse of the mass of the particle.

So, that is why, I considered the scattering of electron by a heavy particle. In principle, you can consider electron electron scattering at higher order and all those things. It is quite, it is almost identical to this process except that there will be more diagrams here. Here, I am considering this scattering with an heavy particle because there are lesser number of diagrams; other than that, there is no reason for us to consider this heavy particle in this thing. The only reason we are considering is that we can ignore these diagrams because they are suppressed by one over the mass of the heavy particle. Now, we will focus on these various diagrams.

The first one is known as the vertex correction. We will see that it will give a correction to the vertex vector here. The second and third ones are the external leg corrections. Then here the final diagram is the vacuum polarisation, a virtual photon creating an electron positron pair and then subsequently annihilating and a virtual photon is getting created. What we will do is that we will first discuss the vertex correction. Then later on, I will discuss all these corrections, the external leg corrections, the vacuum polarisation and so on in great detail.

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So, let us discuss the vertex correction. So, you must remember that when you draw Feynman diagram set at higher orders, you should keep in mind that you should consider all possible Feynman diagrams which are connected. Then you keep only those diagrams which are imputed in the sense that if you consider, let us say you consider this diagram, let us consider all possible correction to the vertex vector. So, we are considering scattering of electron with a heavy particle. The electron absorbs a momentum q and then it is getting scattered. So, this is the lowest order diagram.

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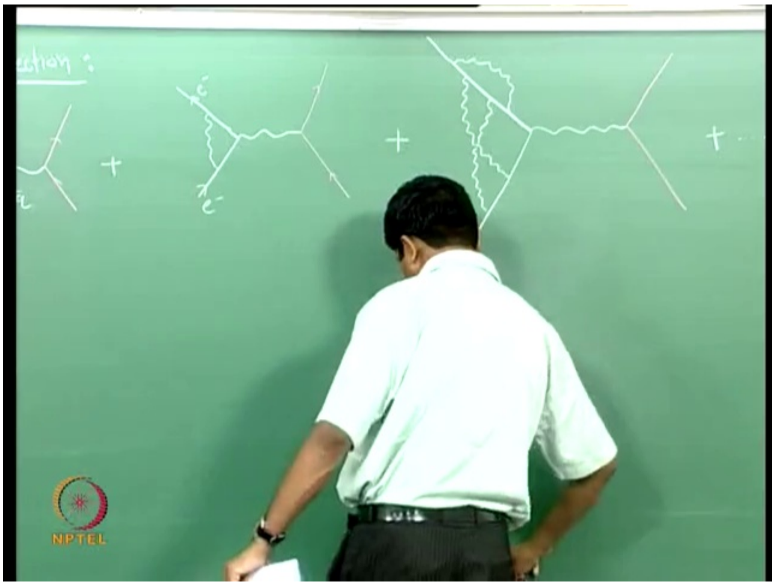

The next order diagram, which will give correction to the vertex function, is this one. The correction to the vertex is this. The next higher order diagram to this process is you have and this is the diagram in the next order and so on. Here, what I did is I considered all the Feynman diagrams, which are connected and then which are imputed in the sense that if you remove any of these internal lines, you see the diagram does not separate into two complete pieces. Especially just to give you an example I am not considering a diagram like this because if you cut here, you see that you just get this one. So, that is the Feynman rule. It basically says that you consider only this class of diagrams. They will give vertex corrections. I will represent this by this big block here.

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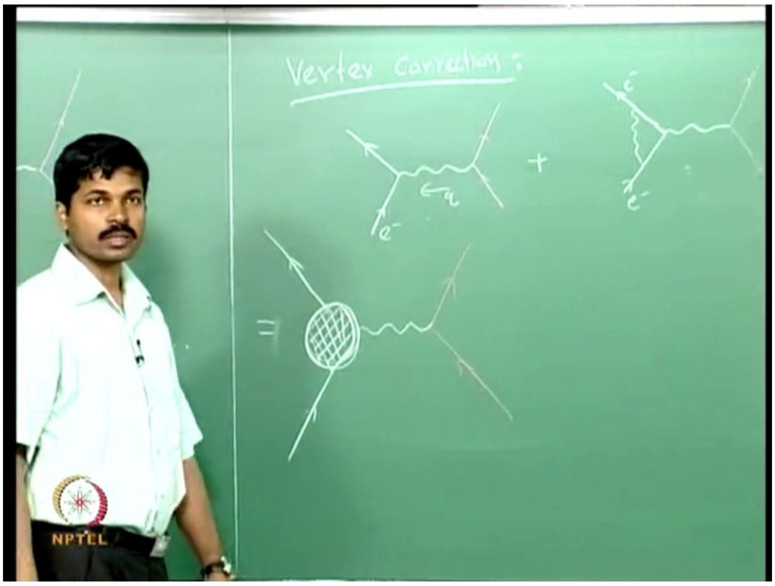

You consider this. So, what happens is that you consider all the corrections to all orders. Then that amounts to certain modification of the vertex function here that I am just representing by this block here. So, I do not know what this function here exactly is, but I know the Feynman rules. So, I know how to compute them order by order. What we will do is we will eventually compute the vertex correction at one loop level, but before that, we will see if we can get some general form for the vertex correction from general arguments.

So, even before doing a one look computation, we will see what general statement that we can make about the vertex function. Is this clear? So, let us see what we can say about the vertex function. You consider this. So, in other words, what you want is the following. You consider a process like this. We do not know what this vertex correction is.

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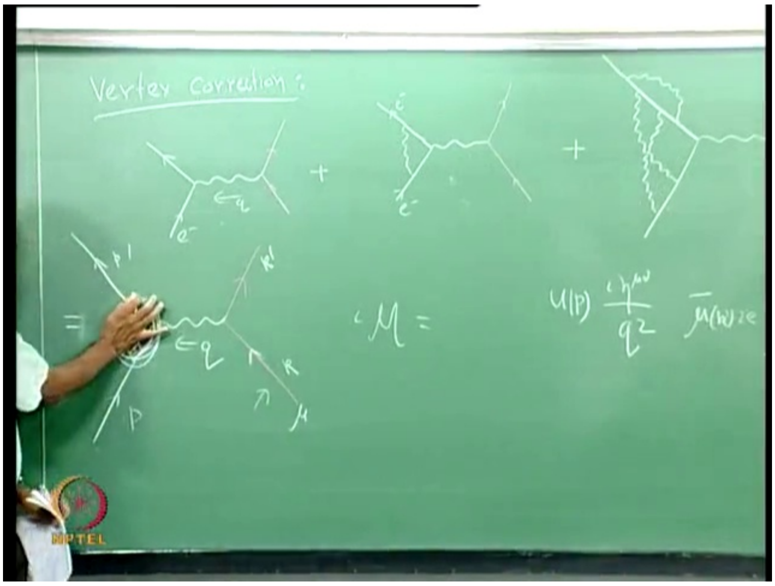

So, the matrix element here, the Feynman amplitude for this process will be given by, first you consider this heavy particle which I will denote, let us say mu, then mu of momentum k, mu of momentum k prime and a virtual photon of momentum q. Here p and p prime are the moment of the incoming and outgoing photon. So, I can use Feynman rules to write down the amplitude for this process. I will denote the Dirac, the solution to the Dirac equation for this heavy particle to be this, to be mu.

So, you have mu here and then i e gamma mu for the vertex function here. You have an outgoing heavy particle mu bar of k prime and then you have a photon propagator, so eta mu nu i by eta mu nu divided by q square. Then you have one incoming electron of momentum p. I will denote this by up. I do not know what the vertex vector here is after I include all this corrections. So, I will denote that to be ie gamma nu with capital gamma to represent all the vertex corrections.

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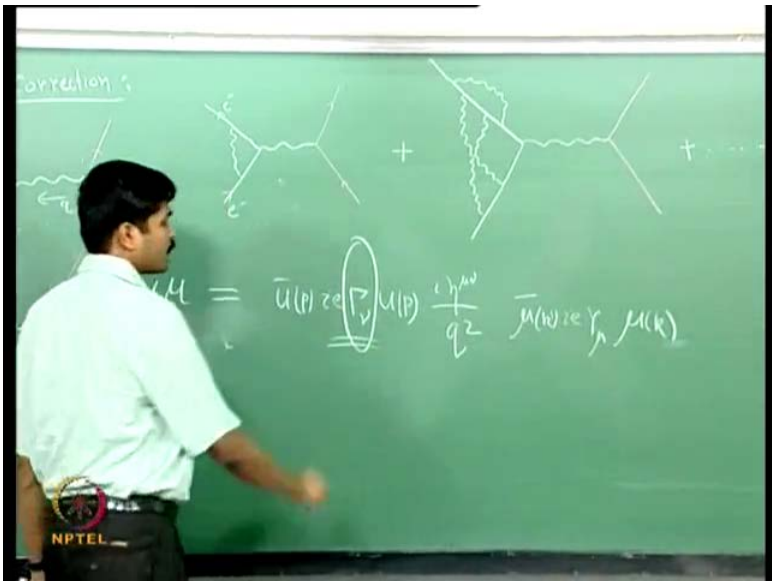

Then, you have the outgoing electron of momentum p prime, u bar of p prime. So, this is the Feynman amplitude which I will denote by iM. The Feynman amplitude for this process is given by this. Our goal is to compute this vertex correction or in other words, to compute what this gamma mu can be. Before doing an explicit loop computation, we can see what form this gamma mu can take place. So, what is the form it can take? First of all, we can use various symmetries. For example, we can use the Lorentz transformation property for this gamma mu.

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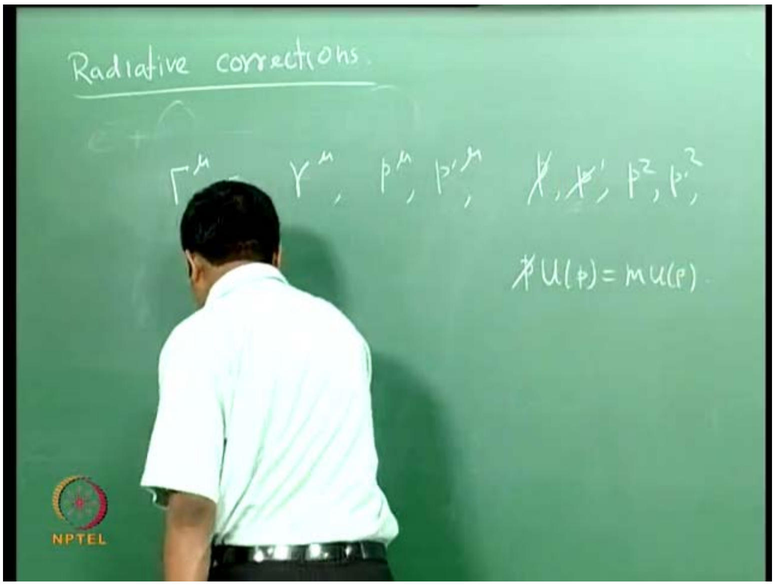

It will say that this gamma mu, which we know to the lowest order, what is the value of this capital gamma mu? It is small gamma mu. So, to lowest order, it is gamma mu. For higher order terms because it is a Lorentz vector here, it can only be, it can involve gamma mu, it can involve p mu, p prime mu and q is of course, p plus p plus q is p prime. So, q is p prime minus p. So, I am not writing q mu because it is not linearly independent than this

In addition to these, it can contain Lorentz scalars. It can also contain p slash p prime slash p square p prime square and all those things, but, you should remember that this is evaluated with the u p and u bar p prime in between. So, even if you have p slash p prime slash and so on, you can use the Dirac equation, which is p slash up is equal to m up. So, these terms will eventually go away.

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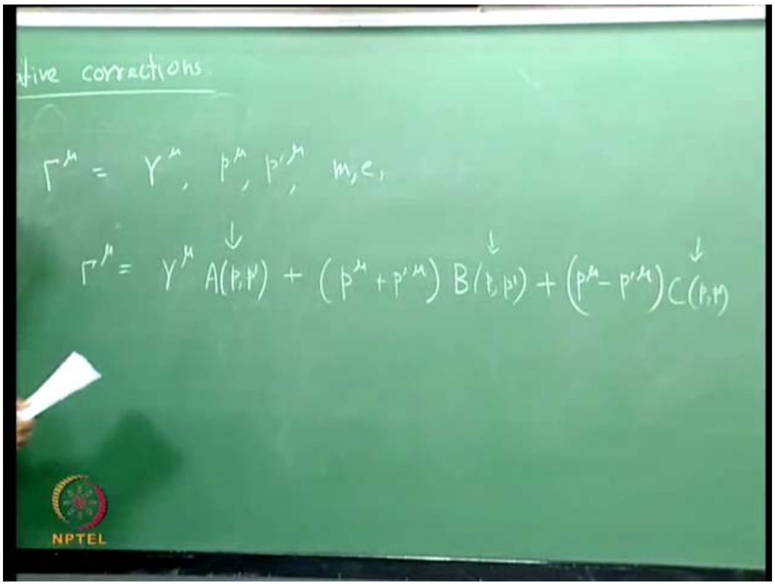

So, it can only depend on these quantities and p square etcetera are also of course, m square. So, there are these constants, m, e, and so on. So, it can depend on these and m, e and then the dot products p dot p prime and so on. This is scalar, which involves p and p prime. We will consider this to be the most, so this basically says that most general form for gamma mu can be this gamma mu times of A, which is a function of p p prime apart from the constants scalar function of p and p prime plus p mu plus p prime mu times B, which again can be of scalar function of p and p prime plus p mu minus p prime mu u times C, which is also a function of p and p prime.

Our goal is to find these quantities A, B and C. If you know A, B, C to all orders, we know what the exact correction of the vertex vector is. Let us see even doing any loop correction, we can say some more something more about these scalar functions, A, B, C.

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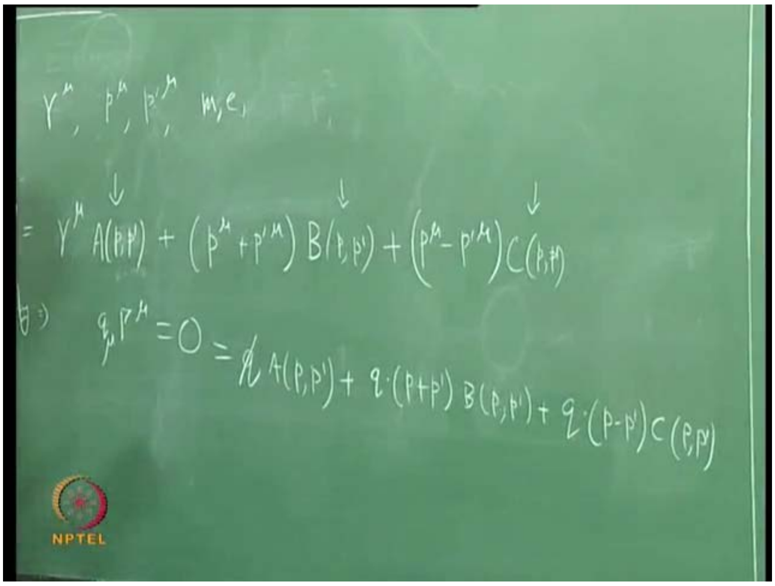

We will use what is known as the word identity which we will prove later. It basically says that q mu gamma mu is equal to 0. So, in addition to Lorentz invariance or in addition to Lorentz transformation, we can use this, which is a consequence of conservation law. This basically implies that q slash A p p prime plus q dot p plus p prime B of p p prime plus q dot p minus p prime C of p p prime, this has to be equal to 0. Now, what is q from this diagram?

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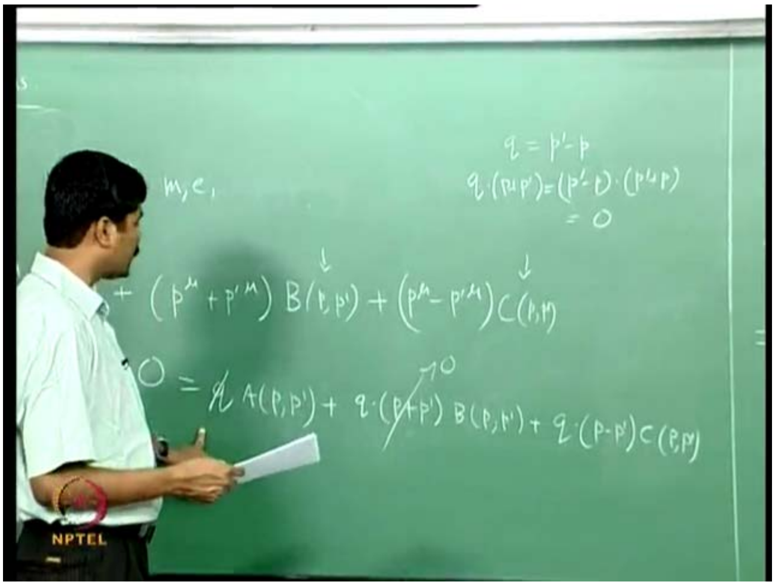

q is basically p prime minus p. So, if you look at the q dot p plus p prime is basically P prime minus p into p prime plus p which is given by it is 0 because p square m square and p prime square is also m square. Both incoming and outgoing electrons are on the shell. So, this q dot this is 0. So, this vanishes. What about this q slash? So, this is an operator equation. So, you should evaluate it within this.

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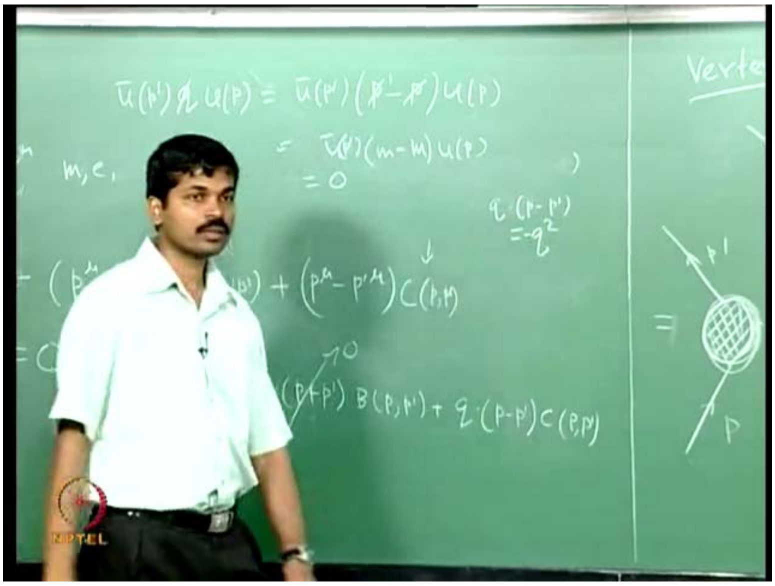

So, if you consider u bar of p prime q slash u of p, then you can see that this is also equal to 0. So, let us just see how. Since q is p prime minus p, this is u bar p prime p prime slash minus p slash u of p, p slash acting on u of p will give you m times u of p. So, this will give me m u of p, whereas p prime slash acting on u bar will give m.

So, this is just u bar p prime. This is 0. Therefore, this also is equal to 0, whereas this quantity here is not 0, q dot p minus p prime is basically q square minus q square, but this is a virtual photon. So, for this, q square need not be 0. In fact, if these are on shell, p prime and p, p and p prime are on shell, then you can show that q square in fact cannot be equal to 0 if you consider any virtual photon. The reason is the following.

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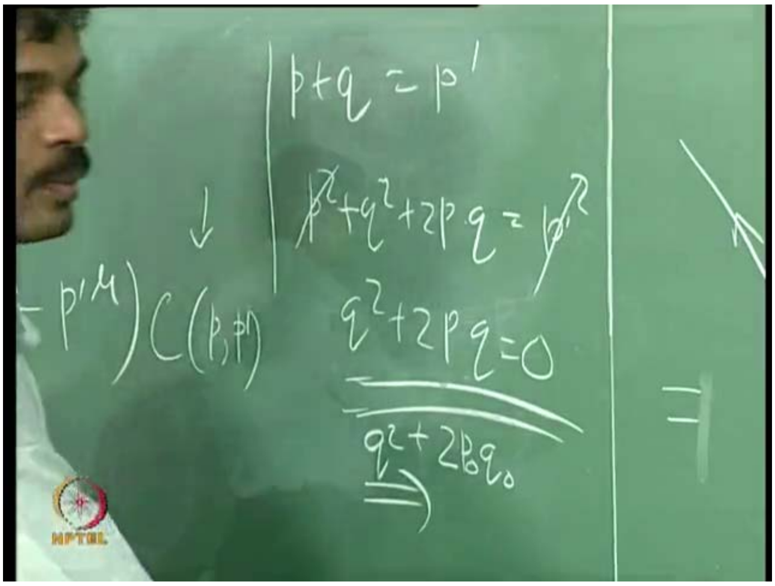

q square is p prime minus p. If you look at this p plus q is p prime, if you square this, p square plus q square plus 2p dot q equal to p prime square, but p prime square is equal to p square because they are on shell. Therefore, q square plus 2p dot q equal to 0. So, this basically implies that if q is not 0, then q square cannot be equal to 0. For example, you consider this in the rest frame of the incoming photon. Then it basically says that this is 2p0q0 plus q square. Therefore, q square has to in fact be negative. If the incoming electron is on shell, then q square is less than less than 0. So, this quantity is not 0.

Since this quantity q dot, this is not 0, therefore, C has to be equal to 0. For this equation to be satisfied, this must be equal to 0. So, this identity q mu gamma mu equal to 0 implies that although you can write it in this most general expression, this coefficient, the scalar function C of p p prime must be equal to 0. So, this basically leaves us the following expression for the vertex correction.

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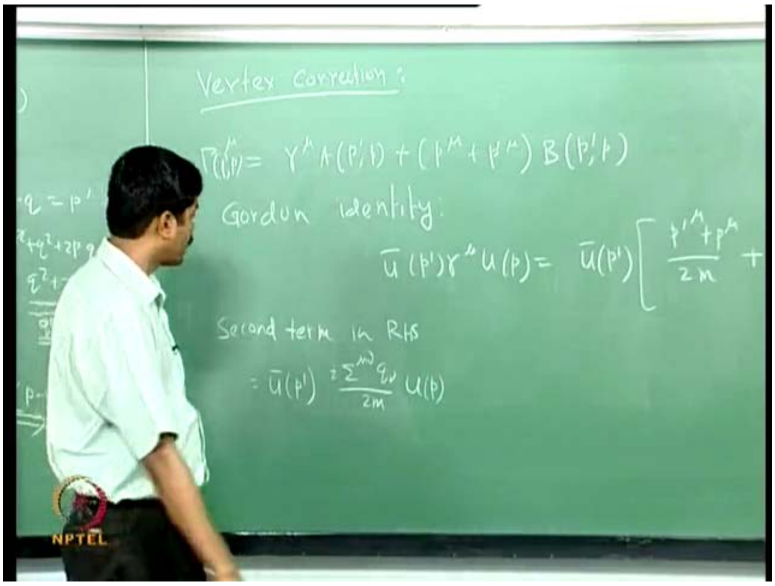

So, this simply implies that gamma mu can in fact be equal to gamma mu A of p prime p plus p mu plus p prime mu B of p prime p, gamma mu of p prime p is equal to this.

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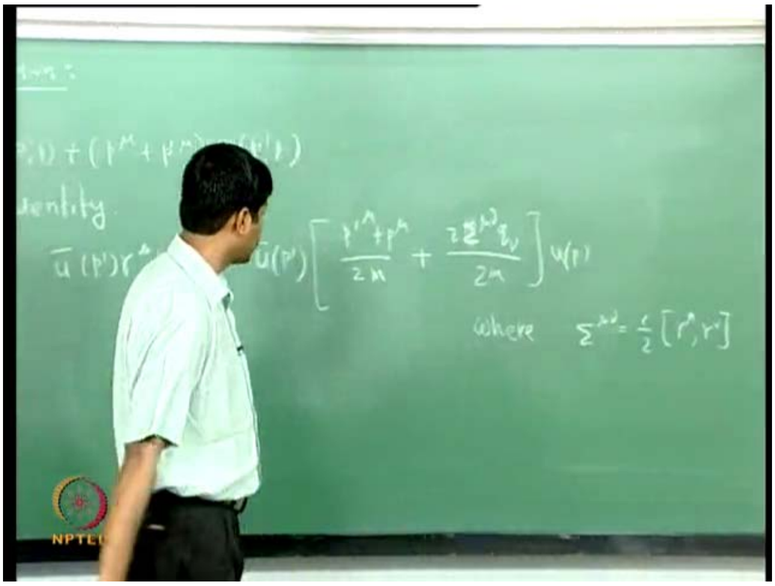

Now, what we can do is we will use something which is known as the Gordon identity, which basically says that u bar p prime gamma mu u of p is equal to u bar p prime p prime mu plus p mu divided by 2m plus i sigma mu nu q nu divided by 2m. Sigma mu nu is the generator for Lorentz transformation, which we have seen earlier is given by i by 2, commutator of gamma, mu gamma nu.

So, it is very easy to prove this identity. You can start from here, the second term in the RHS.

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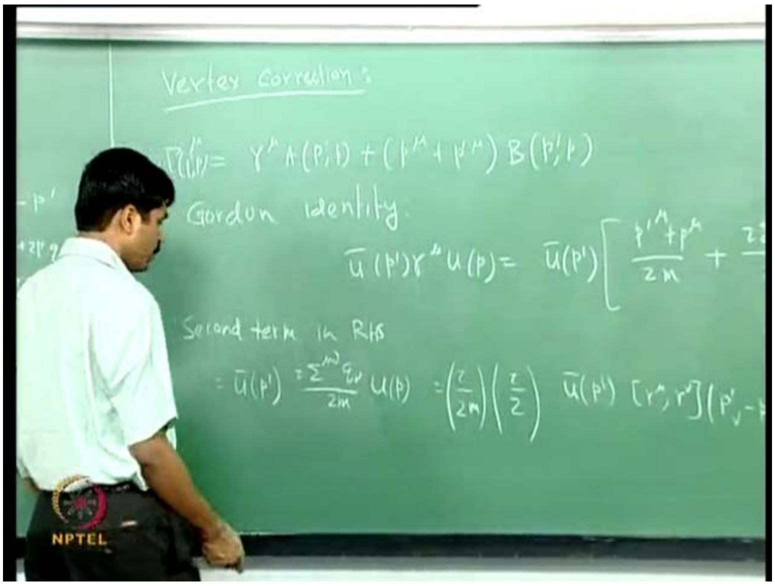

In RHS, this is basically u bar p prime i sigma mu nu q nu over 2m u of p. You have to use the fact that q is p prime minus p. So, i over 2m will give me i over 2 factor, i over 2 and then u bar p prime. Then I will have commutator of gamma mu, gamma nu.

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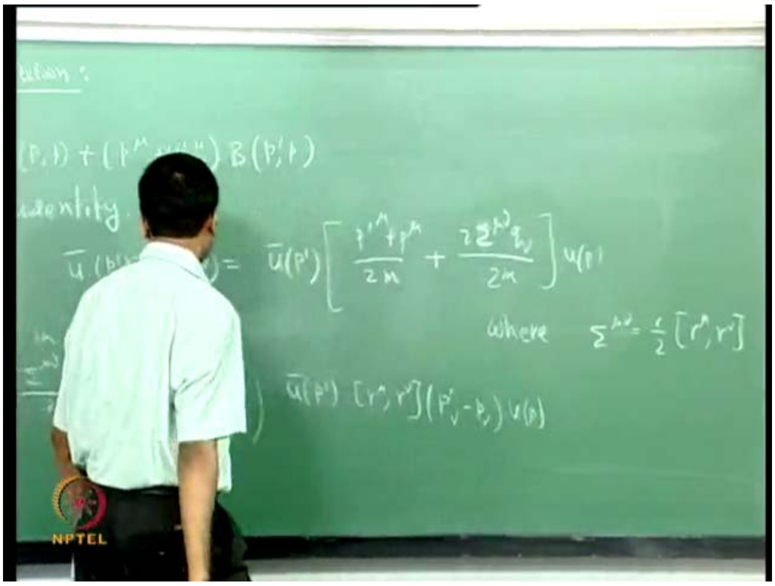

q mu is nothing but p prime nu minus p mu u of p. This is what is the second term.

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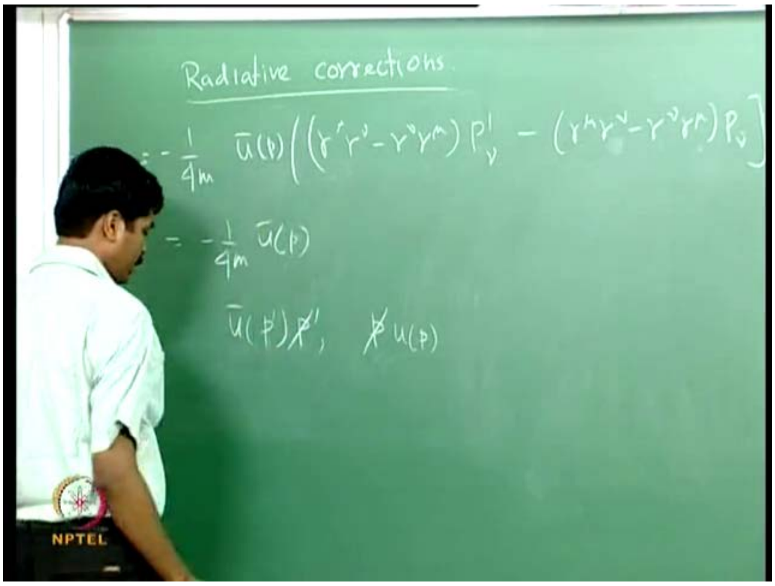

So, this is minus 1 over 4m and u bar p gamma mu gamma nu minus gamma nu mu p prime nu minus gamma mu gamma nu gamma nu gamma mu p mu u of p. Now, what do I do minus 1 over 4m u bar p. Now, you know that I can evaluate this u bar p prime p prime slash. This I know how to evaluate using Dirac equation. Also, I know how to evaluate p slash u of p.

So, what I will do is I will use the Clifford algebra for the Dirac matrices. You bring this gamma nu in the second term to the right so that I get is p slash to the right here which will act on this u of p, whereas in this expression, I will use the Clifford algebra to take this gamma nu to the left so that I will get this p prime slash to the left so that it will act on this u bar p. So, once I do that, what I get here?

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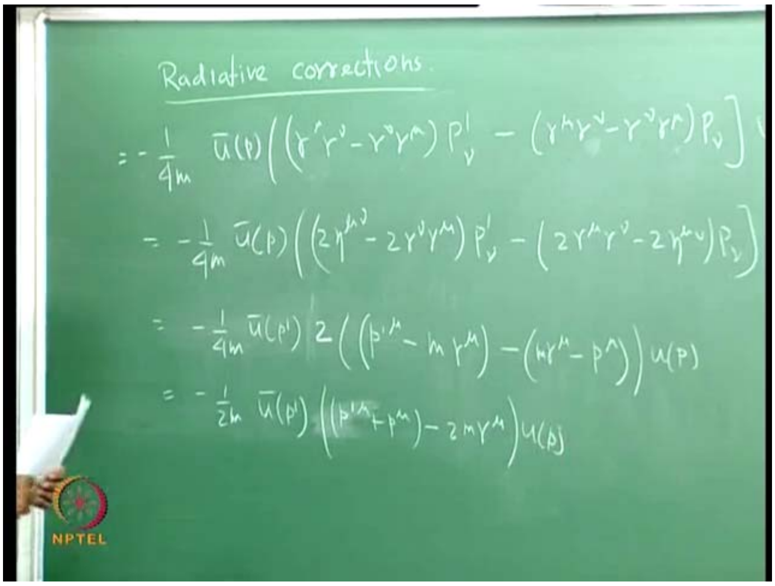

I will get this is just 2 eta mu nu minus 2 gamma nu gamma mu p prime nu, here minus. What I will do I will flip this and then I will write it as 2 gamma mu gamma nu minus 2 eta mu nu p nu u of p. So, now it is very easy, minus 1 over 4m u bar p prime. This I will get overall factor of 2. Then this will give me p prime mu minus m gamma mu from here and here minus, this will give me gamma mu m minus p mu. Once I use the Dirac equation, this is what I will get. It is very trivial. Now, you see that, now if you just minus 1 over 2m u bar p prime, this will give me twice gamma mu p prime mu plus p mu minus 2 m gamma mu u of p. Now, you see that the identity is proved. So, once I know this identity, this is the identity.

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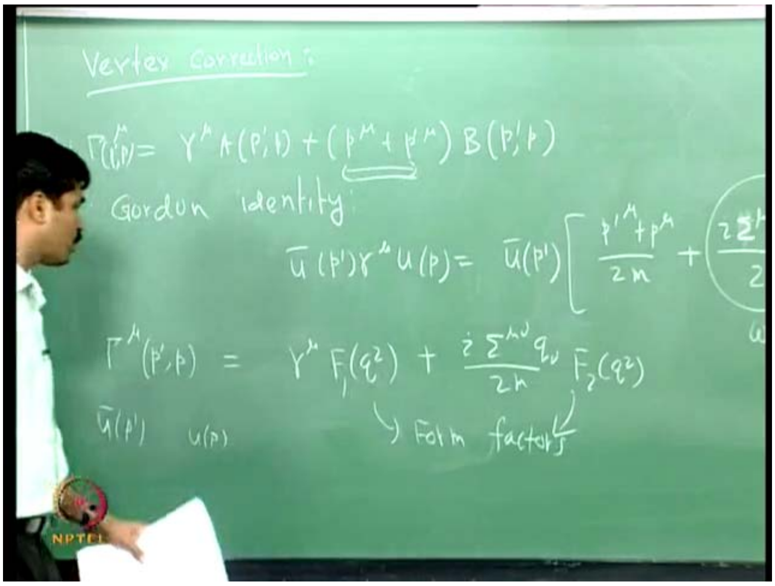

Then, what I can do is in place of p prime plus p mu plus p prime mu, I can just write as this. In other words, if I use this identity, then gamma mu p prime p can be written as gamma mu times F1 of q square plus i sigma mu nu q nu divided by 2m F2 of q square from sum F1 and F2 as a function of A of p prime p and B of p prime p. Remember q square simply means that it is a scalar function of p prime and p. So, the p prime and p dependence can only come through q square, nothing else because rest all are just constants m and so on. Is this clear?

So, if you evaluate this between mu bar p prime and mu of p where you evaluate this and you evaluate this, you will see that this, if you use this identity, then it is same for sum F1 and F2. So, these F1 and F2 are unknown functions, which we have to determine by computing the loops. These are known as the form factors. So, our goal would be to determine these form factors F1 and F2 by computing the loop corrections. What we want to know is even before computing these; can I understand what contribution they give to various physical physically measurable quantities?

So, that is what we will discuss in the next few minutes. In the next lecture, we will explicitly compute the correction to these factors, these form factors. So, let us very quickly go over this F1 of q square and F2 q square. You consider the lowest order term here. We know to lowest order, this gamma mu is nothing but this gamma mu here.

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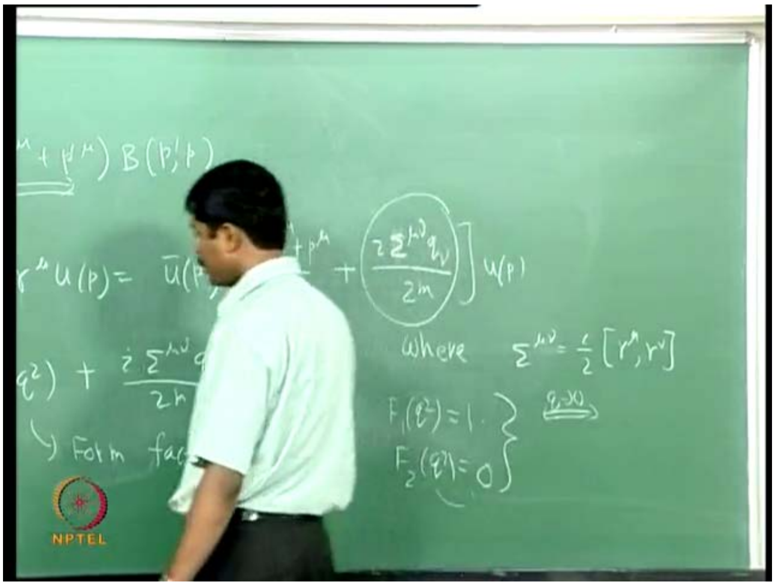

Therefore, the lowest order term for F1 of q square, it is just 1. The lowest order term for F2 q square, it is 0. So, F2 q square to the lowest order, F1 q square is 1 and F2 q square is equal to 0. That is the first thing that we conclude from this general expression. This also suggests that if you consider q goes to 0 limit, non relativistic limit where the momentum transferred from the heavy particle is 0, it tends to 0, then this F1 of q square will not receive any correction in this limit, whereas this can receive any, this can receive nontrivial corrections. Let us consider this scattering of an electron from an external electromagnetic field.

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Let us say you consider some static vector potential so that you have only external magnetic field. The electron is scattered from an external magnetic field. What will be the matrix element? The interaction Hamilton, I know is delta HX interaction or H interaction is basically d cube X e times Z mu, some classical external electromagnetic field. We are not quantising the external field here j mu.

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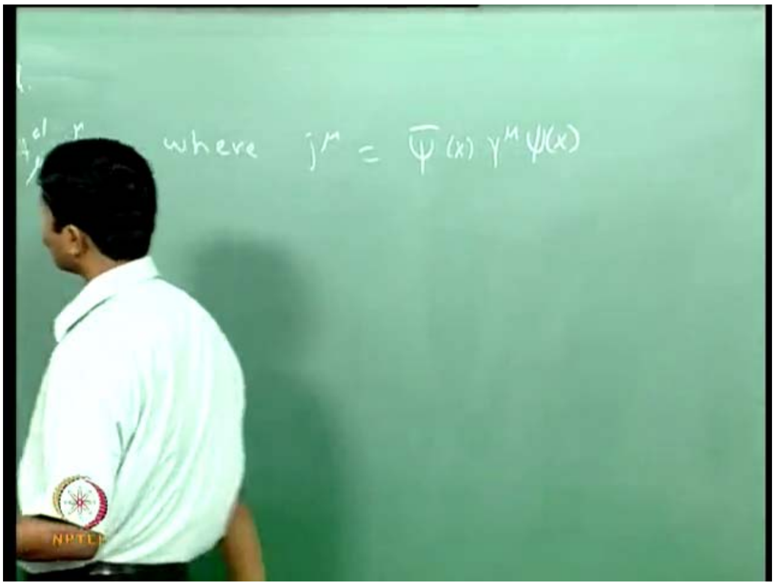

j mu is psi bar of x gamma mu psi of x. You consider this interaction Hamilton. It is very straightforward to compute what is the Feynman amplitude for this process.

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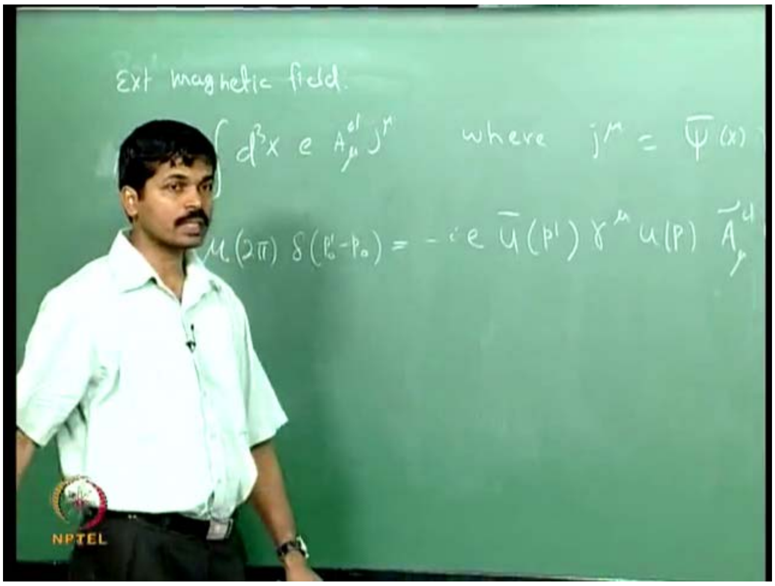

The Feynman amplitude will be given by iM. So, if you consider an electrostatic potential, then you just have delta p 0 prime minus p 0 will be given by minus i e u bar of p prime gamma mu.

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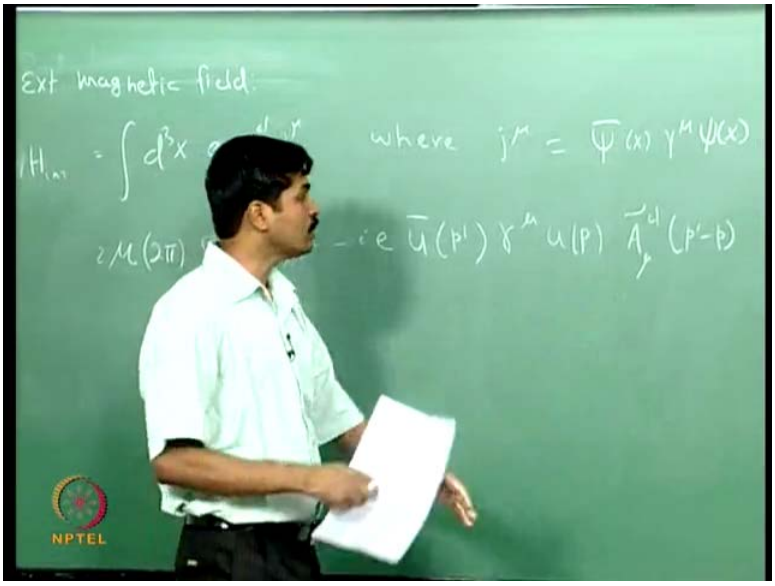

A tilde mu p prime minus p.

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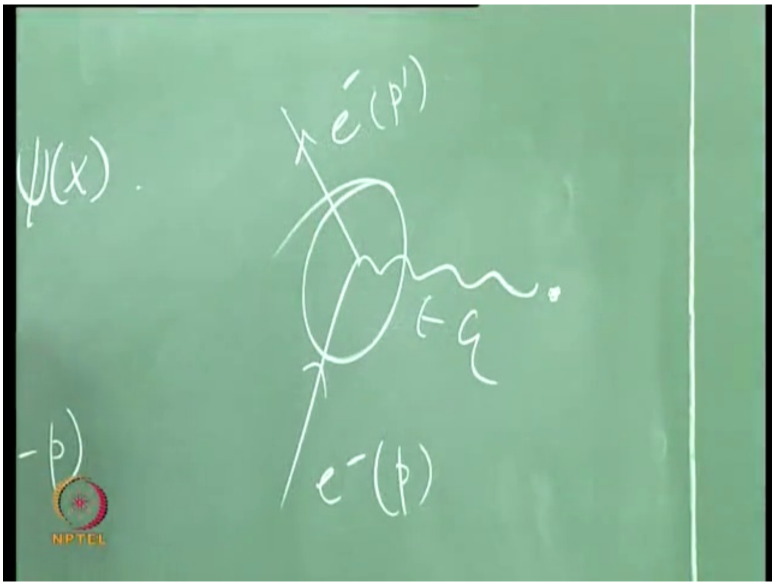

So, suppose you have some external electromagnetic field. The electron is scattered from this by absorbing a virtual photon of momentum q. Then you have an incoming electron of momentum p, you have an outgoing of momentum p prime, q is p prime minus p. A mu tilde is just the Fourier transform of the gauge filled a mu. The Feynman amplitude, you can compute the lowest order term in the S matrix. The Feynman amplitude will look like this. Now, we know how to compute the vertex corrections.

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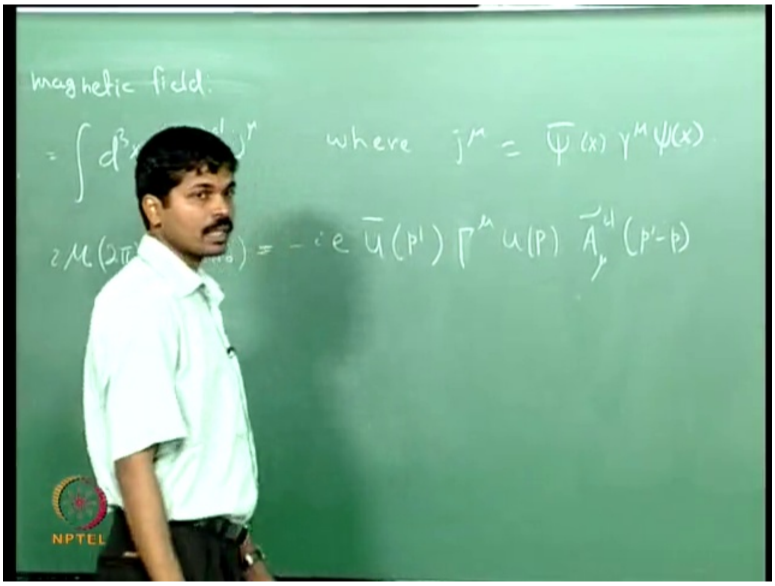

So, if you consider all the vertex corrections, then the gamma mu here will be replaced by the capital gamma mu here. Now, what will do is that we will consider the case when we have a static vector potential.

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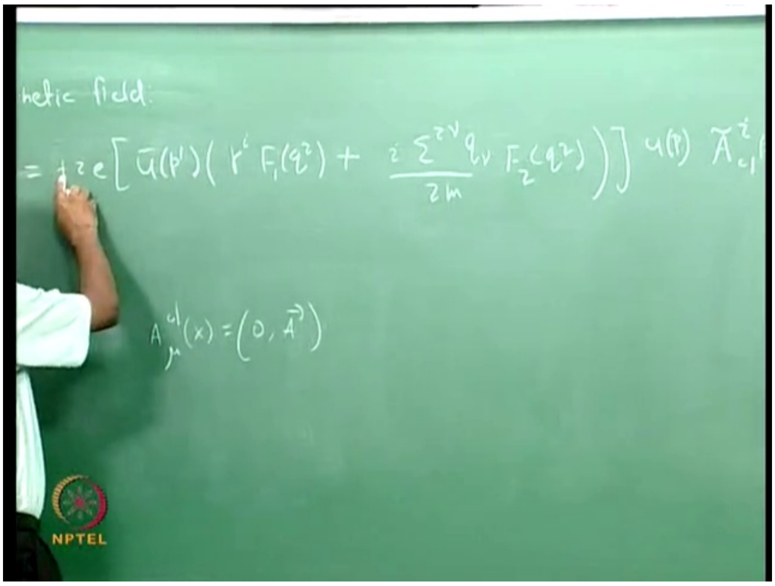

So, A mu suppose is given by 0 and A. Then what is the Feynman amplitude for this process? So, the Feynman amplitude here will simply be given by, I am writing, iM will be plus i e q bar p prime, capital gamma mu will simply be gamma i here, gamma i F1 of q square plus i sigma i mu q nu divided by 2m F2 of q square u of p this times A tilde i.

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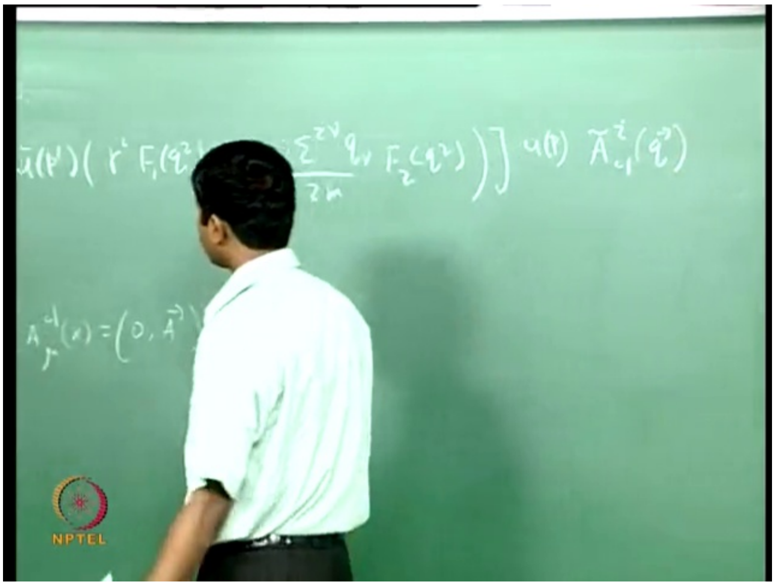

All you have to do is you have to carefully keep track of this iM here. You know in the earlier formulae, I had a minus here. Then this gamma mu and sigma mu nu were contracted with the A mu properly. Now, my A mu is just 0 and A. The zero th component of A mu is 0. This vector Ai, when if raise it, if will just get a minus sign which will make it plus. What we will do is that we will consider this. Then we will consider the non relativistic limit.

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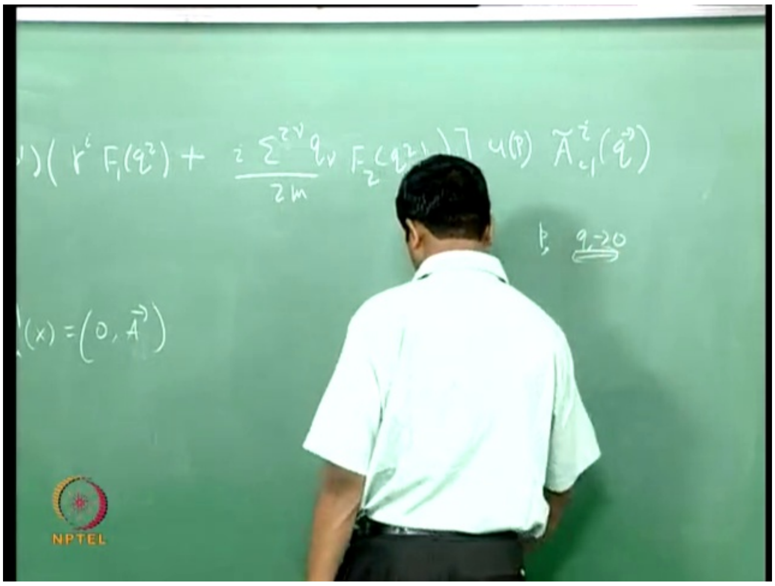

Here, the initial incoming electron is a non relativistic electron and also q is very small. The momentum, the virtual photo, which is observed by the electron, is also of very low energy. In this limit, we can compute this quantity here, the Feynman amplitude and then we will see what this quantity looks like. Let us use the explicit expression for this. So, what we will see? I do not think we have much time to evaluate in this lecture.

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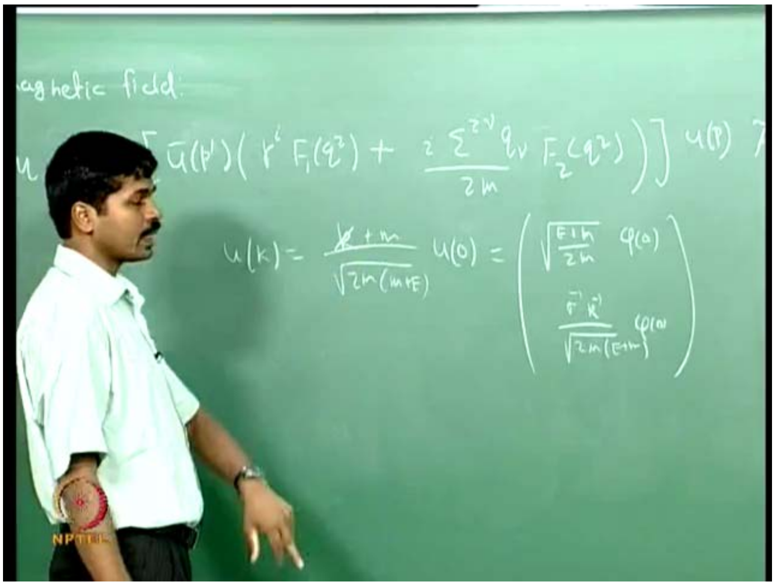

What we will see is that we will consider this explicit form for u which is u of k is k slash plus m divided by 2m, m plus E, which we have obtained by solving the Dirac equation explicitly which is also in terms of the two component spinors. This can be written as E plus m divided by 2m phi 0 and sigma dot k over 2m E plus m phi 0 where phi 0 is a two component spinor. We will use this explicit expression here.

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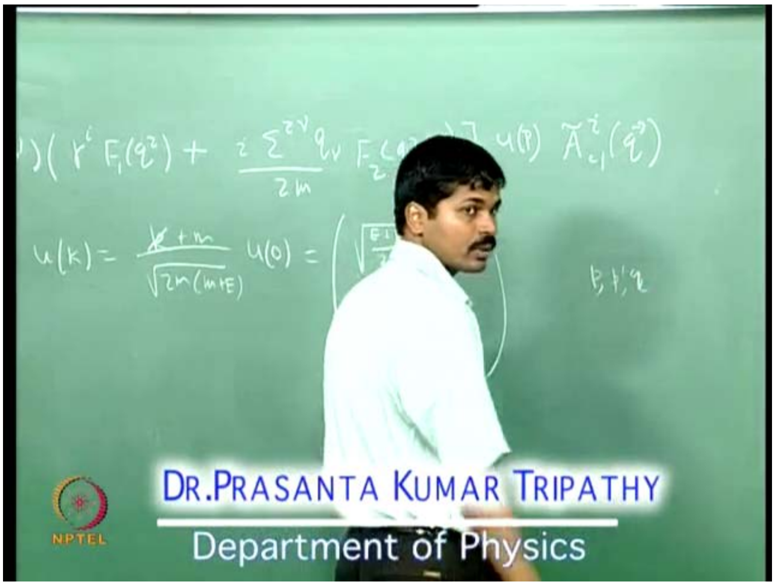

We will put it here. Then we will take the limit where k in other words we will consider limit where p, p prime, and q are small compared to the mass of the electron. So, in this limit, we will evaluate this explicitly.

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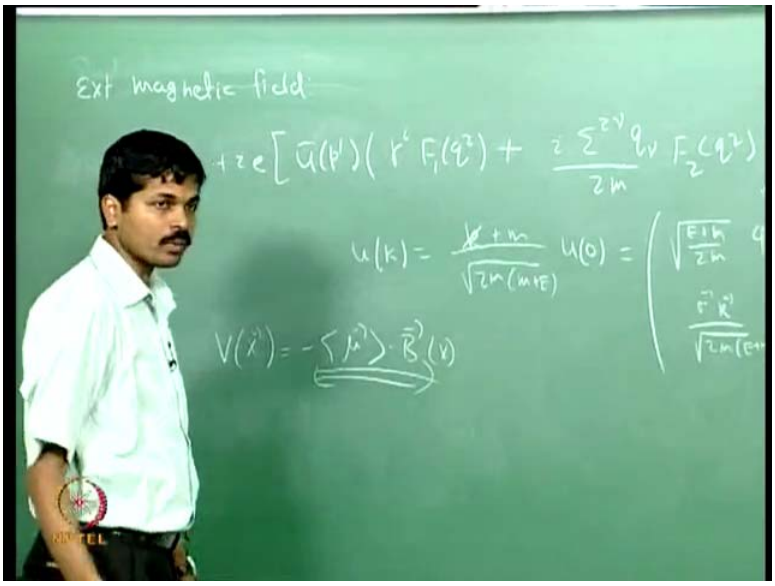

Then, we will show that this matrix element in fact is something, which can be evaluated from a potential, V of X is equal to minus mu dot B of X. Therefore, this basically will give us the correction to the anomalous magnetic moment of the electron. We will see what is the precise form of this and what is the precise form of this average magnetisation from here, so that we can know which of these form factors give you correction to the anomalous magnetic moment and in what way. So, this we will discuss in tomorrow's lecture. Then we will compute and this correction to the one loop order.