Quantum Field Theory

Prof. Dr. Prasanta Kumar Tripathy
Department of Physics
Indian Institute of Technology, Madras

Module - 04
Quantum Electrodynamics
Lecture - 29
Compton Scattering III

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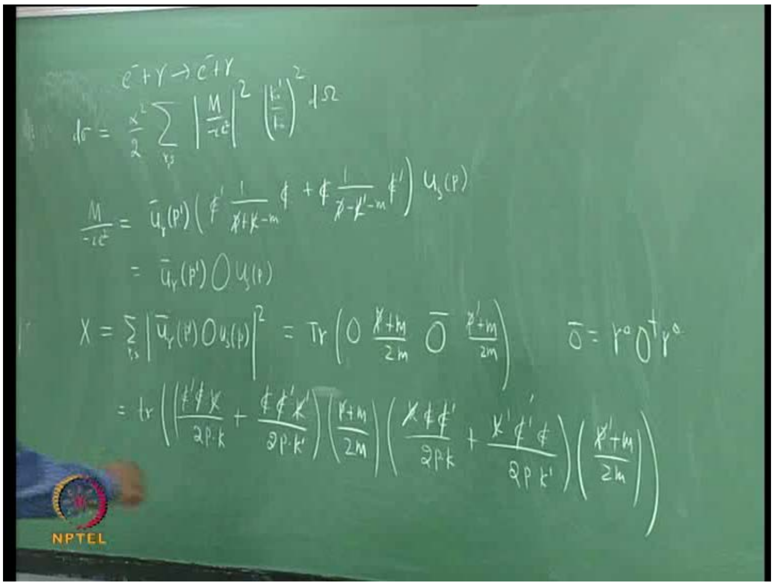

So, let me summarize our discussion so far. We have been computing the cross section for a Compton's scattering. So, e minus plus gamma going to e minus plus gamma and then, we have seen that the differential scattering cross section the sigma is given by alpha square over 2 sum over r s mode m over minus i e square whole square k 0 prime k 0 square d omega, where this quantity here m over minus i e square. We have seen this given by u bar r p prime epsilon prime slash 1 over p slash plus k slash minus m epsilon slash plus epsilon slash 1 over p slash minus k prime slash minus n epsilon prime slash u s p which is of this for u bar r p, prime time's some operator u s p. What we are interested is mode square of this sum d over r s.

So, I will denote this quantity x to be sum over r s mode u bar r p prime o u sp square and what we have seen is that this quantity is nothing, but given by trace o p slash plus m over 2 m o bar p prime slash plus m divided by 2 m where o is given by this quantity here. So, when I substitute this for o and our o bar is gamma 0 overtake a gamma 0.

When we substitute for o what, we get is this quantity is nothing, but some of photons. So, let me do one more step this is trace of epsilon prime slash epsilon slash k slash divided by twice p dot k plus epsilon slash epsilon prime slash k prime slash over twice p dot k prime this times p slash plus m over 2 m k slash epsilon slash epsilon prime slash over twice k dot k plus k prime slash epsilon prime slash epsilon slash divided by twice p dot k prime p prime plus m divided by 2 m.

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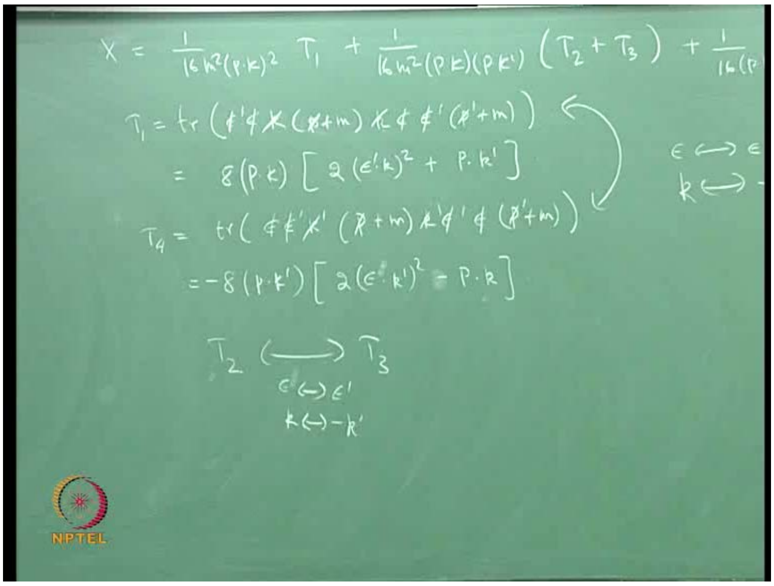

So, this is the quantity I have express it in terms of T 1 T 2 T 3 for x is equal to if I expand this will get four terms. So, I have denoted them is 1 over 16 m square p dot k square T 1 plus 1 over 16 m square p 0 k p 0 k prime times T 2 plus T 3 plus 1 over 16 p 0 k prime square m square T 4. We have evaluate T 1 and then we have shown that T 1 is equal to trace of epsilon prime epsilon slash k slash p slash plus m k slash epsilon slash epsilon prime slash e prime slash plus m we have evaluated the trace and then we have shown that this quantity is given by 8 p 0 k times 2 epsilon prime that k square plus p 0 k prime. We are going to evaluate T 2, T 3 and T 4, but you can notice that, if you just look at the expression T 4 then this is given by first and last term.

So, these is the last two terms multiplied and between these two epsilon slash epsilon prime slash k prime slash then p slash plus m p prime slash epsilon prime slash epsilon slash p prime slash plus m. Now, you can notice one interesting fact. These T 1 and T 4 are related to each other by this efficiency epsilon to epsilon prime and if you access k 2 minus k prime then this term and this term at the same. So, you do not need to evaluate this t for explicitly. All you have to do is to do substitution in p one, then you will get what you want for T 4.

So, this immediately tells us what T 4 is. It is simply 8 p dot k prime with a minus sign and then twice epsilon prime 0 k prime, where plus minus epsilon 0. So, epsilon prime goes to epsilon k goes to minus k prime, but there is a whole square. So, this is twice epsilon 0 k prime and here k prime goes to minus k. Therefore, I got a minus sign. There is an overall minus sign because of the presence of p 0 k. So, immediately I evaluated T 4. Similarly, you can see that T 2 and T 3 are also related by the same symmetry. If you consider exchange of the photon polarization and the momentum with a minus sign k goes to minus k prime then T 2 is related to T 3.

So, it is enough for us to compute T 2, which is given by T 2 equal to trace epsilon prime slash epsilon slash k slash p slash plus m k slash epsilon prime slash epsilon slash p prime slash plus m[n]. So, T 2 is the product of this and T 3 is the product of this. So, these are T 2s and T 3s and you can see clearly that T 2 and T 3 are related by this symmetry that I have given here. So, I will work out from this trace of detail and then from T 2, we will get the expression for T 3 also. Finally, we will put all this values for T1T2 T 3 T 4 in this expression and we will put x in the expression for the differential scattering cross section, finally we will get a differential scattering cross section.

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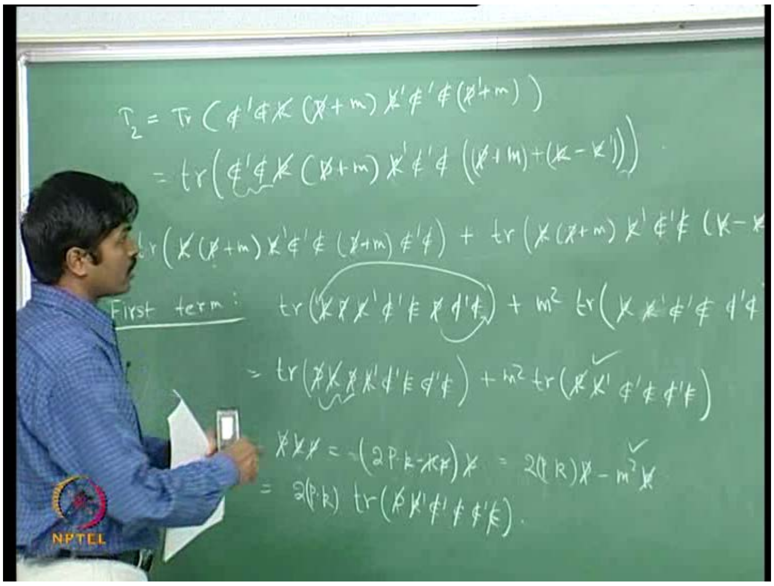

So, let us try to do this trace, you start with these and then you note that p rime plus k prime is equal to p plus k, the large moment of conservation rule. So, p prime is the p rime k minus k prime, which I will substitute here.

So, when I do that, I get trace epsilon prime slash epsilon slash k slash p slash plus m and a k prime slash epsilon prime slash epsilon slash. This quantity is just p slash plus m plus k minus k prime slash. Alright! So, the trace of these things I will take this and use this cyclic property of trace and then I will bring this fact here and then I will split into two terms , in one I will step plus 1 m, the other I will have k slash minus k prime slash. Let us do that.

So, when I do that, I get trace of k slash p slash plus m k prime slash epsilon prime slash epsilon slash and then p slash plus m epsilon prime slash epsilon slash that is the first term and the second term is a trace of k slash p slash plus m k prime slash epsilon prime slash epsilon slash then k slash minus k prime slash epsilon prime slash epsilon slash that the second term we will evaluate each of these term. So, let us consider the first term first. So, the first term again you can see in general, if you expand this you will get four terms, one which does not contain m at all, the other one contains m square and there are two more terms which are linear in m, but term which are liner in m contains all number of gamma matrices you can count as 1 2 3 4 5 6 7 8.

So, they contain even one piece. So, they contain seven gamma matrices trace is 0, so the first term is simply trace of k slash p slash k rime slash epsilon prime slash epsilon slash p slash epsilon prime slash epsilon slash. This is the first term and then the second one contains m square trace k slash k prime slash epsilon prime slash epsilon slash k prime slash epsilon prime slash epsilon slash and again epsilon prime slash epsilon slash. Now, what we can do is you can bring this p slash to this n here, because p d0 epsilon equal top 0 epsilon prime equal to 0 this implies p slash epsilon prime slash is minus epsilon prime slash p slash. So, if you move it once you will get 1 minus sin similarly again p slash epsilon slash is minus epsilon slash p slash.

So, you will get 2 minus sins which is plus 1, therefore what you can do is just pull this p slash to here and then you use the cyclic property of trace and then, finally this p l slash you can write it here, so this becomes trace of p slash k slash p slash k prime slash epsilon prime slash epsilon slash epsilon prime slash epsilon slash, this is the first term and the second term is m square trace of k slash k prime slash epsilon prime slash epsilon prime slash epsilon prime slash epsilon slash, but now you notice this term here. This three terms, the first three term this p slash k slash p slash which is given by p slash k slash is plus price p 0 k minus k slash p slash right this times p slash, therefore the first term you get is twice p 0 k p slash and in the second term you have 2 p slash. What is p slash square? It is simply p square which is equal to m square. So, this is minus m square k slash, now in the second term if you just see minus m square k slash here, this will exactly cancel with this term here. So, this term here cancels this and hence this is simply given by twice p 0 k times trace of p slash k prime slash epsilon prime slash epsilon slash epsilon prime slash epsilon slash. So, the whole things now reduce to some trace which involves 6 gamma matrices. We can further simplify it and then we can write it in terms of the product of all this piece case and epsilon.

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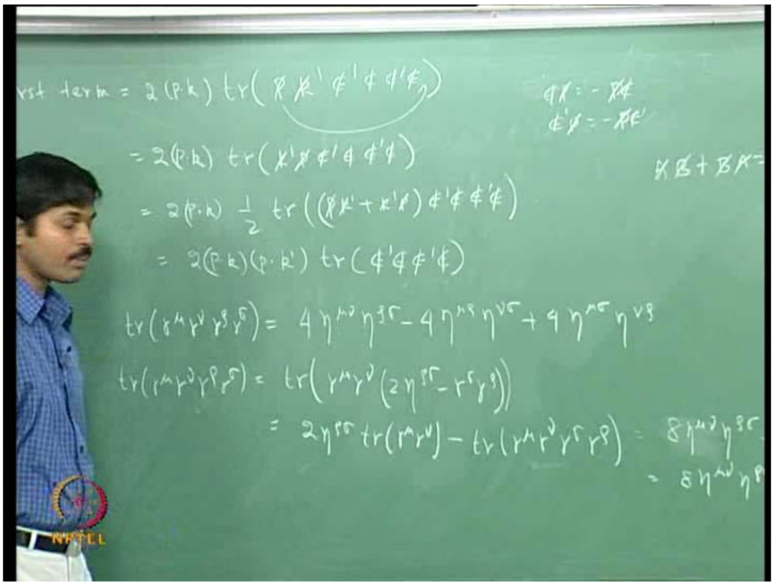

So, let us do that. So, we are evaluating the first term and we have seen that this reduce to twice p 0 k trace of p slash p prime slash epsilon prime slash epsilon slash epsilon prime slash epsilon slash. Now, I can again do is let us say the simplest thing, but I can bring it here and then I will use the fact that p dot epsilon equal to 0, then if I pull it here it will get four times minus 1 which is I plus 1. So, this quantity itself is equal to twice p 0 k trace of k prime slash p slash epsilon prime slash epsilon slash epsilon prime slash epsilon slash is this clear first. I use the cyclic property of trace.

So, I take this p slash here and then I use this property p slash epsilon slash p slash is minus p slash epsilon slash and epsilon prime slash p slash equal to minus p slash epsilon prime slash. So, I use this twice, and then I get this identity. Now, what I can do is take this quantity twice p 0 k times half of some of these two terms half of trace e slash k prime slash plus k prime slash p slash times epsilon prime slash epsilon slash epsilon prime slash epsilon slash, now what is this quantity a slash b slash plus b slash a slash? It is simply twice a 0 b because this will give gamma mu gamma mu gamma mu gamma mu which is twice eta mu nu times a mu n mu which is twice a 0 b. So, what I get is twice p 0 k times half times 2 will give me one p d0 ot k prime trace of epsilon prime slash epsilon prime plus epsilon prime slash. Now, I need to evaluate the trace of this, but anything which contains 4 gamma matrices, it is very easy to evaluate the trace. So, now I will trace gamma mu gamma mu gamma rho gamma sigma is equal to 4 eta mu eta rho sigma minus 4 eta mu rho eta mu sigma is equal to 4 eta mu nu eta rho sigma minus 4 eta mu rho eta nu sigma plus 4 eta mu sigma eta mu rho. So, then it is very easy to use this relation to evaluate this trace. So, let us now prove that the trace of gamma 4 matrices is equal to this and then we will use this relation to evaluate this trace. It is very easy to prove that you just have to pull this gamma sigma here three times and then use cyclic property of trace.

So, let us see that trace of gamma mu gamma nu gamma rho gamma sigma is equal to trace gamma mu gamma nu times twice eta rho sigma minus gamma sigma gamma rho alright. So, this is twice eta rho sigma trace gamma mu gamma nu and then minus trace of gamma mu gamma nu gamma sigma gamma rho. Again I will pull this gamma sigma here, then I will get this to be twice eta by the trace of gamma mu gamma nu is four times eta mu nu.

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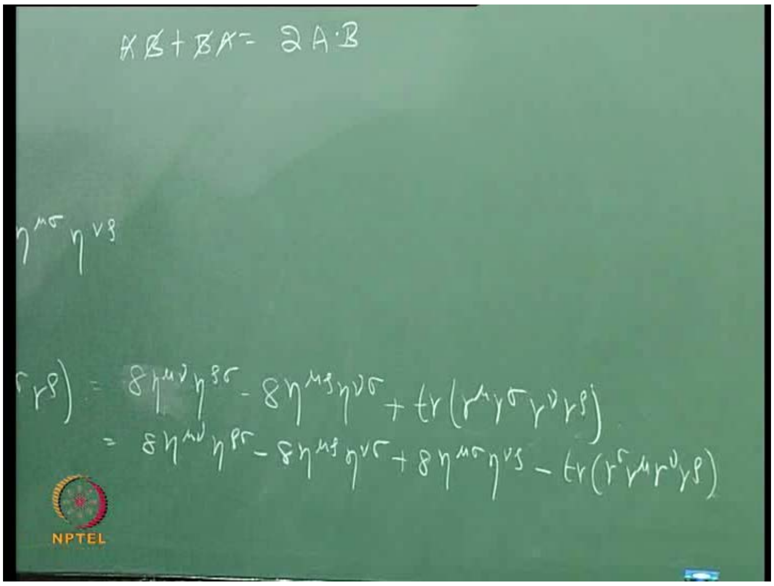

So, what I get is 8 eta mu nu eta rho sigma and when I pull it once I will get twice eta mu sigma times trace gamma mu gamma rho which is minus 8 eta mu rho eta mu sigma and then plus trace of gamma mu gamma sigma gamma mu gamma rho.

Now, in this expression again, I will pull it once then what I get is 8 eta mu nu eta rho sigma minus 8 eta mu rho eta minus 8 sigma plus, this will give you twice eta mu sigma times trace of gamma mu gamma rho which is 8 eta sigma eta mu rho minus trace gamma sigma gamma mu gamma mu gamma rho, but this quantity is same as this. So, if you bring it here this will give twice trace of trace of gamma mu gamma mu gamma rho gamma sigma is equal to 8 times this quantity therefore, the trace of 4 gamma matrices is given by this.

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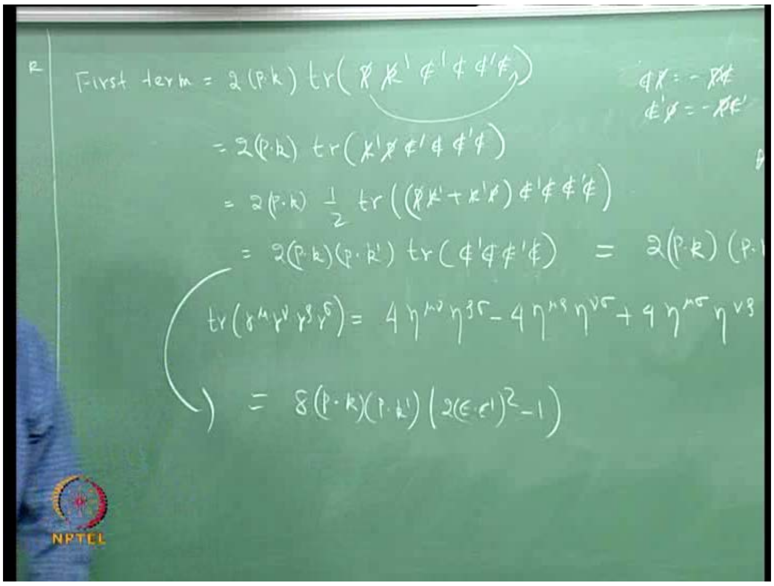

So, now that we have proved this relation, I can use this two- two derive this trace. This will simply given by twice p 0 k p 0 k prime times 4 the first term mu nu rho sigma will give you epsilon 0 epsilon prime whole square. Then you have minus eta mu rho eta sigma will say that first one will have 0 product with the third one, second one will have with forth one. So, minus 4 epsilon 0 epsilon prime dot epsilon prime and finally, the last term will say that, it is again the epsilon prime 0 epsilon whole square. So, plus 4 epsilon prime 0 epsilon whole square, what is this quantity here this is one with epsilon 0 epsilon is minus 1. So, therefore, this term here is evaluate as 8 p 0 k p 0 k prime times twice epsilon 0 epsilon prime whole square minus 1.

So, now that we have evaluated the first term this T 2 has two terms, the first term we have evaluated and then we have shown that this is equal to this, now we will come back to the second term here and then we will evaluate this in more detail.

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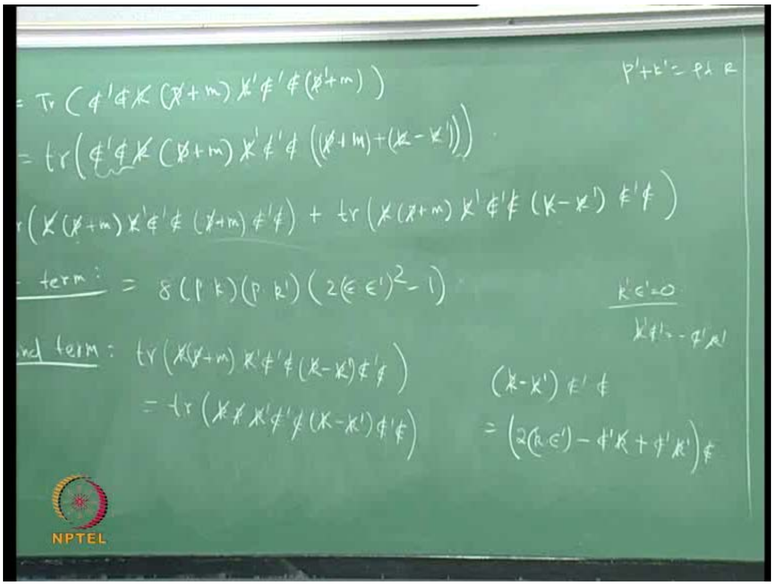

So, let us summaries what we got here. We got the first term to p equal to 8 p 0 k p 0 k prime twice epsilon 0 epsilon prime whole square minus 1, now we need to evaluate the second term. So, let us look at the second term, it is trace till slash plus m k prime slash epsilon prime slash epsilon slash k slash minus k prime slash, then 10 slash come slash again, you see this involves there are two terms.

So, one with p slash contains 8 gamma matrices, but the one with m 47 gamma matrices. So, the turbo noises linear in m will simply vanish therefore, this is trace k slash p slash k prime slash epsilon prime slash epsilon slash k slash minus k prime slash epsilon prime slash epsilon slash. So, now I will focus on the last three terms. Here what I will get is k slash minus k slash minus k prime slash times epsilon prime slash epsilon slash, now you try to move this epsilon slash to the epsilon prime slash. So, this side then what you will get the first term will have twice k dot epsilon prime minus epsilon prime slash k slash and here k prime slash epsilon primes slash is simply epsilon prime slash k prime slash with the minus sign.

So, this is epsilon prime slash k prime slash k prime slash the whole thing multiplied by epsilon slash because k prime dot epsilon prime is equal to 0. We are considering transfer proton therefore, k prime slash epsilon slash is minus epsilon prime slash k prime slash. So, now this is to be inserted here, let us see what you get when you set this.

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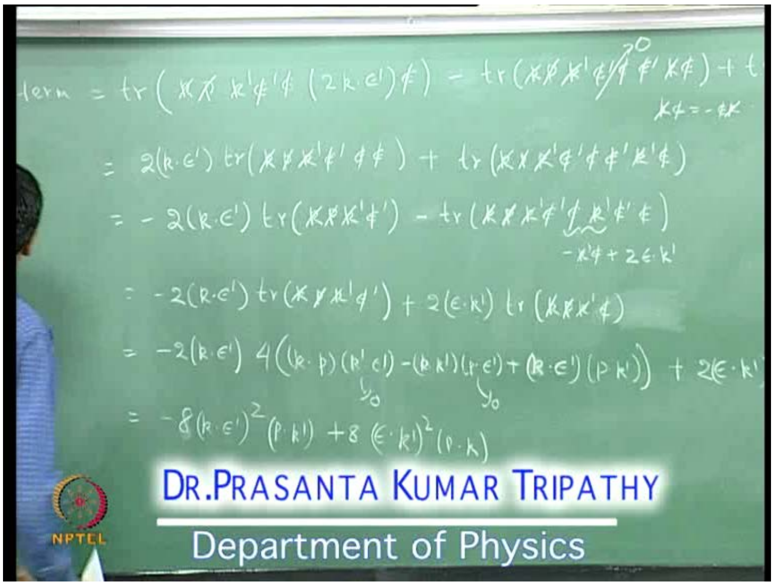

So, k slash p slash p prime epsilon prime then this is twice epsilon prime epsilon slash this is the first term and then you have trace of eta minus trace of k slash p slash k prime slash epsilon prime slash epsilon slash epsilon prime slash epsilon slash and finally, trace of k slash k prime slash epsilon prime slash e slash e prime slash k slash epsilon slash. Now, you just look at the second term here, I can click k and epsilon using this fact k slash epsilon slash is minus epsilon slash k slash, then there is the k slash here one, but there is one k slash here, you use the cyclic property of trace you bring this k slash here that will give you k slash square which is 0.

So, the second term here when it is because of this. Now, we are left with first and last term one prime slash l k prime slash. So, this is twice k 0 epsilon prime trace of k slash p slash k prime slash epsilon prime slash epsilon slash epsilon slash. This is one term and the second term I have seen. So, k slash p slash k prime slash epsilon prime slash epsilon slash epsilon prime slash. So, let us look at this term here there are two epsilon slash. So, therefore, this is the minus 1. So, this term here is minus twice k 0 epsilon prime trace of k slash p slash k epsilon prime slash and then you have this term here which I have to simplify, here this involves of 4 gamma matrices.

So, I know how to do that. Here also what I can do is I can just rearrange things, so that this again reduces to something which involves trace of 4 gamma matrices and then I can evaluate that in strait for manners epsilon dot k prime. So, let us focus this term here and then I will click this k prime epsilon prime. I will get a minus sign. So, minus trace of k slash p slash k prime slash epsilon prime slash epsilon slash and then this is a prime slash epsilon prime slash epsilon slash this is 1, then what I will do is that I will again take this k prime to this side that will give me two terms, one of them will be simply, so minus k prime slash epsilon slash plus twice epsilon 0 k prime that is what I will get from here. Now, you can see the first term which contain minus k prime plus epsilon prime slash it comes with k prime epsilon prime here.

So, what you get here for the first term is something which involves k prime plus epsilon prime slash k prime slash. What is the quantity this is 0, because this again you can flip it then you get k slash prime square which is k square which is 0. So, therefore, this term is simply 0. So, what you get is minus twice k dot epsilon prime trace of k slash p slash k prime slash epsilon prime slash and here you have minus twice epsilon 0 k prime times trace of k slash p slash k prime slash epsilon prime slash and here again epsilon prime slash epsilon slash alright because for this I simply substituted the twice epsilon 0 k prime. Now, you have again epsilon time slash epsilon prime slash square which will give you 1 minus sign. So, this quantity here is plus epsilon slash 2 epsilon j k prime epsilon k prime slash epsilon slash good. So, we have started this term and then we have reduce it to two sum of two terms which involves trace of 4 gamma matrices therefore, we can easily compute them, lets the right the answer for that minus twice k 0 epsilon prime times four times this is k 0 p k 0 prime 0 epsilon prime minus k 0 k prime p 0 epsilon prime plus k 0 epsilon prime p 0 k prime, that is for the first term and similarly for the second term. I have twice epsilon k prime times 4 k 0 epsilon k 0 p k prime 0 epsilon minus k 0 k prime p 0 epsilon plus k 0 epsilon p 0 k prime.

But, now you use to fact that the photon polarization as transverse and then we are working in the laboratory frame. So, that is again orthogonal to p. So, p 0 epsilon and p 0 epsilon prime at 0 therefore, this term is here as 0, which is beta p 0 epsilon prime 0 here and because 14 polarization is transverse k prime 0 epsilon prime 0. Now, here similarly this terms survives, but p 0 epsilon is 0 and k 0 epsilon is again 0. We can photon polarizations is transverse. So, now, I will put all this things together, then what I get is minus twice k 0 epsilon prime, there is the factor of 4. So, 8 k 0 epsilon prime times k 0 epsilon prime square k 0 k prime and the second term is again plus k epsilon 0 k prime square p 0 k. So, this is what we got for the second term.

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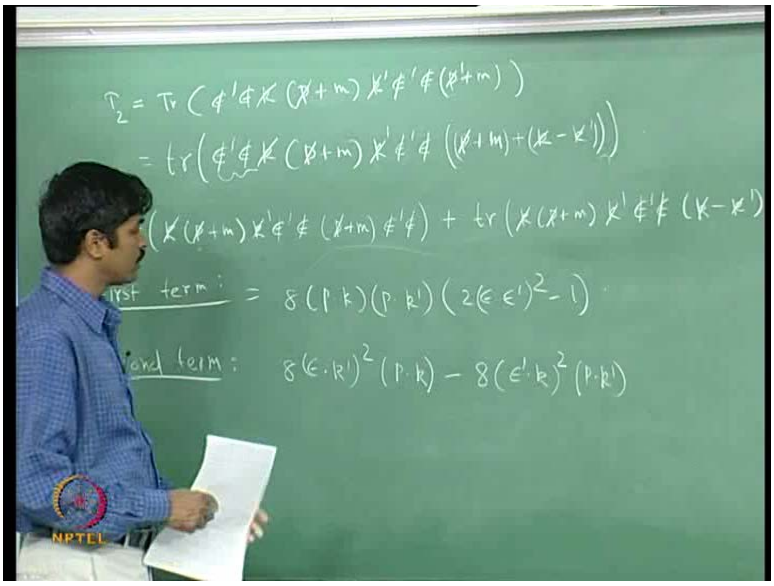

So, the first term is this and the second term is a 8 epsilon 0 k prime square k 0 k minus 8 epsilon prime 0 k square k 0 k prime. So, T 2 is nothing, but some of these two terms therefore, I can simply add them to get T 2.

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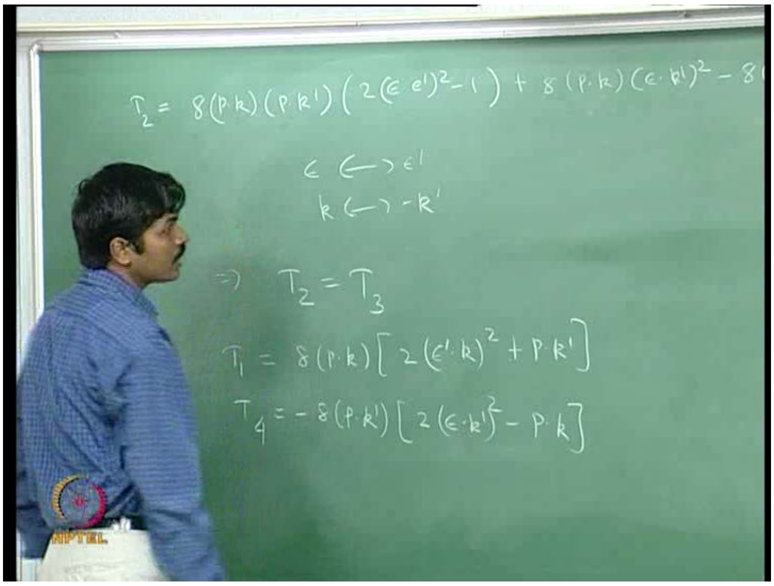

So, 8 p 0 k p 0 k prime times 0 epsilon 0 epsilon prime square minus 1 plus 8 p 0 k epsilon 0 k prime square minus 8 k 0 k prime epsilon prime 0 k square this is our T 2.

Now, I have to find T 3 from here, but you can see that, if you make this transformation method epsilon goes to epsilon prime and k goes to minus k prime, then here it will keep two minus sign. Here to tell k because minus k prime k prime goes to minus k therefore, this whole thing is invariant here epsilon and epsilon prime are inter change this is whole square. So, the factor which is invariant and under this exchange the second term goes to the third term because there is the minus p 0 k prime here and this is epsilon prime 0 k and this term goes to this. So, the second and third term interchange and the first term remain invariant.

Therefore, T 2 is invariant under this transformation, but we have seen earlier that if we make this transformation in the original trace, then we get T3. This simply implies that T 2 is equal to T 3. So, what we have done is we have derived an expression for T 1 T 2 T 3 and T 4. We will put all these things together and then we will see what we get for x. So, let me write the answer for T 1 and T 2. T 1 is equal to 8 p 0 k 8 twice epsilon prime 0 k square plus p 0 k prime and hence T 4 is equal to minus 8 p 0 k prime twice epsilon d0 k prime square minus p 0 k.

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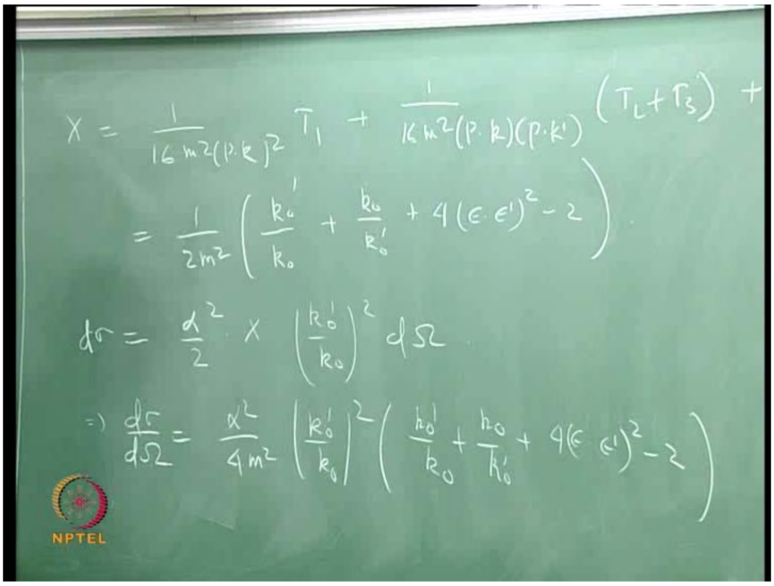

This is your T 1 T 2 T 3 T 4 and your x is 1 over 16 m square p 0 k square T 1 plus 1 over 16 m square p 0 k p 0 k prime T 2 plus T 3 and plus 1 over 16 m square p 0 k prime square T 4. Now, I will substitute for T 1 T 2 T 3 T 4 and then you will see that after little bit of simplification this quantity will reduce to this simple expression 1 over 2 m squares k 0 prime over k 0 plus k 0 over k 0 prime plus 4 epsilon 0 epsilon prime square minus 2. This is what we get for x and d sigma is nothing but alpha square over 2 into x into k 0 prime over k 0 whole square d omega. If you look at what we discussed in the earlier lectures, therefore it simply implies the sigma over d omega is equal to alpha square over 4 m square. I will simply substitute this for x, here then what I get is k 0 prime over k 0 square times k 0 prime over k 0 plus k 0 over k 0 prime plus 4 epsilon 0 prime 2 square minus 2.

So, this is the expression for the differential scale scattering cross section that we got after doing the complicated calculation. I will briefly tell you, what we can do is that we can also average over the photon polarization. We can also assume that our initial photon is un- polarized and we also do not detect the final photon polarization, then all other terms have been now effected except, this one because this involves term photo polarization. what I claim is that if you consider this.

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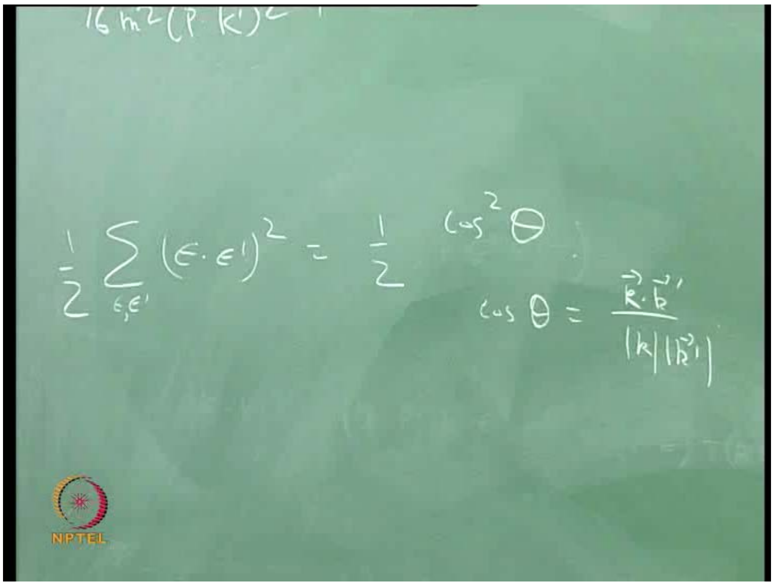

So, you have half epsilon 0 epsilon prime whole square sum of a our epsilon and epsilon prime all possible values of epsilon and epsilon prime this is half cos square theta, where theta is simply equal to cos theta is k 0 k prime k mode k prime its angle, but in the initial and the final photon you can show this and you can substitute it in the expression for the differential scattering from cross section and find this scattering cross section when the electron as well as the photon un- polarized.