Quantum Field Theory
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Module - 4
Quantum Electrodynamics
Lecture - 24
The S-Matrix Expansion in QED II

So, we are discussing the S matrix for the quantum electro dynamics. Then in the last lecture, we were looking at the second order term and the expansion.

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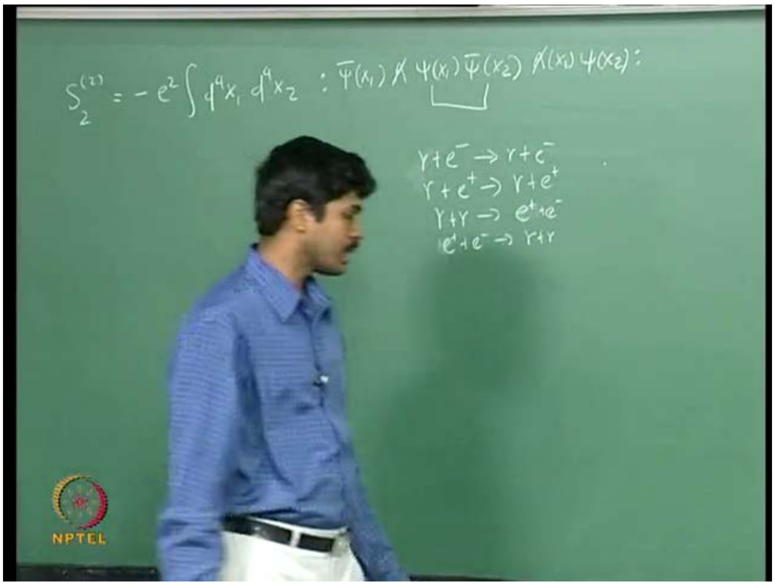

Then we were seeing which term in the S matrix can contribute to what physical process, especially we were looking at this term that I have denoted it as S 2 2, which is given by minus e square d 4 x 1 d 4 x 2 normal order product of psi bar x 1 A slash psi x 1 psi bar x 2 A slash x 2 psi x 2, where psi of x 1 is contracted to psi bar of x 2. Now, this contraction here indicates that there is a virtual fermion which propagates from x 1 to x 2. Then there will be 1 fermion, there will be 2 fermion and 2 photons in the initial and final processes.

So, this indicates that the physical processes that that can give a non zero contribution from this term in this S matrix are the following. You can have the Compton's scattering gamma plus e minus, you can have the m omegas of Compton's scattering with positron resurgence gamma plus e plus, you can have pair creation or pair annihilation, so gamma plus gamma going to gamma e plus plus e minus e plus plus e minus going to gamma plus gamma. So, let us try to understand the Compton's scattering. Then what is the term in the S matrix that gives non zero contribution to the Compton's scattering? Here, this basically says that there is an incoming photon and an incoming electron and there is an outgoing photon and outgoing electron.

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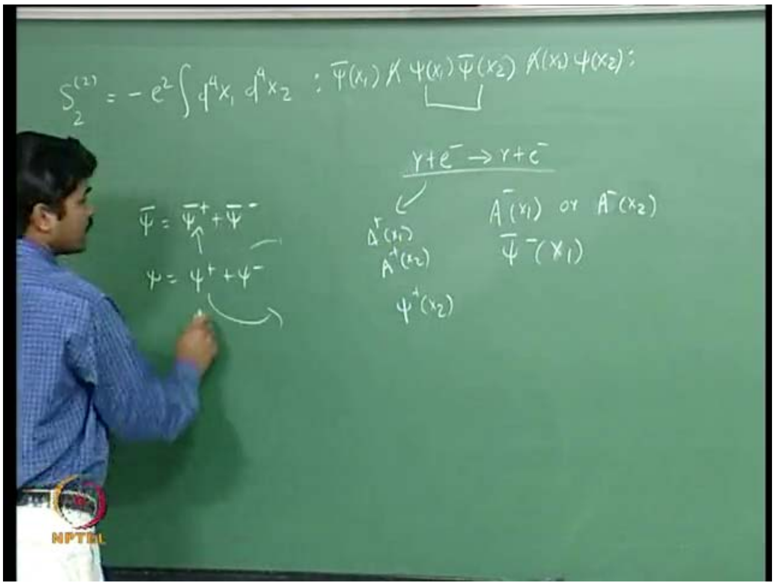

So, outgoing photon and outgoing electron basically indicates that it should contain plus of x 1 or plus of x 2 for the outgoing photon. For the outgoing electron, it should contain psi bar minus of x 1 and for the incoming photon, you have this will destroy a photon rat. So, you need A 1 A minus of x 1 or A minus of x 2 for this one, you can have A plus of x 1 A plus of x 2. You can have psi plus of x 2 psi bar of x 1 will have two terms, psi bar equal to psi bar plus psi bar minus psi bar plus represents the annihilation of a positron, whereas psi bar minus represents the creation of an electron.

So, the only term here that can contribute to this processes psi bar minus where as psi which is psi plus plus psi minus. Psi plus indicates annihilation of an electron, so this whereas psi minus represents creation of a positron. So, for this process, only psi plus will be present. So, what we can have is the following.

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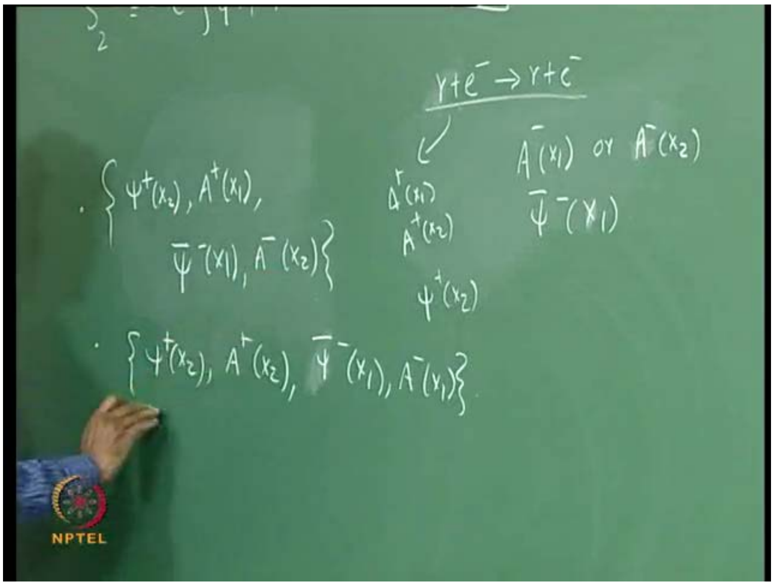

For this process, this S matrix should contain psi plus of x 2 A plus of x 1 and then psi bar minus of x 1 and A minus of x 2; either it should contain this pair or it should, it can also contain psi plus of x 2 A plus of x 2, then psi bar minus of x 1 A minus of x 1. It cannot contain both A plus of x 1 and A plus of x 2 because that that will not a give a finite contribution to this process. So, there are only these two possibilities.

If there any other possibility that can give a non zero contribution to this process, you can inspect quickly and then you can conclude that no other possibilities are there. So, you can either have this set of operators or you can have this set of operators in the S matrix.

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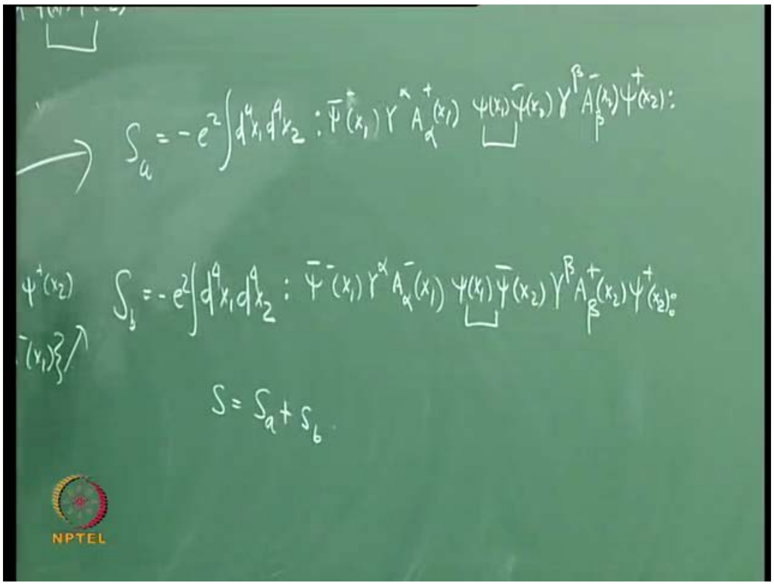

So, I will write therefore, accordingly there will be two terms. This one, I will call Sa, which is minus e square d 4 x 1 d 4 x 2 normal order product of psi bar minus of x 2 gamma alpha A psi bar minus of x 1 gamma alpha A alpha plus x 1 this propagator and then gamma beta A beta minus of x 2 psi plus of x 2, whereas when this set of fields are there, what you will have? I will call this as Sb, which is minus e square d 4 x 1 d 4 x 2 normal order product of psi bar minus of x 1 gamma alpha A alpha minus of x 1 psi of x 1 psi bar of x 2 contraction gamma beta A beta plus of x 2 psi plus of x 2. The amplitude for this process will be S equal to the sum of these two terms, Sa plus Sb.

In other words, if you write psi bar equal to psi bar plus psi bar minus and so on, this term will give many terms. However, if you evaluate the matrix element of this quantity for this process e plus e minus plus gamma going to e minus plus gamma, then these are the only two terms, which will give you non zero contributions. Let us pictorially represent both these terms.

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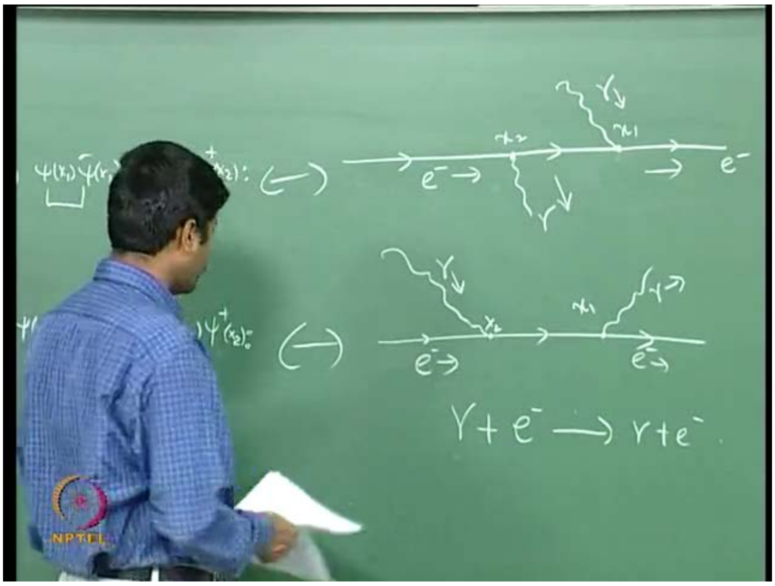

This term Si here, I can write it is some incoming electron which is annihilated at x 2 and then there is a photon gamma. Then there is a fermion propagating from x 2 to x 1. Then you have an outgoing electron e minus with a photon, which is getting absorbed at x 1. So, let us look at A plus of x 2. So, A plus of x 1 here, for example, indicates annihilation of a photon at this waste time point x 1. So, you have correspondingly an incoming photon at x 1, which is getting annihilated where is there is A minus of x 2, which represents creation of a photon at x 2.

So, therefore, there is an outgoing photon at this waste time point x 2. Similarly, you have psi plus of x 2, which indicates annihilation of an electron at x 2. So, you have an incoming electron, which is getting annihilated at x 2, whereas psi bar minus of x 1 indicates creation of an electron at x 1 so you have an outgoing electron which is getting created at x 1. Then there is a propagator psi x 1 psi bar x 2, which describes propagation of an electron from x 2 to x 1. So, pictorially I can represent this term of the S matrix like this. What about this term here?

So, this term again says that you have psi plus of x 2 means that there is an incoming electron, which is annihilated at x 2 and then A plus of x 2. So, there is an incoming photon, which is getting annihilated at x 2. Then it also contains psi bar minus of x 1. So, at x 1, there is an outgoing electron which is getting created and A alpha minus x 1 basically says that there is a photon, which is created at x 1. So, what I did is I represented this incoming electron with an arrow, with a straight line with an arrow. The propagator again, I represented it as an arrow and out outgoing electron is again a straight line with an outward arrow, whereas for the photon, I represent it as a curly line just to make a distinction between electron and photon.

So, photons are represented by curly lines whereas the electrons are represented by straight lines with an arrow. So, these kinds of pictorial representation for a physical process, which gives contribution to the S matrix, are known as the Feynman diagram. So, what I have drawn here for you is the Feynman diagram for the lowest order for the process gamma going to gamma plus e minus going to gamma plus e minus. So, this are the Feynman diagrams for Compton's scattering in the lowest order correspondingly. If somebody gives you a diagram, so given a physical process, you can think of how to pictorially represent this process.

Then, given this diagram, you can surely write down what element of the S matrix will give you a non zero contribution for this process. Now, it is straightforward from whatever we have analyzed till now. If I give you this picture here, then you can write down of course this S matrix element. Similarly, this one here corresponds to this S matrix. So, you consider this process. You know that at lowest order, these are the only two possibilities. So, you draw these two diagrams and then you write the corresponding amplitudes and then you add them.

So, this will give you the amplitude for the full amplitude for this physical process gamma plus e minus going to gamma plus e minus. So, this is just one such term here. It describes this process of Compton's scattering. Similarly, you can you can discuss about pair creation, pair annihilation and so on. Let us see how can it, what happens when I want to describe this process of pair creation?

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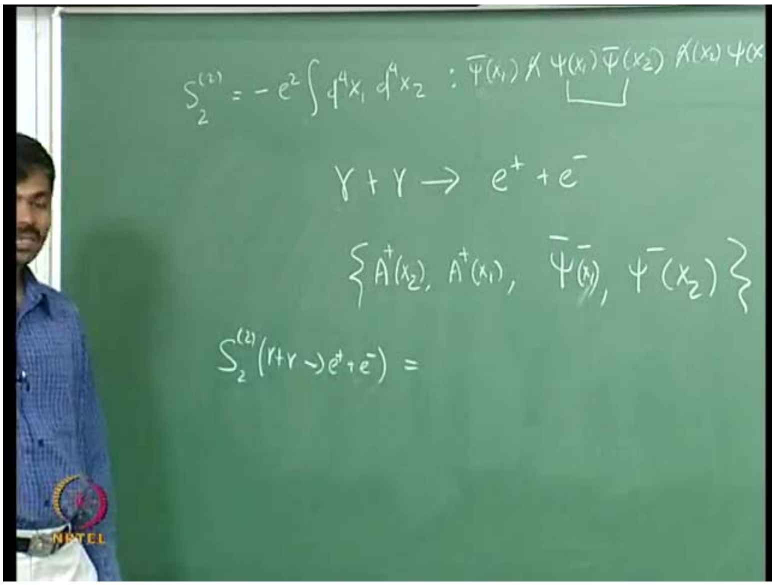

So, so suppose you want to describe this physical process gamma, gamma going to e plus, plus e minus. So, what do you expect from this, from this S matrix element here? What of the terms it should contain? It should of course; there are two incoming, incoming photons. So, you should expect A plus of x 2 and A plus of x 1. It should contain A plus x 1 and A plus x 2, whereas there is this e, there is a outgoing electron and there is an outgoing positron.

So, what do you have for outgoing positron? So, for outgoing positron and outgoing electron, you have psi minus and psi psi minus of x 1 psi bar minus of x 1 and psi psi minus of x 2. So, it should contain this combination of operators definitely. Is there any other combination operator that is possible for this process? You can see and conclude that there is no other combination, which is possible for this process. So, this part of the S matrix S 22 gamma plus gamma going to e plus plus e minus, basically means that the term in the s matrix gives a non zero contribution for this process.

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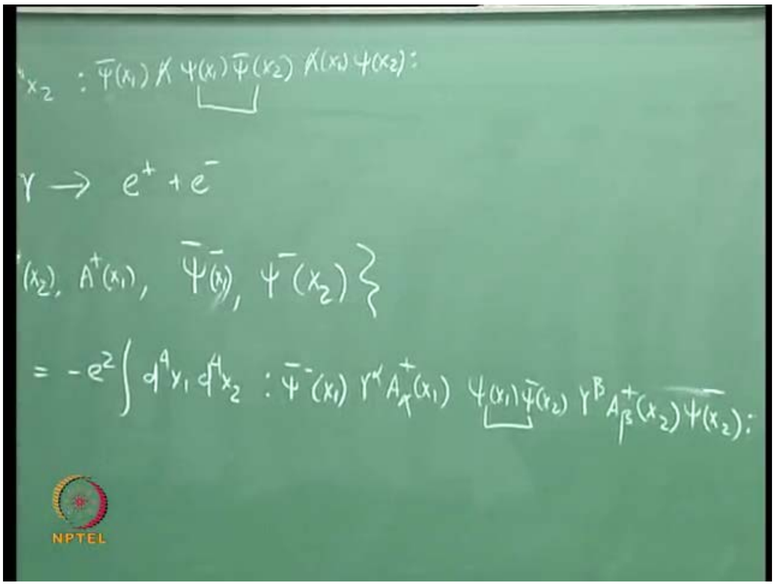

That term must look like integration d 4 x 1 d 4 x 2 and normal order product of psi bar minus of x 1 A slash of x, not A slash gamma alpha A alpha plus of x 1, then psi x 1 psi bar x 2 and gamma beta A beta plus of x 2, then psi minus of x 2. So, this is the term in the S matrix that will give non zero contribution to this process. Pictorially we can again represent this.

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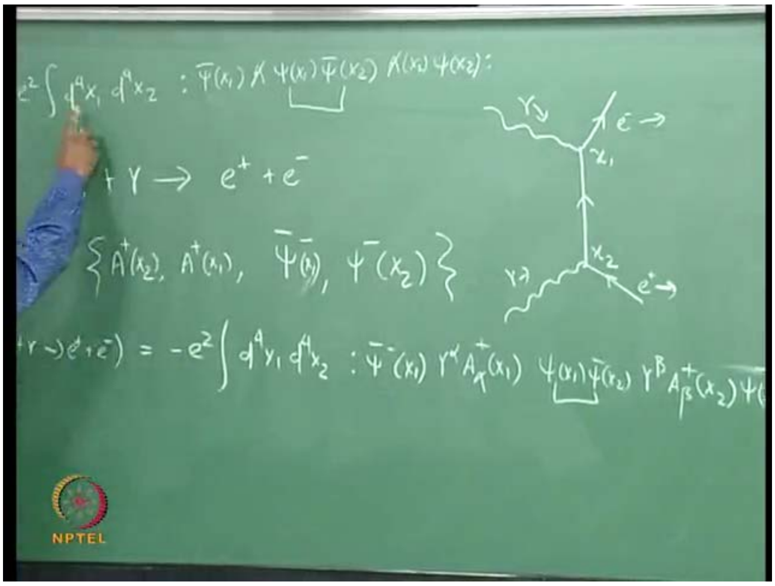

So, you see that is there is an outgoing electron, which is created at x 1. So, this must be this waste time point x 1 from where is an electron is getting created. There is a fermion, which propagates from x 1 to x 2 or x 2 to x 1. There is an incoming photon, which is getting annihilated at x 1. There is an incoming photon, which is getting annihilated at x 2. This term basically see that there is an outgoing positron, which is created at x 2. Therefore, the corresponding diagram looks like that. You may notice that although there is an outgoing positron here, I have drawn this arrow inwards. This is just to make consistent so that this arrow has one direction.

So, whenever there is an outgoing electron, I will draw it as a straight line with an outgoing arrow, but whenever there is an outgoing positron, I will draw, I will represent it with a straight line with an incoming arrow. Similarly, if there is an incoming electron, I will represent it with a straight line with an incoming arrow, whereas if there is an incoming positron, I will represent it with a straight line with an outgoing arrow. That way you can see that in this diagram, this arrow is directed along one direction only. Two arrows never proceed. So, their ends meet in a single point.

So, this is the Feynman diagram for this process. This is the element of S matrix that gives you a non vanishing contribution for this process. So, you can consider other physical processes for arising from this term in the S matrix. Then you can similarly, write down which term in this S matrix gives a non zero contribution. What you will do is we have written the terms that that we got from the wick expansion. So, we will discuss them and then see what the physical processes are which we can understand from the old steps.

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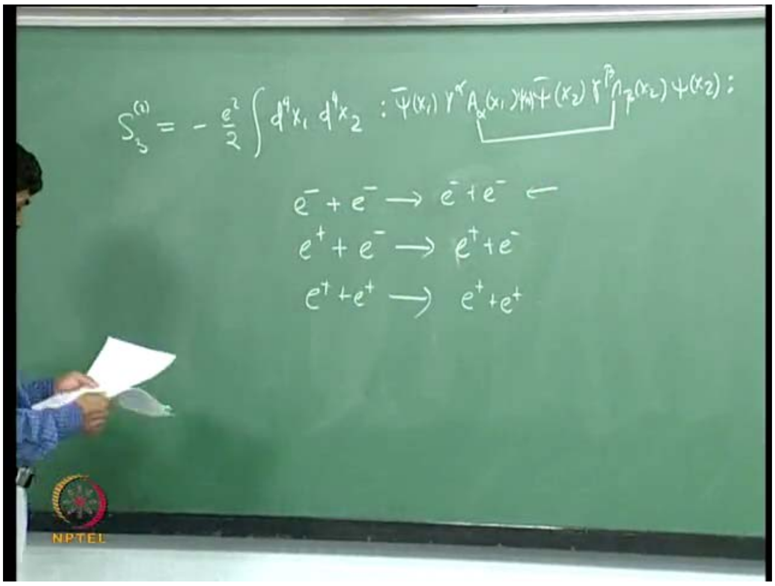

So, let us discuss. Let us look at this term, which I represented as S 2 3. This is given by minus e square over 2 d 4 x 1 d 4 x 2 normal order product of psi bar x 1 gamma alpha A alpha of x 1 psi bar x 2 gamma beta A beta of x 2 psi of x 2 with normal. So, you can denote with this term contracted to this term. You can notice that there is a psi of x 1 here. This is good.

So, you note that here the photons are contracted with each other. So, there is a photon propagator and hence there are no free a mu fields inside the normal ordering. Therefore, this represents that there is no incoming photon or outgoing photon at all. There are only incoming electrons and positrons and outgoing electrons and positrons possible. So therefore, there are these physical processes, which will give non zero contribution. The processes are e minus plus e minus going to e minus plus e minus, which goes with the name of Morelos scattering.

There is also this process, which is called as Wave scattering, which is scattering between electron and positron. There is this positron analogous for the Morelos scattering, which says that e plus plus e plus going to e plus plus e plus. So, these are the processes that can be understood from this term in the S matrix. Let us try to understand the electron electron scattering. So, for electron electron scattering, what do you expect? What are the terms in the S matrix that should be there?

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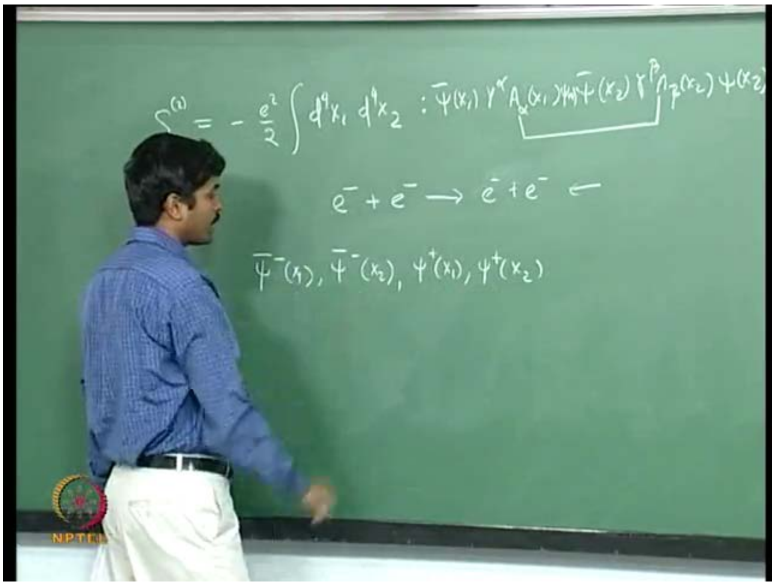

You should have incoming electrons, so two incoming electron and two outgoing electrons. So, you have psi bar minus of x 1 psi bar minus of x 2 and psi plus of x 1 psi plus of x 2 psi plus x 1 and psi plus x 2 will destroy an electron at x 1 and x 2 respectively, whereas psi bar minus x 1 and psi bar minus x 2 will create electrons at x 1 and x 2 respectively.

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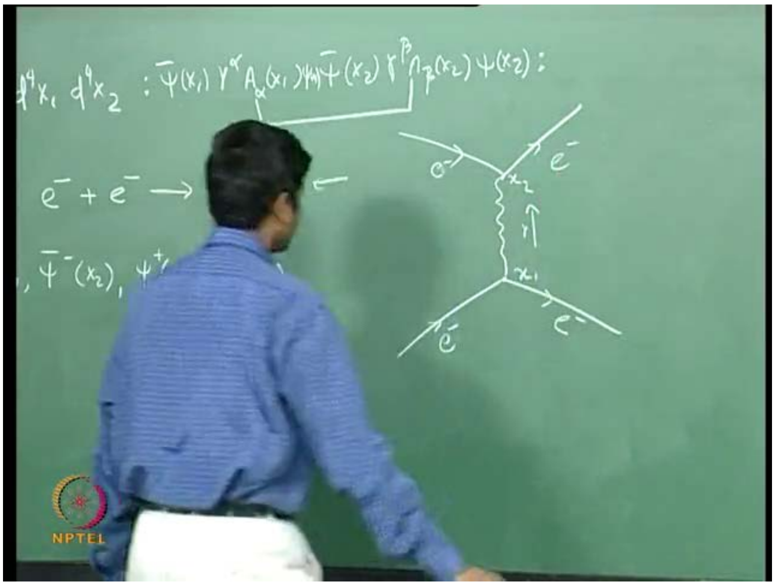

So, pictorially you have an incoming electron, which is destroyed here because of the presence of psi plus x 1. There is an outgoing electron which is created here because of psi bar minus x 1. There is a photon propagator between x 1 and x 2. So, you have a virtual photon propagating from x 1 to x 2. This is a gamma. Then you can see that there is a incoming electron which is getting annihilated at x 2 psi plus of x 2 e minus and e minus. So, this is how it should look. Therefore, this is the way you can write down the S matrix that gives you non zero contribution to this process.

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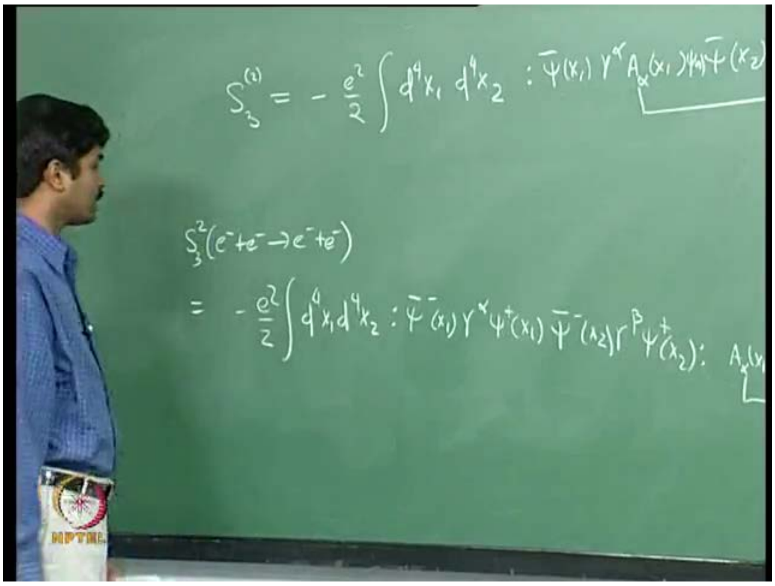

So, S 2 3 e minus plus e minus going to e minus plus e minus is given by minus e square over 2 d 4 x 1 d 4 x 2 psi bar minus of x 1 gamma alpha psi plus of x 1 psi bar minus of x 2 gamma beta psi plus of x 2 with normal ordering, then A alpha x 1 A beta x 2 is contracted, because this is a seen number.

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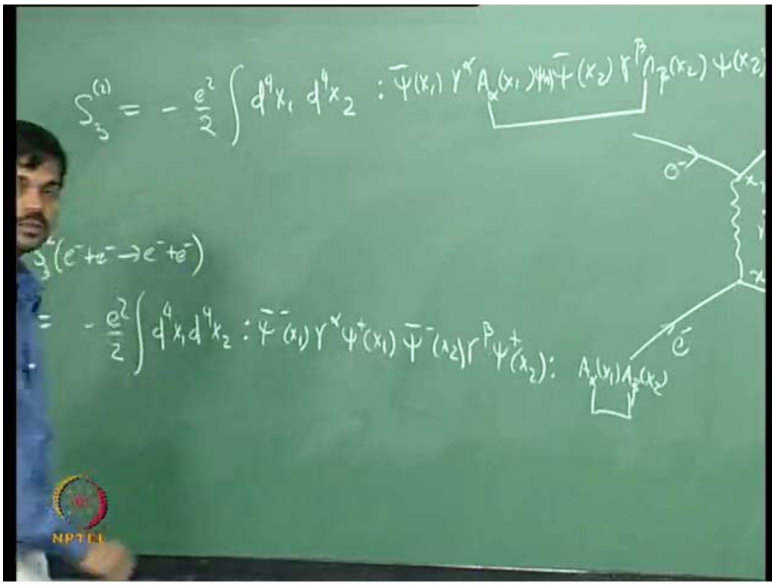

I can take it outside the normal ordering. This is what I get. This is the term which gives you contribution to the S matrix. Now, there is something very important for this in this process that we need to discuss in any more detail.

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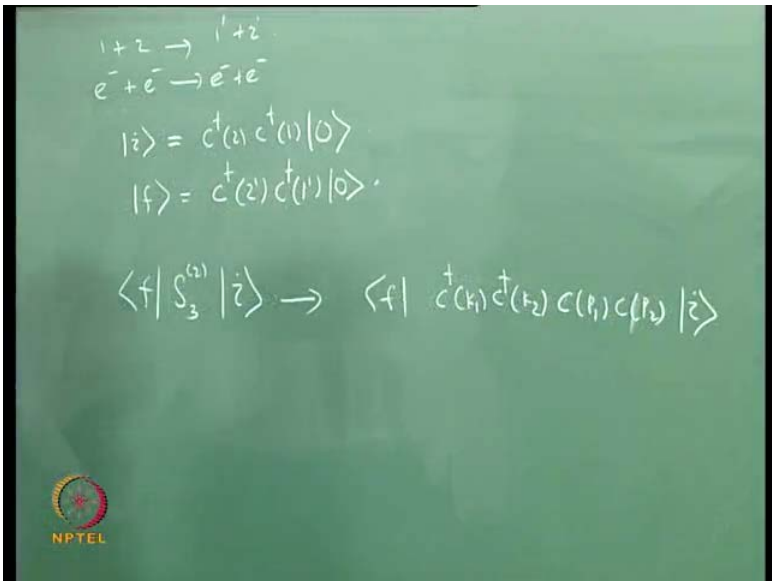

You know the incoming electrons e minus plus e minus going to e minus plus e minus, They are identical particles. So, an interaction can take place between them. Then we need to understand how to compute the amplitude for this process taking care of the x sense interaction. So, let us understand this process here. Let us say this is electron 1 and this is electron 2. So, electron 1 and 2 are going to electron 1 prime and 2 prime. Instead of specifying the momentum and spin etcetera, I am just representing them by 1 and 2, 1 prime and 2 prime respectively.

So, what is the in state? The in state is basically c dagger 2 c dagger 1 acting on the vacuum; whereas the out state is I will call as f is c dagger 2 prime c dagger 1 prime acting on the vacuum. So, if you want to evaluate, if you want to take this term here, then you look at the matrix element of this term for this process S 23, then what are the terms that you expect? First of all, it will contain, you can write it in terms of your components. Then it will contain bunts of p integrations and so on. Then you have gamma alpha and all those things. We will not worry integration over d 4 x 1 and d 4 x 2.

Let us not worry them for a moment. What you will have is you will have in between this, you have four operators which I will denote as two creation and two annihilation operators because of the normal ordering. Now, there will be order like this c dagger k 1 c dagger k 2 c p 1 c p 2. Then there will be bunts of terms, which I will not worry for the time being. So, this is what you will have, this and then other terms will be there in the matrix element. So, what do you get from these terms?

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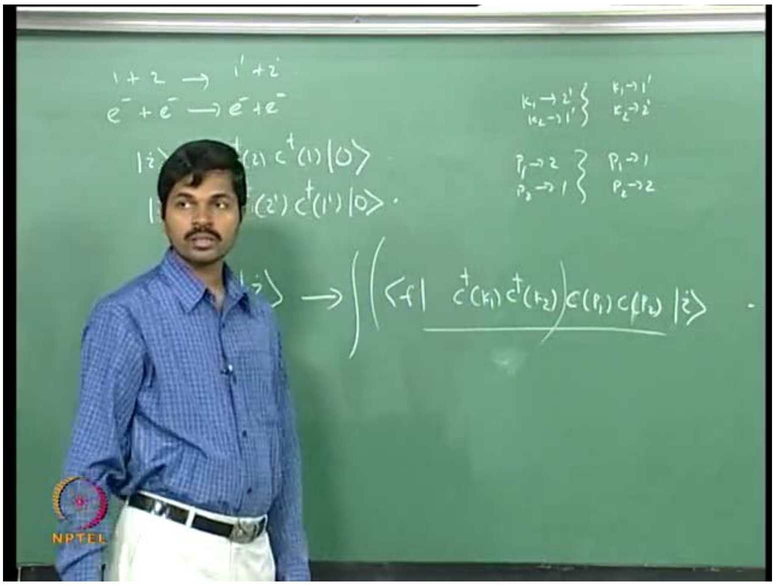

Now, because of this term, the presence of this term here as you can see, it will give a non zero contribution, if p 1 equal to 2 p 2 equal to 1 or it can give a non zero contribution 1 p 1 as 1 p 2 equal to 2. Similarly, you can look at this part here and again you will have a non zero contribution if k 1 is 2 prime k 2 is 1 prime or if k 1 is 1 prime k 2 is 2 prime. So, therefore, when you evaluate it in more detail, which we will do later on, what you will get is you will get four terms here. However, you note that there is this integration over d 4 x 1 d 4 x 2. So, when you change the variable x 1 to x 2, you will see you will see that out of these four terms, two of them will be equal to each other. So, there will be two remaining terms here. So, I will let us say that 1, 2.

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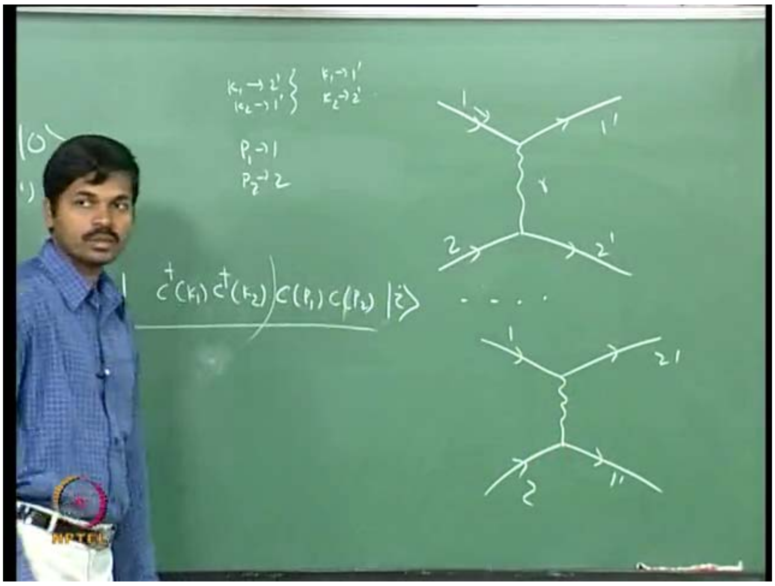

So, the two remaining terms are I will call them p 1 going to 1, p 2 going to 2 and then this. Let us assume that it is already taken care of. Therefore, you have this option where k 1 will be equal to 2 prime and k 2 will be equal to 1 prime with this or you have k 1 going to 1 prime and k 2 going to 2 prime with this. These are the two terms here, which are possible.

So, what you can do is you have correspondingly two diagrams here. So, there is an incoming electron, which I will call as 1 and this incoming electron, which I will denote it as 2. So, 1 2 going to 1 prime 2 prime, there is a photon propagator. You have this possibility here that 1 going to 2 prime, 2 going to 1 prime and there is a photon propagator.

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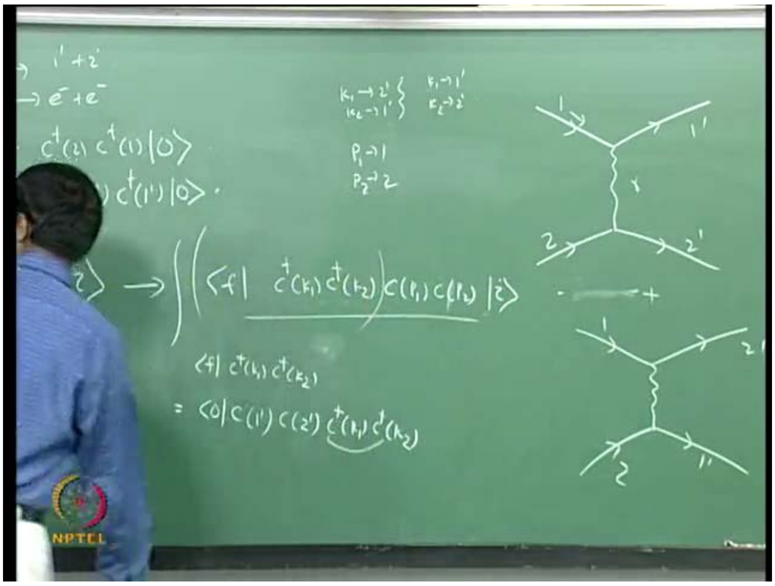

The amplitude will be sum of these two amplitudes.You look at this process here. What is this f c dagger k 1 c dagger k 2 is nothing but this c 1 prime c 2 prime and then c dagger k 1 c dagger k 2. So, therefore, if 2 equal to 1 prime, so whatever contribution, it will give for this process, when k 1 is equal 2 prime and k 2 equal to 1 prime because these operators anti commute. If you concentrate this process where k 1 is 1 prime and k 2 is 2 prime, then this operator needs to move this side and then you consider the anti commutations. Therefore, there is relative minus sign for these things.

So, what you can do is either you can systematically look at this term here, evaluate it is matrix element. Ultimately you will get two terms with a relative minus sign between them or you can look at these two Feynman diagrams here. Compute the amplitude in the user way. Then you add them with a relative minus sign between these two amplitudes to compute the full amplitude for this process e minus plus e minus going to e minus plus e minus. Then this origin of relative minus sign is here because of the fact that this operators actually anti commute instead of commuting with each other. So, there will be other processes, which arise from this term of the S matrix. I will not discuss them in any detail any more.

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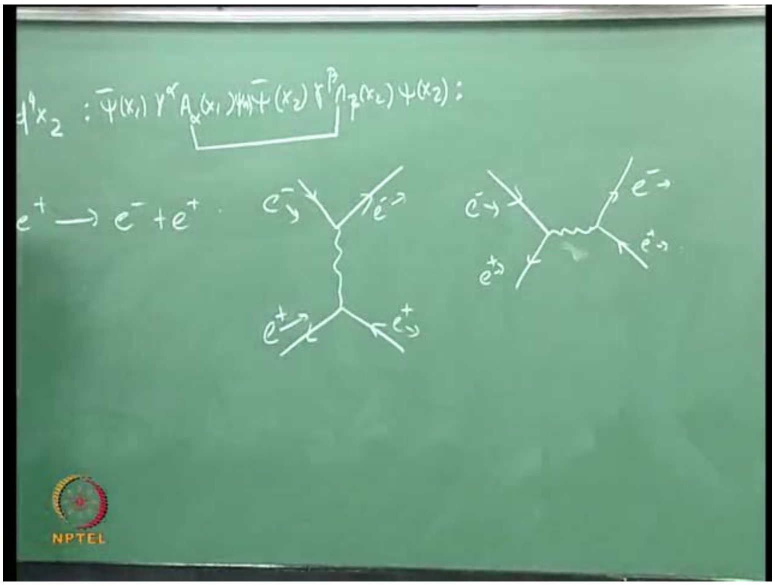

So, I think you will, you can work them out or may be what I will do is that I will discuss this process here e minus plus e plus going to e minus plus e plus. The Feynman diagram for this process is so e minus e plus, there is an incoming. So, this represents an incoming positron. This represents an incoming electron, and then the exchange a virtual photon.

Then, you have an outgoing electron and outgoing positron. So, this is the possibility. This is also a possibility. You have an incoming electron and you have an incoming positron. The annihilate produce a virtual photon and then this virtual photon goes into an outgoing electron and an outgoing positron. So, this is also a possibility. You can see both these possibilities from the S matrix here. Then, the amplitude here will be sum of these two terms. There is no identical particle here, no identical incoming or outgoing particles. So, you need not worry about the extended term at any more. So, what are the other terms in the S matrix?

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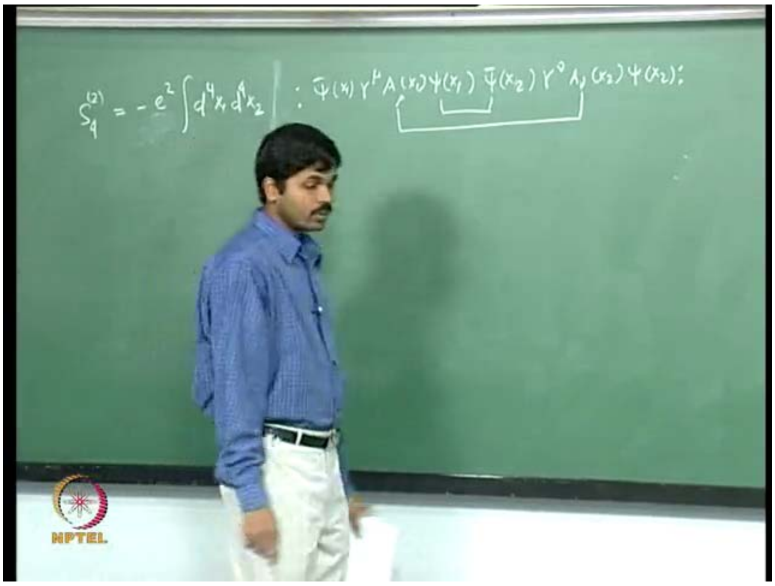

Here this term, which is gamma mu A mu of x 1 psi of x 1 psi bar of x 1 psi bar of x 2 gamma mu A mu of x 2 psi of x 2 over here psi of x 1 propagates with psi of x 1 and A mu of x 1 propagates with A mu of x 2. So, if you look at S24, there will be two such terms. After changing this dummy variable, they will be equal. So, this is the term here. So, what does it represent? It basically says that there is a fermion propagator and there is a photon propagator here. Then you can have one incoming electron and one outgoing electron or one incoming positron and one outgoing positron.

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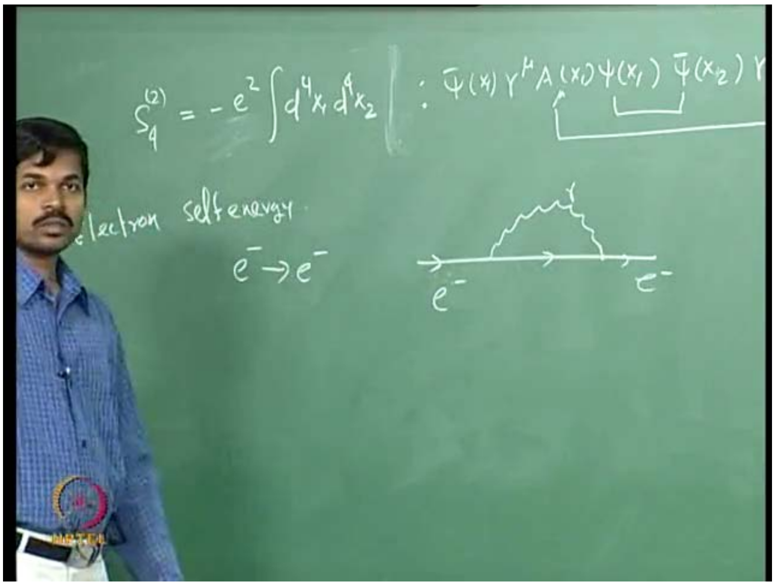

So, pictorially, this can be written as this electron. So, this term represents the electron self energy. This is what is known as, so an incoming electron can emit a virtual photon, which it can subsequently absorb and then result in an outgoing electron. The same thing might happen for a positron. So, this is what, this is the physical process that that is described by this term in the S matrix.

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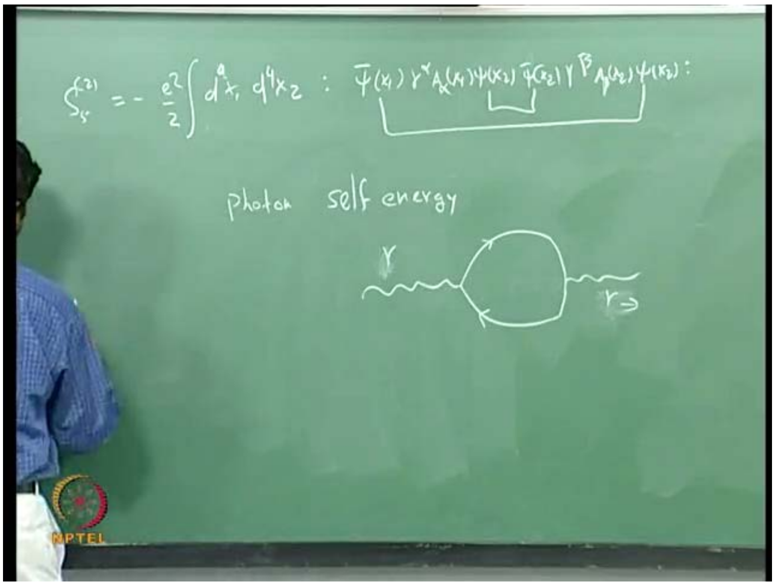

Finally, there is this term, which I denote is S 25, which is minus e square over 2 d square x d 4 x 1 d 4 x 2 and normal order product of psi bar of x 1 gamma alpha A alpha x 1 psi x 2 psi bar x 2 gamma beta A beta x 2 psi x 2, where all the fermions are contracted. So, therefore, there are two. So, there is no outgoing electron or positron at all in this process because all this psi and psi bar are contracted with each other, whereas there is one incoming photon and one outgoing photon.

So, this term represents the photon self energy and the corresponding Feynman diagram, which I can represent is there is an incoming photon. What can happen is that it can emit a virtual air of electron and positron, which subsequently can annihilate. So, there it produces an outgoing photon. So, this is what the Feynman diagram is. It will give a non minus in contribution for this term.

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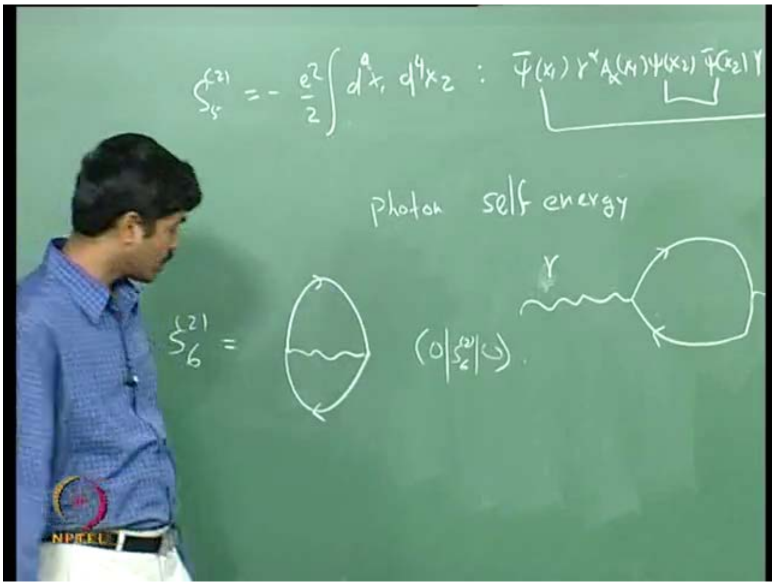

Ultimately, you have at the end; you have S26, where all the fields are contracted with each other. So, there is no outgoing particles or incoming particles at all. Therefore, this term will not give contribution to any physical process, where you study scattering or decay of particle or any such thing. It will just represent a term like this. So, all these are contracted and this is the corresponding Feynman diagram. It will only give a non zero contribution for this term. So, what we will do in the next lecture is we will look at these physical processes that we have discussed in today's lecture. Then we will explicitly compute the amplitude, then the corresponding cross section for all those processes in the next few lectures.