Condensed Matter Physics
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Lecture - 6
Physical Properties of Crystals

In the last few lectures, we have been discussing how the crystal structure of a solid, may be determined using scattering techniques or diffraction techniques. And this gives you the information about the position of atoms and molecules inside the crystal, having d1 that we are now in a position to go on to discussing the physical properties of crystals and the role of symmetry in determining these physical properties. So, this will be the subject of a discussion today.

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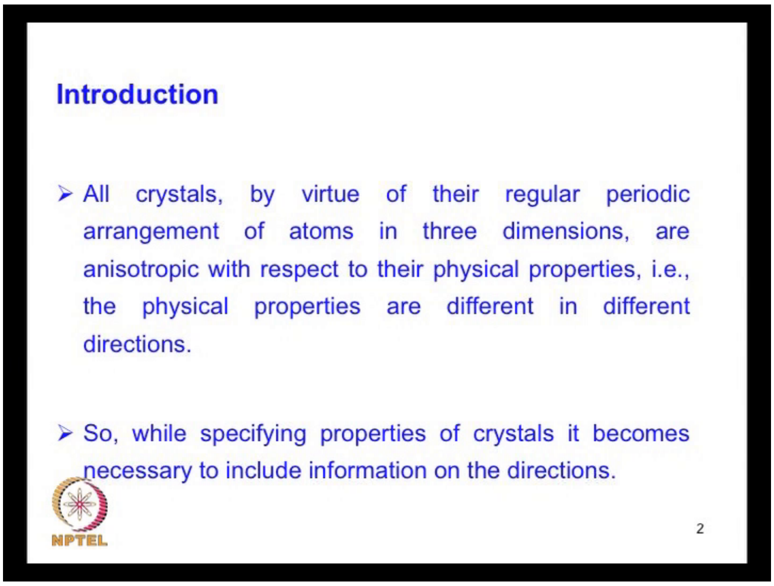

All crystals, by virtual the regular periodic arrangement of atoms in three dimensions, are anisotropic with respect to their physical properties, as we have already mentioned. This means that the physical properties are different in different directions. So, while specifying properties of crystals, it becomes necessary to include information on the directions. For instance, the electrical conductivity for crystal involves two directions.

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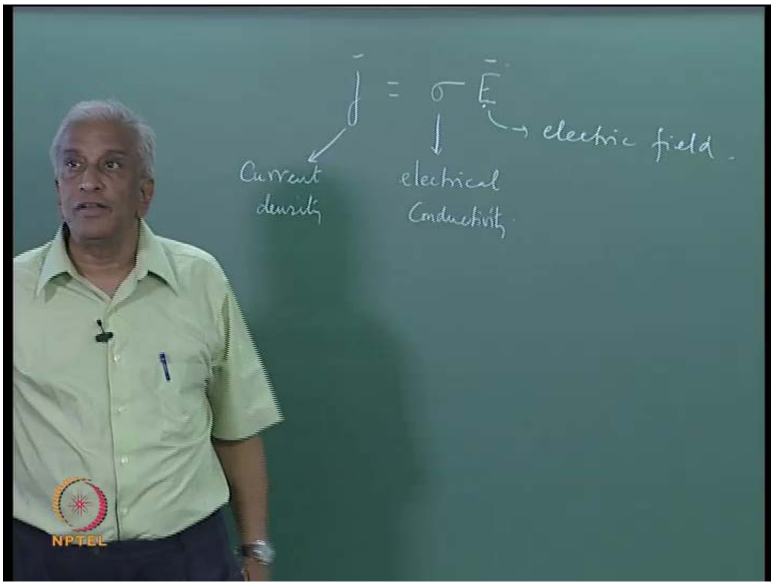

Because the electrical conductivity is given by the general relation j equals sigma e, where this is electrical connectivity which we want to determine, this is the electric field and this is the current density. So, the direction in which the electric field is applied as given by the vector E, and the direction in which the current density produce this measure as given by the vector j, these are the 2 directions. Both of which have to be specified in order to state, what is the value of the conductivity, and if these directions are changed, the value of the conductivity will be different. So, it is necessary to mention these 2 directions of the 2 vector quantities involved in the measurement while specifying the conductivity.

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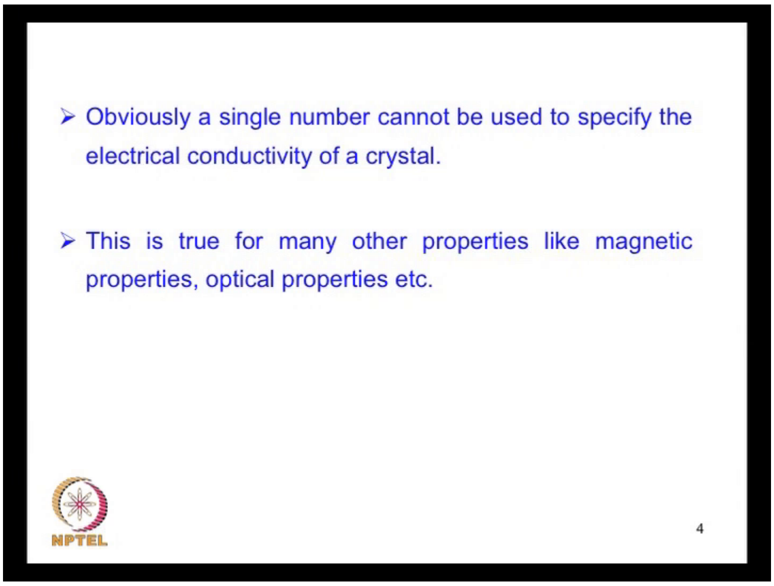

Obviously a single number cannot be used to specify the electrical conductivity of a crystal. This is actually true for many other properties like the the magnetic susceptibility, the refractive index etcetera.

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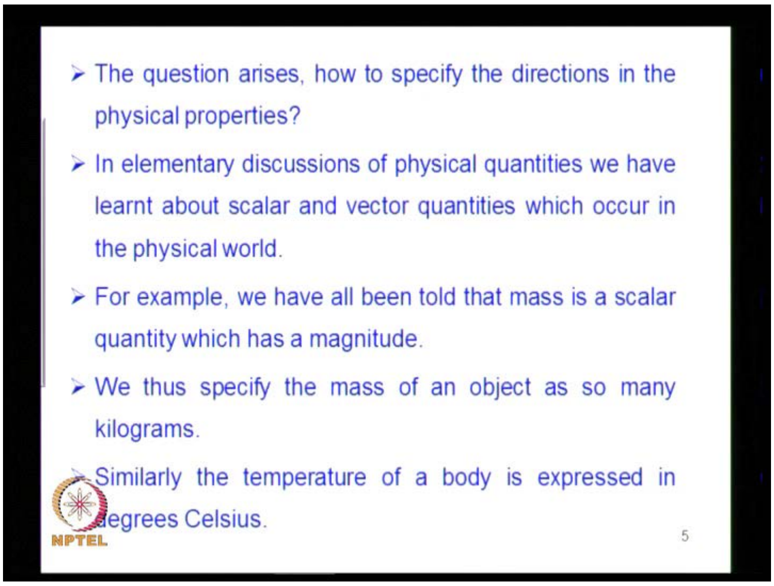

The question then arises, how to specify the directions regarding the physical property measurements and specifications. Now in elementary discussions about physical quantities, we have learnt about scalars and vector quantities, for example, we have all been told that the mass is a scalar.

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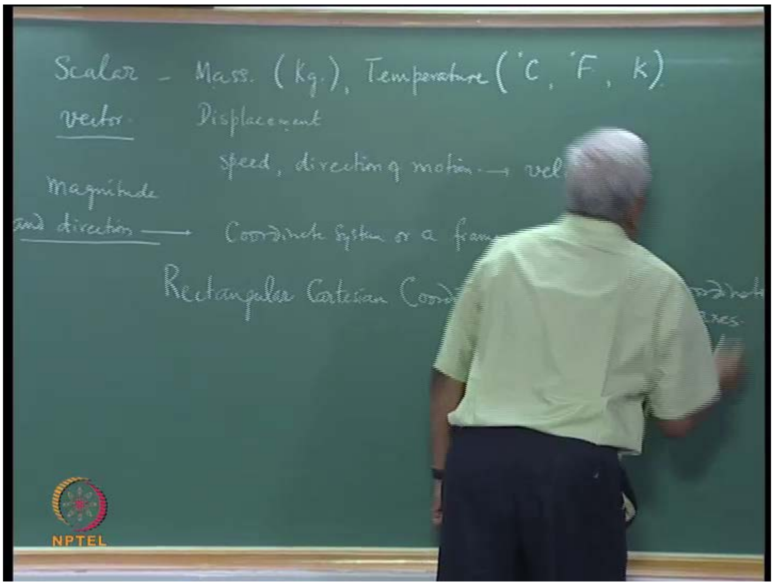

We specify the mass of an object as for example, so many kilograms. Another quantity which comes to the mind for a examples of a scalar quantity is the temperature. We specify the temperature of an object in degrees Celsius or degree Fahrenheit or degree Kelvin and so on.

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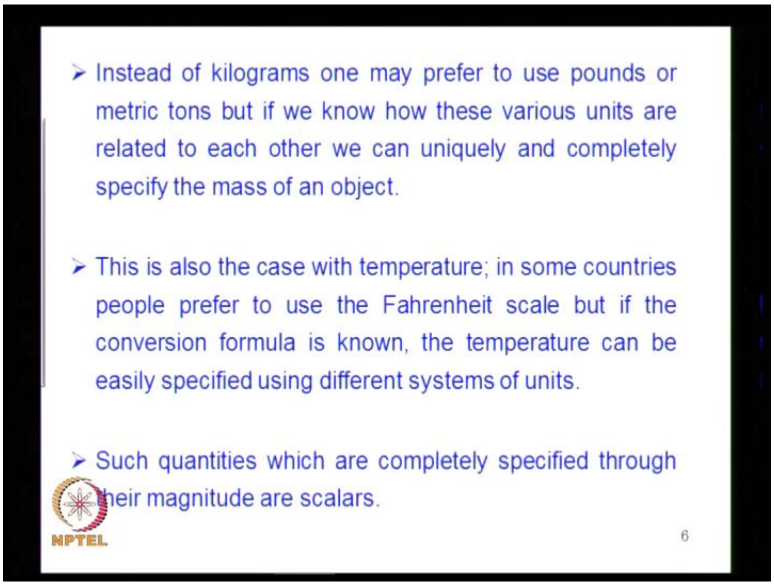

So, there are different units in which these quantities are specified. If instead of kilograms one prefers to use pounds or metric tons, but then we know how these various units are related to each other. So, we can uniquely and completely specifying the mass of an object regardless of which system of units we adopt. This is also the case with the temperature, we may specify the temperature in degree Celsius are in Fahrenheit scale are in the thermodynamic absolute Kelvin scale, but as long as we know how these different values are related to each other, there is no ambiguity regarding what a given object, what is the temperature of heat is. So, such quantities like the mass and the temperature are completely specified once their magnitudes are given, such quantities are example of scalar quantities.

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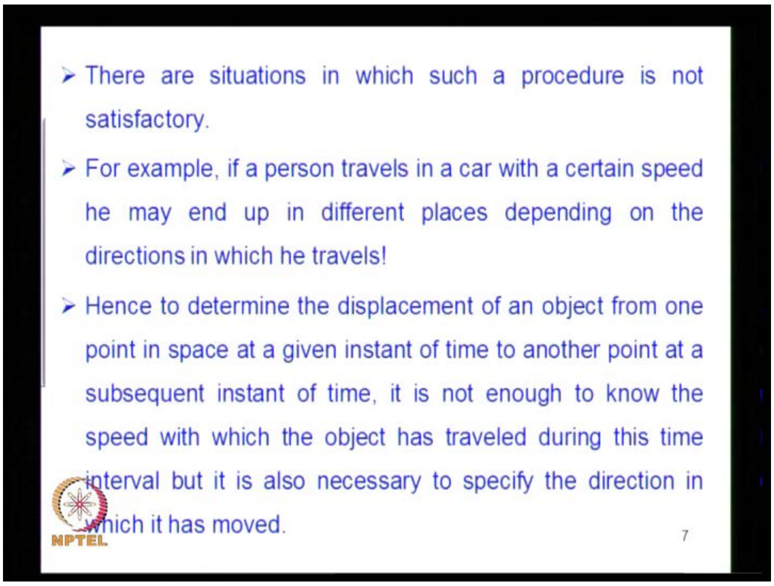

In contrast to this, there are other situations in which such a procedure will not be satisfactory. For example, suppose we think of a person who travels in a car with a certain speed, he will end up in different places depending on the direction in which he travels. So, in order to determine the displacement of an object from one point in space to at a given instant of time to another point in space at a subsequent instant of time, it is not just enough to know the speed with which the object as travel during this interval. But it is also necessary to specify in which direction it is moving, then only we will have the complete information required to determine the displacement.

The displacement in order to specify the displacement, it is not enough if we just specify the speed of the object, but also it is necessary to specify the direction of motion. We may specify this speed along with the information about the direction of motion, we refer to this as the velocity. So, the velocity as a magnitude given by this speed under direction such a quantity is known as the vector. So, in dynamic, we come across several such quantities such as the displacement, velocity, acceleration, force, linear momentum etcetera. So, these are all vector quantities which have not only a magnitude, but also a direction. So, this is something that we are all familiar with.

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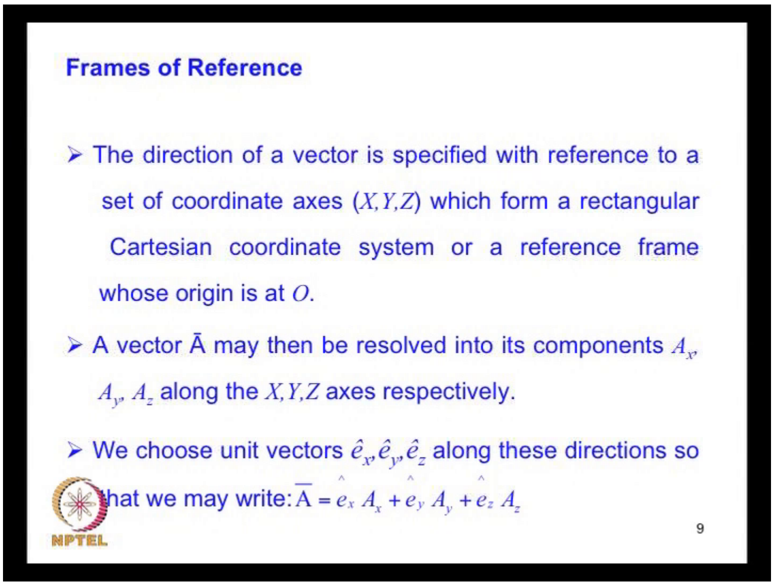

In order to proceed further, in order to be able to talk about quantities like the electrical conductivity, it is necessary for us to know, how to specify this direction with respect to a certain fixed coordinate system or a frame of reference. So, the direction yesterday specified with respect to such a coordinate system. Suppose, we have a rectangular Cartesian coordinates, which with we all are familiar which is specified by a certain origin and then X Y Z; and origin - O and X, Y, Z are the axis - coordinate axis.

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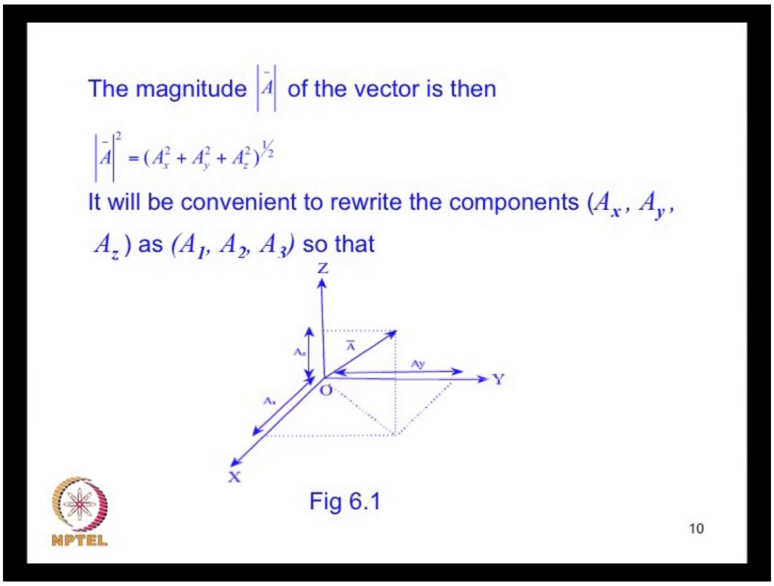

So, we have something like this, we have this given in figure four 1 you have O X, O Y, O Z are orthogonal directions along the three mutually perpendicular standard directions. In three-dimensional base and a specify the coordinates of an object by specifying a vector for example can then a vector a maybe specify by giving its components A x, A y, A z along these three coordinate axis.

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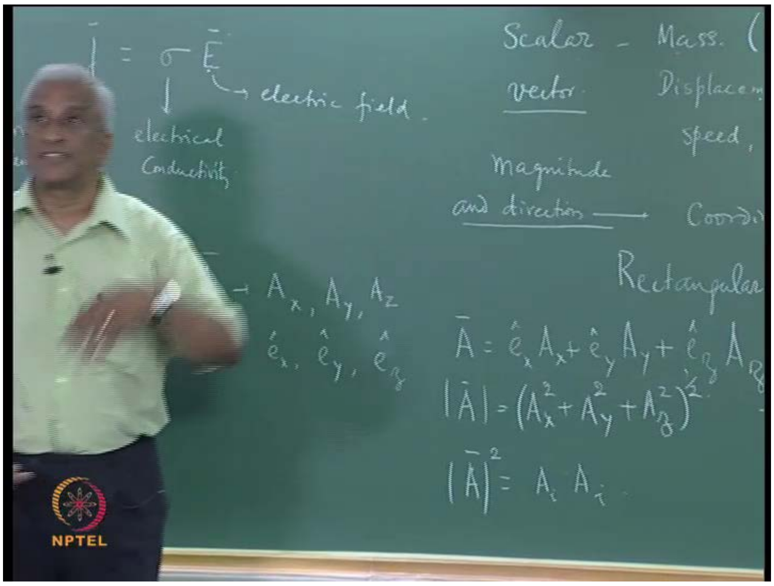

So, that and the unit vector along these are given as e x, e y, e z with respect to e then the vector a may be written as e x A x e y A y plus e z A z. It is fairly well-known, it straightforward to know that the magnitude of this vector is just the squire root of the summer squires of this components a x squire a y squire a z squire half the magnitude. So, in general, far more general representations, it is more convenient to drop these x y and z and replaced them by a 1 a 1 a 3 by which we understand that a symbol subscripts 1 2 three stands for the x y and z components. So, that I can write the same relation as a 1 squires plus a two squire plus a three squire to the power half and this 1 we can write this relation as e 1 A 1 plus e 2 A 2 plus e 3 A 3. The advantages of this rotation will become obvious when we introduced.

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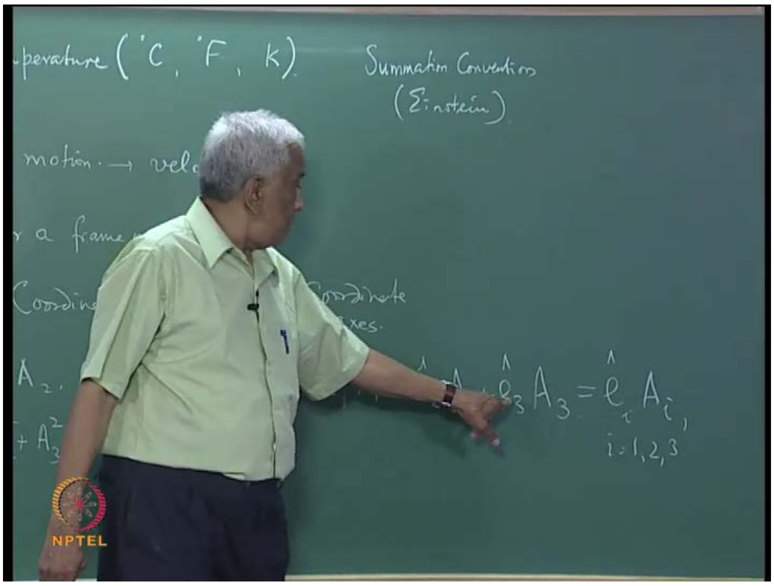

What is known as the summation conventional, which was introduced by Einstein according to this convention whenever a particular subscripts is repeated it implies a summation over that subscript. For example, we can write this using the summation can mention has e i A i, there i is 1, 2 three since i is repeated this means that there is a summation. So, e 1 A 1 plus e 2 A 2 plus e 3 A 3; i range over 1, 2, 3. And similarly, the magnitude square is a i squire or a i i again, since i is repeated this means that there is a summation over this induces. So, I have A 1, a 1 which is A 1 square plus A 2. A 2 which is A 2 square plus A 3. A 3 which is A 3 square giving us the same expression, but we have arrived at a much more compact notation in both cases.

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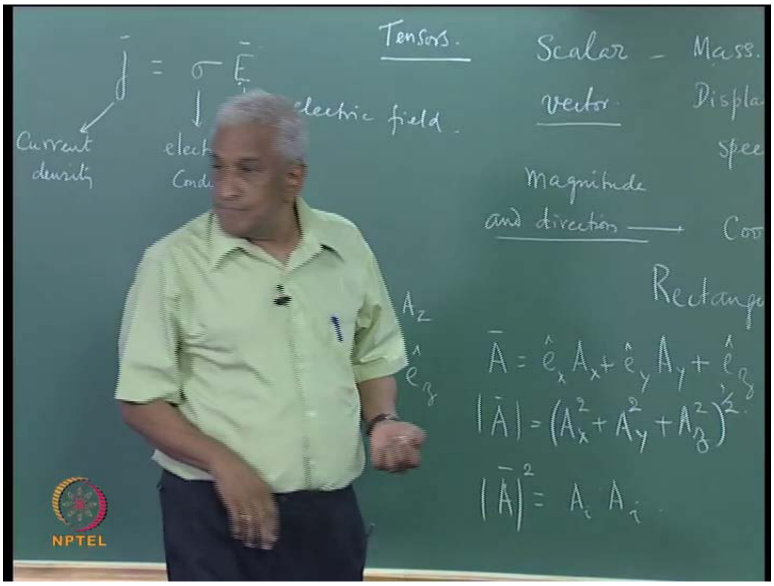

So, this rotation enables us generalize this considerations to more complex physical quantities which are in general known s tensors in order to know what these stencils means letters go back to these two vector quantities which are related to each other just to make life more interesting.

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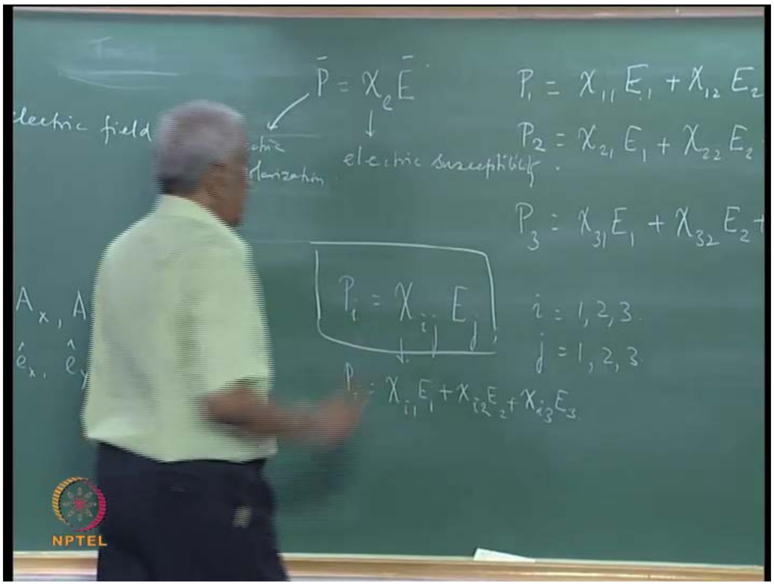

Let us take a different a similar, but different relation which relates the electric polarization p which is a vector to the applied electric field e and the relation these 2 quantities polarization and the electric field are related by a so called electric susceptibility. This is the electric polarization this is very similar to this relation, but I am just for interest sake I am taking a different example. So, in general p and e which are vector will not have the same direction therefore, it is necessary to specify their directions.

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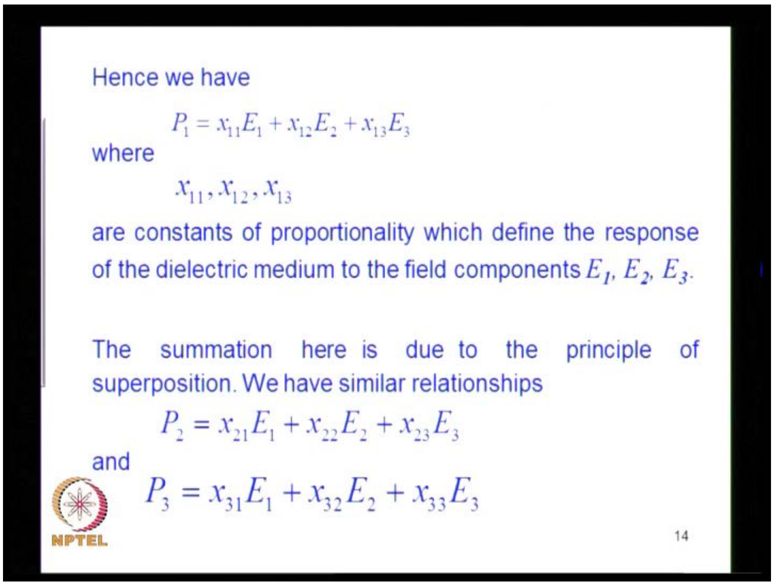

So, then I will have P 1 equals x i E 1 let me write this relation then explain. So, the electric field is applied in a general election such that it as component E 1, E 2 and E 3 along the three coordinate axis in a chosen reference frame. So, the component p 1 of the polarization vector which is along the x direction is given is due to not only E 1, but also E 2 and E 3. Therefore, we have to write this general relationship between p and e in terms of a sum, which includes the contribution due to the polarization produced by the component E 1 and the contribution from E 2 and the contribution from e three using the principle of superposition. So, the contribution sum E 1 to 1 is known as the component x i 1 1 and the contribution which means that this is the contribution due to the electric field company and E 1 for the polarization component P 1.

Similarly the contribution from the component E 2 the polarization component p 1 is x i 1 two. So, there are two subscripts now for each of these quantities 1 subscript indicating which component of the polarization it contributes to and the other component subscript indicating due to which company or the electric field it arises. Similarly, I will have other relations like p two x i 2 1 E 1 x i 2 2 E 2 plus x i 2 3 E 3 and P 3 is x i 3 1 E 1 plus x i 3

2 E 2 x i 3 3 E 3. These are the that gives you all the three components of the polarization vector which arrives from contribution from three components E 1 E 2 e three of the electric field. So, this vector quantities x i 1 1, x i 1 2, x i 1 2, x i 2 1, x i 2 2, x i 2 3, x i 3 1, x i 3 2, x i 3 3. This entire set of quantities together determine the total response to the electric field the polarization produced by the electric field this looks very complicated, but I can use summation convention and write this as a very compact relationship in this way where i runs over 1 2, 3; j also runs over 1, 2 3. So, i use since i have x i i j e j is subscript j there is a reputation. So, there is a summation over a subscript j. So, this really mean as i substitute this is going to give me this is really the following p i equals x i i 1 1 plus x i i 2 E 2 plus x i three e three and i f i give the values i for i from 1 2 3. I will get back all these three relationships. So, this compact relationship is really a very compact way to represent this complexion relationship.

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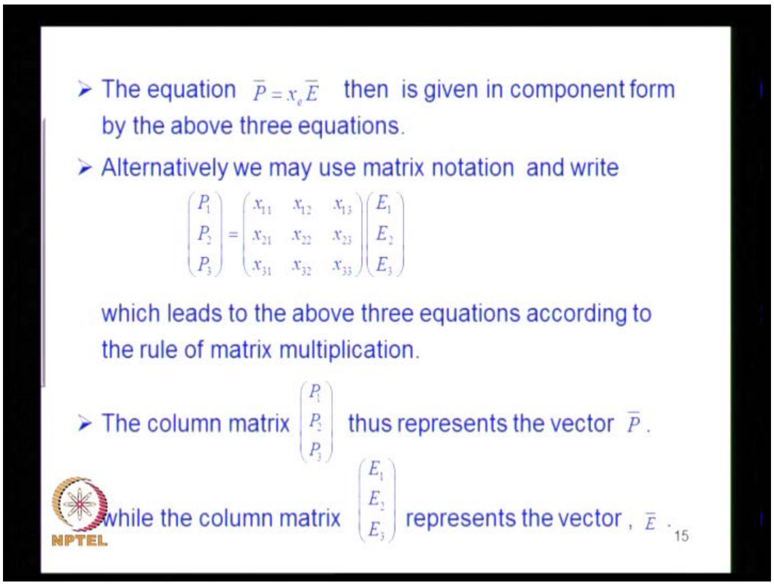

And the entire array of x i together all the nine questions they form the component s of a tensor known as the electric susceptibility tensor similarly the electrical conductivity is also a tensor we could use for example, instead of the algebraic notation

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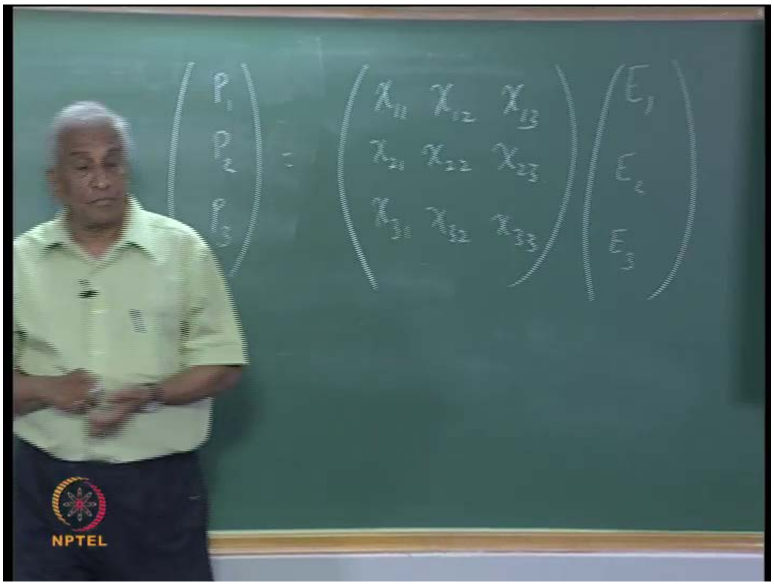

This immediately suggests the possibility to write this in matrix form using the method of matrix multiplication we get back the same relationship. So, the column matrix P 1, P 2, P 3 that gives you the component of column vector p while the column matrix here E 1, E 2, E 3 gives us the components of vector E. And this three by three matrix consisting of the array of numbers x i 1 1 etcetera up to x i 3 3 that gives you a three by three matrix which represents the electric susceptibility tensor.

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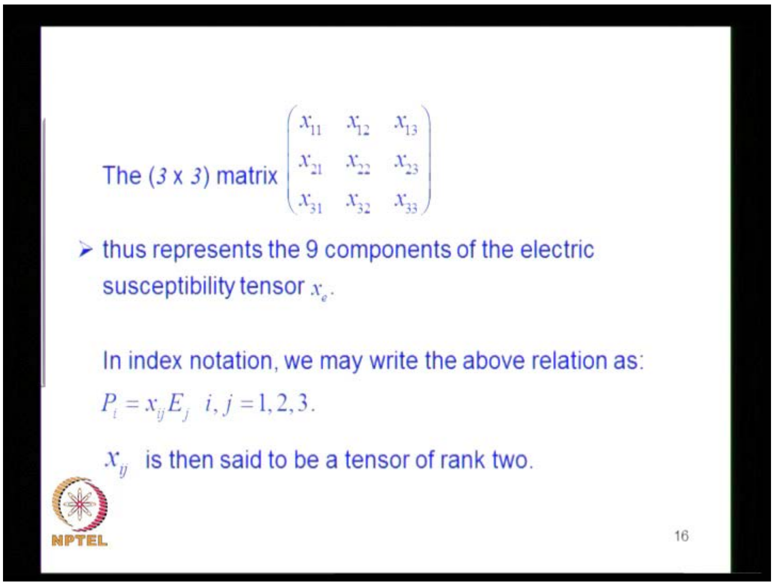

So, that is the general way to represent quantities of this kind. So, you can see already it is not just a tensor it is a tensor of rank two a vector is a tensor of rank 1 a scalar is a tensor of rank zero.

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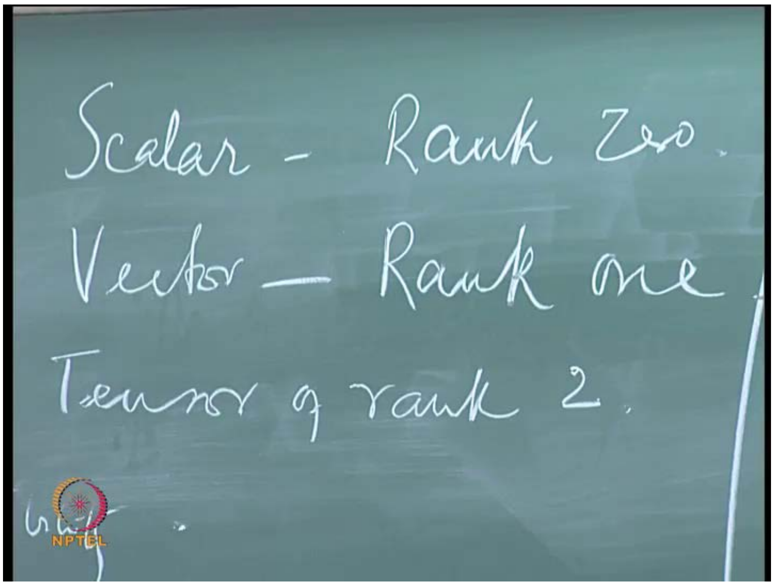

What does it mean, this means in three-dimensional space. If you raise this to the power 3 to the power zero which is 1 scalar as just one component. It is one quantity, if you specify the magnitude of the tensor scalar you have everything about this scalar whereas, this one has three to the power 1 which is pretty component a vector quantity as the components under tensor of rank two three squire nine components has shown here for example. So, it is clear that we can have depending on what are the quantities involved in this relationship you can even have tensor of rank three rank for and so on higher order tensors.

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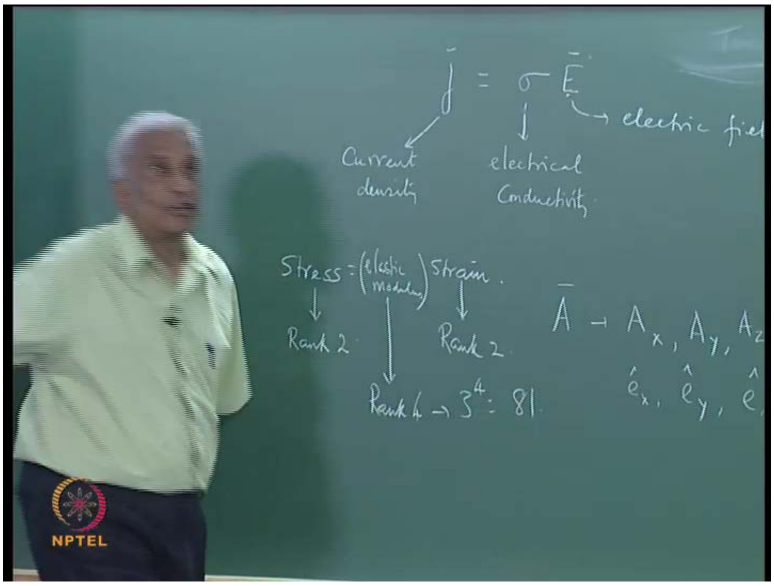

For example, elastic modulus elastic modulus every1 know s is given by a relation connecting this a stress of the strain. So, the relation connecting the stress is a tensor of rank two a strain is also a tensor of rank two. So, the elastic modulus is a tensor of rank four which means that it as three to the power four in general equal to eighty 1 compounds

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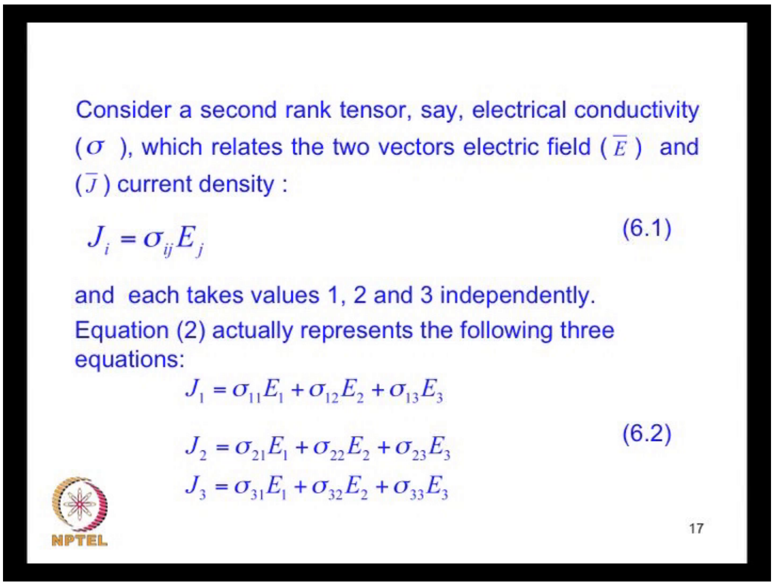

Let us get back to the conductivity tensor specified by this relationship and we know that if we have these conductivity density and then suppose i apply the electric field along a particular coordinate axis. So, then only that component will be non-zero for example, E 1 is non-zero, but E 2 and e three are zero in which case we will simply have J 1, J 2, J 3. So, this will have only three components which are non-zero, but if the electric field is applied along in orbitary direction then all the nine components will be nonzero these nine components are written for example, likewise in the more compact notation J i equal to sigma i J e j. Where i and j run over 1 2 and three the summation convention enables us to skip the summation symbol.

As I already told you, it is very important to specify the directions. These directions are specified with respect to a fixed the reference frame or coordinate system, but very often in real life we may frequently go from one type of coordinate system to another. For example, we may want to know how a given object is moving inside an elevator as compared to the motion relative to the fixed para firma. So, we would like to know how these two are related the elevator provides one set of coordinates while the fixed there is outside gives you another coordinate system or a reference frame and it is very necessary.

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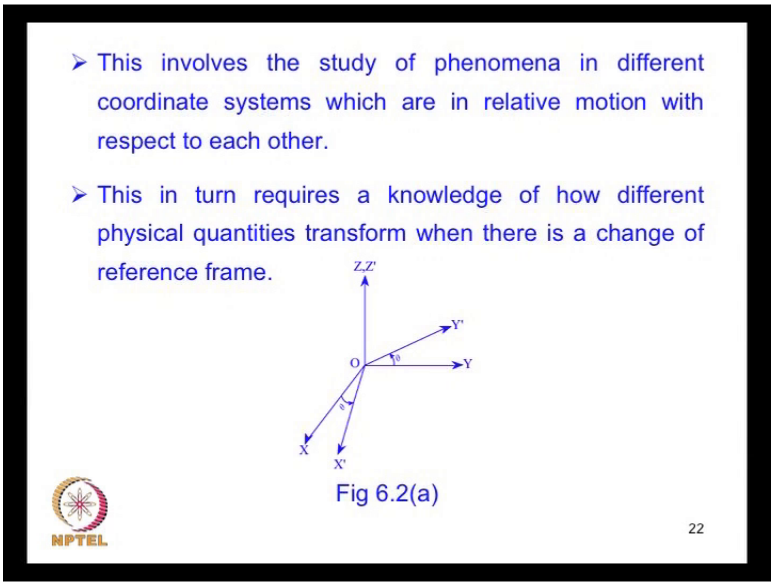

Fig 6.2(a)

For us to know how the different quantities are transformed when we go from one set of coordinates to another this require a knowledge of how different physical quantities transform when there is exchange in the reference frame in this figure shows a change a simple change effected by a simple rotation about this z axis by an amount theta.

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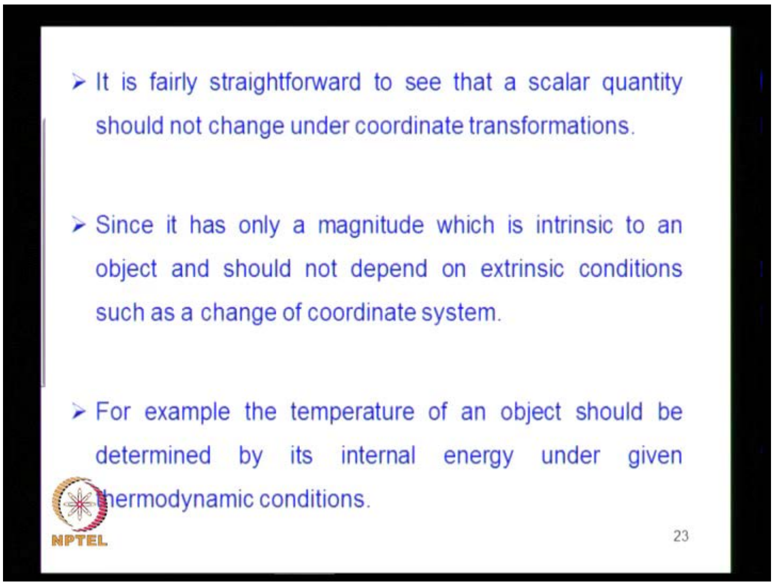

So, it is fairly straightforward to see that if you have way scalar quantity since it is only a magnitude the magnitude should not change under coordinate transformation. So, a scalar quantity is one which remains invariant under coordinate transformations.

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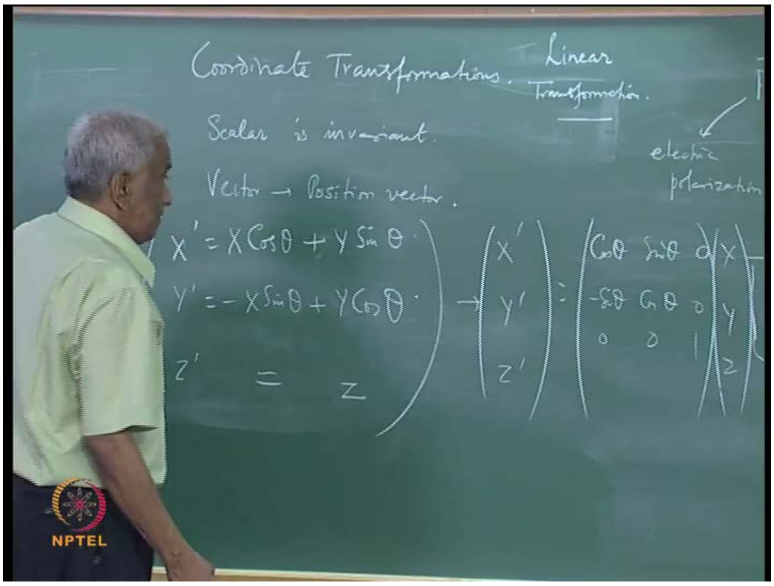

So, we are talking about coordinate transformation now and we are seen that a scalar quantity easy invariant with respect to a changing the reference frame. For example, the moss of an object or the temperature of an object are not determined by what coordinate system we use the specified this.

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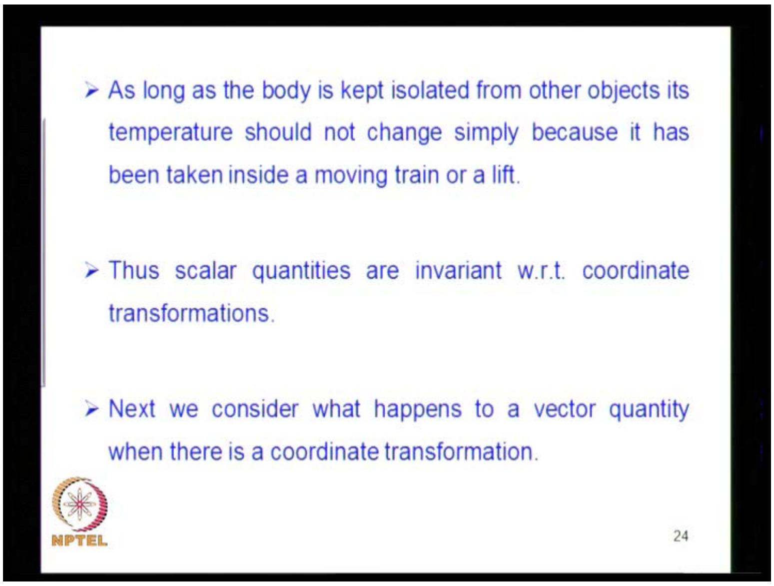

So, that is what we mean by saying that the scalar quantity easy invariant under coordinate transformations, but this is not a case for vector and as the tensor quantities a vector changes in magnitude.

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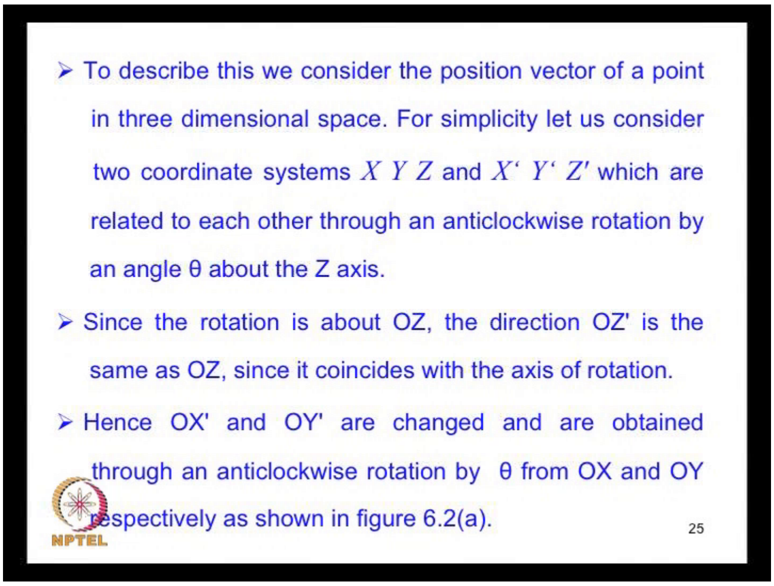

And direction for example, one of the simplest examples is the position vector.

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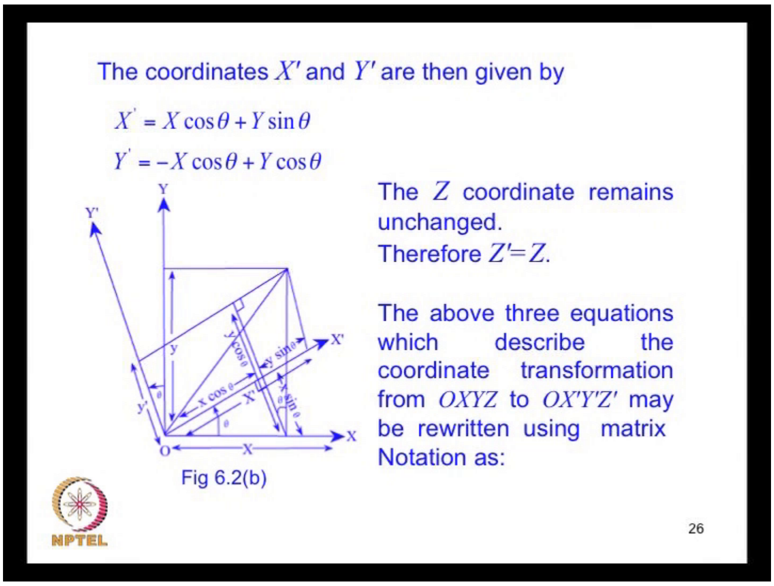

So, be sure the position vector under a coordinate transformation and we specify the position vector in terms of its components x prime y prime and x y in the 2 reference frames which are located by a rotation about this z axis since the rotation is about the z axis the z coordinate remains the same. So, we do not specify the z coordinate it is this same. So, we can simply write z prime equal to z whereas, the x prime and y prime are; obviously, related by things like this equation of this form these are the transformation equations which described it transformation and components x prime y prime and z prime under a coordinate transformation namely rotation by an angle theta about the z axis. So, this can also be straightaway written in matrix form.

So, we can use this basic idea of a coordinate transformation from a basis x, y, z to another basis x prime y prime z prime by a general rotation of what is known as a linear transformation. So, these equation this equation describe such a linear transformation which can in general be written even in a more general form as a 1 1, a 1 2, a 1 3, a 2 1, a 2 2, a 2 3, a 3 1, a 3 2, a 3 3; x, y, z. Where the a i j constitute an array of nine number which specify the direction cosigns of the new coordinate axis with respect to old coordinate a x is or in our compact.

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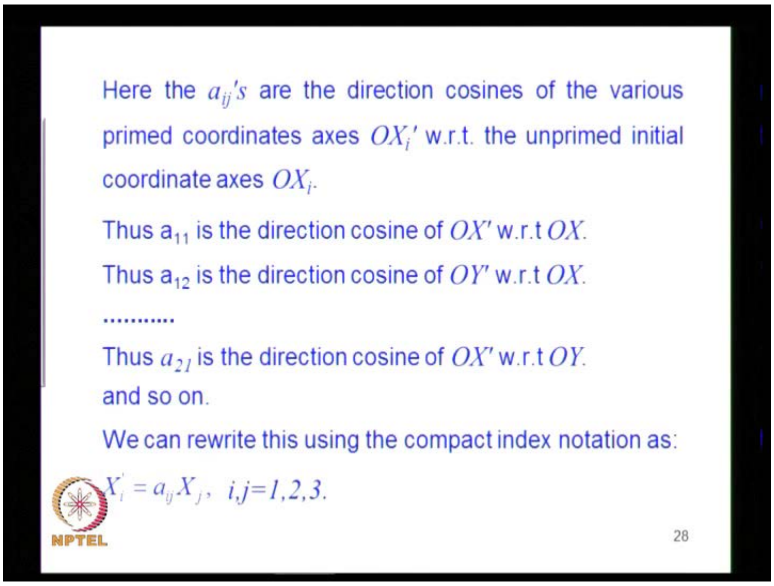

In summation rotation appear subscript notation i can write a i j x j.

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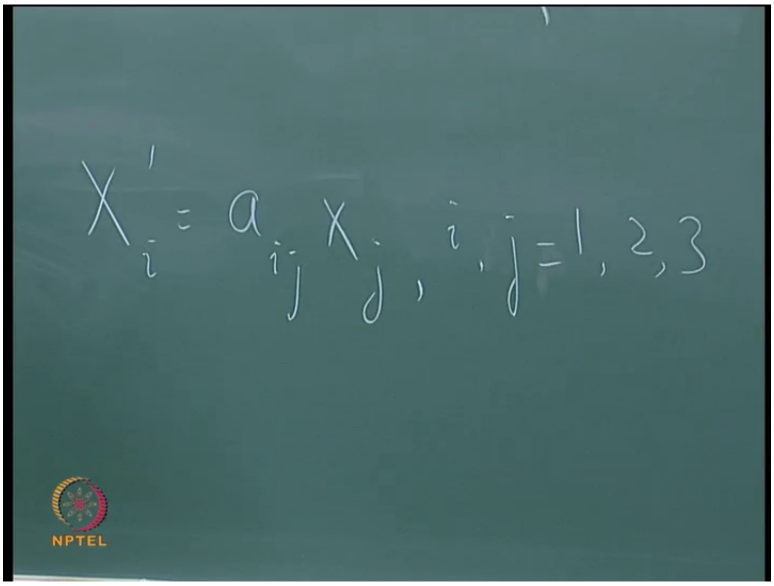

That is a linear transformation from the coordinate bases x j to the bases x i prime where the e a i j is constitute coefficient of the transformation which give you the connection between the x j and x i prime. Again the repeated summation reputation of subscript j implies summation as can be seen here from this from this one. So, this can be used even though we are written it for the components of the position vector this can be straightaway taken over the specified the components of any arbitrary vector or tensor of rank one.

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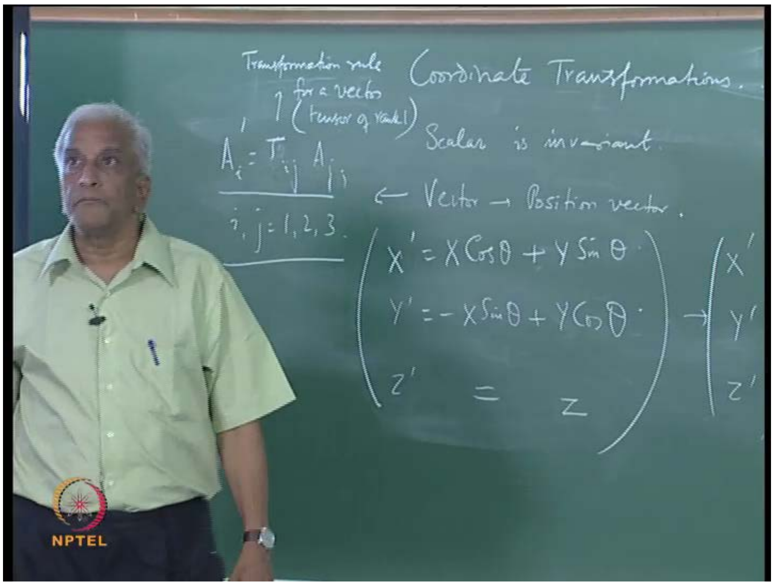

And we can say that the components of any vector A i prime T i j to keep the distinction between the direction cosigns of a position vector, but these are these are the components these are the coefficients which relate the components a i j and a i prime involved in the transformation. So, this defines the transformation role for a vector a components of a vector which is a tensor of rank 1 . So, now, we can generalize to the case of the transformation role governing the any higher order tensor. So, we can here itself for example, t i j as we already saw since it connects the 2 vector quantities k i j should be a second rank tensor. So, we can now generalize and find out what happens to the component t i j of this second rank tensor under a linear transformation of this kind using the same principles we will do this in the next time.