Condensed Matter Physics
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Lecture - 40
Quantum Liquids and Quantum Solids – Worked Examples

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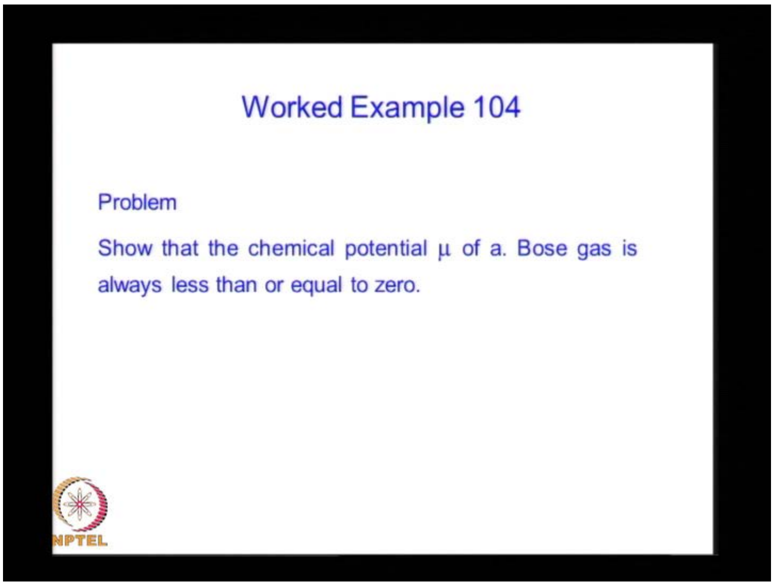

Today, we will work out some examples on the topic of quantum liquids and quantum solids. The first example is one in which we are asked to show that the chemical potential typical symbol is mu of a Bose gas is always less than or equal to zero.

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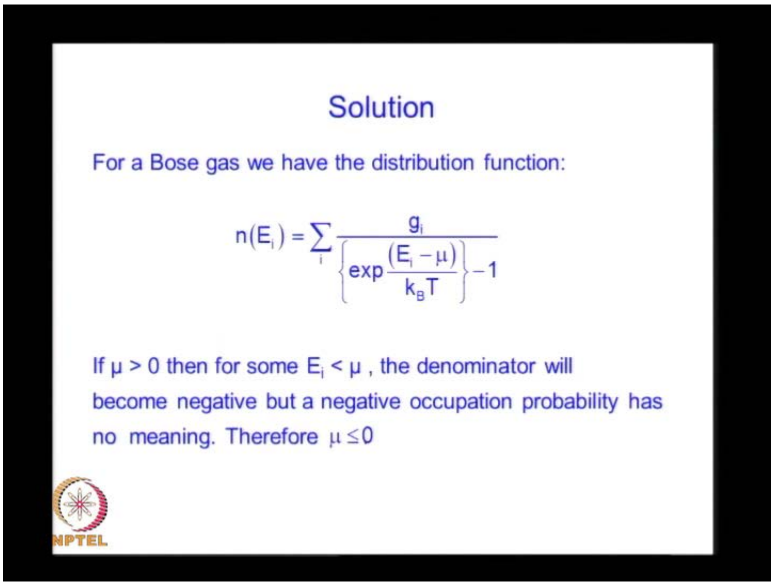

We know that for a Bose gas the distribution function has the typical form, that is the typical form of the distribution function. Suppose mu is greater than zero then what happens, we see that if E i spontaneous state energy level is such that E i is less than mu. If E i is less than mu then n of E i will be sigma i g i by exponential. Now E i is less than mu, therefore this is positive, therefore this is less, so this will be a negative quantity by K B T minus one. And therefore, this is less than one, therefore denominator of this expression is negative, that means n of E i is negative. But n of E i is the occupation probability, so occupation probability cannot be negative, cannot be less zero. Therefore, our assumptions that mu greater than zero is not valid, that is that is regarding the value of the chemical potential for a Bose gas, which should either be negative or utmost equal to zero.

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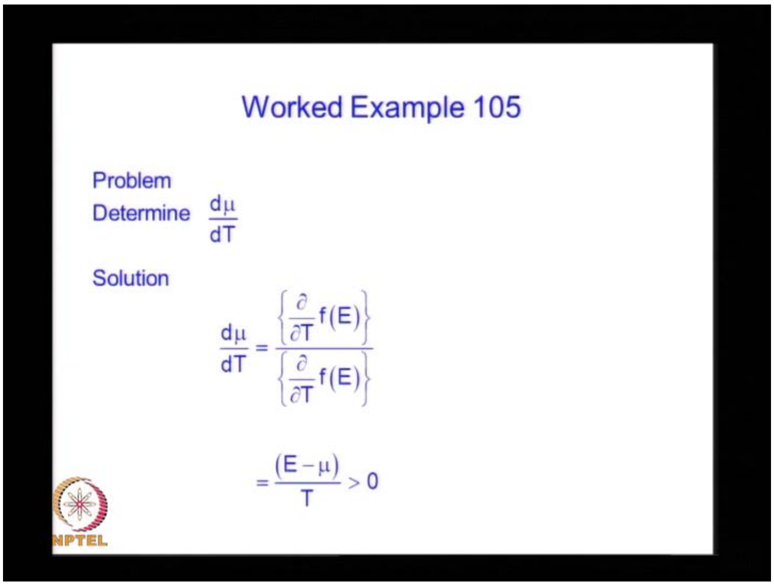

Next, we are ask to determine the temperature dependence of the chemical potential; in other words, the derivative with respect to temperature of mu. In order to do this, we note that we can write this as dou by dou t of f of e divided by… Now the dou by dou t of f of e is sigma i g i then exponential e i minus mu by K b T minus 1 whole square times d by d t of this, so minus one by K b T square of E i minus mu that is (( )) derivative. Whereas, if you differentiate f of e with respect to mu then that will be g i again exponential E i minus mu by K b T minus 1 whole square. Then I have to differentiate with respect to mu, so that gives me again exponential times minus one by K b T. So that will be the values of this derivative, so the ratio of these two which is d mu by d t is this divided by this, which will give me e minus mu by T. And since mu is always less than or equal to zero, this will become positive definite and E is always zero or positive – greater than zero. Therefore, this will be greater than zero. So, that is see how we determine the temperature derivative of the chemical potential.

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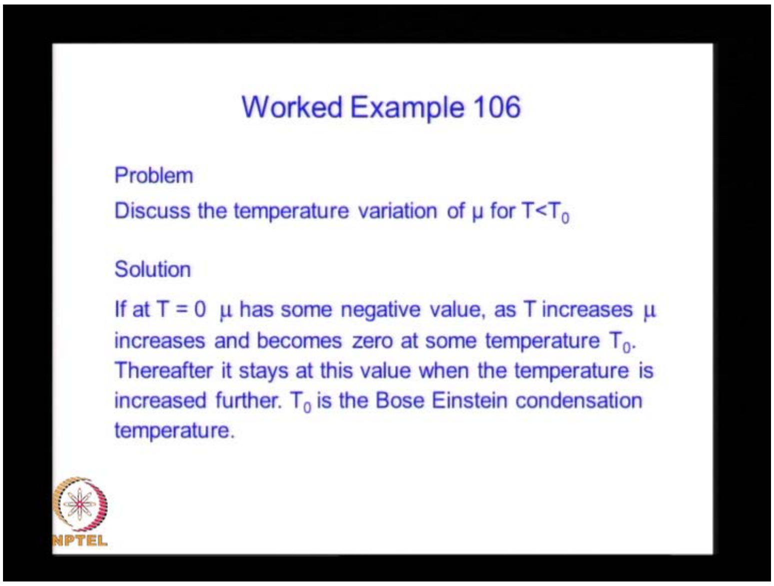

In the next example, we are ask to discuss what happens discuss the temperature variation of mu for zero less than T less than T 0, where T 0 is so called Bose Einstein condensation temperature, which we discussed in the lecture. So, let us say at T equal to 0 k, suppose mu has some negative value then we also know that d mu by d t is positive; that means, as temperature increases mu increases, that is mu increases with temperature. So, as one changes increases the temperature from absolute zero as T increases mu becomes it also increases; that means, it is takes negative values approaching 0. So, eventually mu becomes equal to zero this is what happens at T equal T naught. This is how we defined T naught. Once it reaches T equal to T naught, since d mu by dt is positive so that means, mu as to stay constant at this value for at higher temperatures that is the temperature variation of mu.

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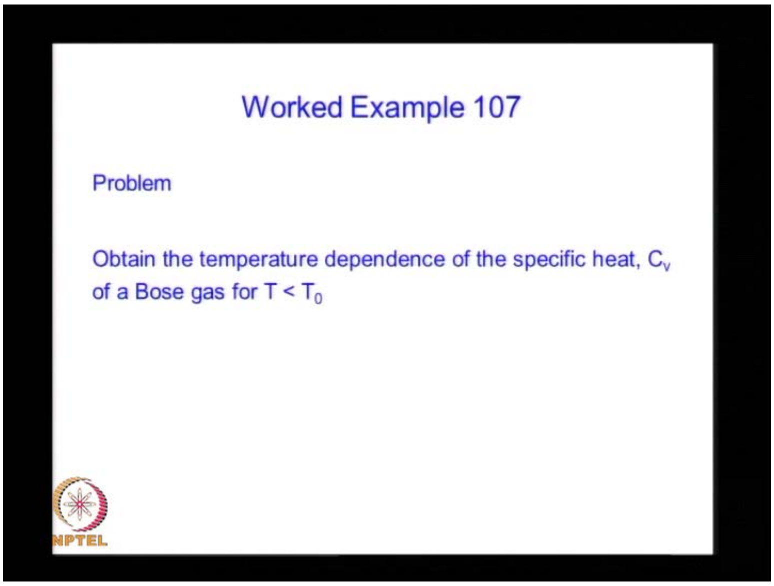

Having got the temperature variation of the chemical potential. We are asked to discuss the temperature variation dependence of the specific heat C v for a Bose gas T less than T naught and also T greater than T naught.

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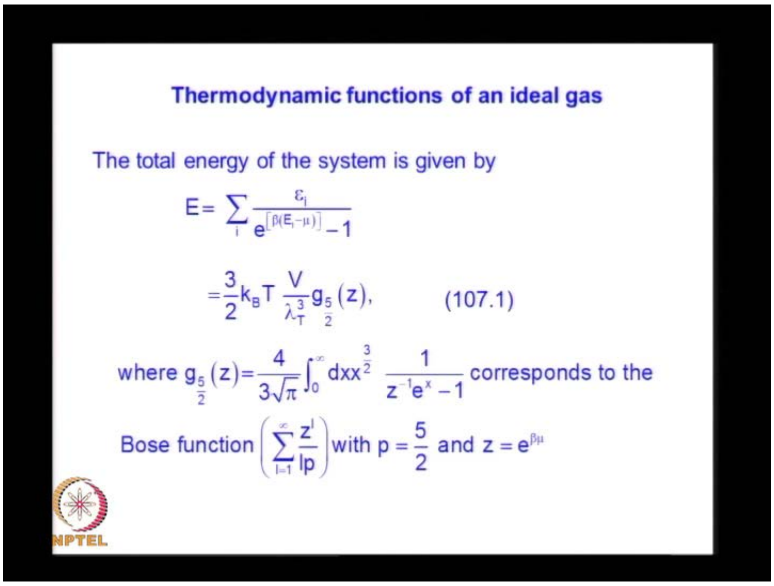

We know that the specific heat is the temperature derivative of the internal energy. Therefore, we first determine the internal energy and then differentiate the expression with respect to temperature to obtain the specific heat. The energy is given by 3 by 2 k B T V by lambda cube is an expression that we discussed in the lecture g 5 by 2 of z.

Where g 5 by 2 of z is the following expression 4 by 3 root pi integral zero to infinity d x x to the power 3 by 2 into 1 by z to the power minus 1 exponential x minus one. This is the Bose function, which we discussed in terms of the Riemann-Zeeman zeta function in the lecture and has the value sigma z to the power l by l to the power p; p equal to 5 by 2 and the summation is over l and z is e to the power beta mu.

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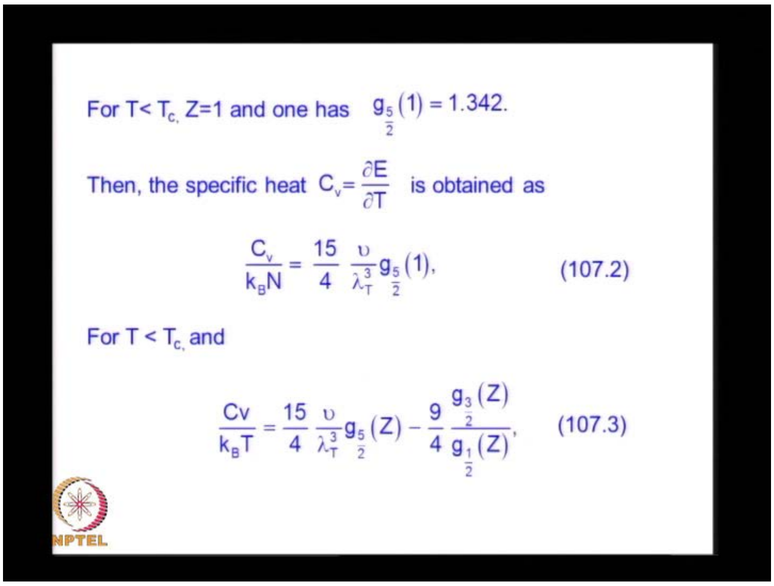

So, let us first discuss what happens for the temperature range T greater than T naught; here z equals 1, since mu is zero. Therefore, g 5 by 2 of 1 has the value 1.342. Therefore, the specific heat, hence we can substitute this here and the specific heat which is de by dt is such that this is N k B into 15 by 4 V by lambda cube into g 5 by 2 of 1 for k less than T naught. In the same way, we can get c v equals N k B into 15 by 4 V by lambda cube into g 5 by 2 of z minus 9 by 4 g 3 by 2 of z divided by g half z, this is got by simple differentiation of the expression for the average energy total energy.

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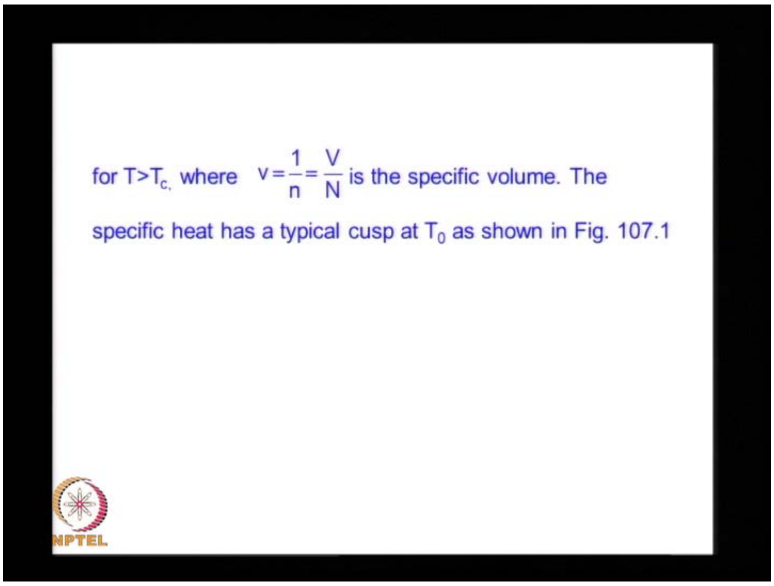

So, this is how we obtain the specific heat and well we can write small v in both cases where v is V by N - the specific volume.

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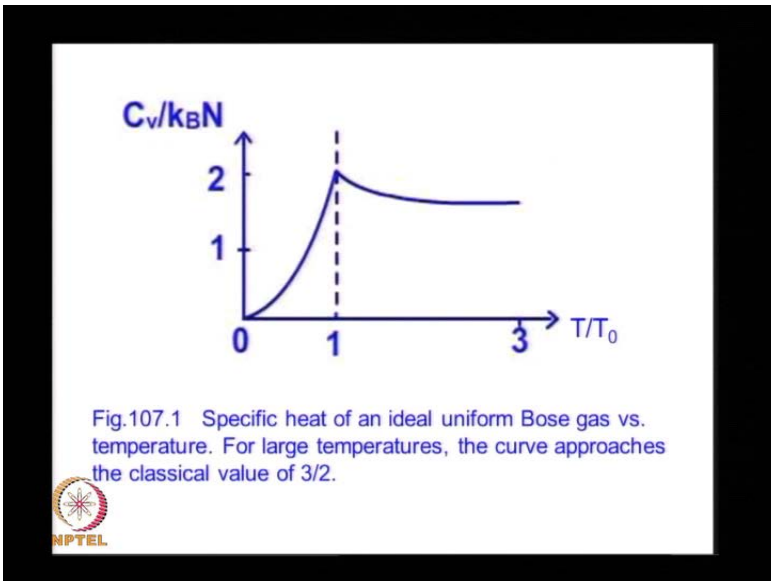

Fig.107.1 Specific heat of an ideal uniform Bose gas vs. temperature. For large temperatures, the curve approaches the classical value of 3/2.

So, the variation of the specific heat is shown in figure and looks like this. So, let us plot it as C v by N k B and T-by-T naught. So, this has when T equal to T naught, it has the value one. So, this goes somewhat like this and then comes down. So, this has a cusp at T equal to T naught, this is a theoretical variation of the specific heat of an ideal Bose gas, whereas the observed specific heat variation at the lambda point of liquid helium two is more like this is more or less right, but this goes like this. So, that is more like the observed variation of liquid helium two for around T lambda.

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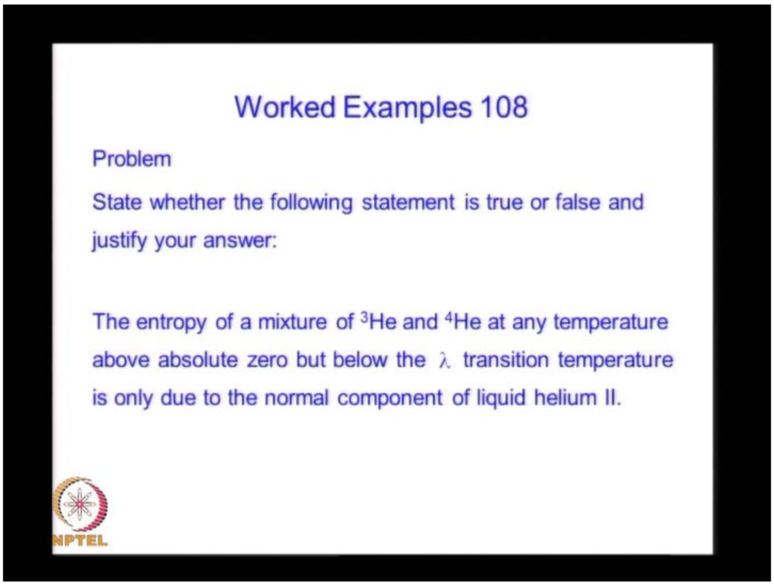

Next example is in the form of a true or false question for a given statement. We are ask to state whether the given statement is true or false and justify the answer. What is the statement? The entropy of a helium 3, helium 4 mixture at T greater than zero k, but plus you can write it as for zero less than T less than T lambda. This entropy is only the entropy of the normal component, normal component of liquid helium 4. This is the statement.

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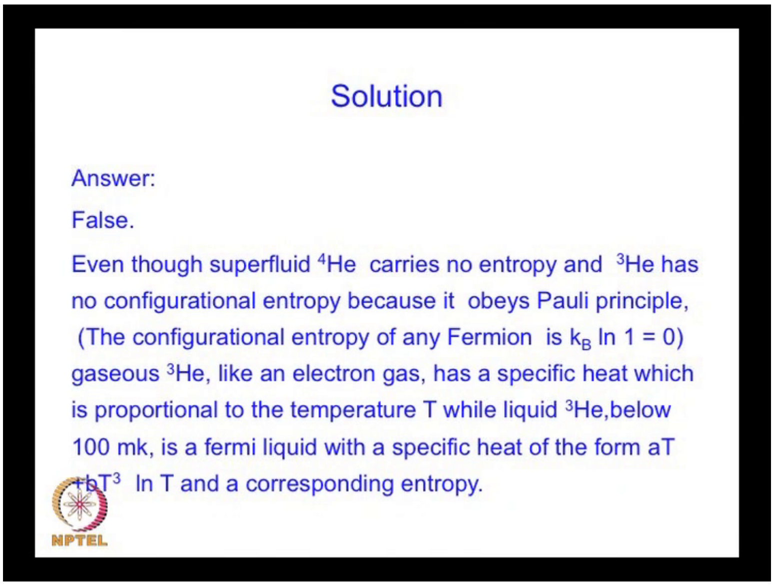

Now, the answer is that the statement is true. Now we have to justify, the justification is we know that in the two fluid model according to liquid helium 4 is a mixture of a super fluid and a normal component we discussed this is rho s plus rho n. Now the super fluid carries no entropy, we saw this when discussing the thermo mechanical effect. Now coming to helium 3, it is a Fermion, because of the odd number of nucleons three. Therefore, for a Fermion, it obeys Pauli-exclusions principle according to which each state has is either occupied or unoccupied.

So, the occupation probability is one or zero. Therefore for the occupied states, the entropy of helium 3 is given by the Boltzmann formula k B log 1 which of course, is zero. Therefore, the entropy of the helium component helium 3 is zero, and the entropy of the super fluid fractions of the helium four is also 0. So, the entropy which is of this mixture is only due to this normal component of liquid helium 4 that is the answer.