Condensed Matter Physics
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Lecture - 40
Quantum Fluids and Quantum Solids

During our discussion of the cohesion in molecular solids, such as the inner gas solids solid organ, crypton, xenon, neon, etcetera. I made a mention about helium and remarked that even though helium is also air inner gas, but the behavior or solid helium is different substantially different from that of the other rare gas solids. So, today we will take this up for detailed discussion.

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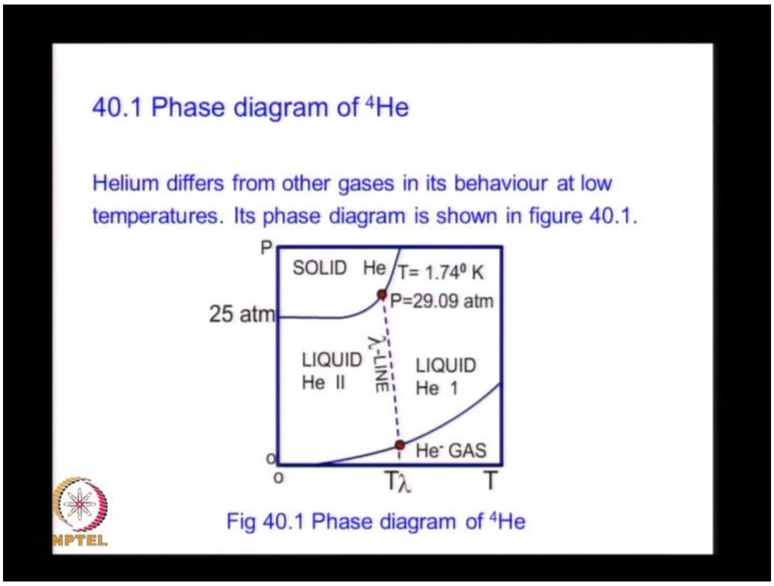

Now, this discussion is going to be in general on quantum fluids and quantum solids, and we will see that helium is a very important prototype or the behavior of quantum fluids and quantum solids. Helium defers from the other gases especially in its behavior at low temperatures, it phase diagram is shown in figure 41 1 can readily see that helium does not fully defined at any temperature down to absolute 0 under ambient pressure.

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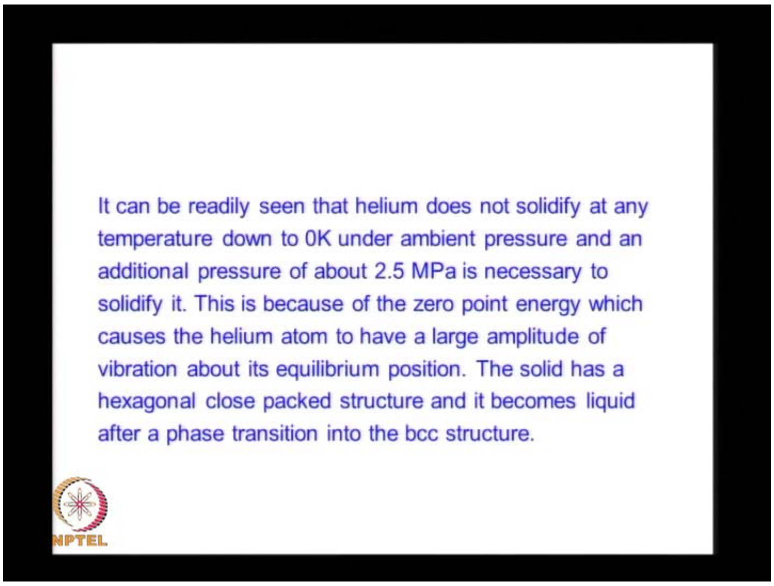

And additional pressure of about 2.5 mega pascals, which is 25 atmospheres, is necessary to solidify it this is because of the 0 point energy, which causes the helium atom to have a large amplitude of vibration about its equilibrium position in the solid the solid as a hexagonal close packed structure. And it becomes liquid after a phase transition from the x c p into the bcc or body centered cubic structure.

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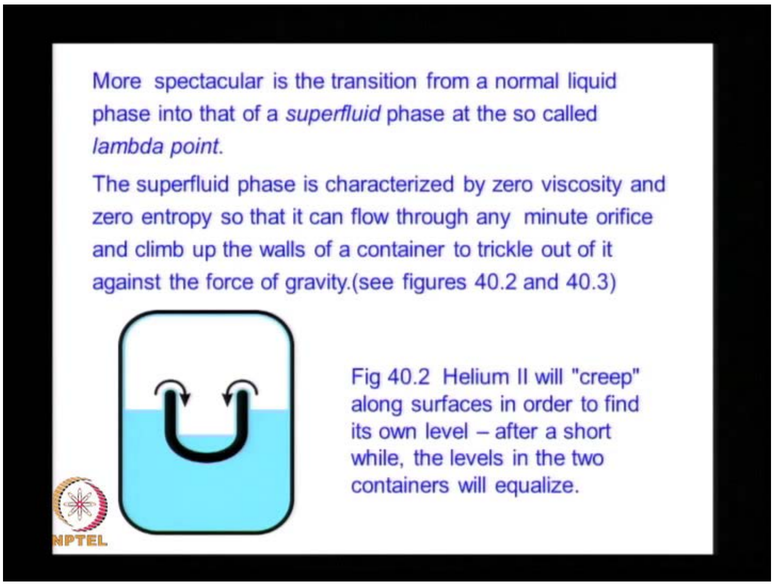

Much more spectacular is the transition from a normal liquid phase into that a super fluid phase as one can see in the phase diagram, you got liquid helium two, and liquid helium one two faces in the liquid state, and separated by what is known as a lambda line. The transition from normally liquid phase into that of a super fluid phase takes place at so-called lambda point, we will talk about it little later.

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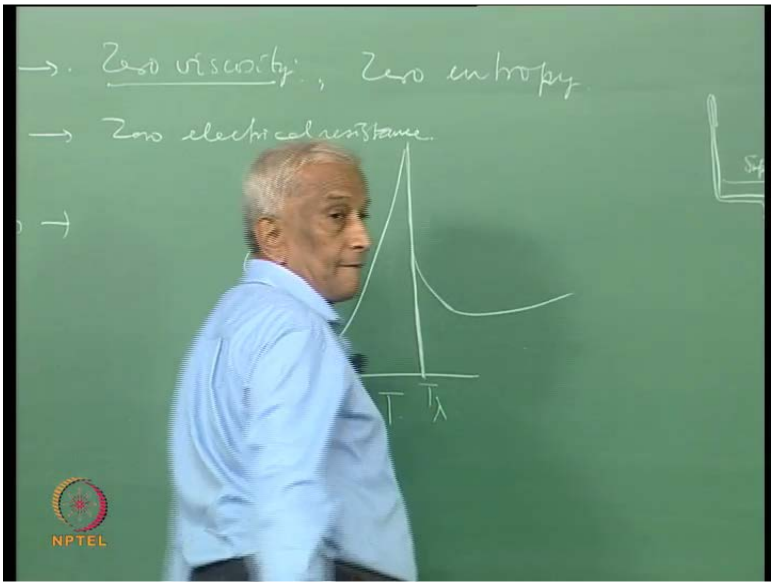

Let us now look at the super fluid phase, which is characterized by 0 viscosity. So, super fluid helium is characterized why is it called supers fluid 0 viscosity in this respect it is super fluid is somewhat similar to that of a super conductor, the behavior of a superconductor which is which may be regarded as a charge fluid. So, it has 0 electrical resistance as we have already seen, and in the ordinary fluid state in the mass transport the analog of electrical distance is the viscous resistance to flow of the liquid. So, this viscous resistance vanishes in the case of a super fluid, just like the electrical resistance vanishes in the case of a superconductor. In addition it has also 0 entropy in a thermodynamics sense, because of this 0, viscosity you can flow-through any minute of his and all so climbed up the walls of a container.

So, if you have super fluid helium then it can climb up the walls and little out flying clime down, then you can see droplets coming up. So, the liquid will spontaneously coming out of the container. So, it will climb up the walls of a container to trickle out of fight against the force of gravity this is shown in figure 42, where you have two containers. And the helium two will creep along surfaces in order to find its own level after a short level, there are two containers here and the level in the two containers will equalize after sometime.

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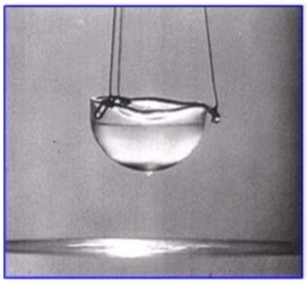

Fig 40.3 The liquid helium is in the superfluid phase. As long as it remains superfluid, it creeps up the wall of the cup as a thin film. It comes down on the outside, forming a drop which will fall into the liquid below. Another drop will form – and so on – until the cup is empty.

The next picture 43 shows liquid helium is in the super fluid phase, and as long as this super fluid it creeps up the ball of the cup as a thin film it comes down on the outside. As you can see in the photograph forming a draw, which will fall into the liquid below another drop will form, and another, and so on. Until the cup is will empty itself it will look almost miraculous we talked about the lambda transmission from the normal liquid phase into the super fluid phase lambda.

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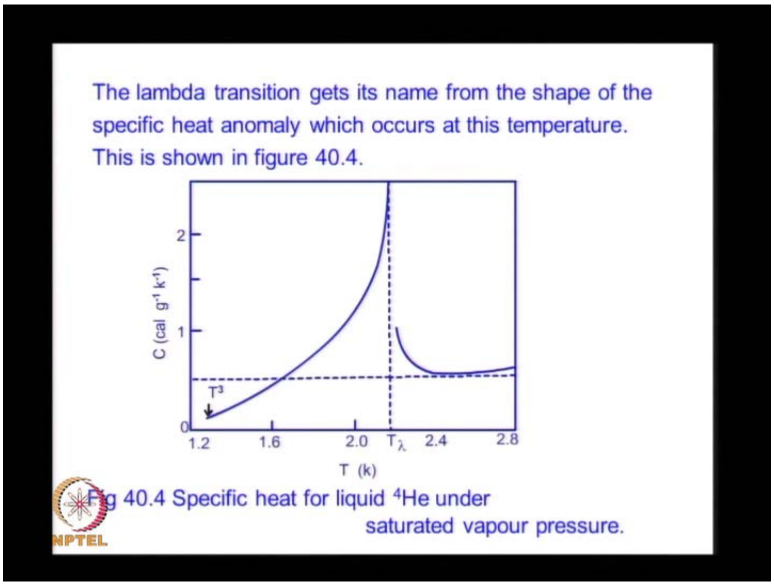

Now, this is called the lambda transition because of the shape of this specificate normally which occurs at this temperature. So, you have this specificate and we have something like the shape is somewhat like this that is T lambda and. So, that is the roughly the kind of shape this is shown exactly in figure forty four. So, the resemblance to the figure lambda is what gives you the name of lambda transition why should the lambda transition occur and anther for this was given by London in terms of what is known as Bose Einstein condensation, what is a Bose Einstein condensation?

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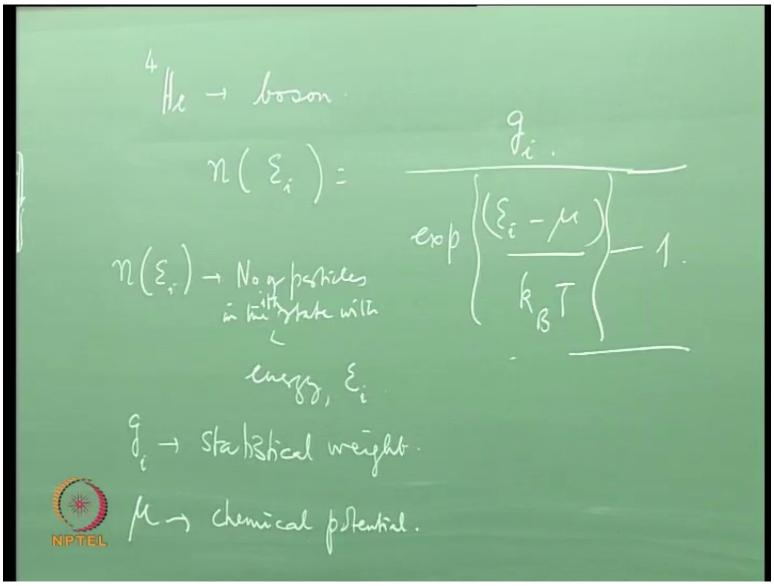

Now, helium fore is a Bose on helium fore is a abundant of helium. So, this is the Bose on which forms satisfies the Bose Einstein distribution. So, the distribution function as the form now g i N of e i is the number of particles in the state with energy in I th state with energy e i when g i is the statistical weight of the state mu is so-called chemical potential. So, that is the distribution function and looking at this distribution function one find very interesting characteristics.

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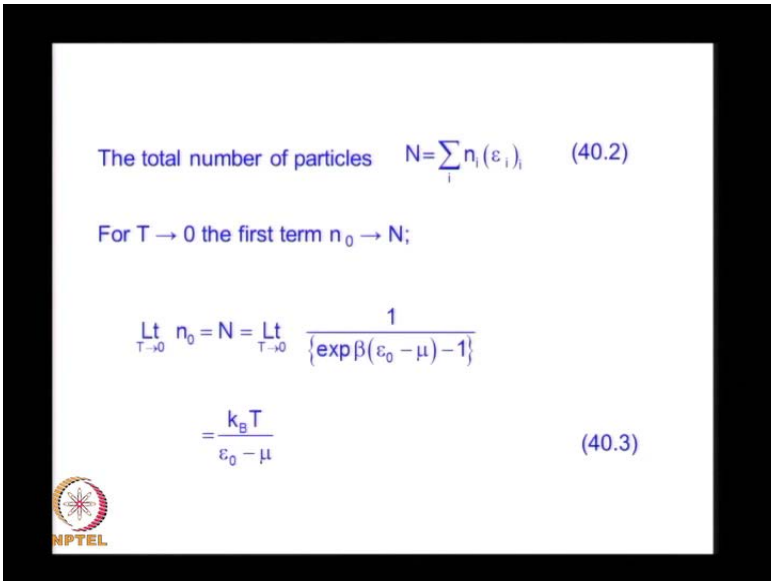

The total number of particles in this case helium atoms is N which is sigma N i e i. Now, if you look at this this distribution function it as some peculiar features as T tends to 0.

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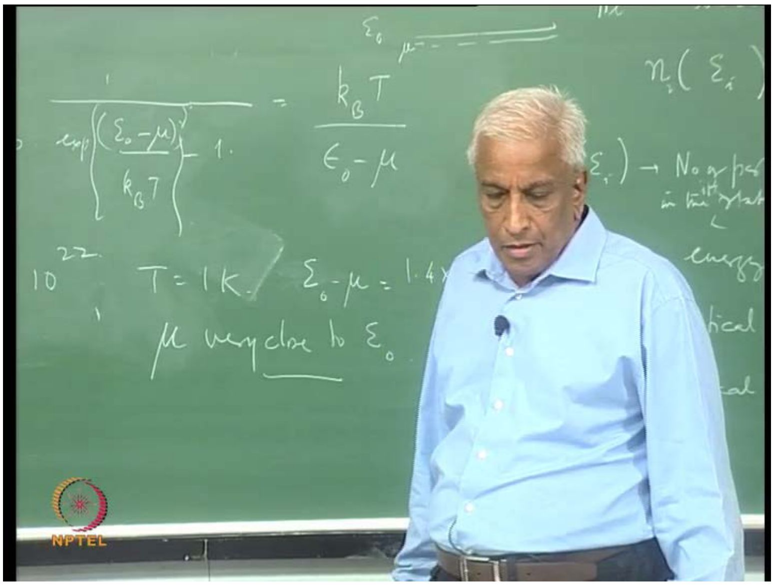

We find that N I N 0 tends to N in the grounds state 0 corresponds to the ground state of energy is zero. So, in the limit T tending to 0 N I N 0 equals N, which is the same as limit T tending to 0 one by exponential e 0 minus mu n minus one. So, this is this limit. So, far N equal to ten to the power twenty two the order of at T equal to one k.

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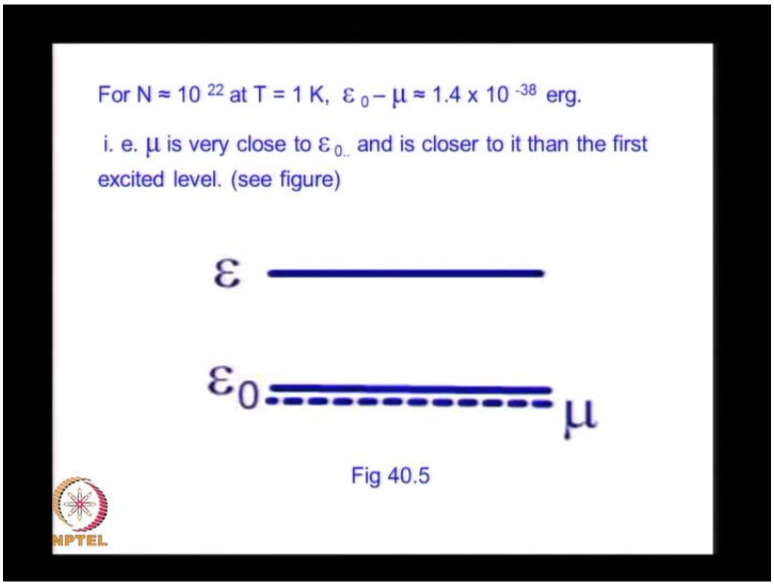

We can easily verify by substituting hear that e 0 minus mu is 1.4 into 10 to the power minus 38, which is extremely small number, that is mu is very close to epsilon naught the ground state energy. This situation is shown in figure 45 where mu is shown to be extremely close to epsilon naught and closer to it, then the first excited level the first excited level lies here mu is somewhere here. So, this is epsilon naught this is epsilon one this is mu hence most of the particle tends to occupy the ground T tending to zero.

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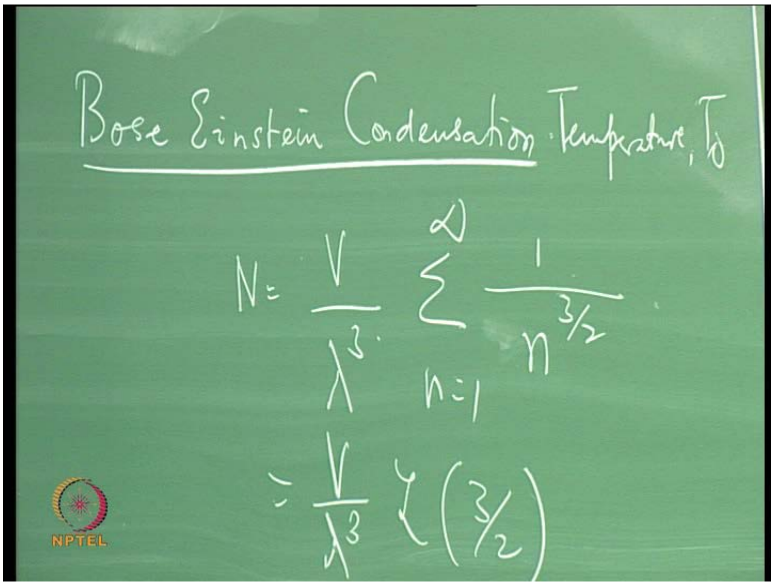

This is of course, possible for a Bose on assembling and this situation is known as Bose Einstein in condensation the temperature, at which this happens namely the Bose Einstein condensation temperature. Let us say T naught one has to defined evaluate these N equals v by lambda q into sigma, that is the same as this is written in terms of as the Riemann zeta function.

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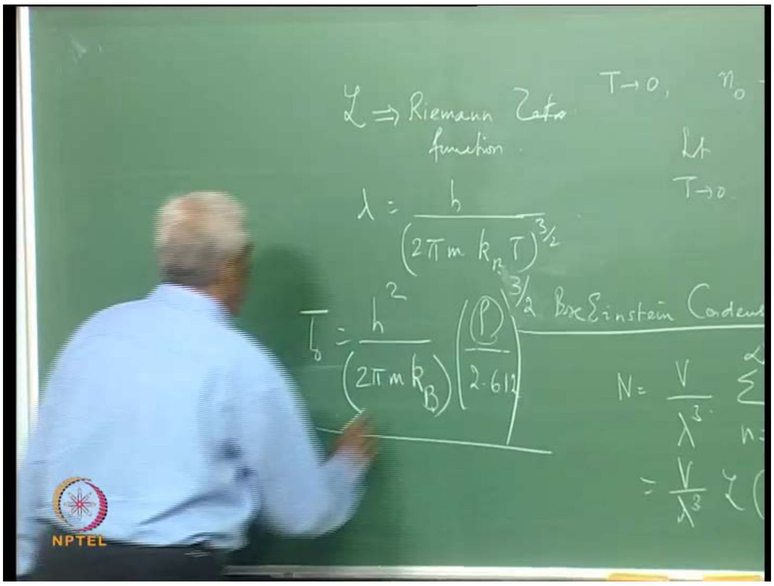

Zeta is the Riemann zeta function and lambda is the is defined as h by two. So, in terms of these this zeta function as this known to have the value2.612. So, this can be written as where lambda is also known to have this value. So, inverting back we can write T naught.

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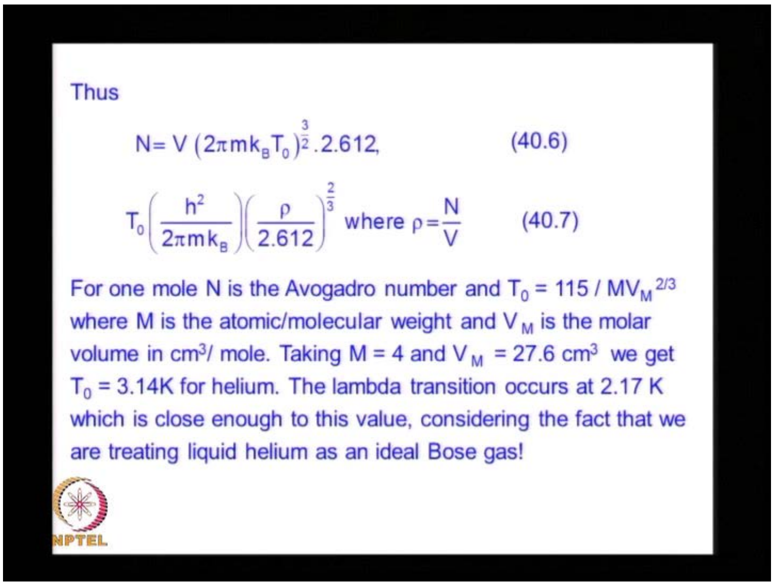

The Bose Einstein condensation temperature as, so that is the defining equation for the Bose Einstein condensation temperature three by 2 here row is N by v the density. So, far one more N is alligator number.

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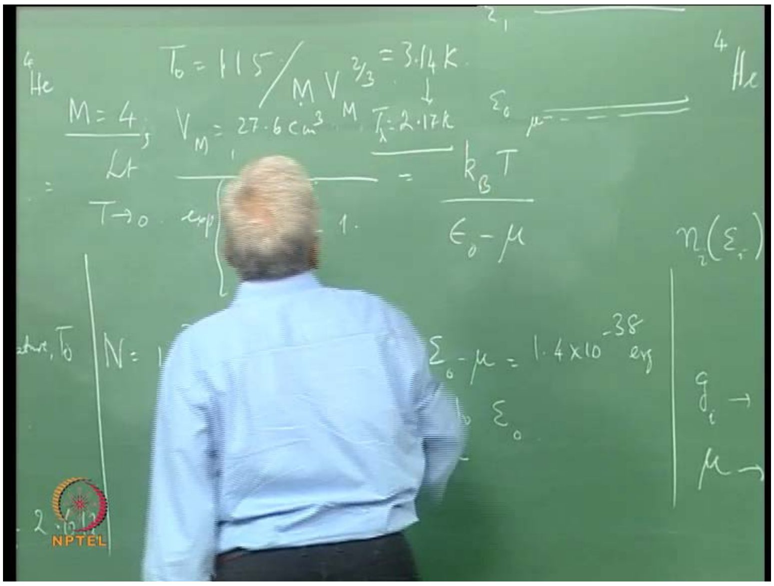

And therefore we can calculate T naught and it turns out T naught equal 1.1 15 times and m v m this is two third were m is the mass of the helium atom. For example, in the case of helium. So, m is 4 for helium and the molar volume v m is in centimeter cube 27.6. So, that we can calculate T naught for helium 4, and this turns out to be three point one 4 kelvin the actual observed value for the lambda transition is 2.17 k. So, the bose einstein condensation temperature calculated value is 3.14 k assuming that the helium gas is a Bose an ideal Bose Einstein gas which is far from true, it is not a gas it is a liquid. So, it does realize. Qualifying as an ideal Bose Einstein gas it is an approximation considering this approximation this is an extremely good agreement.

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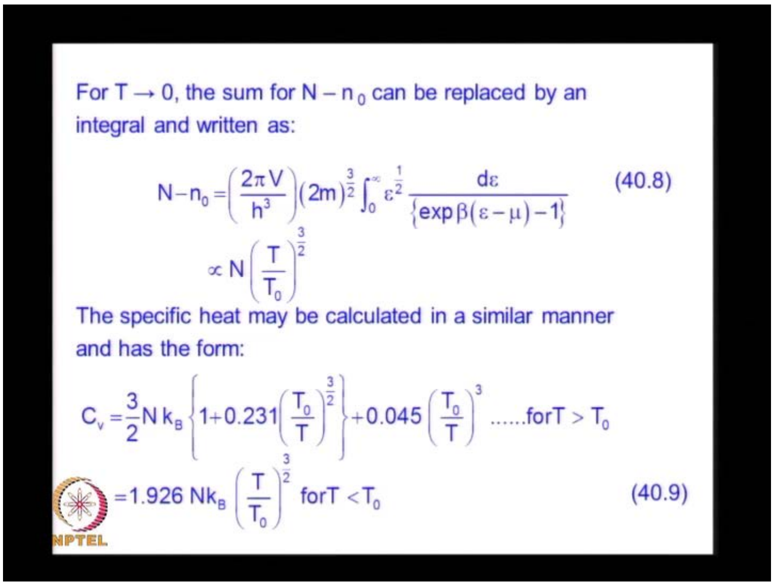

So, the Bose Einstein condensation picture of the lambda transition from the normal state in to the super fluid state has the is accepted.

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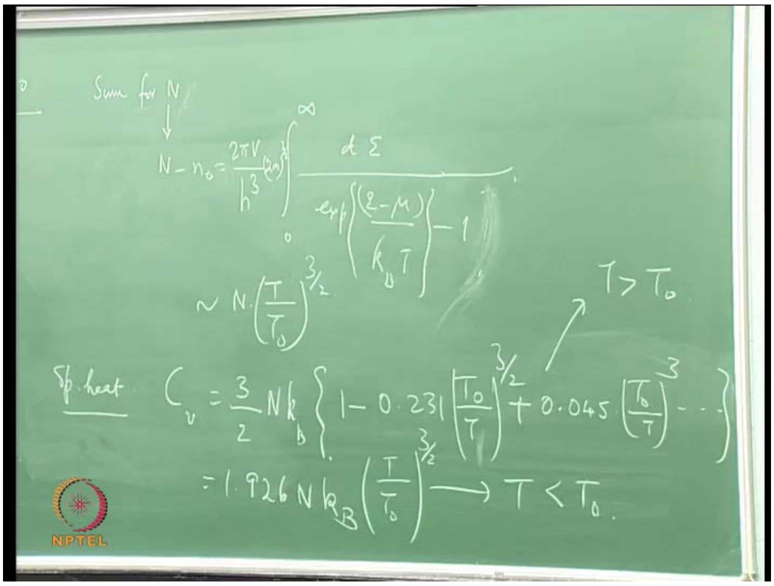

Therefore, so far in the limit of T tending to 0, we can calculate the rest of the sum for N now we are taken out the ground state condensation. So, N minus N naught can be written as an integral integral the states are from 0 to infinity the standard density states and here you have d e by exponential. So, that integral goes as N times T by T naught three by two now using the same picture the specifications can also we calculate and compare with this experiment the specificate is behavior is very exact accurately measure both above and below the lambda transition. So, c v as the value three half N k v into 1 minus 0.231T by T naught 3 by 2 T naught by T plus plus 0.045 T naught by T is squire this is for T greater than T naught, this is a high temperatures. And at low temperatures this as the value it is T B y T naught for T less than T naught.

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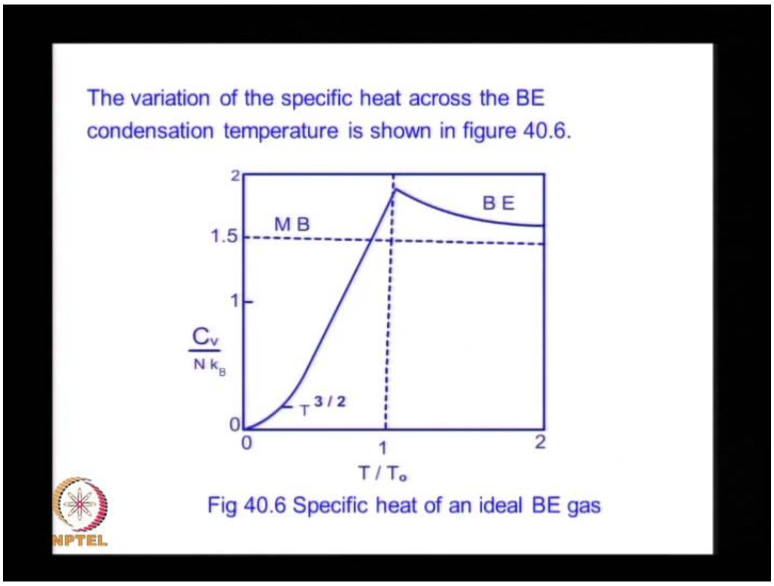

So, the calculated values as they specificate variational specificate across the Bose Einstein condensation temperature is shown in figure 46, and it as at the Bose Einstein condensation temperature unlike the lambda transition, but still the correspondence then is close.

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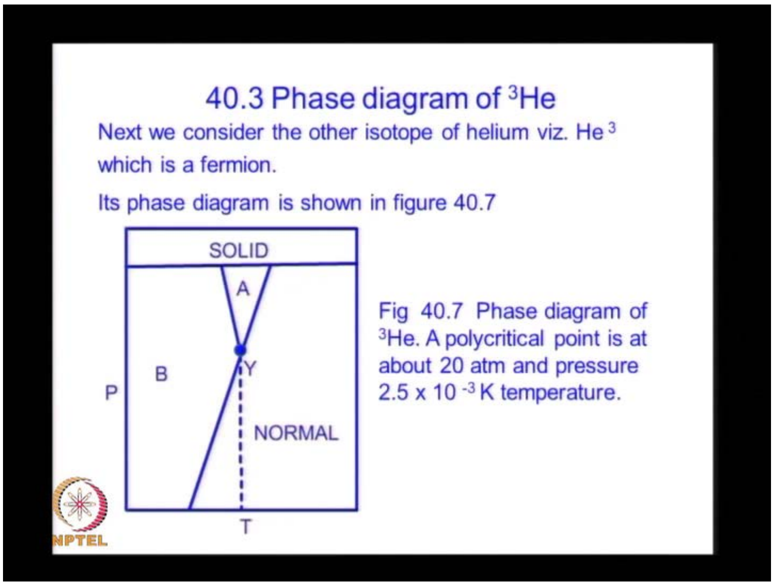

Next we gone to considered the other isotope of helium three the phase diagram of helium three is shown in figure 47. Now helium three is a rare isotope and you can see that there is a poly critical point at y at the point y shown in the figure, which is at about 20 atmospheres pressure and 2.5 into 10 to the power minus 3 kelvin 2.5 mile kelvin temperature. So, the poly critical point occurs here.

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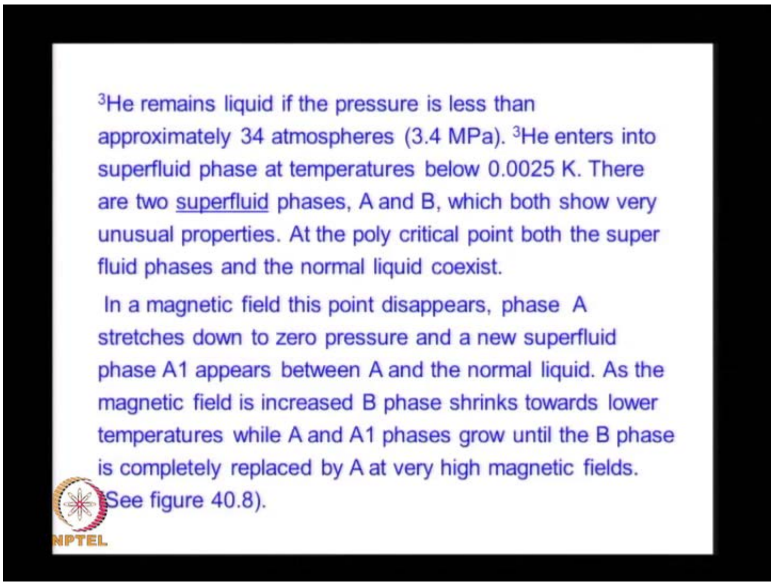

So, if the pressure is less than approximately 34 atmospheres, you can see there is a solid phase helium three entered into a super fluid phase at temperatures below 0.0025 k two point mile Kelvin, there are two super fluid phases a and B, which both show very unusual properties at the poly critical point why both the super fluid. And the normal liquid coexist can see this is the normal liquid, and you have two super fluid faces all three coexistent y, that is why it is called a poorly critical point.

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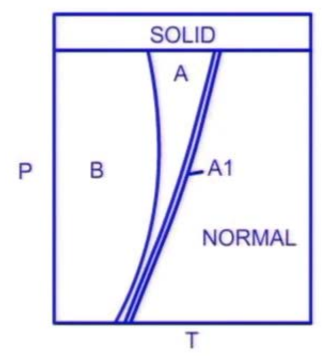

Fig 40.8 Phase diagram of ^3He when the magnetic field is present

Once you apply a magnetic field the poorly critical point why disappears as you can see in figure 48 and the phase a stretches down to 0 pressure unlike in figure forty seven in the absence of a magnetic field. And a new super fluid phase a one appears between a and the normal liquid as the magnetic field is strength is increased the B phase stretch shrinks towards lower temperatures well A, and A 1 phases grow until the B phase is completely replace by the a at very high magnetic fields.

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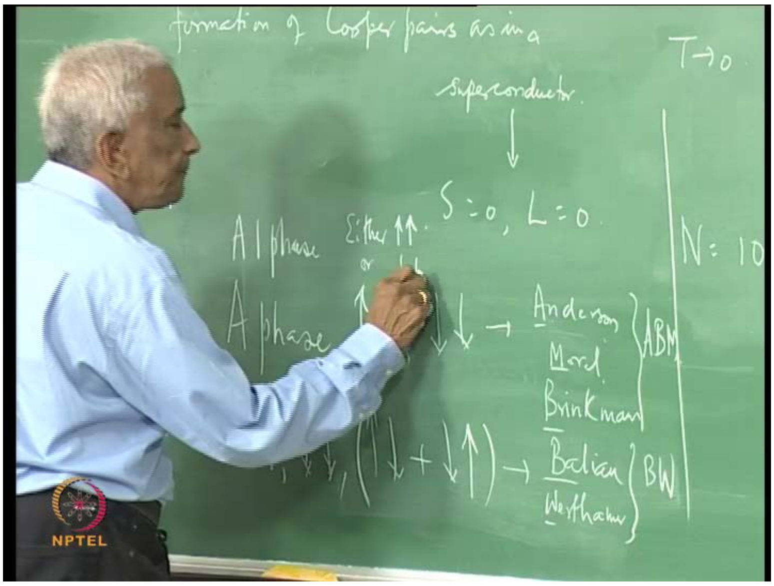

Now, helium three is a fermion unlike helium fore which is a Bose on. So, the bose einstein condensation picture cannot be used to explain the occurrence of super fluidity the occurrence A and B phases etcetera in helium three. It is more like formation of super phase as in the superconductor airs of helium atom joined together through interactions as in the name superconductor, once you form a pair of these fermions then it becomes a Bose Einstein.

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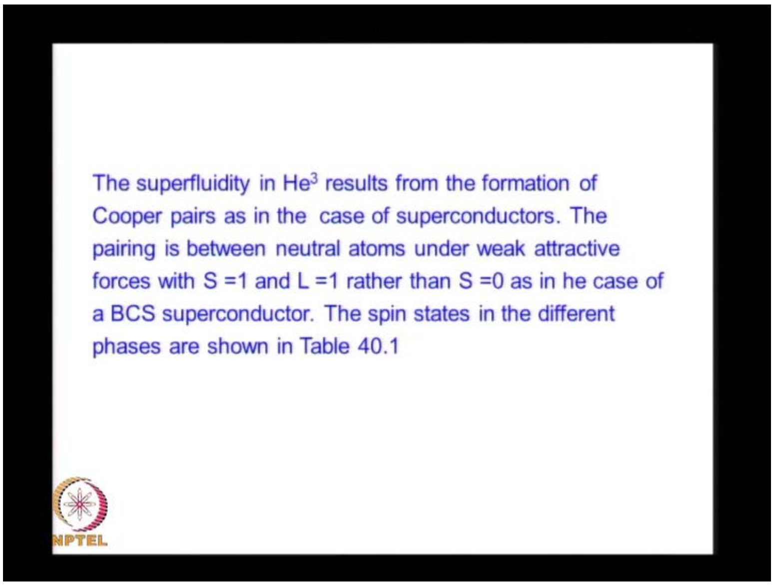

The paring is between neutral helium three atoms under week attractive forces with S equal to one, and L equal to one unlike in the case of an ordinary superconductor in S ways superconductor their S is 0 L is 0.

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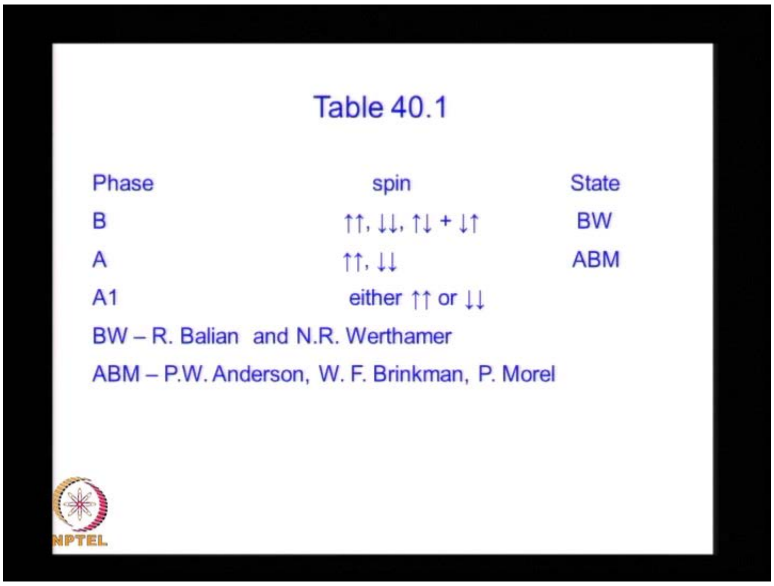

The spin states in the different phases A and B, and A 1 are shown in table 41, if a phase has the spin states spin up spin up or spin or down o spin down. Now that is a picture proposed by Anderson, morel, Brinkman. So, it is known as a B m phase the B phase as all possibilities, it is a true triplet state which we discussed any case of ferromagnetic will be either this or it can be. So, all three states in that was a picture proposed by Balian and brinkman. So, it is called B w ways a phase is either this or this that is the overall simplified picture of the super fluid phase transition in the liquid helium three which take place at mile kelvin temperature much lower than that of liquid helium 4.

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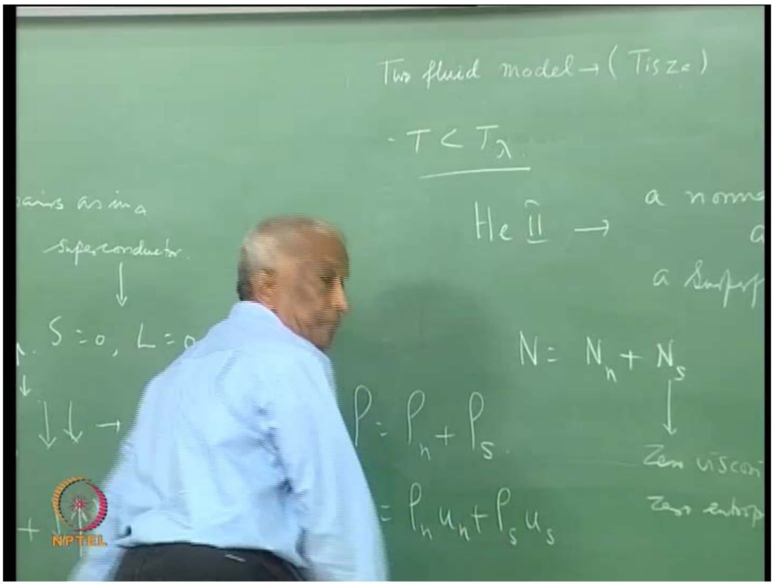

We now go on to look at bit closely at the normal and super fluid phase are helium 4 based on the two fluid model proposed by Tisza he suggested that for T less than T lambda the super fluid helium two consist of a normal component as well as and a super fluid compound. So, that is the basis of the two fluid model proposed by Tisza. So, N equals N N plus N S, where N S is the particles number of particles in the normal phase and N S is the number of particles in the super fluid phase.

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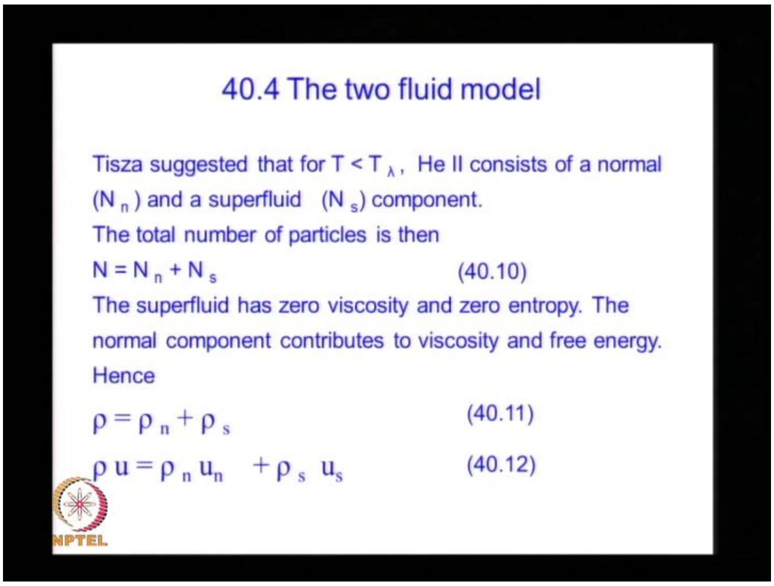

The super fluid has 0 viscosity and 0 entropy while be normal fluid contributes to viscosity. Therefore we can write the density as and also these p row u is row N u N plus row S u S; these are the basic equations of the two fluid model, now a person called and Nonikashvili conducted a very interesting experiment.

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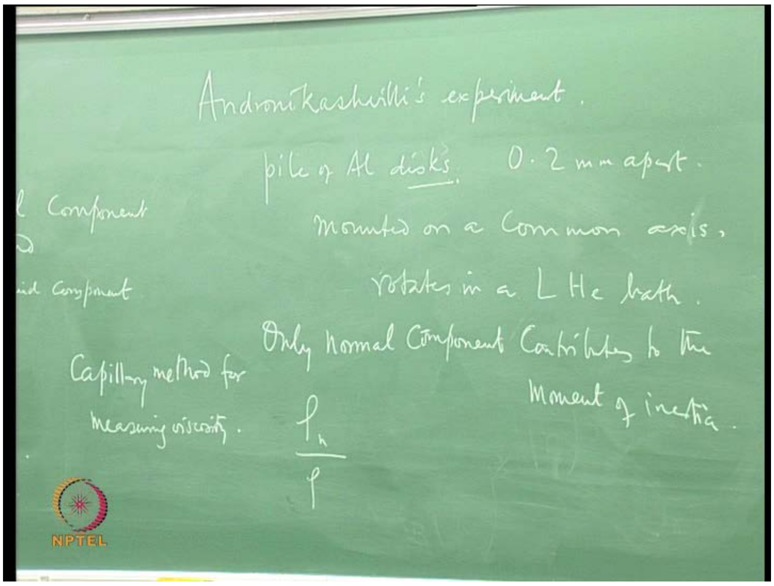

The experimental arrangement used by him is shown in figure 49, it consist of a pile of aluminum disks space two point millimeter part very close and mountain on a common axis.

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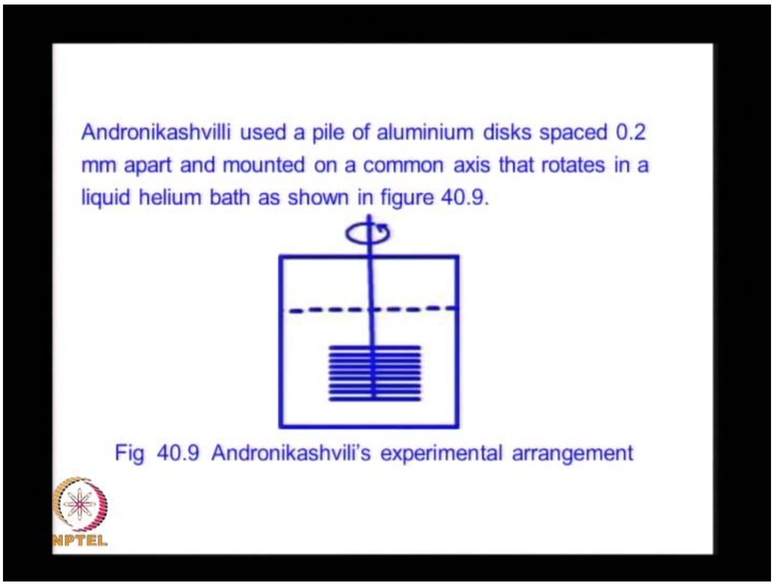

And this assembly rotates rotation in a liquid helium bulk, this is shown in the figure, this means that only the normal compound can rotate between the disks. And therefore, it contributes to the moment of inertia the actual dynamical moment of the inertia which determines the p rate of rotates. So, the ratio the experimental and the geometrical movements of inertia is the ratio row N by row the ratio of the normal component density to the total density of fluid well. There is a another method of measuring viscosity using a capillary method for measuring viscosity in contrast to the this arrangement which is a kind of oscillating visco meter, where the period of oscillation determine by this moment of inertia the period becomes longer, because they are damping due to the normal component and this viscosity.

Now if you use polis ally differential equation in order to determine the viscosity using a capillary only the super fluid can pass through the capillary, whereas in the dampening as the disk here it is only the normal component. So, the two experiments are different. So, give rise to totally different result which can be understood in terms of the two fluid module a very remarkable consequent of this is typified by thermo mechanical.

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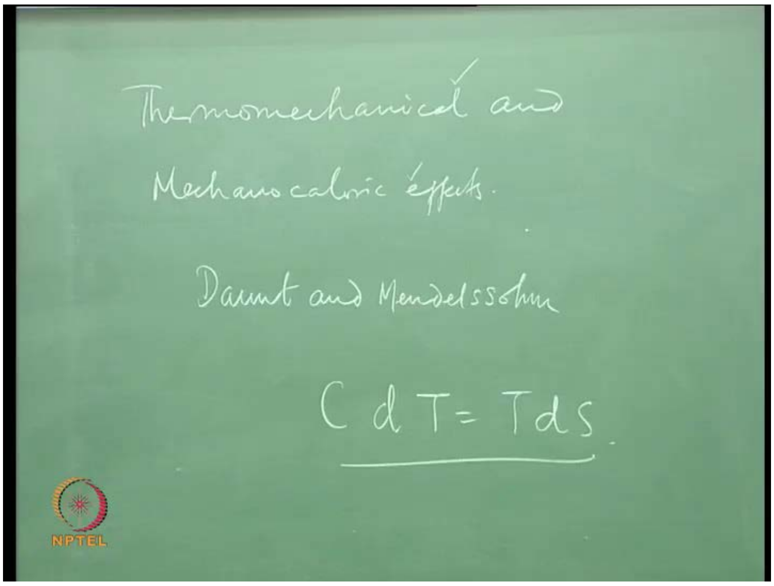

And Mechano caloric effects reverse of each other daunt and Mendelssohn immersed a heater immersed a heater in a helium two bath a, which is surrounded by another bath b.

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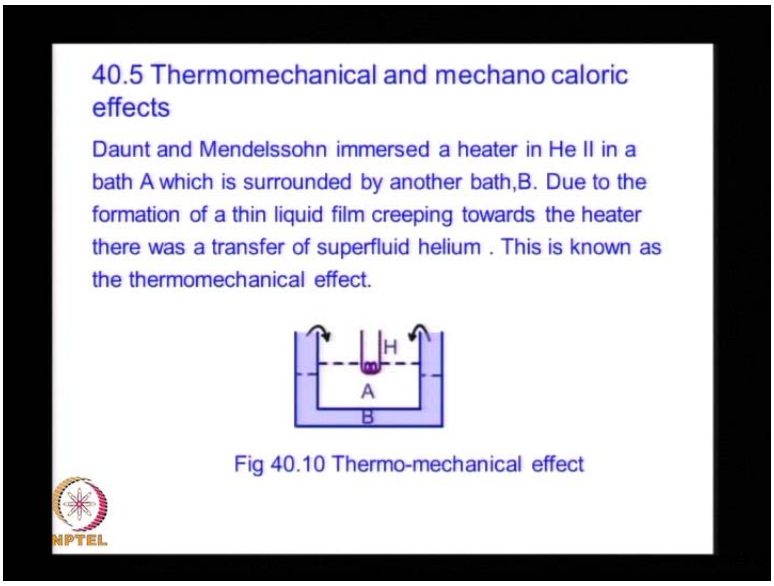

So, due to the formation of a thin liquid film creeping towards the heater there was the transfer of super fluid helium, this is known as the thermal mechanical effect this is illustrated in figure 40 10.

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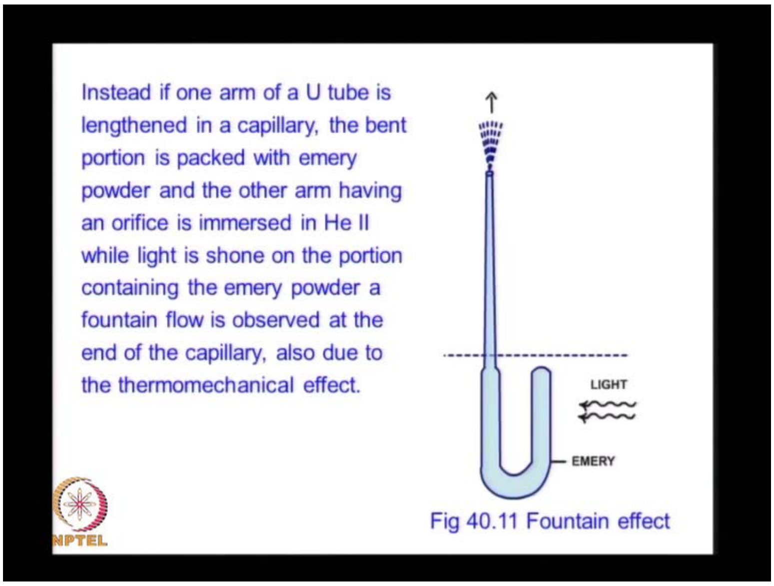

Instead if one arm of a U tube is length ended in a capillary, I shown in the figure 40 11 the bend portion is packed with emery powder. And the other arm having an orifice immersed in a bath of helium two while light is shone on a portion containing the emery powder a fountain flow is observed at the end of the capillary also again due to the thermal mechanical effect.

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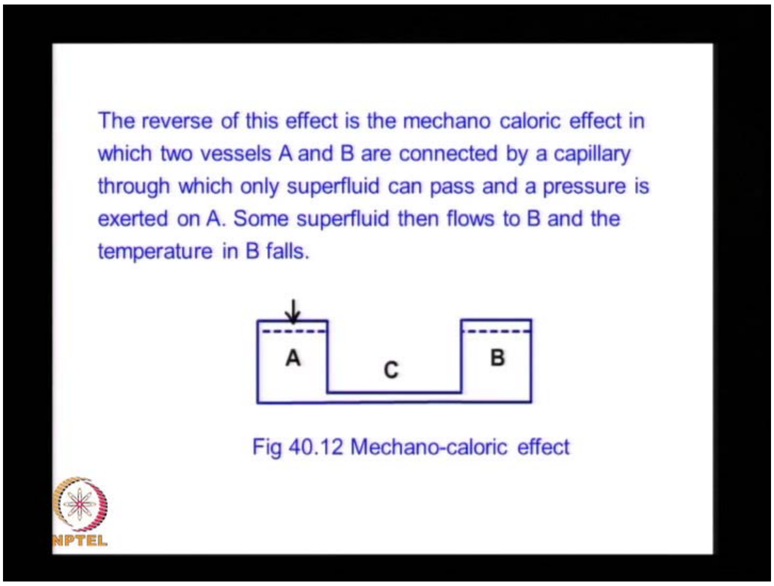

The rivers of this effect is the Mangano caloric effects which is shown in figure 40 12, where two vessels A and B are connected by a capillary through which only super fluid can pass and A pressure is exerted on A. Then some super fluid will flows to B and the temperature in B will fall that is the mechanical effect.

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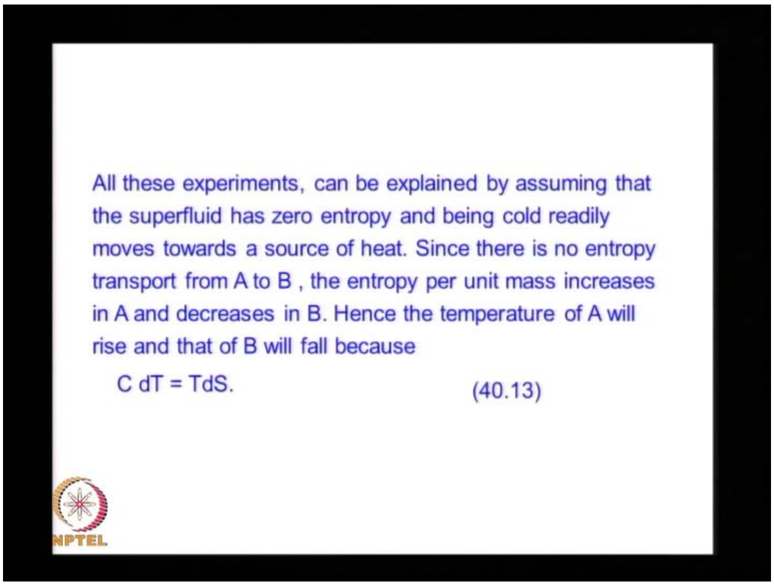

All these experiments can be explained by assuming that the super fluid has 0 entropy and being cold readily moves towards a source of heat. Since there is no entropy transport from A to B the entropy per unit mass increases in A and decreases in B, hence the temperature of a will rise and that of B will fall, because c d T equals T d S were c is the specific heat.

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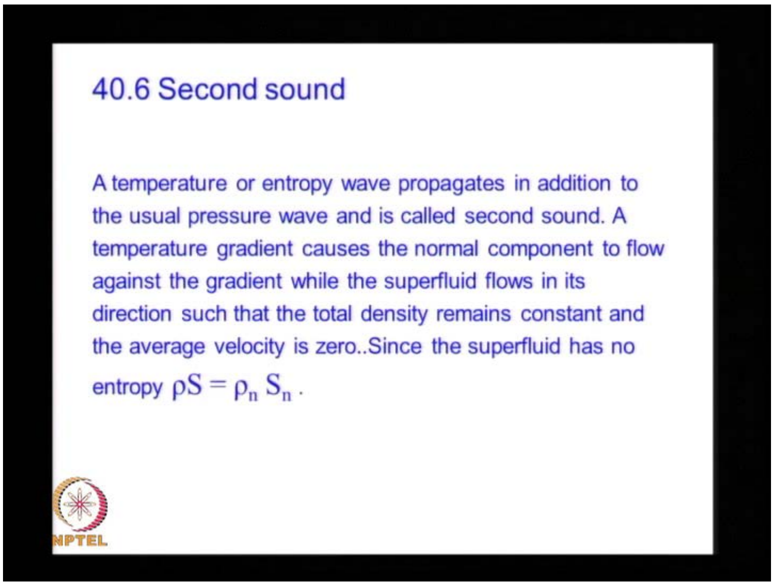

Another very striking f a of this phenomena of super fluidity is that of the propagation of second sound in liquid helium two. We all are similarly with ordinary sound which is the first sound the second sound is a temperature or entropy wave, which propagates in addition to the usual pressure wave. This is called the second sound a temperature gradient causes the normal component to flow again this gradient while the super fluid flows in the direction gradient such that the total density remains constant and the average velocity is 0.

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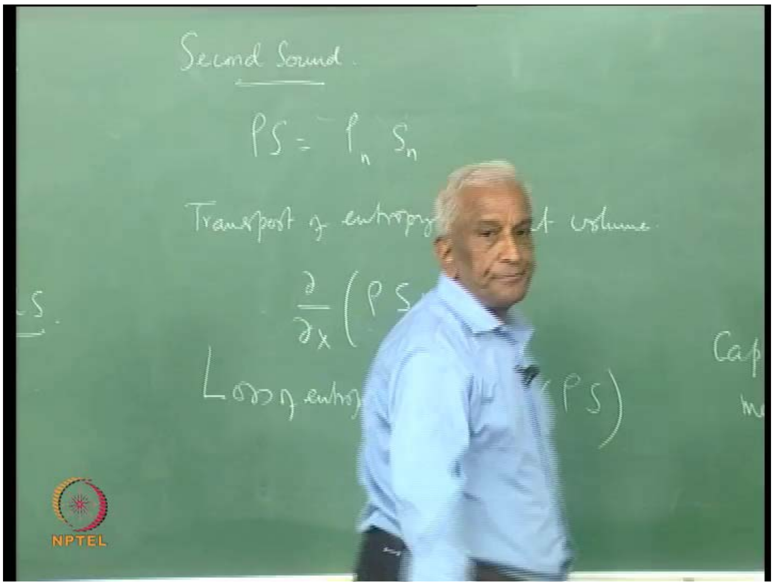

Since the super fluid as no entropy row S is row N S n. So, the net transport of entropy per unit volume maybe written in the form of a partial derivative, if the flowing along the x direction dou by dou x of row S.

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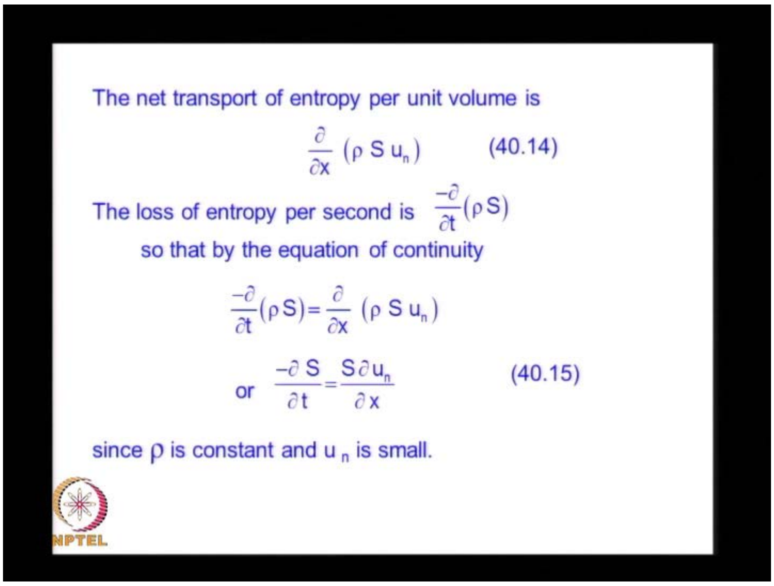

The loss of entropy because of this transport per second is minus dou by dou T of row S. So, the question of continuity demands that these two are equal.

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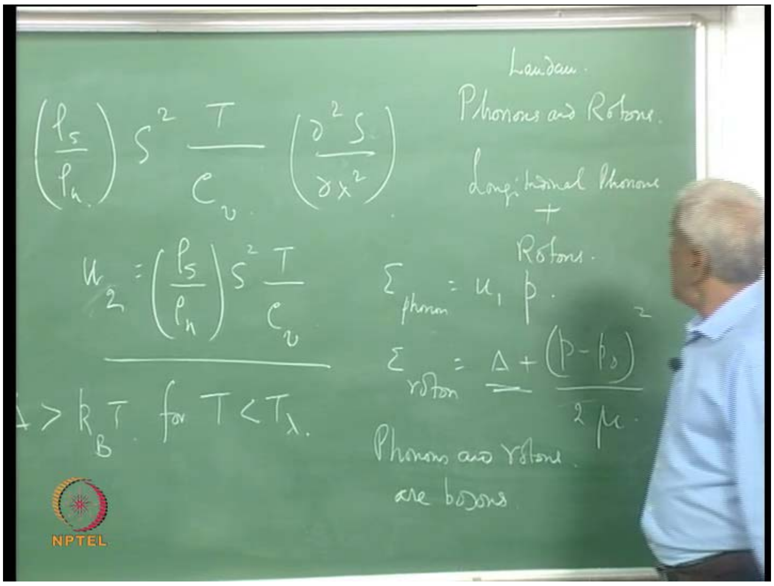

And a little bit of algebra shows using the kinetic energy density, we can write the equation of motion as for these entropy waves as the wave equation. So, that this second sound it is just what we have here.

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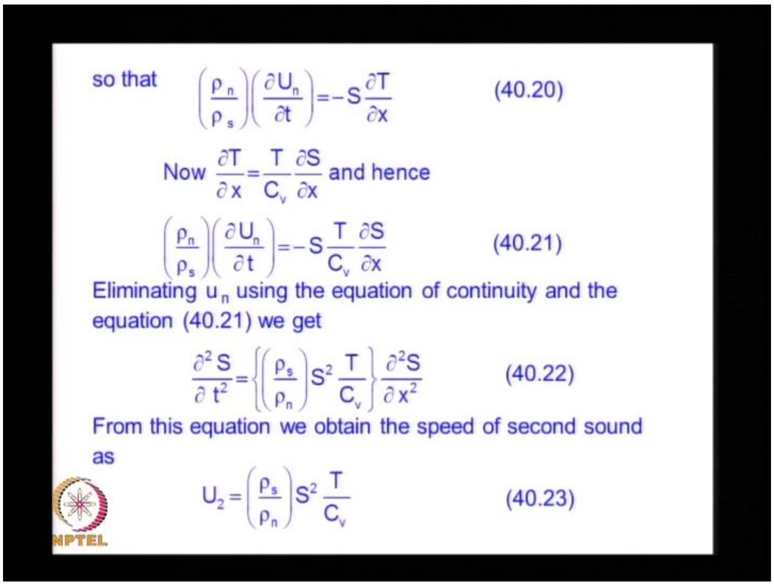

So, this second sound can be detected using a heater, and this p of these sub boundaries can be determined again that can give value for the relative ratio of the super fluid normal fraction at any given temperature. Now in contrast to this 2 4 model lambda emphasize the collective aspect.

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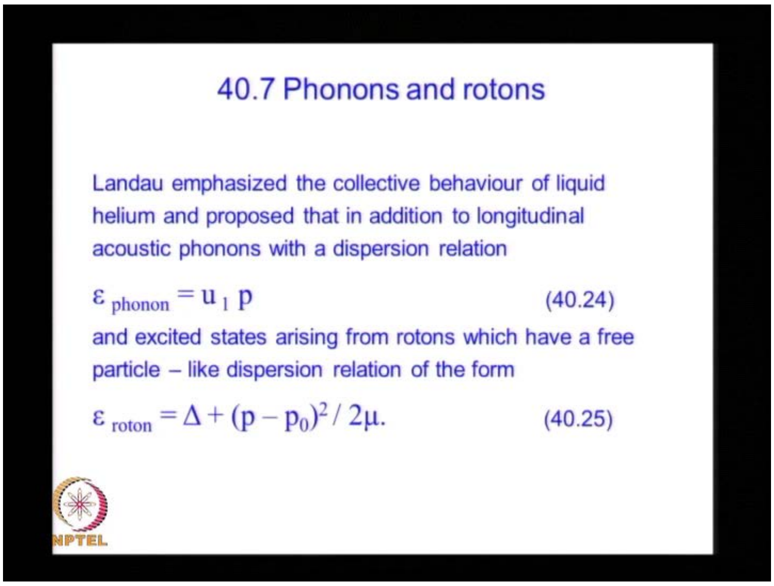

And he said that in addition to longitudinal phonons there are also rotons the longitudinal phonons as the dispersion relationship the energy momentum relationship, whereas the rotons as the relationship it is a particle like exaltation with an energy gap delta.

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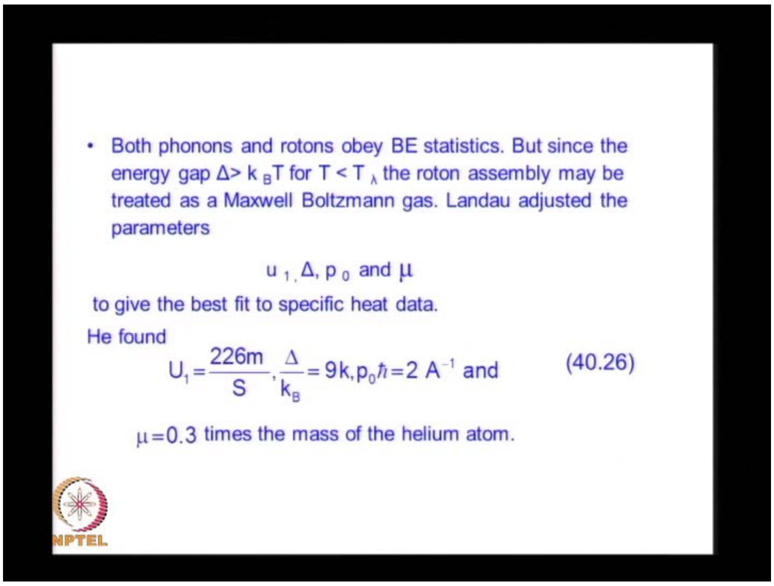

So, both phonons and rotons obey bose einstein statistics and. So, on bose on since its energy gap delta is greater than k B T for T less than T lambda. So, rotons assembly may be treated as a Boltzmann gas, then you can calculate the excited spectrum by taking U 1 delta p naught and mu as adjustable parameters.

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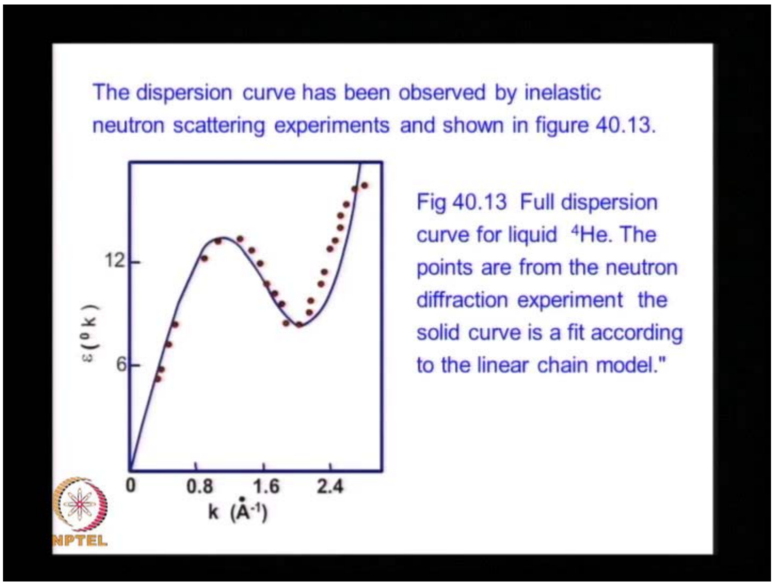

So, he got the best fit specifically heat data using this excited spectrum which is shown in figure forty thirteen all. So, along red inelastic neutron scattering data which give you the full dispersion curve a agreement between the experiment and theory it is quite remarkable.

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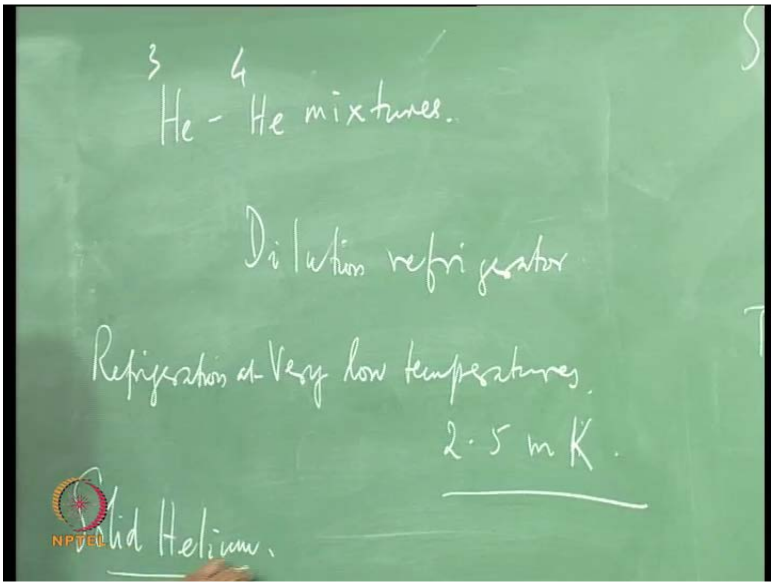

Next, we considered another interesting aspect namely that of helium three helium 4 mixtures, what happens when you mix helium three un helium 4.

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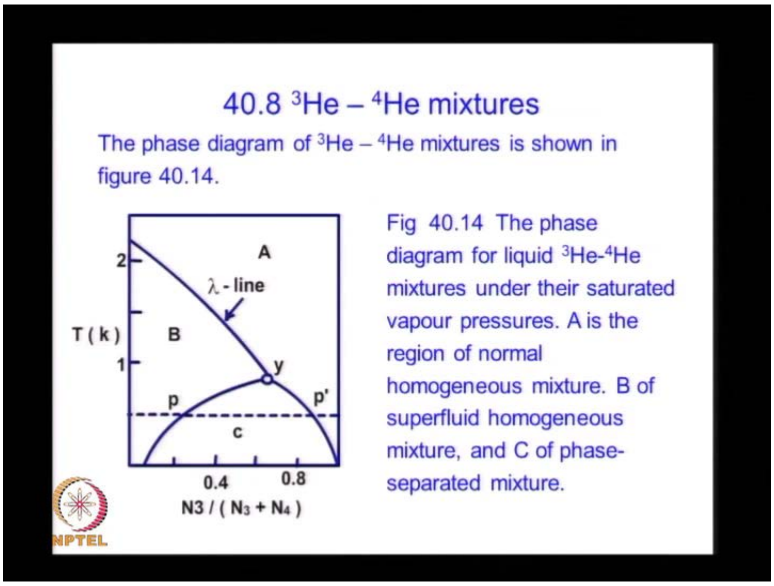

The phase diagram of such mixtures is shown in figure 40 14 a is the region of normal homogeneous mixture B is a region of super fluid homogeneous mixture, and c of a phase separate mixture.

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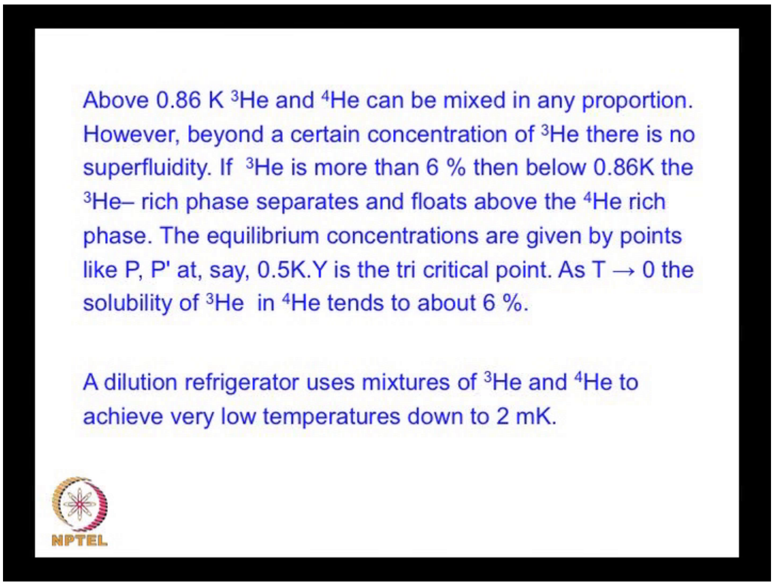

So, you can see that above 0.86 k helium three and helium 4 can be mixed in any proportion; however, beyond a certain concentration of helium 3, there is no super fluidity if helium 3 is more than 6 percent then the low 0.86 k helium rich phase separates and floats above the helium four. So, this is above 0.6.

So, these region I am talking about below 0.86 k the equilibrium concentration are given by point such as p, and p dash at say point k, why is it right critical point as T tends to 0 the solubility of helium 3. In helium 4 tends to about 6 percent based on this interesting behavior of helium three of helium 4 mixtures a dilution refrigerator as been invented which dilutes helium three helium 4 mixture by driving the helium three atoms across the face boundary. So, this delusion is useful in achieving very low temperatures refrigeration at very low temperatures of the order of say down to 2.5 mille Kelvin.

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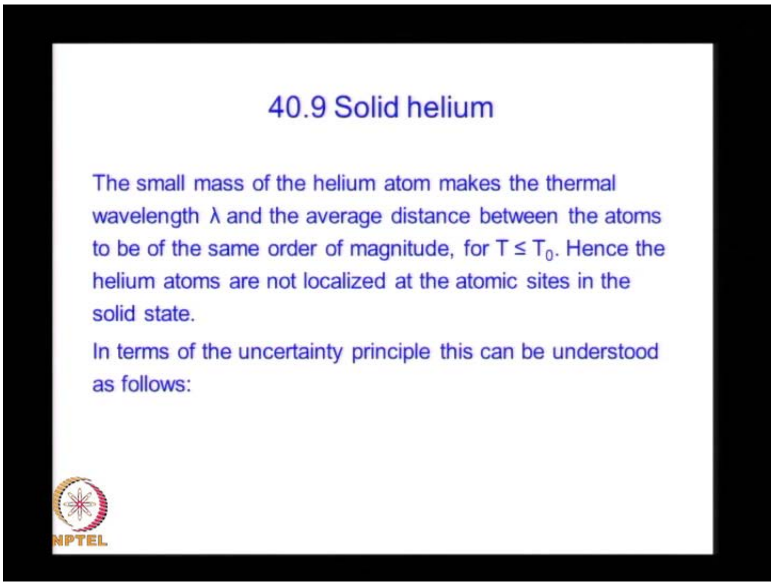

We now turn to a brief discussion on solid helium that is the prototype of quantum solid the small mass of the helium atom makes the thermal wavelength lambda, and the average distance between atoms that and to be of the same order of magnitude for T less than or equal to T naught. Hence the helium atoms are not localized at the atomic sites in the solid-state interns of the uncertainty principle this can be understood and follows.

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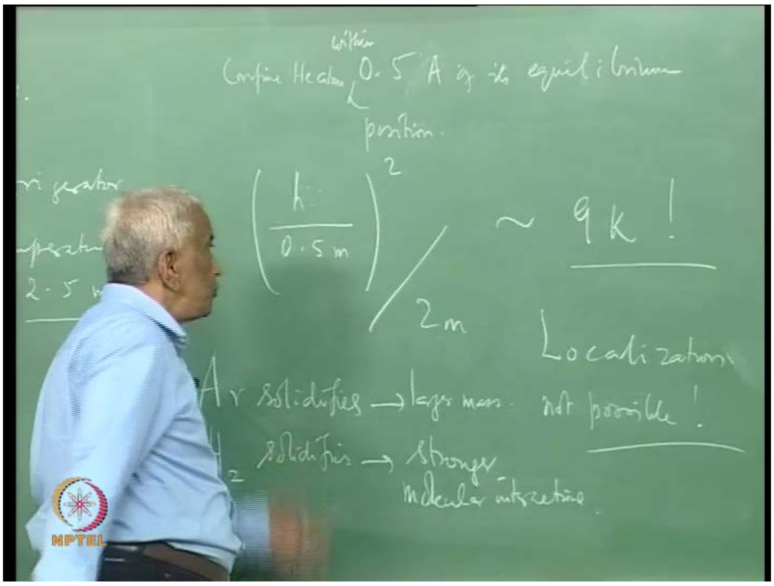

If we want to confine the helium atom within a distance is a order of say 0.5 Armstrong's from its equilibrium position this confine helium atom within of its equilibrium position. In order to do this the uncertainty principle says that we must have h cross that would be the order of the momentum, and squire divided by two m that would be the uncertainty in energy. And this in temperature in units is high as nine k for helium.

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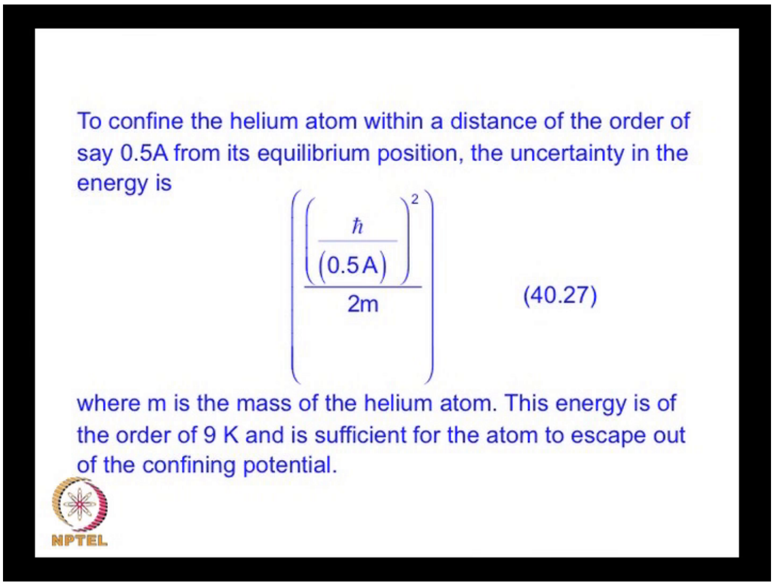

So, this energy is efficient for the helium to a escape out of the confining potential, so localization not possible. So, that is the key to the understanding and behavior of solid helium.

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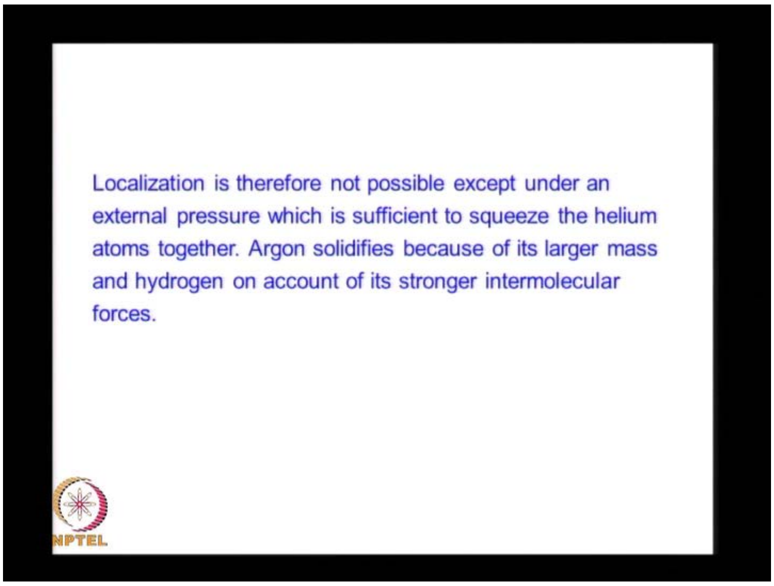

So, except under an external pressure which is sufficient to squeeze the helium atoms together, whereas argon solidifies under ambient pressure in contrast to helium this is because of its larger mass. And hydrogen also fully defies, because of its stronger interactions stronger molecular interactions, and like these two helium cannot fully defined by itself except under external pressure.

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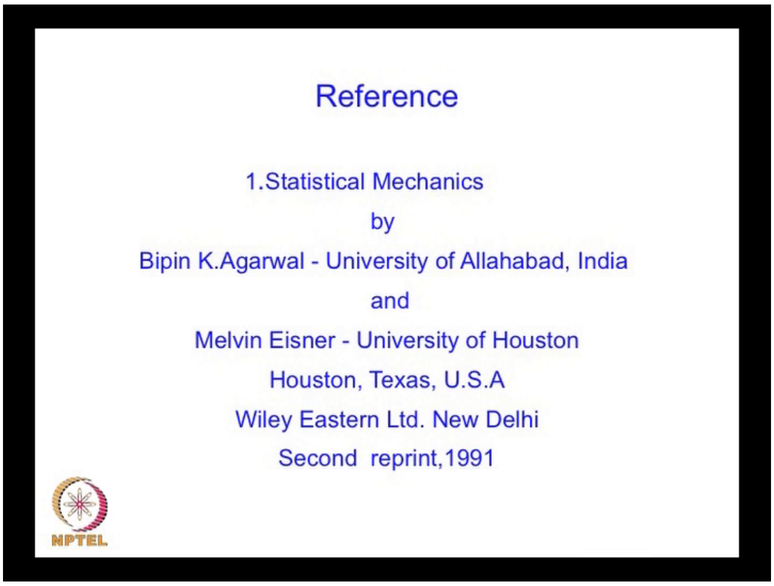

The concept, which I have discussed here can be found in the book on statistical mechanics by Agarwal, Agarwal and Einstein