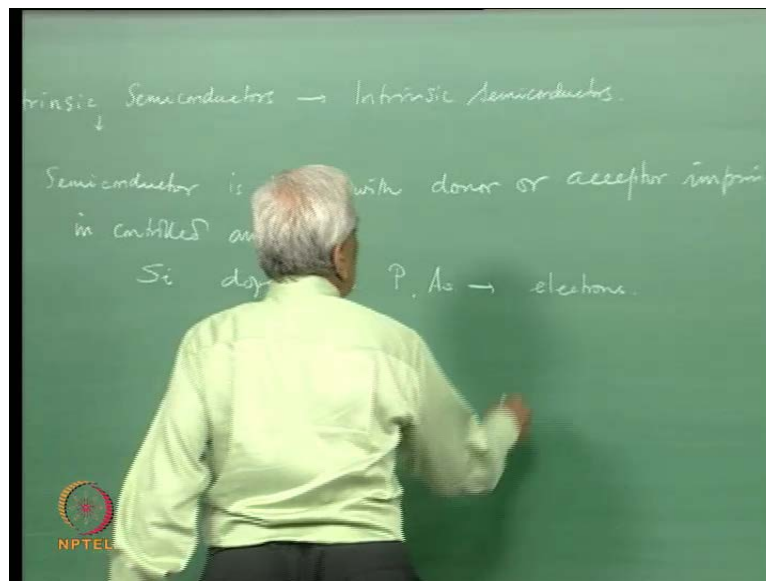


Condensed Matter Physics
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Lecture - 37
Semiconductors (Continued)

Last time, we discussed the carrier concentration and conduction in intrinsic semiconductors, but the crucial thing about semiconductors is the possibility to precisely control the carrier concentrations through controlled doping of a very pure sample of semiconductor. In other words, the entire interest from the point of view of devices and semiconductor electronics is based on the fact that there exist extrinsic semiconductors contrasted with intrinsic semiconductors which we discussed last time.

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We will discuss the carrier transport in an extrinsic semiconductor in this lecture. It has already been mentioned that an extrinsic semiconductor is one in which a semiconductor, which is extremely pure is doped with donor or acceptor impurities in controlled amounts. For example, a sample of silicon is doped with an impurity like phosphorus, arsenic, etcetera to give a donor impurity which modifies the behavior of the semiconductor completely and which gives the possibility to control this behavior and use it in various kinds of physical situations. So, similarly there can be acceptor impurities which also inject controlled amounts of holes. This injects controlled amounts of electrons whereas, 1 acceptor impurity provides a hole.

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Donor Energy Levels

In the n-type extrinsic semiconductor, the fifth electron of the donor atom does not take part in the covalent tetrahedral bonding and is loosely attached to the donor atom. The donor atom is now positively charged as it has lost one electron and this electron can be thought of as moving in the field of the positive ion. The dielectric medium between the positive ion and the electron is that of silicon.



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So, we will discuss this mechanism of how to calculate the carrier concentration which is the basic information required for technical exploitation of these materials.

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Donor energy levels.


n type semiconductor

has fifth electron is loosely bound to Si atom.

Donor is positively charged.

Medium is Si.

like. Atom in Q.M. Ground state energy $\sim -13.6 \text{ eV}$



In order to do this, we will start considering the energy levels of a donor impurity in an extrinsic semiconductor this is called an n type semiconductor. So, the fifth electron in a pentavalent impurity is not able to get tetrahedral bounded to the silicon arbiters know that is because the tetrahedral bounding allows four electron. So, the fifth electron is loosely bound to the silicon atom the donor such as phosphorous or arsenic is now

positively charge because it is donated on electron. This loosely bound fifth electron can be thought as moving in the never hood of this positively charged atom or ion to be more specific the dielectric medium in which this motion takes place is that of silicon. So, we consider this in the framework favor of the wee known theory of the hydrogen atom or hydrogen like atoms in quantum mechanics.


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The radius and energy of this orbital can be worked out using simple quantum mechanics (of hydrogen like atom).

According to quantum theory of hydrogen-like atom the ground state energy is given by

$$E_H = -\frac{me^4}{2(4\pi\epsilon_0\hbar)^2} = -13.6 \text{ eV} \quad (37.1)$$

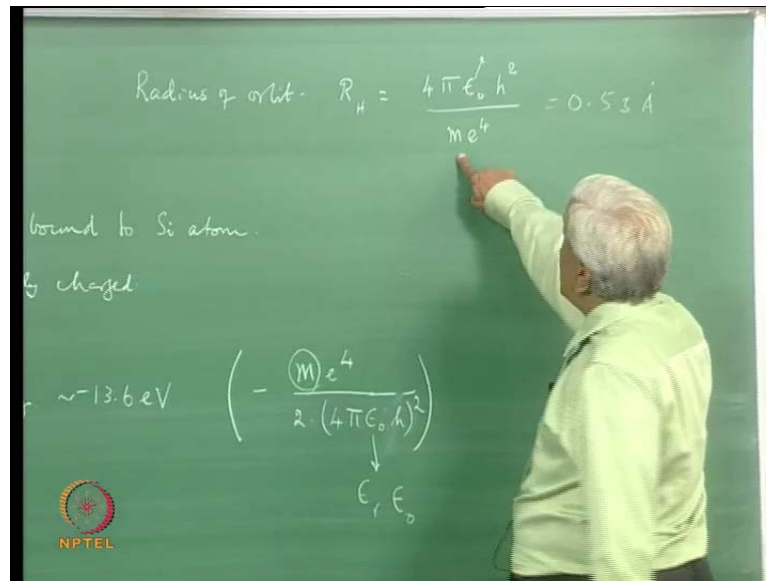
where ϵ_0 is permittivity of free space. The radius of the ground state electron orbit is given by

$$r = \frac{4\pi\epsilon_0\hbar^2}{me^2} = 0.53\text{\AA} \quad (37.2)$$


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And borrow the results for this which is well known the ground state energy of such a hydrogen like atom is 13.6 electron volts negative because it is a binding energy and this is got from the expression minus m E to the power 4 by 2 into 4 pi epsilon naught x square. Now, in this case the moss of the electron is known to be the effective moss m star in the semiconductor and epsilon naught is replaced by the relative dielectric constant epsilon r of silica and the radius of this orbit of this electron that the radius is given by.

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And that works out to be 0.53 in a hydrogen like atom now these things again are modified by the replacement of the dielectric permittivity by epsilon R epsilon naught, where epsilon R is the characteristic relative dielectric constant of silicon and also the replacement of m the electronic mass by the effective mass in the semiconductor.

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Adopting the same equations to donor impurity atom in n-type semiconductor, the ionization energy of the donor atom

$$E_d = -\frac{m_e^* e^4}{2(4\pi\epsilon_0\epsilon_r h)^2} = \frac{13.6}{\epsilon_r^2} \frac{m_e^*}{m} eV \quad (37.3)$$

where ϵ_r is the dielectric constant of the semiconductor.

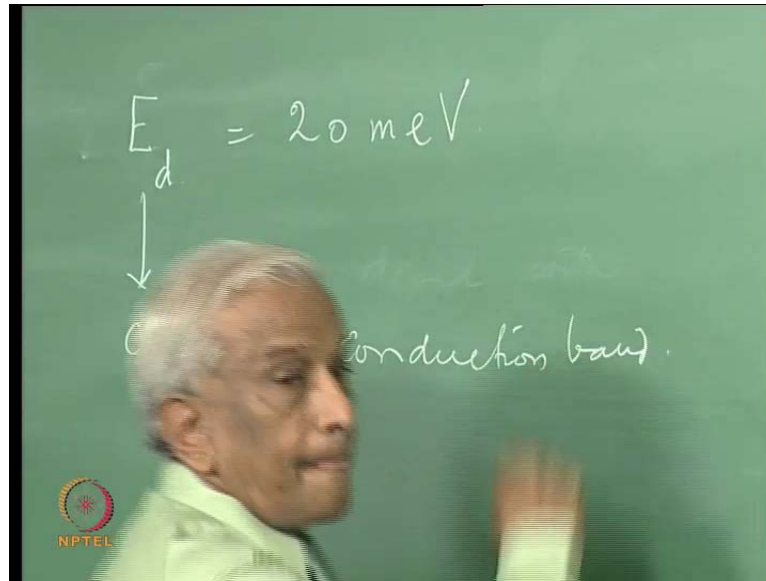
For Si $\epsilon_r = 11.7$ and $\frac{m_e^*}{m} = 0.2$. so that

$$E_d = -0.2 eV = 20m eV \quad (37.4)$$

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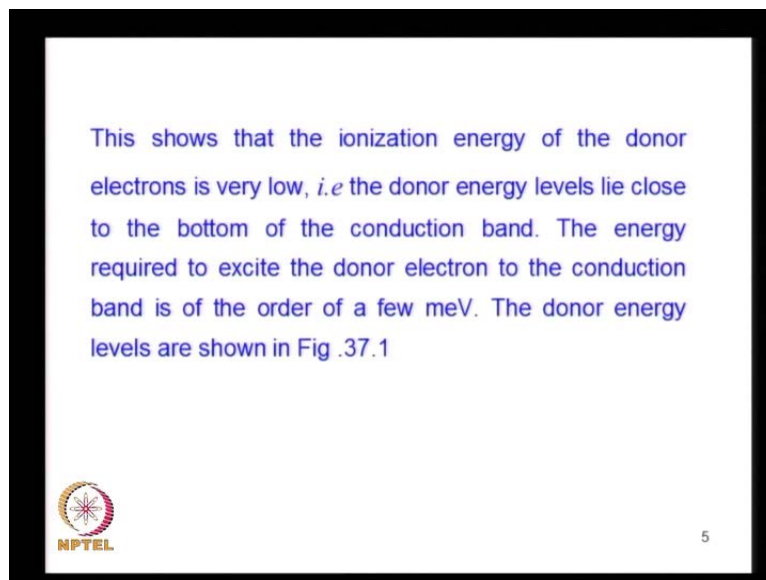
So, there donor energy can therefore, be readily calculated using these expressions and using known values at the effective mass and the dielectric constant.

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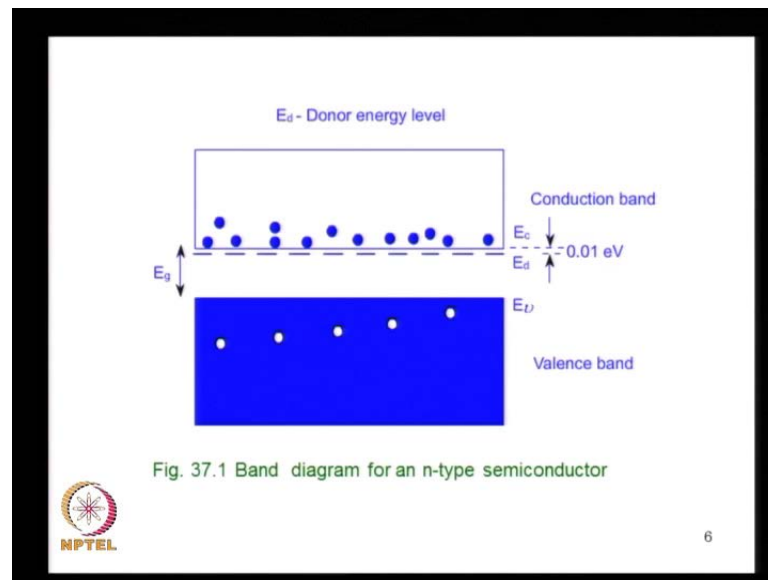
So, this works out to be E_d the energy of the donor atom using the well known result for hydrogen atom the 13.6 electron volts this works out to be 20 million electron volt.

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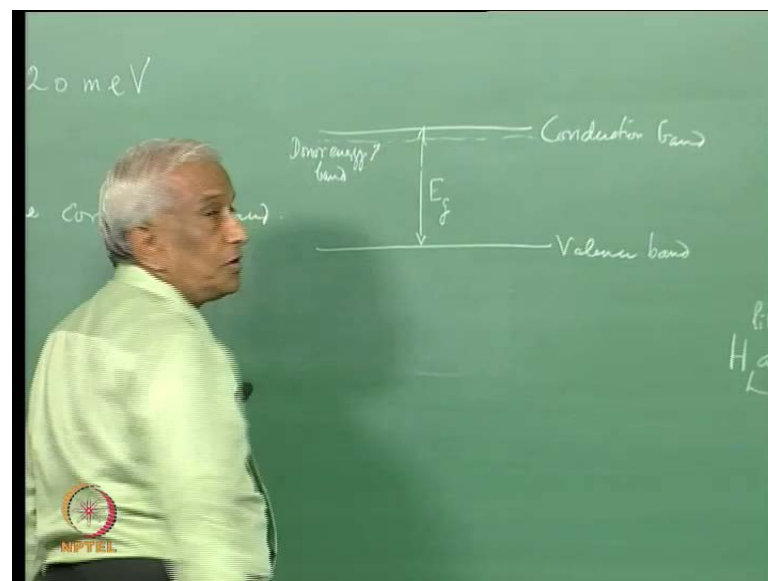
Which is an extremely small quantity and the bore orbit is also rather large. So, the donor energy levels lie very close they are very close they are very close to the conduction band.

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So, this is shown in the figure you have the conduction band.

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The bottom of the conduction band and the top of the valance band with an energy gap here and the donor energy level band likes very close donor energy band it is an extremely small quantity the distance between them.

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The radius of the orbit:

$$r_d = \frac{4\pi\epsilon_r\epsilon_0\hbar^2}{m_e^*} = \epsilon_r \left(\frac{m}{m_e^*} \right) r_H = 31\text{\AA} \quad (37.5)$$

The radius of this loosely bound electron is quite large so that the donor orbitals overlap at relatively low concentrations, forming an impurity energy band near the conduction band as shown in Fig.37.1. Since the donor energy levels are very close to the conduction band, even at low temperatures a large fraction of the donor level electrons is thermally excited into the conduction band.




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And the radius of this orbit is also very large. So, that is a big overlap between the orbits of the neighboring ions and therefore, they form a donor band. So, this is a band so close to the bottom of the conduction band that thermal excitation can provide a very convenient way of exciting the electrons from the donor levels into the conduction band.

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This fraction is much larger than the fraction of electrons excited into the conduction band due to the intrinsic process discussed in the last lecture. In n-type semiconductors, the number of electrons in the conduction band exceeds the number of holes in the valence band, hence the electrons are called the *majority carriers* and the holes are called the *minority carriers*.

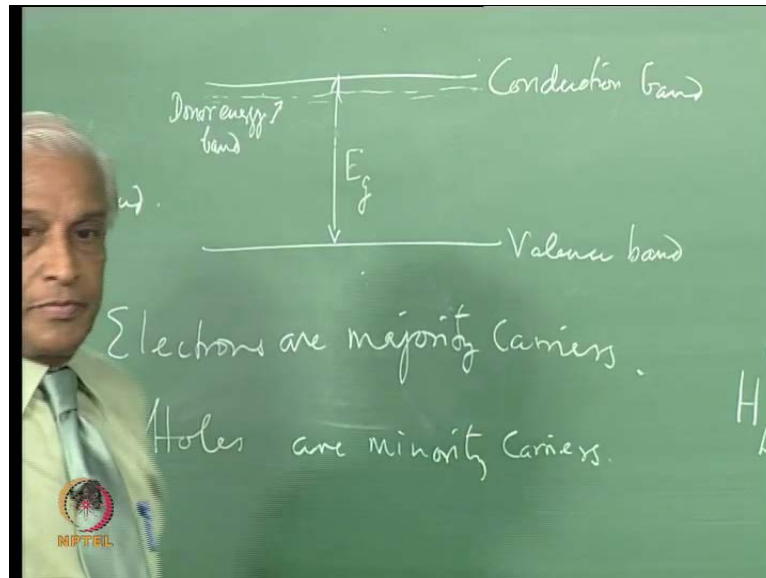


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They facilitate conduction. So, the fraction of electrons thus excited thermally or otherwise from the donor band to the conduction band is relatively large in comparison

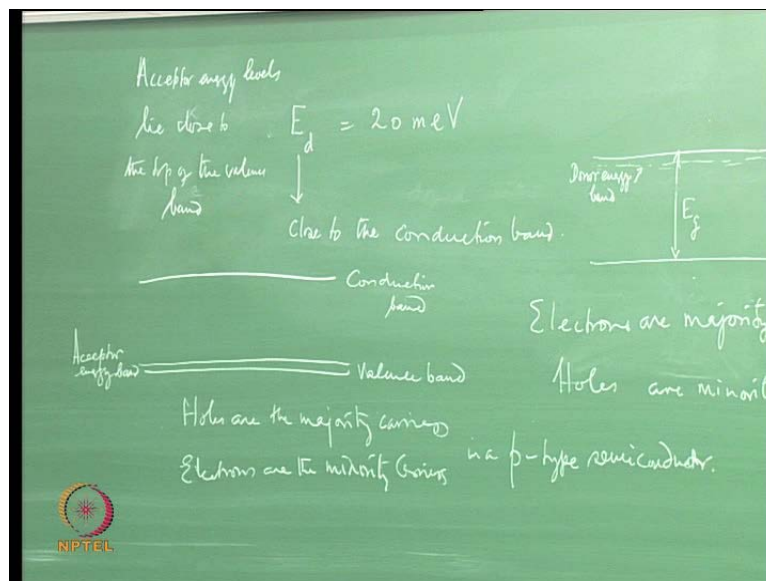
to the fraction which is excited across the energy gap from the valance band. So, and these are due to the whole states in the valance band.

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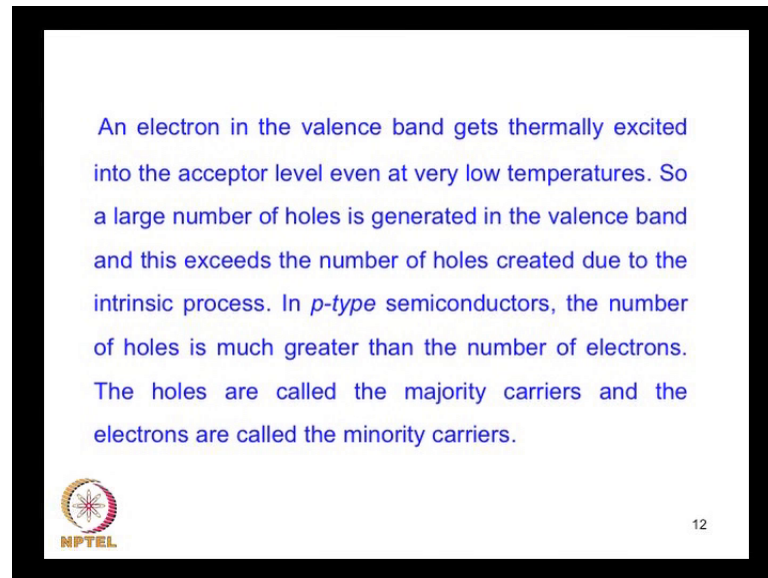
So, the electrons from the donor are the so called majority carriers, because they are in very large numbers; whereas, the holes from the valance band are minority carriers in an n type semiconductor by following the same argument.

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One can also see that the acceptor energy levels lay very close to the top of the valence band. So, this picture gives like this is the conduction band this is the valence band and we have acceptor energy band.

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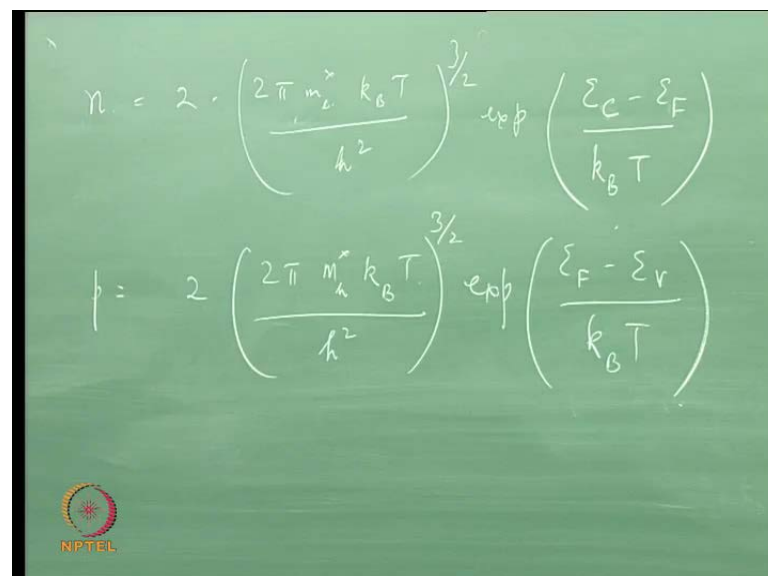

An electron in the valence band gets thermally excited into the acceptor level even at very low temperatures. So a large number of holes is generated in the valence band and this exceeds the number of holes created due to the intrinsic process. In *p-type* semiconductors, the number of holes is much greater than the number of electrons. The holes are called the majority carriers and the electrons are called the minority carriers.



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Which lies extremely growth to the top of the valence band and because of this we have the majority carriers are holes in this case while electrons are the minority carriers in a *p* type semiconductor. So, this is the overall situation and now we have already see in that electron concentration.

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$$n = 2 \cdot \left(\frac{2\pi m_e^* k_B T}{h^2} \right)^{3/2} \exp\left(\frac{\epsilon_C - \epsilon_F}{k_B T} \right)$$
$$p = 2 \cdot \left(\frac{2\pi m_h^* k_B T}{h^2} \right)^{3/2} \exp\left(\frac{\epsilon_F - \epsilon_V}{k_B T} \right)$$


The electron concentration is given as n and that is $2 \times 2 \pi m^* k_B T$ by square hole to the power $3/2$ exponential $E_f - E_c$. Similarly, the hole concentration is given by a similar expression where it is effective mass of the hole T is the temperature.

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$$np = N_{\text{eff}}^C N_{\text{eff}}^V e^{-E_g/k_B T}$$

$$n + N_A^- = p + N_D^+ \rightarrow \text{charge neutrality}$$

acceptor atoms
(-vely ionized)
Donor atoms
(positively ionized)

So, the product of these two becomes n times p is considering these two taking the product of these two expressions. Now these are the partition functions which are given by the p exponential factors here.

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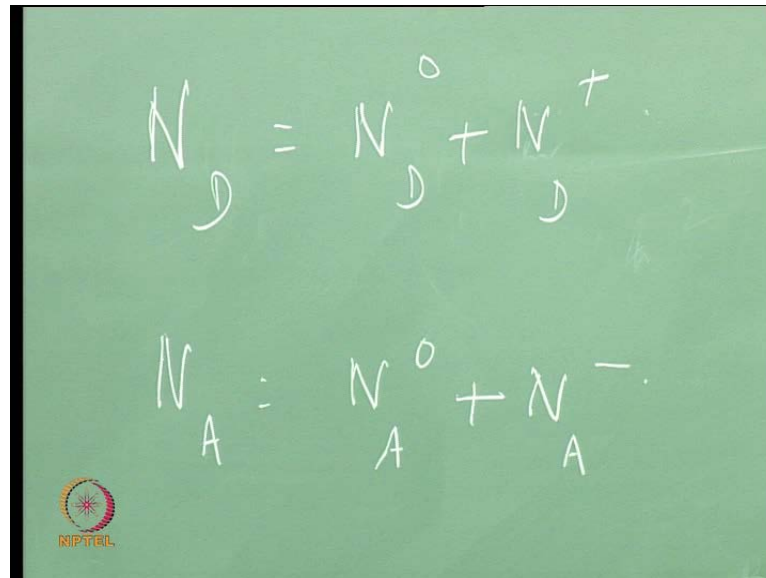
Therefore, $np = N_{\text{eff}}^C N_{\text{eff}}^V e^{-E_g/k_B T}$ (37.8)

where, $N_{\text{eff}}^C = 2 \left(\frac{2\pi m_e^* k_B T}{h^2} \right)^{3/2}$ (37.9)

and $N_{\text{eff}}^V = 2 \left(\frac{2\pi m_h^* k_B T}{h^2} \right)^{3/2}$ (37.10)

Now, the charge in locality requires that N plus N_a minus equals p plus N_d plus where N_a is the acceptor atoms which are negatively ionized x N_d is the donor atoms negatively ionized here whereas, the donor atoms you can positively ionized this is charge neutrality.

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Now N_d concentration of donor atoms can be written as plus N_D plus and similarly N_A is N_A^0 the neutral acceptor atoms plus N_A^- . Now it is rather difficult to discuss the general case in the both donors and acceptors.

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Now, $N_D = N_D^0 + N_D^+$ and (37.11-a)
 $N_A = N_A^0 + N_A^-$ (37.11-b)
 where N_D^0 and N_A^0 are the neutral donor and acceptor concentrations respectively.

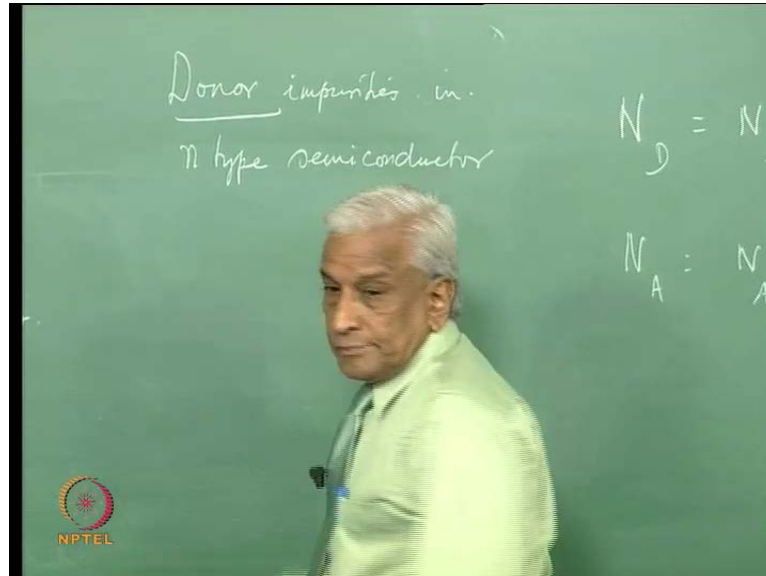
The general case in which the donors and acceptors are considered simultaneously can only be treated numerically. We deal here only with the case of a pure n- type semiconductors in which only donors are available.



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A simultaneous present is can be considered only numerically we do not want to do the here be deal with

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Only the case of a donor impurities, which is theoretically easier to deal with in an n types semiconductor.

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The number of electrons occupying the donor sites is given, to a good approximation, by,

$$n_D = N_D^0 = N_D \left\{ 1 + \exp\left(\frac{E_D - E_F}{k_B T}\right) \right\}^{-1} \quad (37.12)$$

Similarly the number of holes occupying acceptor sites is given as

$$P_A = N_A^0 = N_A \left\{ 1 + \exp\left(\frac{E_F - E_A}{k_B T}\right) \right\}^{-1} \quad (37.13)$$

Free electrons in the conduction band can arise only from donors or from the valence band.



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The concentration of donor electrons is N_D^0 substituting for and the plus this is N_D by one plus exponential E_f minus. Similarly, we can write P_A a similar expression at P_A^0 we are talking about the ionized impurities which involve only the spacing between the

donor level and the bottom of the conduction band and the acceptor level at the top of the valence band.


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$$n = N_D^+ + p \quad (37.14)$$

We also assume that the main contribution to the conductivity comes from ionized donors i.e., $N_D^+ \gg n_i$

Then,

$$n = N_D^+ = N_D - N_D^0 \quad (37.15)$$

$$= N_D \left[1 - \frac{1}{1 + \exp\left[\frac{E_D - E_F}{k_B T}\right]} \right] \quad (37.16)$$


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So, with this we can also say that can be is in gender very large compare to m that is the ionized impurity atom concentration is large the main contribution to conduct to becomes only from this with that we can an expression for n which is the difference between N_D^0 and N_D minus.


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Here E_d is the energy separation of the donor level from the bottom of the conduction band.

Thus,

$$n = N_D^{\text{eff}} e^{(E_F - E_D)/k_B T} \cdot e^{-E_d/k_B T} \quad (37.20)$$

We also have,

$$n = \frac{N_D \exp(E_D - E_F)/k_B T}{1 + \exp(E_D - E_F)/k_B T} \quad (37.21)$$


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And therefore, we can do a little algebraic manipulation to get a closed expression in terms of.

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$$= N_D \frac{1}{\exp((E_F - E_D) / k_B T) + 1}$$

$$= N_D \frac{1}{\left(\frac{n \cdot e^{E_D / k_B T}}{N_{\text{eff}}^C} \right) + 1}$$

Thus, $n + \frac{n^2}{N_{\text{eff}}^C} e^{E_D / k_B T} = N_D$ (37.22)

The physically meaningful solution of this quadratic equation is $n \simeq \frac{2N_D}{1 + \sqrt{1 + 4 \frac{N_D}{N_{\text{eff}}^C} e^{E_D / k_B T}}}$ (37.23)

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So, that gives the concentration and which with a little bit of manipulation again as you break a manipulation lives to a simple quadratic equation for the concentration this quadratic equation has you form.

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$n + \frac{n^2}{N_{\text{eff}}^C} \exp\left(\frac{E_D}{k_B T}\right) = N_D$

$n = N$

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So, for which this quadratic equation can be solved to get a solution for N which has this form so that gives the concentration.

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Case 1: When


$$\frac{4N_D}{N_{\text{eff}}^C} e^{E_d/k_B T} \gg 1, \quad n = \sqrt{N_D N_{\text{eff}}^C} e^{-E_d/k_B T} \quad (37.24)$$

This is the freeze out region.

Case 2:

$$\frac{4N_D}{N_{\text{eff}}^C} e^{E_d/k_B T} \ll 1, \quad n = N_D = \text{Constant.} \quad (37.25)$$

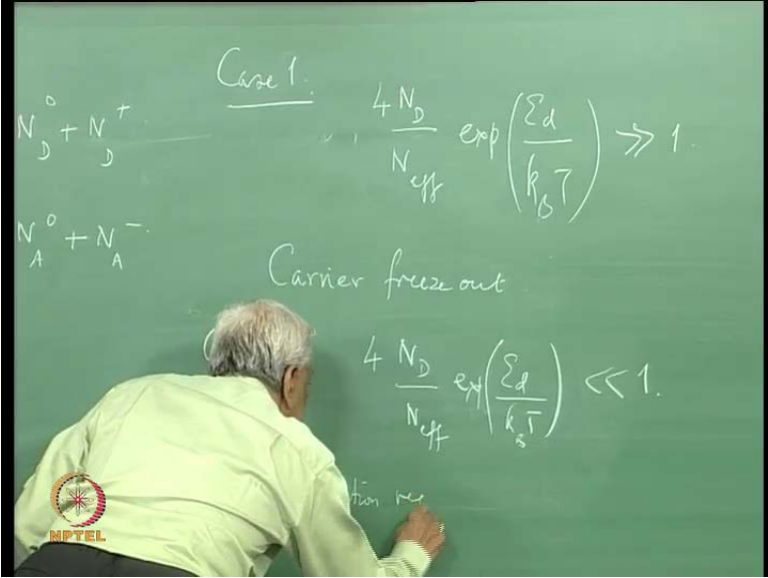
This is the saturation region.



23

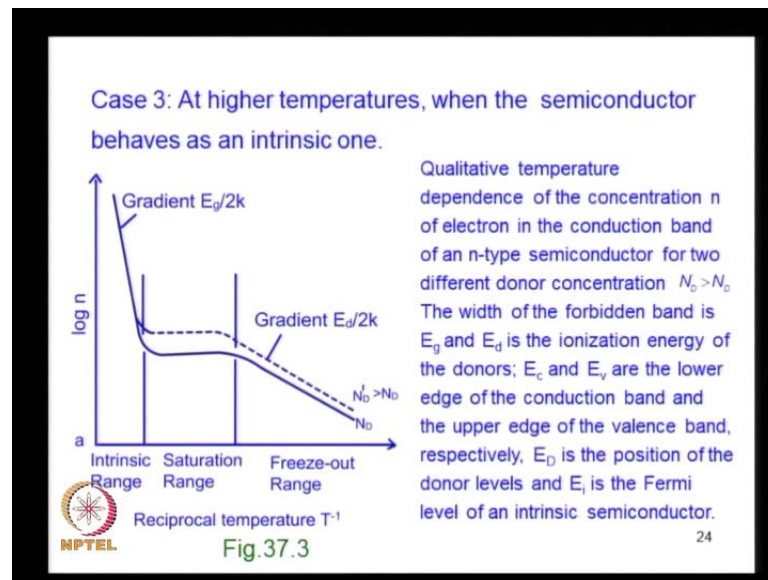
And we can discuss in particular three main cases is one when this quantity under the square root sign is such that.

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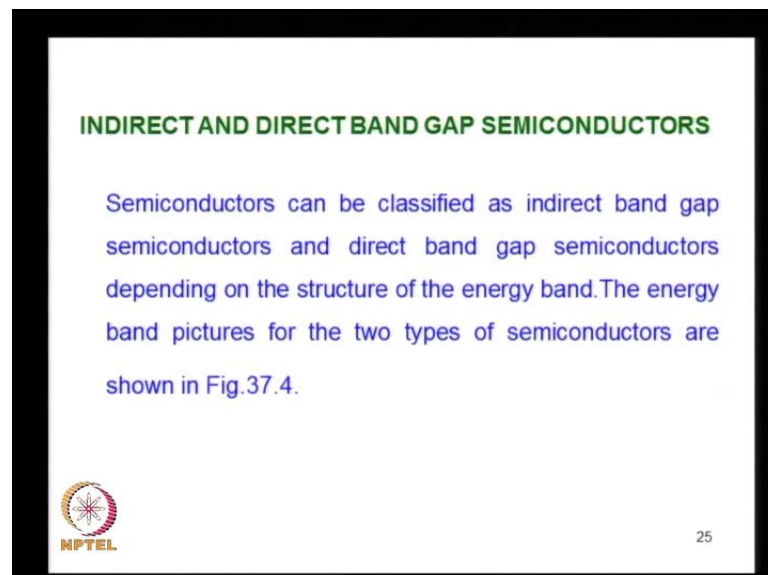
Four N_D by $N_{\text{effective}}$ exponential E_d by $k_B T$, this quantity is very large in comparison to one when I can ignore this and write this in terms of this is the so called carrier freeze out region. Whereas, the second case is when this factor is small in comparison to one, so that this can be neglected and we have a constant concentration. So, this is the saturation region and then third case.

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Case 3 is at still higher temperatures the factor T enters here the conduction becomes intrinsic. So, the figure show the three regions one is the intrinsic range as a function of one by T logarithm of the concentration is plotted and you have an intrinsic range then this saturation range where the concentration is constant then a freeze-out range.

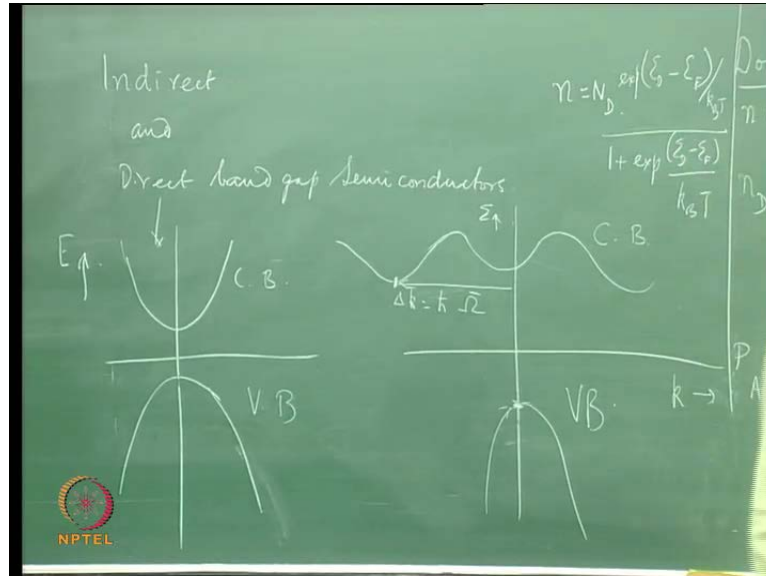
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So, this gives the basic mechanism and basic results of carrier transport in a pure n type of semiconductors this can be treated analytically similarly for p type semiconductor also

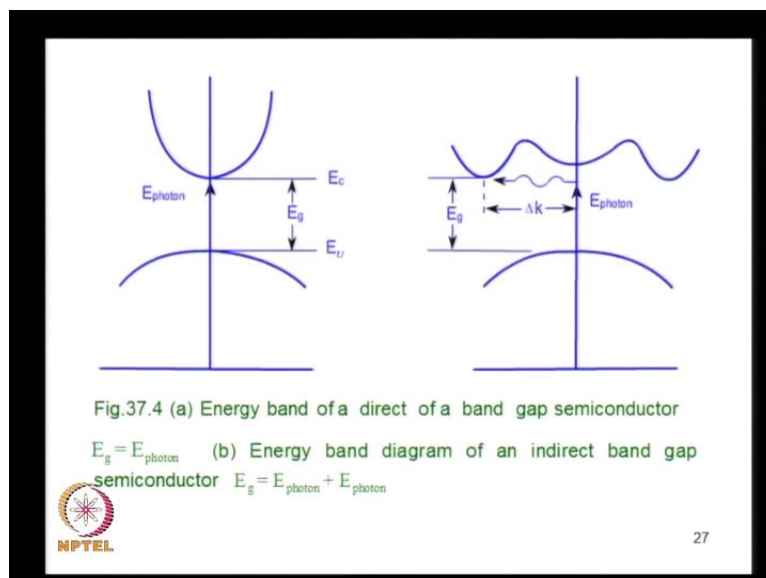
can be treated analytically, but as I already said and both are present then this cannot be treated analytically, but it has to be done only numerically.

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You know have to consider an important distinction between indirect and direct band gap semiconductors.

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These are illustrated in the next figure in the direct band gap semiconductor the the conduction band the bottom to the conduction band. So, this is the conduction band this is the valance band. So, this bottom of the conduction band lies energetically at the same

value as the top of the valance band, so that the band gap is all the energy needed to promote a carrier from the valance band into the conduction band. Whereas, an indirect band gap is one in which and you have a slightly lower structure here and whereas, we have the valance band here the conduction band energy structure like this. So, the bottom of the conduction band lies here while the top of the valance band likes here there do not occur at the same k value. So, one needs to promote the carrier from here to this and then translate it to this by an amount Δk or $h \text{ cross } \omega$.

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In a direct band gap semiconductor, the lowest energy of the conduction band at the same k value as the highest energy of the valance band as shown in Fig 37.4(a). In an indirect band gap semiconductor, the k value corresponding to the lowest energy of the conduction band is different from that of the highest energy of the valence band as shown in Fig.37.4(b).

The threshold frequency for optical absorption in a direct band gap semiconductor is given by

$$h\omega_0 = E_g \quad (37.26)$$


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So, for conduction one has to get a threshold frequency for conduction is just the energy gap for a direct band gap for a direct band gap semiconductor, for example, gallium arsenide you such a material.

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In an indirect band gap semiconductor, the threshold frequency is given by

$$h\omega_0 = E_g + h\Omega \quad (37.27)$$

where $h\Omega$ is the energy of an emitted phonon, i.e. the minimum photon energy required for absorption is greater than the energy gap. The excess energy increases the lattice vibration energy by $h\Omega$ i.e. a phonon of energy $h\Omega$ is generated. The generated phonon has wave vector equal to the difference in k value Δk shown in Fig. 37.4(b).




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Whereas in the case of an indirect band gap we have to have an indirect band gap.

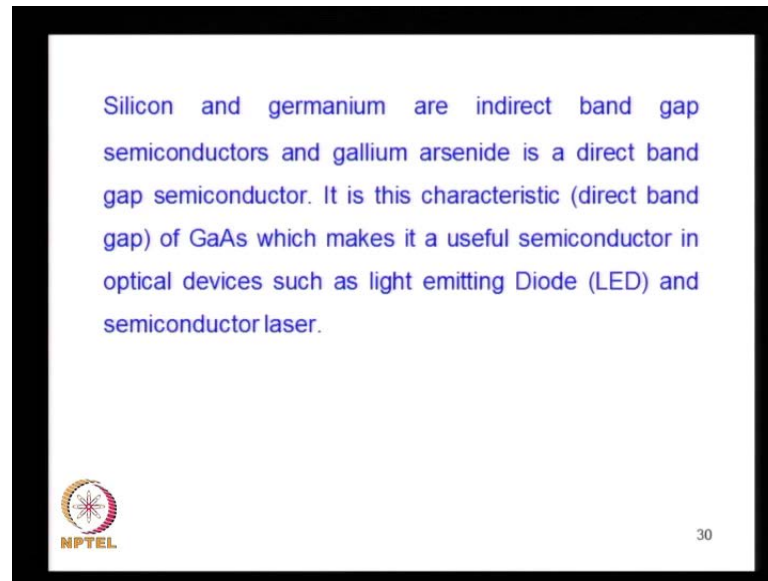
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Indirect band gap
Semiconductor (Si)


$$h\omega = E_g + h\Omega$$


Semiconductor such as silicon one has to have their omega is given by the condition for the wave vector.

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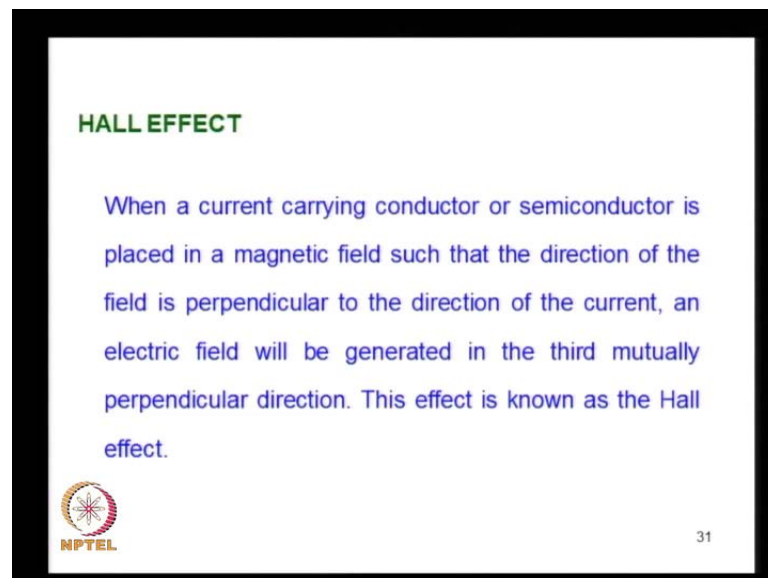
Silicon and germanium are indirect band gap semiconductors and gallium arsenide is a direct band gap semiconductor. It is this characteristic (direct band gap) of GaAs which makes it a useful semiconductor in optical devices such as light emitting Diode (LED) and semiconductor laser.



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So, it is because of these the gallium arsenide is a useful conductor and for example, optoelectronic devices led's semiconductor lasers and so on.

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HALL EFFECT

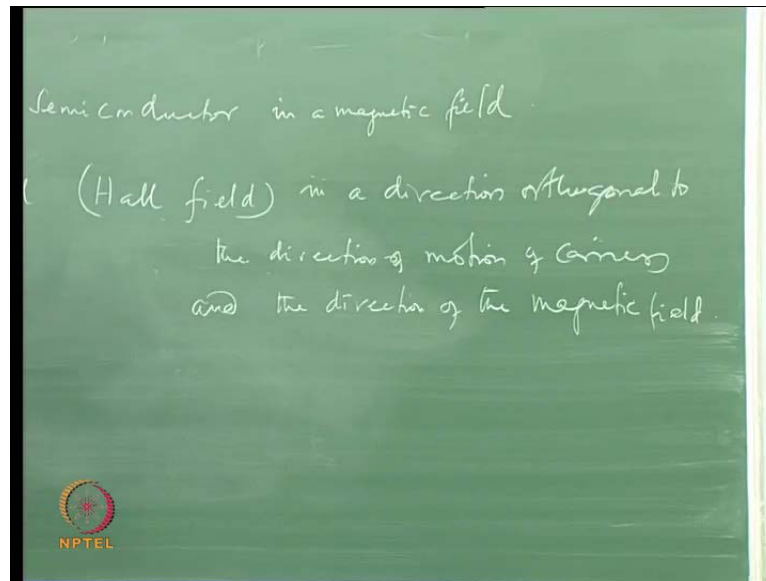
When a current carrying conductor or semiconductor is placed in a magnetic field such that the direction of the field is perpendicular to the direction of the current, an electric field will be generated in the third mutually perpendicular direction. This effect is known as the Hall effect.



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Next we pass on to a consideration of hole effect hall effect is a very well known phenomenon especially in semiconductor.

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So, then a current carrying conductor or semiconductor is placed a magnetic field let us discuss a semiconductor placed a magnetic field. So, because of the Lorentz force acting on a moving charge practically magnetic field you get a lateral displacement of the carriers and therefore, there is a its field which is set up in the direction perpendicularly directional motion of the charge carriers and the direction of the magnetic field. So, this field is known as the hall field and the setting up of this is known as the Hall Effect. So, this is in a direction orthogonal to the direction of the motion of the carriers and the direction of the magnetic field. So, that is the effect which is known as the classical Hall Effect.

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Theory of Hall Effect in a Conductor (Solid with One Type of Charge Carrier)

Figure 37.6 shows a specimen placed in a magnetic field of strength B applied in the Z-direction. A current I is passed through the crystal in the Y-direction by applying an electric field E along Y-direction



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If you have a conductor with one type of charge carriers the situation is extremely simple to deal with

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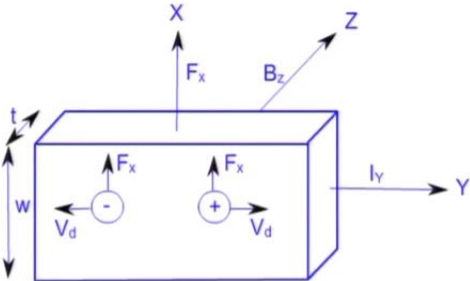



Fig 37.6 Hall effect in a semiconductor. The current is in the Y-direction and the magnetic field is in the Z-direction. The charge carriers experience a force in the positive X-direction. Note that both electron and holes experience the force in the positive X-direction



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Let us say consider a configuration which you shown in the figure here where the field and the current configuration are shown.

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The charge carriers in the specimen experience a force qE along Y, where q is the charge of the carriers. Due to this force the charges acquire an average drift velocity $\langle v_y \rangle$. The current density J_y is given by

$$J_y = nq \langle v_y \rangle \quad (37.28)$$

where n is the concentration of charge carriers. Since a magnetic field B is applied in the Z-direction, the charges moving in the Y-direction with velocity $\langle v_y \rangle$, experience a force in the X-direction given by


$$\vec{F} = q\vec{v} \times \vec{B}$$
$$F_x = q \langle v_y \rangle B_z \quad (37.29)$$


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And one can straight away using the Lorentz source law f equals $q v$ cross b where b is that of the current and you have a b direction. So, the force the lateral force can be readily calculated.

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Due to this force, the charge get deflected in the X-direction. The deflection causes the build-up of charge towards one face leaving an excess of opposite charges at the opposite face of the specimen. This accumulation of charges will continue until the electrostatic field set up by this charge separation balances the magnetic force on the carriers. The electrostatic field set up at equilibrium, along the X-direction, is called the *Hall Field*. The condition for equilibrium, i.e condition for the balancing of the two forces is given by

$$qE_H = q \langle v_y \rangle B_z$$
$$E_H = \langle v_y \rangle B_z \quad (37.30)$$


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And therefore, the Hall field can be calculated also, the hall field is given is if the direction of motion X...

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$$\therefore \frac{E_H}{j_y B_z} = R_H, \text{ Hall constant}$$
$$j_y B_z = \frac{1}{nq}$$

in a magnetic field.

in a direction orthogonal to direction of motion of carriers

The B field is along the z direction the hall field is directed along the y direction. So, this will be $j_y B_z$ by n . Let us write q here to denote the charge in a general way. So, this E_H by $j_y B_z$ is known as Hall constant and that readily seen to be just 1 by nq .

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Hall effect experiment

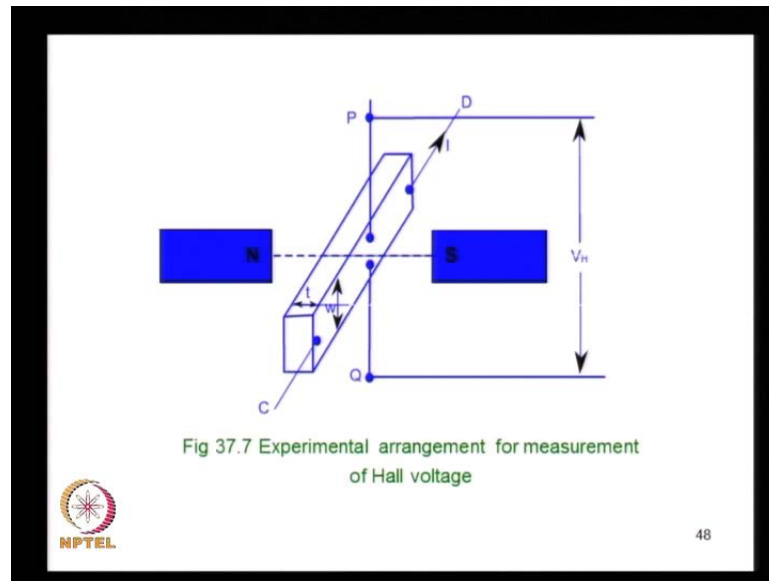
A rectangular thin specimen of a semiconductor crystal which may be n or p types is mounted on a nonmagnetic plate.

Wires are soldered to the crystal for passing current across its length and for measuring the Hall voltage across its width. The specimen is placed in a magnetic field as shown in Fig.37.7. The current is passed along CD and Hall voltage is measured across PQ.

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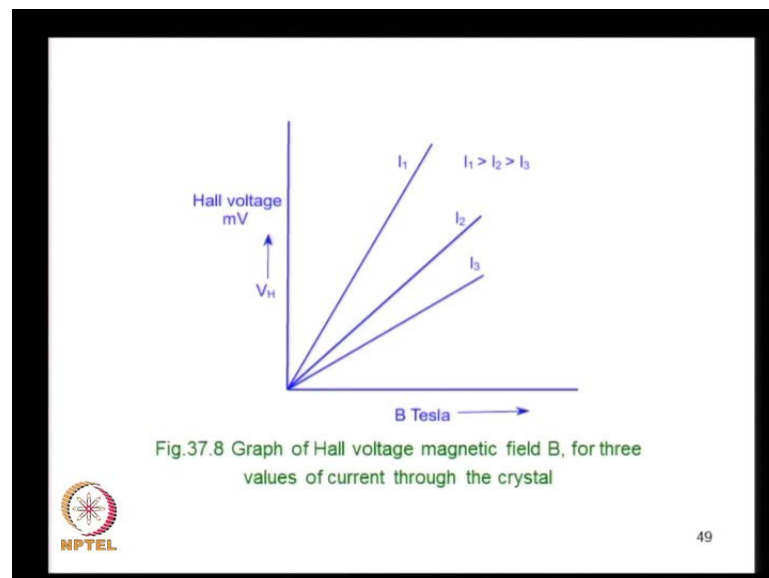
So, this proportionally constant is what is seen here and therefore, the Hall voltage can straight away be calculated.

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So, this is the typical experimental configuration for measuring the Hall coefficient and contacts made along to pass the current.

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And a magnetic field is supplied perpendicular to the direction of flow the current, and then the contracture made also in the third direction to measure the Hall voltage, and this gives the Hall voltage as a function of the applied magnetic field, in different values of the magnetic field.

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The slope of B versus $E_H = -\frac{me^3}{2(4\pi\epsilon_0\hbar)^2} = -13.6eV$


Since I_y and t are known in the experiment, the Hall coefficient may be calculated.

R_H is related to the concentration of charge carriers by

$$n = 1/R_{Hq} = 1/R_H |e|$$

Hence the density of charge carriers can be determined.

n gives the density of majority carriers in an extrinsic semiconductor.



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The hall constant is related to the carrier concentration, so 1 by R_H and q magnitude of q .

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If the conductivity σ of the semiconductor is known, the mobility μ of the charge carriers can be calculated using the relation.

$$\mu = \sigma / n|e|$$

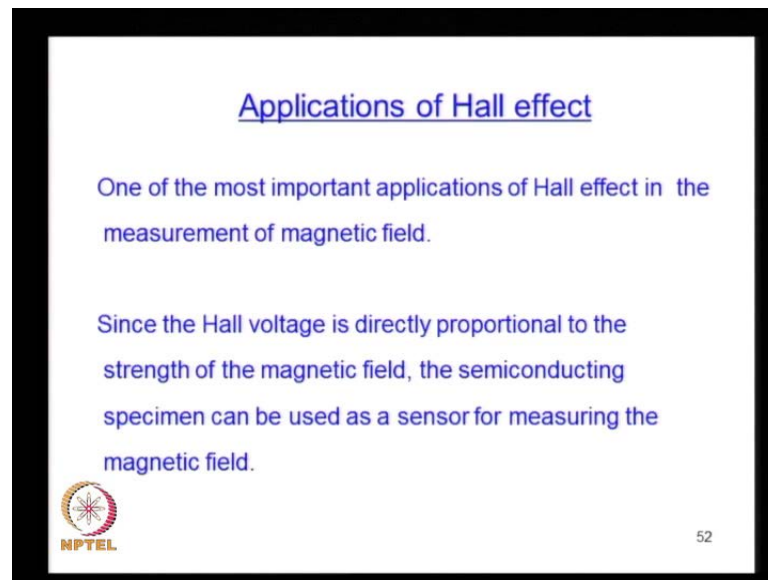
The sign of the majority charge carriers can be determined from the polarity of the Hall voltage.



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So, one can straight away measure the carrier density by measuring the Hall constant. And since the mobility is just therefore, if we know the electrical conductivity and the concentration one can determine the mobility as well.


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Applications of Hall effect

One of the most important applications of Hall effect in the measurement of magnetic field.

Since the Hall voltage is directly proportional to the strength of the magnetic field, the semiconducting specimen can be used as a sensor for measuring the magnetic field.



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So, the Hall effect is used extensively in the measurement of the magnetic field, it is used as a magnetometer and to measure sign and concentration of charge carrier, and the mobility. These are the important applications of Hall Effect measurements.